P S G College of Technology
Dept. of Applied Maths & Computational Sciences
III Sem M.Sc. Software Systems
18XW31 MATHEMATICAL FOUNDATIONS OF COMPUTING
* Relations & Functions* PS 4

1) For the relations on {1,2,3,4},check if it is reflexive, symmetric, asymmetric, or transitive. Represent the relation using digraph, matrix, and table.
   a) {(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)}
   b) {(2,4),(4,2)}
   c) {(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)}
2) Determine whether the relation R on the set of all real nos. is reflexive, symmetric, anti-symmetric, and / or transitive, where (x,y) belongs to R if and only if,
   a) x!=y b) xy >=1
   c) x=y+1 or x=y-1 d) x is a multiple of y
   e) =y^2 f) x ≡ y (mod 7)
3) Let R1={(1,2),(2,3),(3,4)} and R2={(1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)} be the relations from {1,2,3} to {1,2,3,4}.Find
   a) R1 union R2 b) R1 intersection R2
   c) R1-R2 d) R2-R1
4) Let R1 and R2 be the "divides" and "is a multiple of" relations on the set of all positive integers,respectively. That is,R1={(a,b)/a divides b} and R2={(a,b)/a is a multiple of b}.Find
   a) R1 union R2 b) R1 intersection R2
   c) R1-R2 d) R2-R1
5) let R be the relation {(1,2),(1,3),(2,3),(2,4),(3,1)} , and let S be the relation {(2,1),(3,1),(3,2),(4,2)}.Find S composition R.
6) Let R be the relation on {1,2,3,4} containing the ordered pairs(1,1),(1,2),(1,3),(2,3),(2,4),(3,1),(3,4),(3,5),((4,2,(4,5),(5,1),(5,2), and (5,4).Find
   a) R^2 b) R^3 c) R^4 d) R^5
7) Let A={1,2,3,4,5,6,7,8,9,10}. Let R={(1,2), (1,4), (1,6), (1,8), (1,10), (3,5), (3,7), (4.6), (6,8), (7,10)} be a relation on A. Let S={(2,4), (3,6), (5,7), (7,9), (8,10), (8,9), (8,8), (9,9), (3,8), (4,9)} be a second relation on A. Find a) R o S b) S o R
8) Let A = { 1, 2, 3}, B = {u, v}, R1 = { (1,u), (2,u), (2,v), (3, u) }, R2 = { (1, v), (3,u), (3, v) } Find a) $R1 \cup R2$ b) $R1 - R2$ c) M_{R1}, d) M_{R2} e) $R1 \cap R2$, f) $R1 \oplus R2$.
9) List the triples in the relation { (a,b,c) / a,b,c are integers with 0 < a<b<c<5.
10) List the 4-tuples in the relations { (a,b,c,d) / a,b,c,d are positive integers with abcd=6}
11) Find all the primary keys and a composite key with two fields containing Airline field for Table 8.

12) What is the result when you apply selection operator C=Destination=Detroit on Table 8.

TABLE 8 Flights.

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08:47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

TABLE 9 Part_needs.

Supplier	*Part_number*	*Project*
23	1092	1
23	1101	3
23	9048	4
31	4975	3
31	3477	2
32	6984	4
32	9191	2
33	1001	1

TABLE 10 Parts_inventory.

Part_number	*Project*	*Quantity*	*Color_code*
1001	1	14	8
1092	1	2	2
1101	3	1	1
3477	2	25	2
4975	3	6	2
6984	4	10	1
9048	4	12	2
9191	2	80	4

13) Display the projection $P_{1,2,4}$ to the table 8.

14) Construct the table obtained by applying the join operator J2,3 to the relations in tables 9 and 10.

15) Let R be a relation {(a,b) / a divides b} on the set of integers. What is the symmetric closure of R?

16) A = {3, 5, 6, 7}; B = {4, 5, 9} R = { (x, y) | x ∈ A, y ∈B, x<y};

S = { (x, y) | x ∈ A, y ∈ B, |x-y| = 2 }What are the elements of R, S ?

Find a) $R \cap S$ b) $R \cup S$ c) R – S d) S – R

17) Determine whether the relation R on the set of all web pages is reflexive, symmetric, anti-symmetric and transitive where (a, b) ∈ R iff
   a. Everyone who has visited webpage 'a' has also visited webpage 'b'
   b. There are no common links found on both webpage a and webpage b.
   c. There is a webpage that includes links to both webpage a and b.
18) Let X, Y, Z be three sets as X = {1, 3, 5} Y = {2, 4, 8} Z = {2, 3, 6}; Let R : X→ Yand S : Y → Z be the relations "less than" and "less than or equal to". Find $R \circ S$? Give its Graphical representation.
19) Use Algorithm on Zero-One Matrix method to compute transitive closure using Boolean product of these relations on {1,2,3,4}
   a) {(1,2), (2,1), (2,3), (3,4), (4,1)} b) {(2,1),(2,3),(3,1),(3,4),(4,1),(4,3)}
20) Use Warshall's algorithm to compute the transitive closure of the relations on {a,b,c,d,e}
   a) { (a,c), (b,d), (c,a), (d,b), (e,d) }b) { (b.c), (b,e), (c,e), (d,a), (e,b), (e,c) }
21) Let R be a relation that contains the pair (a,b) if a and b are cities such that there is a direct non-stop airline flight from a to b. When is (a,b) in a) R^2 ? b) R^3 ? c) R* ?
22) Let R be the relation on the set of all people who have visited a particular web page such that x R y iff person x and person y have followed the same set of links starting at tjis web page (going from web page to web page until they stop using the web). Show that R is an equivalence relation.
23) Which of these relations on (0,1,2,3} are equivalence relations OR partial orderings?
   a) {(0,0),(1,1),(2,2),(3,3)}
   b) {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)}
24) Determine if relation represented by $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is an equivalence relation ?
25) List the ordered pairs in the equivalence relations produced by the partitions of {0}, {1,2}, {3,4,5} of {0,1,2,3,4,5}
26) Is the pair 5,25 comparable in the poset (Z+, |) ?
27) Draw the Hasse diagram for the "greater than or equal to" relation on {0,1,2,3,4,5}.
28) What is the covering rel. of the partial ordering {(a,b) | a divides b} on {1, 2, 3, 4, 6, 12} ?
29) Answer the following for the poset ({3,5,9,15,24,45}, |)
   a. Find the maximal and minimal elements
   b. Find the greatest element and least element, if available.
   c. Find all upper bounds of {3,5}. Find the least upper bound of {3,5}, if it exists.

d. Find all lower bounds of {15, 45}. Find the greatest lower bound of {15,45} if it exists.

30) Find the computable total ordering for the poset ({1,2,4,5,12,20}, |).

31) Evaluate the expressions:. a) floor($\log_2 17$) b) gcd(14 mod 6, 18 mod 7)

32) Find the composition fo g and g o f and find an integer x such that f(g(x)) ≠g(f(x))

(a) f(x) = ceiling(x/2) and g(x) = 2x (b) f(x)=gcd(x,10) and g(x) = x mod 5

33) Let f(x) = x^2 and g(x,y)= x+y. Find compositions that use f and g for expressions:

(a) $(x+y)^2$ (b) $x^2+y^2+z^2$

34) Find a definition of the function max4 that calculates the maximum value of 4 numbers. Use only composition and the function max that gives the maximum of two numbers.

35) Are the function Injective, surjective or bijective ?

(a) f : R → Z defined by f(x)= $\lceil x+1 \rceil$

(b) f : N_8→N_8 defined by f(x)= 2x mod 8

(c) f : N x N →N defined by f(x,y)=2x+y

(d) Let (0,1)=$\{x \in R \mid 0 < x < 1\}$ $and\ R^+\ denote\ the\ set\ of\ positive\ real\ numbers; f:$ $\{0,1\} \rightarrow R^+\ defined\ by\ f(x) = \frac{x}{1-x}$

36) Find the inverses :

(a) f : N_{26}→N_{26} defined by f(x) = (x+5) mod 26

(b) f : N →N defined by f(x)= If x is odd than x -1, else x +1.

(c) f : Z → N defined by f(x) = |x +1|

(d) f : R^+→ (0,1) defined by f(x)=1/(x+1)

37) Let f and g be bijections from A to A such that g(f(x))=x for all x ϵ A. Prove that f(g(x)) = x for all x ϵ A.

38) Let S={one, two, three, four, five, six, seven, eight, nine} and let f : S → N_9 be defined by f(x)=(3 |x|) mod 9, where |x| means the number of letters in x. For each of the following gaps construct a hash table that contains the strings of S by choosing a string for entry in the table by the order that is listed in S. Resolve collisions by linear probing with the given gap and observe whether all strings can be placed in the table.

(a) Gap = 1 (b) Gap = 2 (c) Gap = 3

39) Repeat (8) for S = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} and the function f : S → N_7 defined by f(x) = (2(x) +3) mod 7.

40) Find integers a and b such that the function f : N_{12}→N_{12} defined by f(x) = (ax+b) mod 12 is bijective and f^{-1} = f.

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