Mathematical Foundations of Computing

Propositional Calculus : 2

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Converse, Inverse, Contrapositive

- Consider the proposition $p \rightarrow q$
  - Its <u>converse</u> is the proposition $q \rightarrow p$
  - Its <u>inverse</u> is the proposition $\neg p \rightarrow \neg q$
  - Its <u>contrapositive</u> is the proposition $\neg q \rightarrow \neg p$

Eg.

- If it snows, the traffic moves slowly.
- p : it snows;   q : the traffic moves slowly
- **Converse**: If the traffic moves slowly, then it snows.
- **Inverse**: If it does not snow, then the traffic does not move slowly.
- **Contrapositive**: If the traffic does not move slowly, then it does not snow.

Examples to work: C, I and CP

- If I am not the President of US, then I will walk to work.
- If I have enough money, then I will buy a car and I will buy a house.
- If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.

Tautology, Contradiction and Satisfiability

- Propositions are synonymous with statements, logical expressions, logical formulas, well formed formulas (wff),
- For any statement formula, we construct a truth table.
- Each row in the TT is called as an interpretation.
- An interpretation is an assignment of truth vale(T or F) to a proposition.
- A compound statement that is always true is a **tautology**
- If a formula is uniformly false, then it is a **contradiction**.
- If a formula is neither a tautology nor a contradiction then it is a **contingency**.
- Examples
  - tautology : $p \vee \neg p$
  - contradiction : $p \wedge \neg p$
  - contingency : p → q

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$\neg p \vee p$	$\neg p \wedge p$
0	0	1	1	1	1	0
0	1	1	1	1	1	0
1	0	0	0	0	1	0
1	1	1	0	1	1	0

Satisfiability

- A formula is satisfiable if there is at least one assignment of truth values to its variables that makes the whole formula true.
- If all assignments are true, then it is said to be valid. (Tautology)
- If all assignments are false, then it is invalid or unsatisfiable.(contradiction)

a	b	c	$b \to c$	$a \vee (b \to c)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

Construct a truth table

- Eg1.
- (P V Q) ^ ~ (P ^ Q)

P	Q	$(P \vee Q)$	$(P \wedge Q)$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge \sim(P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Construct a truth table

- Eg2.
- P ←→ (Q V R)

P	Q	R	$Q \vee R$	$P \Leftrightarrow (Q \vee R)$
T	T	T	T	**T**
T	T	F	T	**T**
T	F	T	T	**T**
T	F	F	F	**F**
F	T	T	T	**F**
F	T	F	T	**F**
F	F	T	T	**F**
F	F	F	F	**T**