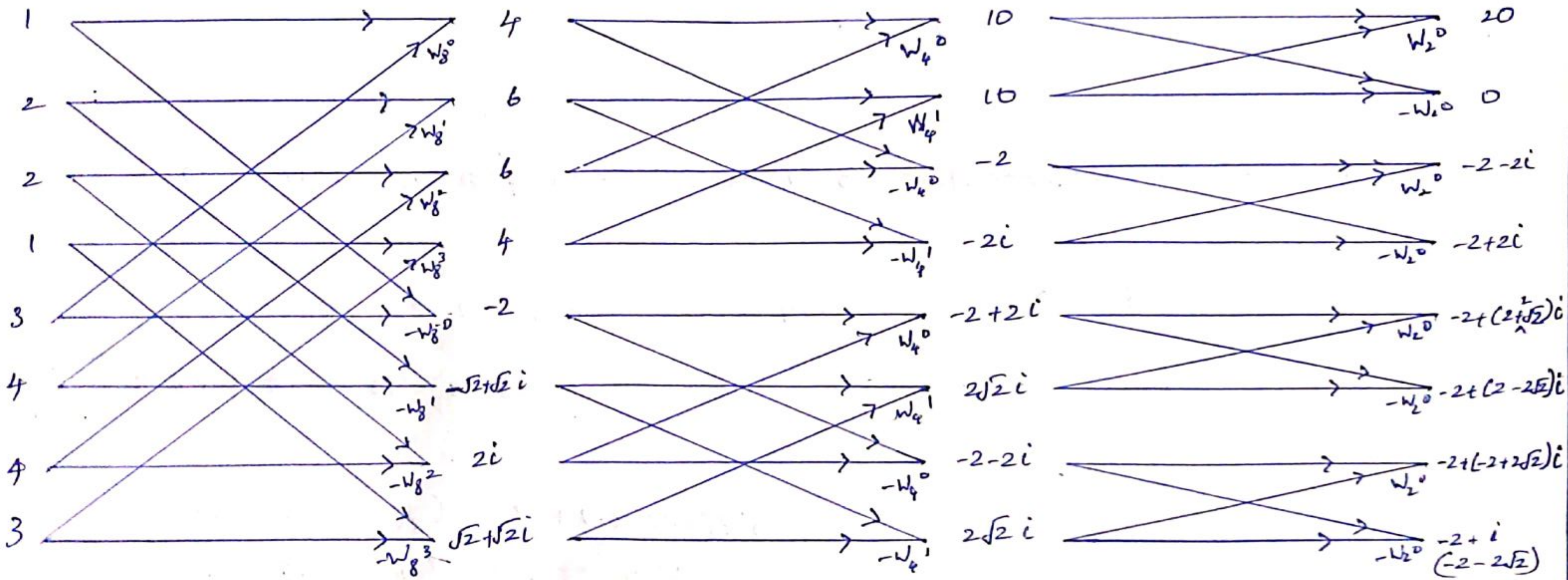
I] DFT Using DIF-FFT

1) $x(n) = \{1, 2, 2, 1, 3, 4, 4, 3\}$



$x(n) = \{1, 2, 2, 1, 3, 4, 4, 3\}$

~~$\{1, 2, 3, 4\}$ $\{2, 1, 4, 3\}$~~

~~$\{4, 3\}$ $\{2, 4\}$ $\{2, 4\}$ $\{1, 3\}$~~

Decimation:

$\Rightarrow \{20, 0\}, \{-2-2i\}, -2+2i\},$

$\{-2 + (2+2\sqrt{2})i, -2+(2-2\sqrt{2})i\},$

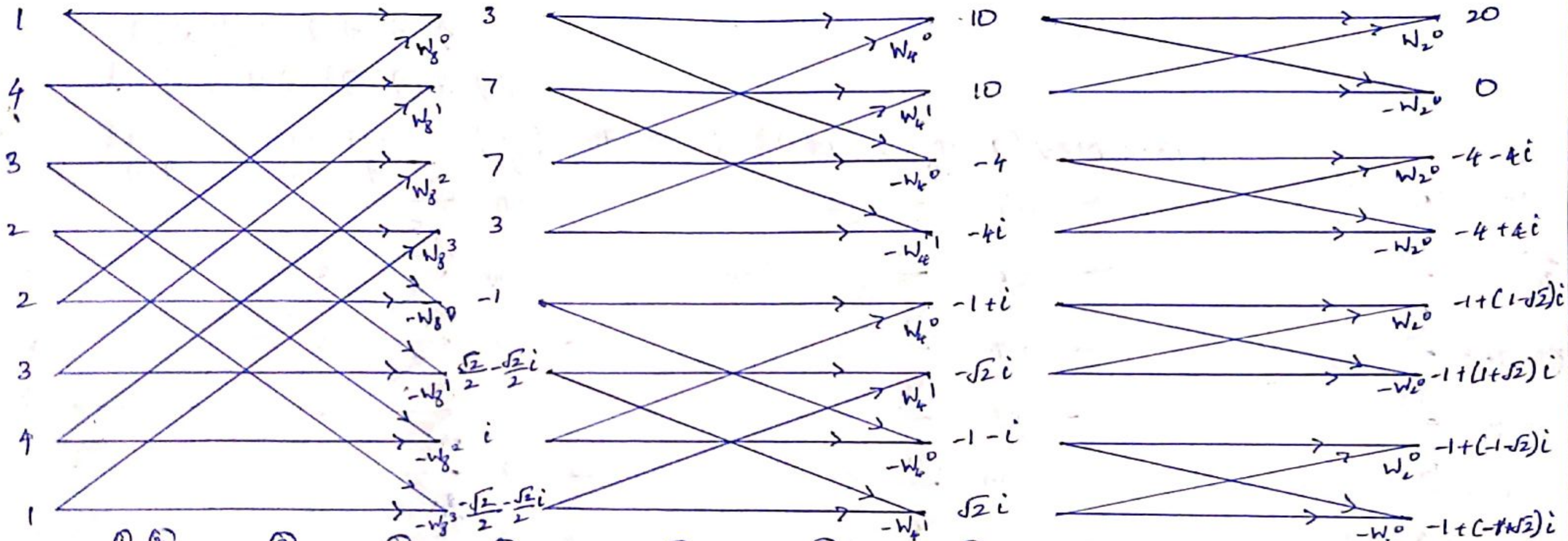
$\{-2+(-2+2\sqrt{2})i, -2+(-2-2\sqrt{2})i\}$

$\Rightarrow \{2.0, -2-2i, 0, -2+2i\},$

$\{-2+(2+2\sqrt{2})i, -2+(-2+2\sqrt{2})i, -2+(2-2\sqrt{2})i, -2+(-2-2\sqrt{2})i\}$

$\Rightarrow \{20, -2+(2+2\sqrt{2})i, 2-2i, -2+(-2+2\sqrt{2})i, 0, -2+(2-2\sqrt{2})i, -2+2i, -2+(-2-2\sqrt{2})i\}$

2) $x(n) = \{1, 4, 3, 2, 2, 3, 4, 1\}$



Decimation: ① ② ③ ④ ⑤ ⑥ ⑦ ⑧

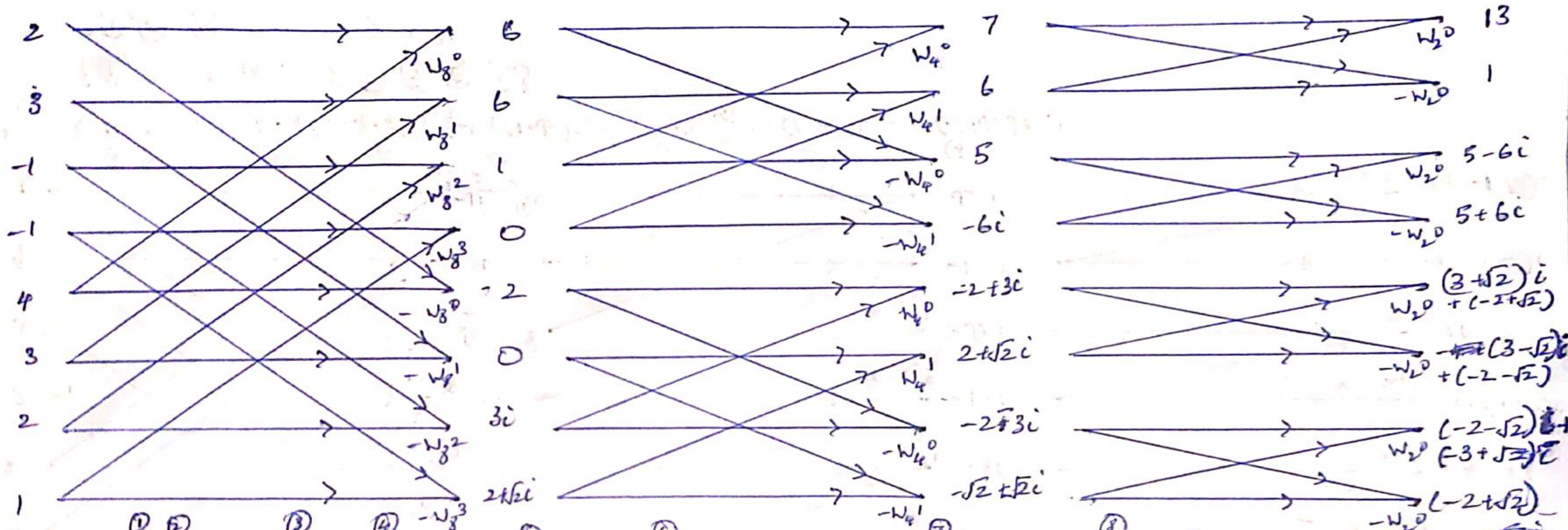
$\{10, 0\}$, $\{-4-4i, -4+4i\}$, $\{-1+(1-\sqrt{2})i, -1+(1+\sqrt{2})i\}$, $\{-1+(1-\sqrt{2})i, -1+(-1+\sqrt{2})i\}$

$\{①, ③, ②, ④\}$ $\{⑤, ⑦, ⑥, ⑧\}$

$\{①, ⑤, ③, ⑦, ②, ⑥, ④, ⑧\}$

$\Rightarrow \{20, -1+(1-\sqrt{2})i, -4-4i, -1+(-1-\sqrt{2})i, 0, -1+(1+\sqrt{2})i, -4+4i, -1+(-1+\sqrt{2})i\}$

I 3) $x(n) = \{2, 3, -1, -1, 4, 3, 2, 1\}$



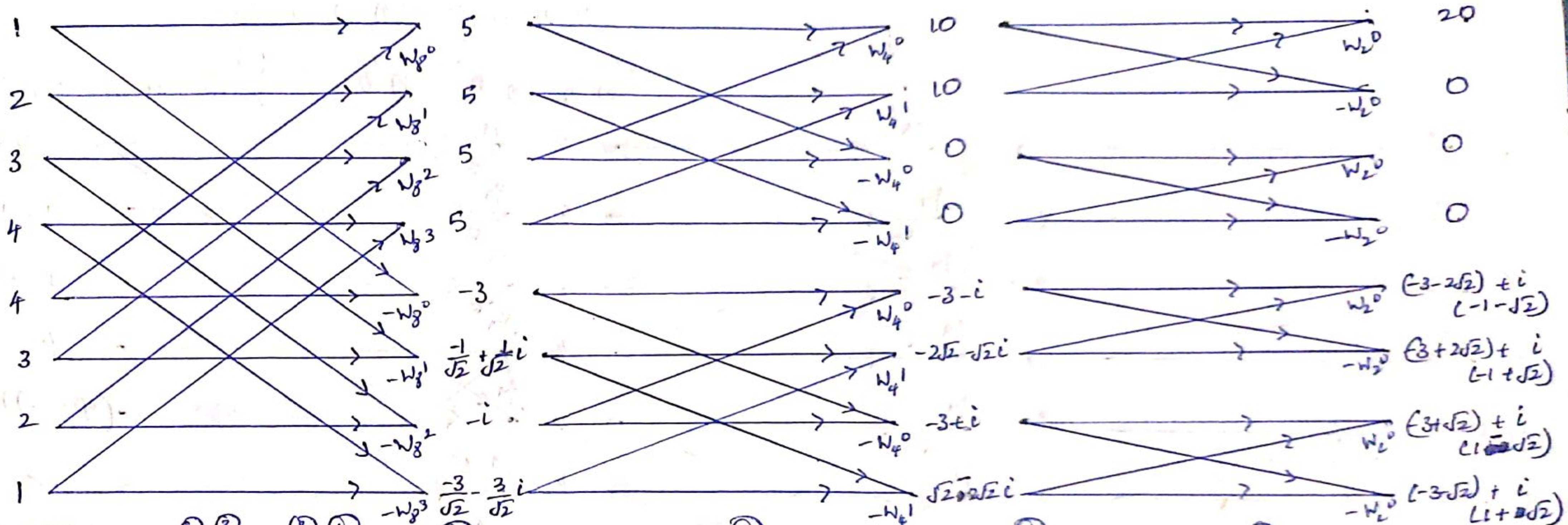
Decimation : $\{13, 1\}$ $\{5-6i, 5+6i\}$ $\{(3+\sqrt{2})i + (-2+\sqrt{2}),\ (3-\sqrt{2})i + (-2-\sqrt{2})\}$ $\{(-2-\sqrt{2}) + (-3+\sqrt{2})i,\ (-2+\sqrt{2}) - i(-3-\sqrt{2})\}$

$\{①, ③, ②, ④\}$ $\{⑤, ⑦, ⑥, ⑧\}$

$\{①, ⑤, ③, ⑦, ②, ⑥, ④, ⑧\}$

$x(k) \Rightarrow \{13,\ (3+\sqrt{2})i + (-2+\sqrt{2}),\ 5-6i,\ (-2-\sqrt{2}) + (-3+\sqrt{2})i,\ 1,\ (3-\sqrt{2})i + (-2-\sqrt{2}),\ 5+6i,\ (-2+\sqrt{2}) - i(-3-\sqrt{2})\}$

I 4) $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$



Decimation: ① ② ③ ④ ⑤ ⑥ ⑦ ⑧

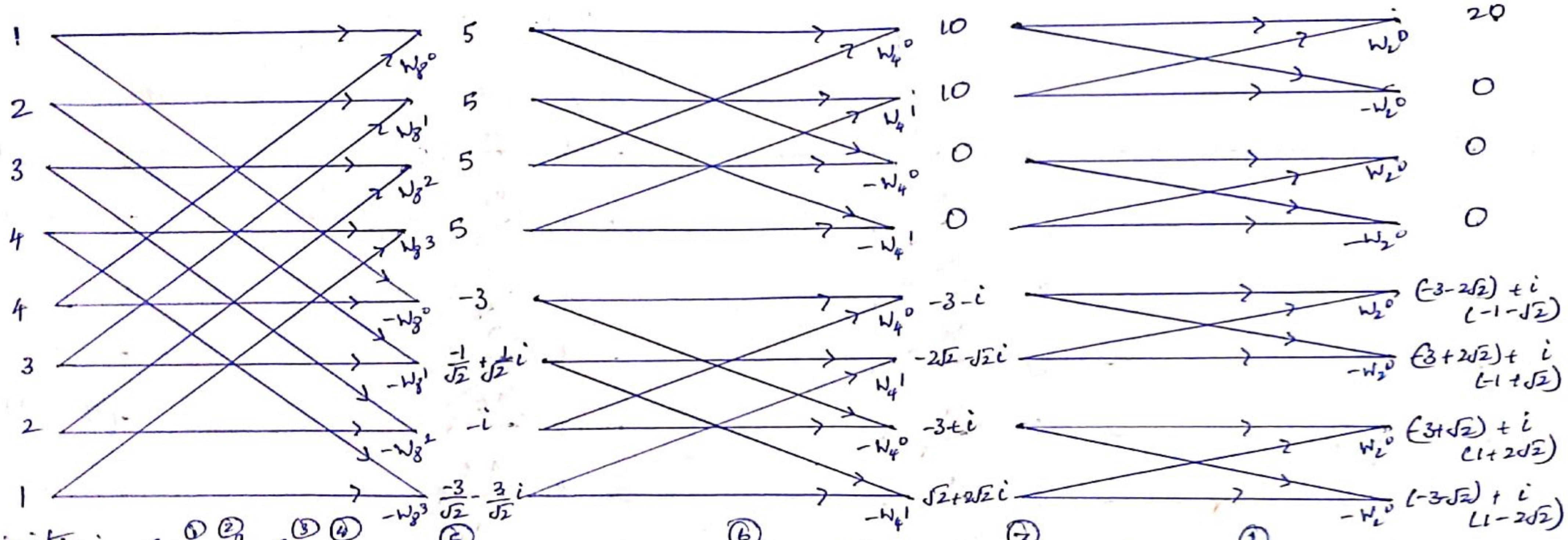
$\{20, 0\}$ $\{0, 0\}$ $\{(-3-2\sqrt{2}) + i(-1-\sqrt{2}), (-3+\sqrt{2}) + i(-1+\sqrt{2})\}$ $\{(-3+\sqrt{2}) + i(1-\sqrt{2}), (-3-\sqrt{2}) + i(1+2\sqrt{2})\}$

$\{①, ③, ②, ④\}$ $\{⑤, ⑦, ⑥, ⑧\}$

$\{①, ⑤, ③, ⑦, ②, ⑥, ④, ⑧\}$

$x(t) \Rightarrow \{20, (-3-2\sqrt{2}) + i(-1-\sqrt{2}), 0, (-3+\sqrt{2}) + i(1-\sqrt{2}), 0, (-3+\sqrt{2}) + i(-1+\sqrt{2}), 0, (-3-\sqrt{2}) + i(1+2\sqrt{2})\}$

I 4) $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$



Decimation: ① ② ③ ④ ⑤ ⑥ ⑦ ⑧

$\{20, 0\}$ $\{0, 0\}$ $\{(-3-2\sqrt{2}) + i(-1-\sqrt{2}), (-3+\sqrt{2}) + i(-1+\sqrt{2})\}$ $\{(-3+\sqrt{2}) + i(1+2\sqrt{2}), (-3-\sqrt{2}) + i(1-2\sqrt{2})\}$

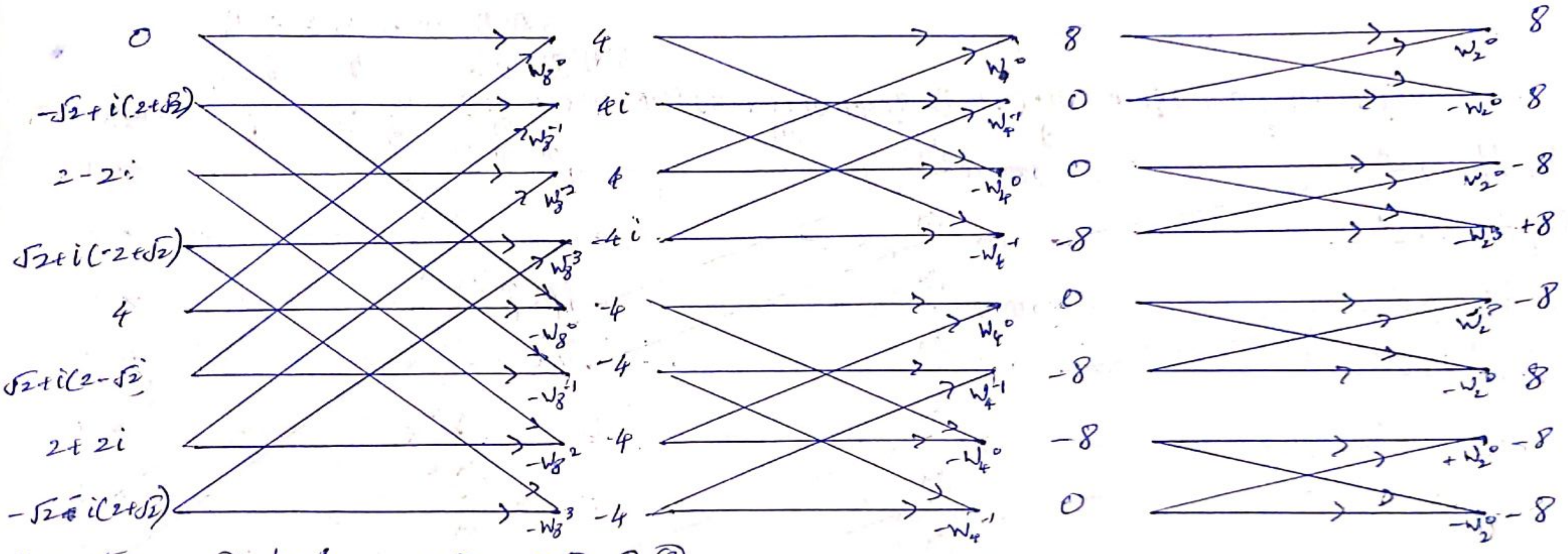
$\{①, ③, ②, ④\}$ $\{⑤, ⑦, ⑥, ⑧\}$

$\{①, ⑤, ③, ⑦, ②, ⑥, ④, ⑧\}$

$\Rightarrow \{20, (-3-2\sqrt{2}) + i(-1-\sqrt{2}), 0, (-3+\sqrt{2}) + i(1+2\sqrt{2}), 0, (-3+\sqrt{2}) + i(-1+\sqrt{2}), 0, (-3-\sqrt{2}) + i(1-2\sqrt{2})\}$

IDFT using DIF-FFT:

1) $\{x(k)\} = \{0, -\sqrt{2}+i(2+\sqrt{2}), 2-2i, \sqrt{2}+i(-2+\sqrt{2}), 4, \sqrt{2}+i(2-\sqrt{2}), 2+2i, -\sqrt{2}-i(2+\sqrt{2})\}$



Decimation: Div by 8

① ② ③ ④ ⑤ ⑥ ⑦ ⑧

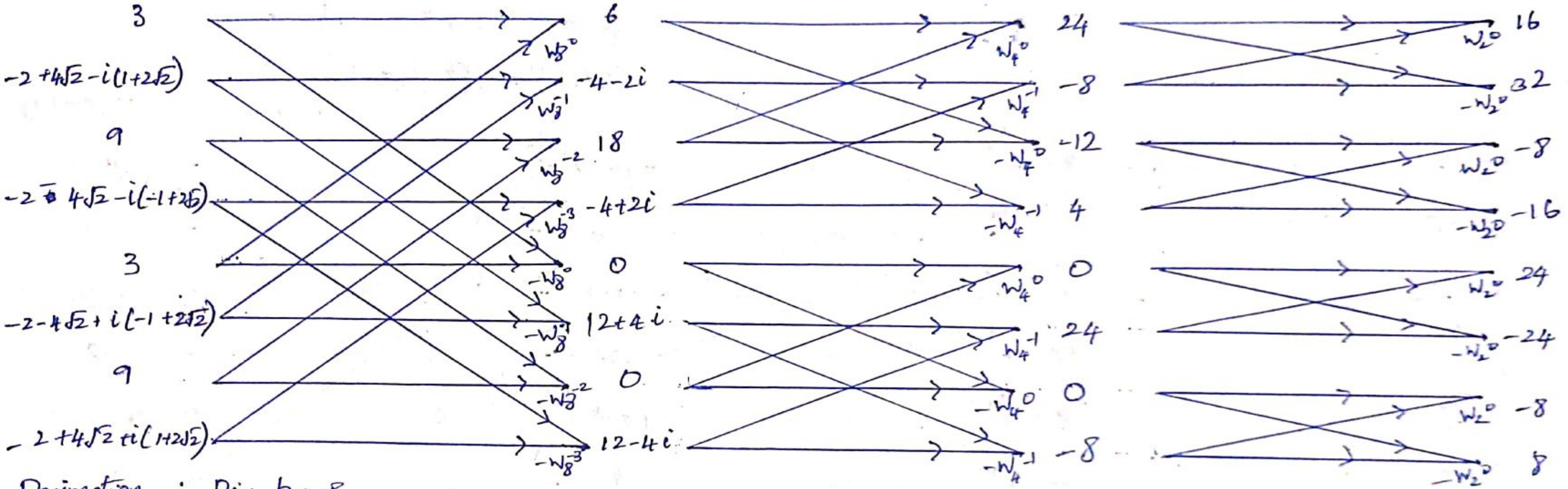
$\Rightarrow \{1,1\}\{-1,1\},\{-1,1\},\{-1,-1\}$

$\{①,③,②,④\}\{⑤,⑦,⑥,⑧\}$

$\{①,⑤,③,⑦,②,⑥,④,⑧\}$

$\Rightarrow \{1,-1,-1,-1,1,1,1,-1\}$

2) $\{x(k)\} = \{3, -2+4\sqrt{2}-i(1+2\sqrt{2}), 9, -2\mp 4\sqrt{2}-i(-1+2\sqrt{2}), 3, -2-4\sqrt{2}+i(-1+2\sqrt{2}), 9, -2\mp 4\sqrt{2}+i(1+2\sqrt{2})\}$



Decimation : Div by 8,

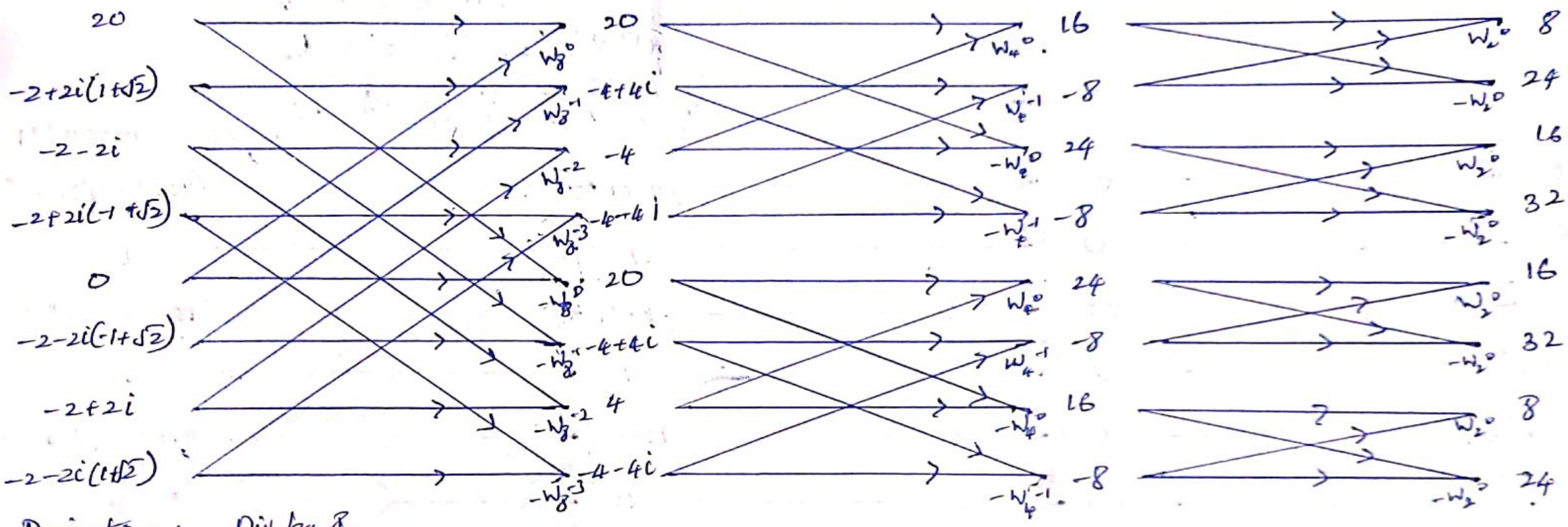
$\overset{①}{\{2}, \overset{②}{4}\} \{-1, -2\} \{3, -3\} \{-1, 1\}$

$\{①, ③, ⑤, ⑦\}, \{②, ④, ⑥, ⑧\}$

$\{①, ⑤, ③, ⑦, ②, ⑥, ④, ⑧\}$

$\Rightarrow \{2, 3, -1, -1, 4, -3, -2, 1\}$

3) $\{x(k)\} = \{20, -2+2i(1+\sqrt{2}), -2-2i, -2+2i(-1+\sqrt{2}), 0, -2-2i(-1+\sqrt{2}), -2+2i, -2-2i(1+\sqrt{2})\}$



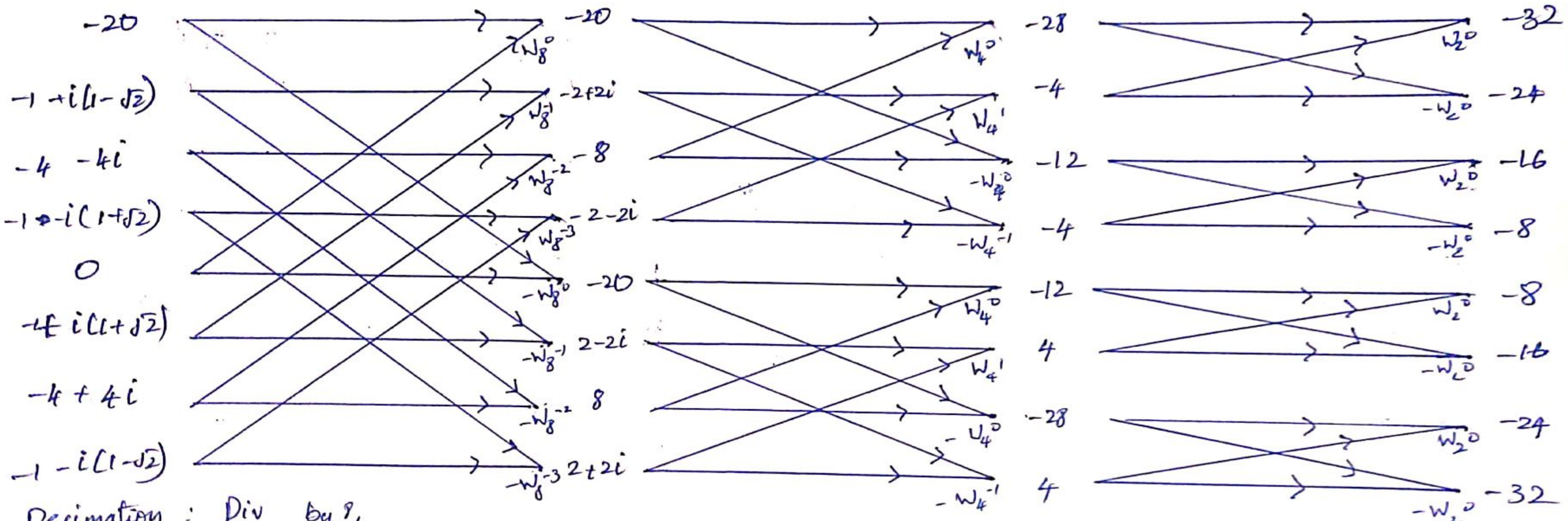
Decimation : Div by 8,

$\Rightarrow \{1,3\}\{2,4\}\{2,4\}\{1,3\}$

$\Rightarrow \{1,2,3,4\}\{2,1,4,3\}$

$= \{1,2,2,1,3,4,4,3\}$

4) $\{x(k)\} = \{-20, -8+i(1-\sqrt{2}), -4-4i, -1-i(1+\sqrt{2}), 0, -1+i(1+\sqrt{2}), -4+4i, -1-i(1-\sqrt{2})\}$



Decimation : Div by 8,

$\Rightarrow \{-4, -3\}\{2, -1\}, \{-1, -2\}, \{-3, -4\}$

$\Rightarrow \{-4, -2, -3, -1\}\{-1, -3, -2, -4\}$

$\Rightarrow \{-4, -1, -2, -3, -3, -2, -1, -4\}$