

PSG College of Technology
Dept of Applied Maths & Computational Sciences
III MSc (SS) : Mathematical Foundations of Computing (18XW31) : PS1 :
Propositional Calculus

1. Write in symbolic form:

P: The food is good Q: The service is good. R: The rating is three star.

- a. Either the food is good, or the service is good, or both.
- b. The food is good while the service is poor.
- c. It is not the case that both the food is good and the rating is three star.
- d. It is not true that a three star rating always mean good food and good service.

2. Write in symbolic form:

P: It is raining.

Q: There are no clouds in the sky.

R: We are in Manchester.

S: We have no umbrella

- a. (i) $R \rightarrow P$ (ii) $\neg S \rightarrow (\neg P \wedge Q)$ (iii) $(\neg Q \wedge S) \rightarrow P \rightarrow R$
- b. Give formal renditions of each of the following:
 - (i) If it is raining and there are no clouds in the sky then we are in Manchester.
 - (j) If we are in Manchester then whenever there are clouds in the sky it must be raining.

3. Write in symbolic form:

P : I bought a lottery ticket this week.

Q : I won the million dollar jackpot.

- a. $\sim P$ b. $P \vee Q$ c. $P \rightarrow Q$ d. $P \wedge Q$ e. $P \leftrightarrow Q$ f. $\neg P \rightarrow \neg Q$
 g. $\neg P \wedge \neg Q$ h. $\neg P \vee (P \wedge Q)$

4. Write in symbolic form:

- a. You get an A on the final, but you don't do every exercise in this book, nevertheless, you get an A in this class.
- b. Getting an S on the final and doing every exercise in this book is sufficient for getting an A in this class.
- c. Your guarantee is good only if you bought your CD player less than 90 days ago.
- d. You get promoted only if you have connections and you have connections only if you get promoted.
- e. For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
- f. For you to win the contest it is necessary and efficient that you have the only running ticket.
- g. The trains run late on exactly those days when I take it.
- h. Students who have taken calculus or computer science, but not both, can enroll in this class.
- i. You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.
- j. If you read the newspaper every day, you will be informed and conversely.
- k. if not A then not B.
- l. A unless B.
- m. A if and only if B.
- n. If A and B then C.
- o. If A then both B and C.
- p. unless B, not A.
- q. If A then either B or C.
- r. If A then if B then C.

5. Write the negation in grammatically correct English:

- a. Today is Friday or Saturday
- b. It is not raining and it is not snowing
- c. If it is raining, then we will stay home and if it is snowing then we will go out.
- d. The printer is slow only if the file is damaged.

6. Find the converse, inverse & contra positive of the following:
- If x is negative and $x^2 = 4$, then $x = -2$.
 - A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
 - I come to class whenever there is going to be a quiz.
 - When I stay up late, it is necessary that I sleep until noon.
 - A necessary condition for the computer program to be correct is that it will not produce error message during translation.

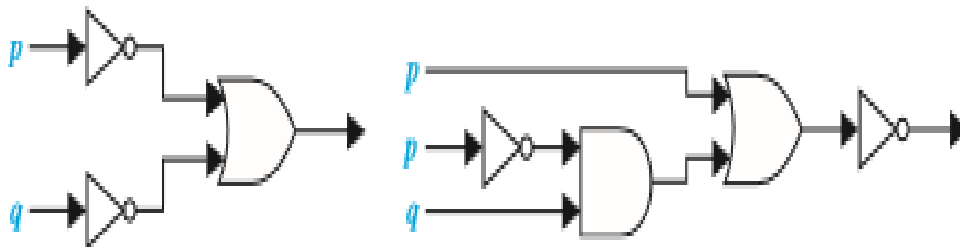
7. Construct truth table for the following:

- $(P \vee \neg Q) \rightarrow R$
- $(P \vee Q) \rightarrow (P \wedge Q)$
- $(P \rightarrow Q) \vee (Q \wedge R)$
- $(P \rightarrow Q) \rightarrow (Q \rightarrow P)$

8. Find the output of the circuits:

a. Fig.1

b. Fig. 2



9. Check if the following are a tautology or not:

- $[\neg P \wedge (P \vee Q)] \rightarrow Q$
- $[P \wedge (P \rightarrow Q)] \rightarrow P$
- $(P \wedge Q) \rightarrow (P \vee Q)$
- $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$
- $\neg(Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$

10. Verify the following equivalences:

- $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow \neg Q)$
- $\neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$
- $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$
- $(\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P) \Leftrightarrow T$
- $P \rightarrow [(P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)] \Leftrightarrow (Q \vee \neg P)$

11. Check whether the following formulae are valid, satisfiable (or) unsatisfiable

- $(A \vee \neg B \vee C) \wedge (\neg A \vee B \vee \neg C)$
- $\neg(A \wedge B) \Leftrightarrow ((\neg A \wedge \neg B))$
- $A \wedge (\neg B \vee \neg A) \wedge B \wedge (\neg B \vee A)$

12. Find the DNF of the following:

- $\neg(P \rightarrow (Q \wedge R))$
- $(Q \vee (P \wedge R)) \wedge \neg((P \vee R) \wedge Q)$
- $(\neg P \rightarrow R) \wedge (P \leftrightarrow Q)$

13. Find the CNF of the following :

- $\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$
- $Q \vee (P \wedge R) \wedge \neg((P \vee R) \wedge Q)$
- $\neg((P \vee \neg Q) \wedge \neg R)$

14. Find the PDNF of the following:

- $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$
- $(P \wedge \neg Q) \vee (Q \wedge \neg P) \vee (R \wedge P)$
- $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$

15. Find the PCNF of the following:

- $P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge (\neg P \vee Q)$
- $P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$
- $P \vee (\neg P \wedge \neg Q \wedge R)$