

P S G College of Technology
Dept. of Applied Maths & Computational Sciences
III M.Sc [SS] – 18XW31 – MFOC – Grammar, Automata, PDA, TM – P Sheet 6

- 1) Let $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$ Determine whether $G = (V, T, S, P)$ is a type 0 grammar but not a type 1 grammar, a type 1 grammar but not a type 2 grammar, or a type 2 grammar but not a type 3 grammar if P , the set of productions, is
 - a) $S \rightarrow aAB, A \rightarrow Bb, B \rightarrow A.$
 - b) $S \rightarrow ABa, AB \rightarrow a.$
 - c) $S \rightarrow aA, A \rightarrow a, A \rightarrow b.$
 - d) $S \rightarrow aA, aA \rightarrow B, B \rightarrow aA, A \rightarrow b.$
- 2) Find the language generated by the following grammars :
 - a) Let $G = (\{B, S\}, \{x, y, z, 0, 1\}, S, \{S \rightarrow xBz, B \rightarrow y \mid 0B1\})$ Derive the word $x00y11z$.
 - b) Let $G = (N, T, P, S)$ where $N = \{B, S\}$ $T = \{a, b, x, y, z\}$,
 $P: \{S \rightarrow xBz, B \rightarrow aybb \mid aBb \mid Bb\}$
 - c) Let $G = (N, T, P, S)$ where $N = \{D, J, K, S\}$ $T = \{a, b, c, d, e, f\}$
 $P: \{S \rightarrow DK, D \rightarrow J \mid aDd, J \rightarrow \lambda \mid bJc, K \rightarrow \lambda \mid eKf\}$
 - d) Let $G = (\{S, A, B, C\}, \{0, 1, 2\}, S, \{S \rightarrow C, C \rightarrow 0CAB, C \rightarrow \lambda, BA \rightarrow AB, 0A \rightarrow 01, 1A \rightarrow 11, 1B \rightarrow 12, 2B \rightarrow 22\})$
 - e) Determine whether the word $cbab$ belongs to language generated by $G = (\{A, B, C, S\}, \{a, b, c\}, S, \{S \rightarrow AB; A \rightarrow Ca; B \rightarrow Ba; B \rightarrow Cb; B \rightarrow b; C \rightarrow cb; C \rightarrow b\})$
- 3) For the following grammars and target strings, decide whether or not the word is generated by the grammar:
 - a) $S \rightarrow SS, S \rightarrow a, S \rightarrow bb;$ $w = abba$
 - b) $S \rightarrow XS, X \rightarrow XX, X \rightarrow a, S \rightarrow b.$ $w = baab$
 - c) $S \rightarrow AB \mid CD \mid a \mid b, A \rightarrow a, B \rightarrow SA, C \rightarrow DS, D \rightarrow b;$ $w = bababab$
- 4) Define a grammar for each of the following languages.
 - a) $L = \{a^{n+1}bc^n / n \in \mathbb{N}\}$
 - b) $L1 = \{a^n / n \in \mathbb{N}\} \cup L2 = \{bc^n / n \in \mathbb{N}\}$
 - c) $L = \{a^{n+1}(bc)^n \mid n \in \mathbb{N}\}.$
 - d) $L = \{a^i b^j c^k d^m / i = k \text{ and } j = m\}$
- 5) Let $G = (V, T, S, P)$ be the phrase-structure grammar with $V = \{O, 1, A, B, S\}$, $T = \{O, 1\}$ and set of productions P consisting of $S \rightarrow OA, S \rightarrow 1A, A \rightarrow OB, B \rightarrow 1A, B \rightarrow 1$.
 - a) Show that 10101 belongs to the language generated by G .
 - b) Show that 10110 does not belong to the language generated by G .
 - c) What is the language generated by G ?
- 6) Give NFAs with the specified number of states recognizing the following languages.
 - a. The language $\{\omega \mid \omega \text{ ends with } 00\}$ with three states.
 - b. The language $\{0\}$ with two states.
 - c. The language $0^*1^*0^*0$ with three states.
 - d. The language $\{\lambda\}$ with one state.
 - e. The set of all strings where pairs of adjacent 0s must be separated by at least one 1, except in the last four characters.
 - f. The set of all strings that do not contain the substring 000 .
- 7) Give state diagrams of DFAs recognizing the following languages. The alphabet is $\{0, 1\}$.
 - a. $\{\omega \mid \omega \text{ begin with a 1 and ends with a 0}\}$
 - b. $\{\omega \mid \omega \text{ contains at least three 1s}\}$
 - c. $\{\omega \mid \omega \text{ contains the substring } 0101, \text{ i.e., } \omega = x0101y \text{ for some } x \text{ and } y\}.$
 - d. $\{\omega \mid \omega \text{ has length at least 3 and its third symbol is a 0}\}.$

- e. $\{\omega \mid \omega \text{ starts with 0 and has odd length, or starts with 1 and has even length}\}$
 - f. $\{\omega \mid \omega \text{ doesn't contain the substring 110}\}$.
 - g. $\{\omega \mid \text{the length of } \omega \text{ is at most 5}\}$.
 - h. $\{\omega \mid \omega \text{ is any string except 11 and 111}\}$.
 - i. $\{\omega \mid \text{every odd position of } \omega \text{ is a 1}\}$.
 - j. $\{\omega \mid \omega \text{ contains at least two 0s and most one 1}\}$.
 - k. $\{\lambda, 0\}$.
 - l. $\{\omega \mid \omega \text{ contains an even number of 0s, or exactly two 1s}\}$.
 - m. The empty set.
 - n. All strings except the empty string.
- 8) The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q1	q1	q2
q2	q1	q3
q3	q2	q4
q4	q3	q5
q5	q4	q5

- 9) Show that each of the following grammars is ambiguous:
- a) $S \rightarrow SabS \mid \lambda$.
 - b) $S \rightarrow SbS \mid A \quad A \rightarrow a \mid aA$.
- 10) Let $s=(0,1)$. Give RG corresponding to the set
- a) $(00, 010, 0110, 01110, \dots)$
 - b) $(0, 001, 000, 00001, 00000, 0000001, \dots)$
- 11) Construct PDA for the following:
- a. $L = \{a^m b^{m+n} c^n \mid m \geq 0, n \geq 1\}$ by final state
 - b. $L = \{a^n b^{2n} \mid n \geq 1\}$ by empty stack.
 - c. $L = \{a^{2n} b^{3n} \mid n \geq 0\}$ by final state.
 - d. $L = \{ww^R \mid w \in \{a, b\}^+\}$
- 12) Construct TM for the following:
- a. $L = \{a^m b^n \mid m, n \geq 1\}$.
 - b. L on $\{a, b\}$ given by $L = \{w \mid w \text{ is a multiple of 3}\}$
 - c. $L = \{0^n 1^n \mid n \geq 1\}$
 - d. To accept all bit strings that consist of an even number of 1s.
 - e. $L = \{1^n 2^n 3^n \mid n \geq 1\}$
 - f. $L = \{0,1\}^*$ ending with 010.
 - g. To find an even palindrome over $\{0,1\}$
 - h. To copy a string of 1s. Input: B11111B; Output : B1111111111B
 - i. To do addition. Input: B111011B Output: B11111B
 - j. To do proper subtraction. Input: B1111011B Output: B11B
 - k. To perform $n \bmod 2$.
 - l. To convert a number to its binary equivalent. (from 11111 to 101)
 - m. To perform string concatenation.

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