

P S G College of Technology
Dept. of Applied Maths & Computational Sciences
III M.Sc [SS] – 18XW31 – Mathematical Foundations of Computing – Counting – PS 5

1. Generating Acronyms: How many three-letter acronyms are possible if every acronym must contain at least one vowel and at least one consonant with no repeating letters?
2. How many license plates consisting of 3 English letters followed by 3 digits contain no letter or digit twice ?
3. A department contains 10 men and 15 women. How many ways are there to form a committee with 6 members if it must have the same number of men and women?
4. How many strings of 8 English letters are there
 - a) That contain no vowels if letters can be repeated ?
 - b) That contain no vowels if letters cannot be repeated ?
 - c) That contain at least one vowel If letters can be repeated ?
 - d) That contain exactly one vowel if letters can be repeated ?
5. A tree diagram to determine the number of subsets of $\{3, 7, 9, 11, 24\}$ with the property that the sum of the elements in the subset is < 28 .
6. Three married couples have got 6 seats in a row for a performance of a comedy.
 - (a) In how many ways can they be seated?
 - (b) In how many ways can they be seated if each couple is to sit together with the husband to the left of his wife?
 - (c) In how many ways can they be seated if each couple is to sit together?
 - (d) In how many ways can they be seated if all the men are to sit together and all the women are to sit together?
7. In how many ways can 8 people A, B, C, D, E, F, G and H be seated in a row if
 - (a) there are no restrictions on seating arrangement;
 - (b) persons A and B must not sit next to each other;
8. In how many ways can six coupons for free lunches at different restaurants be distributed among 10 students
 - (a) If none is to receive more than one coupon ?
 - (b) if there is no restriction on the number of coupons that each student can receive?
9. How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if n has to exceed 50,00,000 ?
10. How many strings can be made by reordering the letters of “darickswaihongchan”?
11. In a class of 20 students, 5 of them will get Grade A, 10 of them Grade B, 3 of them Grade C and 2 will be fail. How many grade distributions are possible among 20 ?
12. How many ways can we distribute a standard deck of 52 playing cards into 4 sets of 13 cards each?
13. Suppose there are 1 red ball, 1 blue ball and 1 green ball in a box. Five students are invited to come out one by one to draw a ball from the box and put it back. How many combinations of colors are possible? (Note: “GRBBR”, “RBRGB” are regarded same)
14. In a lottery, 5 winning nos. are selected out of 15 numbers. How many possibilities?
15. A common form of poker involves hands (sets) of five cards each, dealt from a deck consisting of 52 different cards. How many different 5-card hands are possible?
16. A club has 25 members:

- a) How many ways are there to choose 4 members of the club to serve on an executive committee?
 - b) How many ways are there to first choose a president, and then a vice president, treasurer and secretary of the club?
17. A museum has enough open wall space to display either four $4' \times 3'$ paintings or two $8' \times 4'$ paintings. In their collection, there are 23 paintings of size $4' \times 3'$, and 5 paintings of size $8' \times 4'$. How many display options are there for the museum?
 18. A clothing company is developing a new shirt design. The shirt will be white and have three stripes down the right side. The colors of the stripes will be green, yellow, and orange. How many ways can these strips be ordered on the shirt?
 19. How many solutions are there to the equation $x_1 + x_2 + x_3 = 17$ and x_1, x_2 and x_3 are non-negative integers with
 - a) $x_1 > 1, x_2 > 2$ and $x_3 > 3$?
 - b) $x_1 < 6$ and $x_3 > 5$?
 20. How many ways are there to distribute six objects to five boxes if
 - a) Both objects and boxes are labeled?
 - b) The objects are labeled and boxes are unlabeled?
 - c) The objects are unlabelled and boxes are labelled?
 - d) The objects and boxes are unlabelled?
 21. A witness to a hit-and-run accident tells the police that the license plate of the car in the accident, which has the format XX 99 XX 9999, has TN 38 AS and contains both the digits 1 and 2. How many different license plates can fit this description?
 22. Find a RR for the number of bit strings of length n that contain 3 consecutive 0's. What are the initial conditions? How many bit strings of length 7 contain 3 consecutive 0's?
 23. Solve the RR with the initial conditions given:
 - a) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$ and $a_1 = 0$
 - b) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_0 = 4$ and $a_1 = 1$
 24. The Lucas numbers satisfy the RR $L_n = L_{n-1} + L_{n-2}$ and the initial conditions $L_0 = 2$ and $L_1 = 1$. Show that $L_n = f_{n-1} + f_{n+1}$ for $n = 2, 3, 4, \dots$ where f_n is the n th Fibonacci number. Find an explicit formula for the Lucas numbers.
 25. Find the solution to $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3, 4, 5, \dots$ with $a_0 = 3$, $a_1 = 6$ and $a_2 = 0$.
 26. Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ for $n = 3, 4, 5, \dots$ with $a_0 = 7$, $a_1 = -4$ and $a_2 = 8$.
 27. Solve the RR $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ for $n = 3, 4, 5, \dots$ with $a_0 = 5$, $a_1 = -9$ and $a_2 = 15$.
 28. What is the general form of the solutions of a linear homogenous RR if its characteristic equation has roots $-1, -1, -1, 2, 2, 5, 5, 7$?
 29. Consider the non-homogeneous linear RR $a_n = 2a_{n-1} + 2^n$. Show that $a_n = n2^n$ is a solution of this RR. Find all solutions of this RR and find the solution with $a_0 = 2$.
 30. Find all solutions of the RR $a_n = 2a_{n-1} + 2n^2$. Find the solution of the RR with initial condition $a_1 = 4$.

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