Mathematical Foundations of Computing

Propositional Calculus : 3

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Propositional Equivalences: Introduction

- In order to manipulate a set of statements for the sake of mathematical argumentation, an important step is to replace
  - one statement with
  - another equivalent statement
  - (i.e., with the same truth value)
- Let us discuss
  - Terminology
  - Establishing logical equivalences using truth tables
  - Establishing logical equivalences using known laws (of logical equivalences)

Logical Equivalences: Definition

- **Definition**: Propositions p and q are <u>logically equivalent</u> if $p \leftrightarrow q$ is a <u>tautology</u>.
- Informally, p and q are equivalent if whenever p is true, q is true, and vice versa
- Notation: $p \equiv q$ (p is equivalent to q), $p \leftrightarrow q$, and $p \Leftrightarrow q$
- Alert: $\equiv$ is not a logical connective

$\equiv$

An equivalent for implication

Is there an expression that is equivalent to $p \rightarrow q$ **but uses only the operators** $\neg$, $\wedge$, $\vee$**?**

Consider the proposition $\neg p \vee q$

Implication		
p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$\neg p$	$\neg p \vee q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	F	T

Logical Equivalences: Example 1

- Show that (Exercise 25 from Rosen)

p	*q*	*r*	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Equivalence Laws

Equivalence	Name of Identity
p$\wedge T \equiv p$ p$\vee F \equiv p$	Identity Laws
p$\wedge F \equiv F$ p$\vee T \equiv T$	Domination Laws
p$\wedge p \equiv p$ p$\vee p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Law
p$\wedge q \equiv q \wedge p$ p$\vee q \equiv q \vee p$	Commutative Laws
(p$\wedge q) \wedge r \equiv p \wedge (q \wedge r)$ (p$\vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
p$\wedge(q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ p$\vee(q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Ditributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
p$\wedge(p \vee q) \equiv p$ p$\vee(p \wedge q) \equiv p$	Absorption Laws
p$\wedge\neg p \equiv F$ p$\vee\neg p \equiv T$	Negation Laws

Conditional Equivalence laws

- Contrapositive: $P \rightarrow Q \equiv \sim Q \rightarrow \sim P$
- Conditional : $P \rightarrow Q \equiv \sim P \vee Q$
- Biconditional :
- $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

Construct a truth table

- Eg2.
- P ←→ (Q V R)

P	Q	R	$Q \vee R$	$P \Leftrightarrow (Q \vee R)$
T	T	T	T	**T**
T	T	F	T	**T**
T	F	T	T	**T**
T	F	F	F	**F**
F	T	T	T	**F**
F	T	F	T	**F**
F	F	T	T	**F**
F	F	F	F	**T**