

P S G COLLEGE OF TECHNOLOGY
DEPT OF APPLIED MATHEMATICS & COMPUTATIONAL SCIENCES
18XW31 MATHEMATICAL FOUNDATIONS OF COMPUTING : ASSIGNMENT TOPICS 1
Proof Methods PS3

1. What is a conjecture ? When does it become a proof? 19PW01
2. What is Fermat's last theorem ? Relate it to proposition 19PW02
3. What is Goldbach Conjecture ? Relate it to proposition 19PW03
4. Let n be an integer. Prove that if n^2 is odd, then n is odd. 19PW04
5. If $a \mid b$ and $a \mid c$ then $a \mid b+c$ (Direct Method) 19PW05
6. Every odd integer is a difference of 2 squares. (eg. $7=4^2 - 3^2$) (direct method) 19PW06
7. Prove that $\sqrt{2}$ is irrational by contradiction. 19PW08
8. Use a proof by contradiction to prove that at least one of the numbers $a_1; a_2; \dots; a_n$ is greater than or equal to the average of these numbers, $(a_1 + a_2 + \dots + a_n) / n$. 19PW09
9. If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (Try Cases) 19PW10
10. Suppose $a, b \in \mathbb{Z}$. If both ab and $a+b$ are even, then both a and b are even. (Contrapositive) 19PW11
11. The product of any 5 consecutive integers is divisible by 120. ($3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 2520 = 120 \cdot 21$) (Any Method) 19PW13
12. There exist irrational numbers x and y such that x^y is rational. (constructive proof) 19PW14
13. Prove or disprove : If A, B and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$. (counter example) 19PW15
14. Conjecture a formula for the sum of first n +ve odd integers and prove by Mathematical Induction. 19PW16
15. Given a chocolate bar with $m \times n$ squares of chocolate, task is to divide it into mn individual squares. Only allowed to split one piece of chocolate at a time using a vertical or a horizontal break. For example, suppose that the chocolate bar is 2×2 . The first split makes two pieces, both 2×1 . Each of these pieces requires one more split to form single squares. This gives a total of three splits. Use strong induction to conclude : To divide up a chocolate bar with $m \times n$ squares, we need at most $mn - 1$ splits. Hint: Use strong induction on k , the number of squares in the chocolate bar ($k = mn$). 19PW17

Additional Problems

1. Prove by mathematical induction $1.2.3+2.3.4+ \dots n.(n+1).(n+2)=1/4.n.(n+1).(n+2).(n+3)$
2. Find a formula for $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{2.4} \dots + \frac{1}{n.(n+1)}$ by examining the values of this expression for small n. Prove the formula you conjectured.
3. Use M. I to prove the inequality $n < 2^n$ for every positive integer n.
4. What is a harmonic ? P. T $H_{2^n} \geq 1 + \frac{n}{2}$
5. P. T. $\overline{\bigcap_{j=1}^n A_j} = \bigcup_{j=1}^n \overline{A_j}$ where $A_1, A_2 \dots A_n$ are subsets of universal set and $n \geq 2$.