→ Difference equations:

1) $y_{n+1} + y_n = 1$, $y_0 = 0$

$\Rightarrow y_{n+1} + y_n = 1$

$Z\{y_{n+1}\} + Z\{y_n\} - Z\{1\} = 0$

~~$z Z\{y_{n+1}\} + Z\{$~~

$z Z\{y_n\} - z y_0 + Z\{y_n\} - \frac{z}{z-1} = 0$

$z Z\{y_n\} + Z\{y_n\} = \frac{z}{z-1}$

$Z\{y_n\} = \frac{z}{(z-1)(z+1)}$

$\Rightarrow y_n = Z^{-1}\left[\frac{z}{(z-1)(z+1)}\right]$

$X(z) = \frac{z}{(z-1)(z+1)}$

$\frac{X(z)}{z} = \frac{1}{(z-1)(z+1)} = \frac{1}{2}\left[\frac{(z+1)-(z-1)}{(z+1)(z-1)}\right]$

$= \frac{1}{2}\left[\frac{1}{(z-1)} + \frac{1}{(z+1)}\right]$

$X(z) = \frac{1}{2}\left[\frac{z}{(z-1)} - \frac{z}{(z+1)}\right]$

Apply Inverse,

$Z^{-1}[X(z)] = \frac{1}{2}\left[1-(-1)^n\right]$

$\Rightarrow y_n = \frac{1}{2}\left[1-(-1)^n\right]$

2) $y_{n+2} + 6y_{n+1} + 9y = 2^n$; $y_0 = 0, y_1 = 0$

$$y_{n+2} + 6y_{n+1} + 9y_n - 2^n = 0$$

$$z\{y_{n+2}\} + 6z\{y_{n+1}\} + 9z\{y_n\} - z\{2^n\} = 0$$

$$z^2 z\{y_n\} + 6z z\{y_n\} + 9z\{y_n\} - \frac{z}{z-2} = 0$$

$$(z^2+6z+9)(z\{y_n\}) = \frac{z}{z-2}$$

$$\Rightarrow z\{y_n\} = \frac{z}{(z-2)(z+3)^2}$$

$$y_n = z^{-1}\left[\frac{z}{(z-2)(z+3)^2}\right]$$

$$X(z) = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z+3)^2}$$

$$\frac{1}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2}$$

$$A = 1/25 \quad B = -1/25 \quad C = -1/5$$

$$\frac{X(z)}{z} = \frac{1}{25(z-2)} + \frac{-1}{25(z+3)} + \frac{-1}{5(z+3)^2}$$

$$X(z) = \frac{z}{25(z-1)} + \frac{-z}{25(z+3)} + \frac{-z}{5(z+3)^2}$$

Apply Inverse,

$$z^{-1}[X(z)] = \frac{1(2^n)}{25} - \frac{1(-3^n)}{25} - \frac{1}{5}(-3)^{n-1}n$$

3) $y_{n+2} + y_n = 5.2^n$, $y_0 = 1$, $y_1 = 0$

$$Z\{y_{n+2}\} + Z\{y_n\} = Z\{5.2^n\}$$

$$z^2 Z\{y_n\} - z^2 y_0 - z y_1 + Z\{y_n\} = 5 \cdot \frac{z}{z-2}$$

$$z^2 Z\{y_n\} - z^2 + Z\{y_n\} = \frac{5z}{z-2}$$

$$(z^2+1)\, Z\{y_n\} = \frac{5z}{z-2} + z^2$$

$$Z\{y_n\} = \frac{5z + z^3 - 2z^2}{(z-2)(z^2+1)}$$

$$X(z) = \frac{5z + z^3 - 2z^2}{(z-2)(z^2+1)}$$

$$\frac{X(z)}{z} = \frac{5 + z^2 - 2z}{(z-2)(z^2+1)}$$

$$\frac{5 + z^2 - 2z}{(z-2)(z^2+1)} = \frac{A}{(z-2)} + \frac{Bz + C}{(z^2+1)}$$

$A = 1 \quad B = 0 \quad C = -2$

$$\frac{X(z)}{z} = \frac{1}{(z-2)} - \frac{2}{(z^2+1)}$$

$$X(z) = \frac{z}{z-2} - \frac{2z}{z^2+1}$$

Apply Inverse,

$$Z^{-1}[X(z)] = 2^n - 2\sin\left(\frac{n\pi}{2}\right)$$

4) $y_{n+2} - 76y_{n+1} + 12y_n = 2^n$; $y_0 = 0, y_1 = 0$

$$z\{y_{n+2}\} - 76z\{y_{n+1}\} + 12z\{y_n\} - z\{2^n\} = 0$$

$$z^2 z\{y_n\} - z^2 y_0 - z y_1 - 76[z z\{y_n\} - z y_0] + 12 z\{y_n\} - \frac{z}{z-2} = 0$$

$$z^2 z\{y\} - 76 z z\{y_n\} = \frac{z}{z-2}$$

$$z z\{y_n\} = \frac{z}{(z-2)(z^2-76z+12)}$$

$$y_k = z^{-1}\left[\frac{z}{(z-2)(z-(38+2\sqrt{358}))(z-(38-2\sqrt{358}))}\right]$$

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z-(38+2\sqrt{358}))(z-(38-2\sqrt{358}))}$$

By Partial fractions,

$$\Rightarrow 1 = A(z-(38+2\sqrt{358}))(z-(38-2\sqrt{358})) + B(z-2)(z-(38-2\sqrt{358})) + C(z-2)(z-(38+2\sqrt{358}))$$

$z = 38+2\sqrt{358}$;

$$B = \frac{1}{(36+2\sqrt{358})(4\sqrt{358})}$$

$z = 38-2\sqrt{358}$

$$C = \frac{1}{(36-2\sqrt{358})(-4\sqrt{358})}$$

$z = 2$,

$$A = \frac{1}{(-36-2\sqrt{358})(-36+2\sqrt{358})}$$

$$X(z) = \frac{z}{(-36-2\sqrt{358})(-36+2\sqrt{358})(z-2)} + \frac{z}{(36+2\sqrt{358})(4\sqrt{358})(z-(38+2\sqrt{358}))}$$

$$+\frac{3}{(36-2\sqrt{358})(-4\sqrt{358})}$$

$$(3-(38-2\sqrt{358}))$$

Apply Inverse,

$$y_n = Z^{-1}[X(3)] = \frac{2^n}{(-36-2\sqrt{358})(-36+2\sqrt{358})} + \frac{(38+2\sqrt{358})^n}{(36+2\sqrt{358})(4\sqrt{358})}$$

$$+ \frac{(38-2\sqrt{358})^n}{(36-2\sqrt{358})(-4\sqrt{358})}$$

5) $y_{n+2} + 5y_{n+1} + 6y_n = 2^n$; $y_0 = 0$, $y_1 = 0$

$$Z\{y_{n+2}\} + 5Z\{y_{n+1}\} + 6Z\{y_n\} - Z\{2^n\} = 0$$

~~$z^2 Z\{y_n\} + 5zZ\{$~~

$$z^2 Z\{y_n\} - z^2 y_0 - zy_1 + 5[zZ\{y_n\} - zy_0] + 6Z\{y_n\} - \frac{z}{z-2} = 0$$

$$(z^2+5z+6)\, Z\{y_n\} = \frac{z}{z-2}$$

$$X(z) = Z\{y_n\} = \frac{z}{(z-2)(z+2)(z+3)}$$

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z+2)(z+3)}$$

$$\frac{1}{(z-2)(z+2)(z+3)} = \frac{A}{(z-2)} + \frac{B}{(z+2)} + \frac{C}{(z-3)}$$

$$A = 1/20 \quad B = -1/4 \quad C = 1/5$$

$$\frac{X(z)}{z} = \frac{1}{20(z-2)} + \frac{-1}{4(z+2)} + \frac{1}{5(z-3)}$$

$$X(z) = \frac{z}{20(z-2)} - \frac{z}{4(z+2)} + \frac{z}{5(z+3)}$$

$$Z^{-1}[X(z)] = \frac{2^n}{20} - \frac{(-2)^n}{4} + \frac{(-3)^n}{5}$$

$$\Rightarrow y = \frac{2^n}{20} - \frac{(-2)^n}{4} + \frac{(-3)^n}{5}$$