1. Show that C follows from the premises using truth table technique:
   a. $H_1 : P \to Q; H_2 : Q \to R, C : P \to R$
   b. $H_1 : \neg P \to Q; H_2 : \neg(Q \wedge \neg R), H_3 : \neg R; C : \neg P$

2. Prove the following using direct method:
   a. $A \wedge B, \ (A \leftrightarrow B) \to (C \vee D) \Rightarrow D \vee C$
   b. $P \to Q, \ (\neg Q \vee R) \wedge \neg R, \ \neg(\neg P \wedge S) \Rightarrow \neg S$
   c. $\neg J \to (M \vee N), \ (H \vee G) \to \neg J, \ H \vee G \Rightarrow M \vee N$
   d. $P \vee Q, P \to R, Q \to S \Rightarrow S \vee R$

3. Prove the following using indirect proof:
   a. $P \to Q, Q \to R, \neg(P \wedge R), P \vee R \Rightarrow R$
   b. $S \to \neg Q, S \vee R, \neg R, \neg R \leftrightarrow Q \Rightarrow \neg P$
   c. $\neg(P \to Q) \to \neg(R \vee S), (Q \to P) \vee \neg R, R \Rightarrow P \leftrightarrow Q$
   d. $P \to (Q \to R), \neg S \vee P, Q \Rightarrow S \to R$

4. Check whether the following specifications are consistent or not:
   a. The system is in multi-user state if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multi-user state, then it is in interrupt mode. The system is not in interrupt mode.
   b. The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed. The router does not support the new address space.
   c. If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer.
   d. If Miranda doesn't take a course in discrete mathematics, then she will not graduate. If Miranda does not graduate, then she is not qualified for the job. If Miranda reads the book, then she is qualified for the job. Miranda doesn't take a course in discrete mathematics but she reads the book.

5. Check for validity of arguments:

a. I play basketball and football. If today isn't Saturday, then I play basketball and football.If today is Friday OR today is Saturday, then I don't play football. Therefore, I don't play football.
b. Frank bought a personal computer or a video cassette recorder (VCR).If he bought a VCR, the he likes to watch movies at home. He doesn't like to watch movies at home. Therefore, Frank bought a personal computer.

c. If the band could not play rock music or the refreshments were not delivered on time, then the New year's party would have been cancelled and Alicia would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music.

6. Express the following sentences using quantifiers:
   a. At least one router is functioning normally if the throughout is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.
   b. No rabbit knows calculus.
   c. There is a student who took a discrete math course, but doesn't know Python.
   d. Every student in this class has taken some course in every department in the school of mathematical sciences.
   e. There is a building on the campus of some college in the United States in which every room is painted white.
   f. Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.
   g. Sanjay has chatted with everyone except Joseph.
   h. There is no one in this class who knows French and Hindi.
   i. Not all college graduates are smart.
   j. People who play golf are kind to animals.
   k. Everyone knows someone who plays golf.
   l. No bird eats only worms.
   m. Every committee member is rich and famous.

7. Indicate the free and bound variables. Also show the scope of the quantifiers.
   a. $(\forall x)\,(P(x) \wedge R(x)) \rightarrow (\forall x)\,P(x) \wedge Q(x)$
   b. $(\forall x)\,(P(x) \wedge (\exists x)Q(x)) \vee ((\forall x)\,P(x) \rightarrow Q(x))$
   c. $(\forall x)\,[P(x) \leftrightarrow Q(x) \wedge (\exists x)\,R(x)] \wedge S(x)$

8. Prove the following implications :
   a. $(\forall x)(\forall y)\,[P(x, y) \rightarrow Q(x, y)] \wedge \neg Q(a,b) \Rightarrow \neg P(a,b)$
   b. $\neg[(\exists x)\,P(x) \wedge Q(a)] \Rightarrow (\exists x)P(x) \rightarrow \neg Q(a)$
   c. $(\exists x)\,P(x) \wedge (\forall x)[P(x) \rightarrow Q(x)] \Rightarrow (\exists x)\,Q(x)$

9. Prove the following:
   a. $\forall x\big(P(x) \rightarrow Q(x)\big) \Longleftrightarrow \forall x\,P(x) \rightarrow \forall x\,Q(x)$
   b. $(\forall x(P(x)) \wedge A \Longleftrightarrow \forall x\,(P(x) \wedge A\,)$
   c. $(\exists x\,(P\,(x)) \vee A \Longleftrightarrow \exists x\,(P(x) \vee A)$
   d. $\forall x\,(P(x) \wedge \exists x\,Q\,(x) \Longleftrightarrow \forall x\,\exists y(P(x) \wedge Q(y))$
   e. $\forall x\,P\,(x) \vee \forall x\,Q\,(x) \Longleftrightarrow \forall x\,\forall y\,(P(x) \vee Q(y))$

10. Infer the following:
    a. Jack owns a dog. Every dog owner is a animal lover. No animal lover kills an animal. Either Jack or curiosity killed the cat, who is named Tuna. Did curiosity kill the cat ?
    b. John likes all kinds of food. Apples are food. Chicken is food. Anything anyone eats and is not killed by is food. Bill eats peanuts and is still alive. Sue eats everything Bill eats. Prove John likes peanuts.