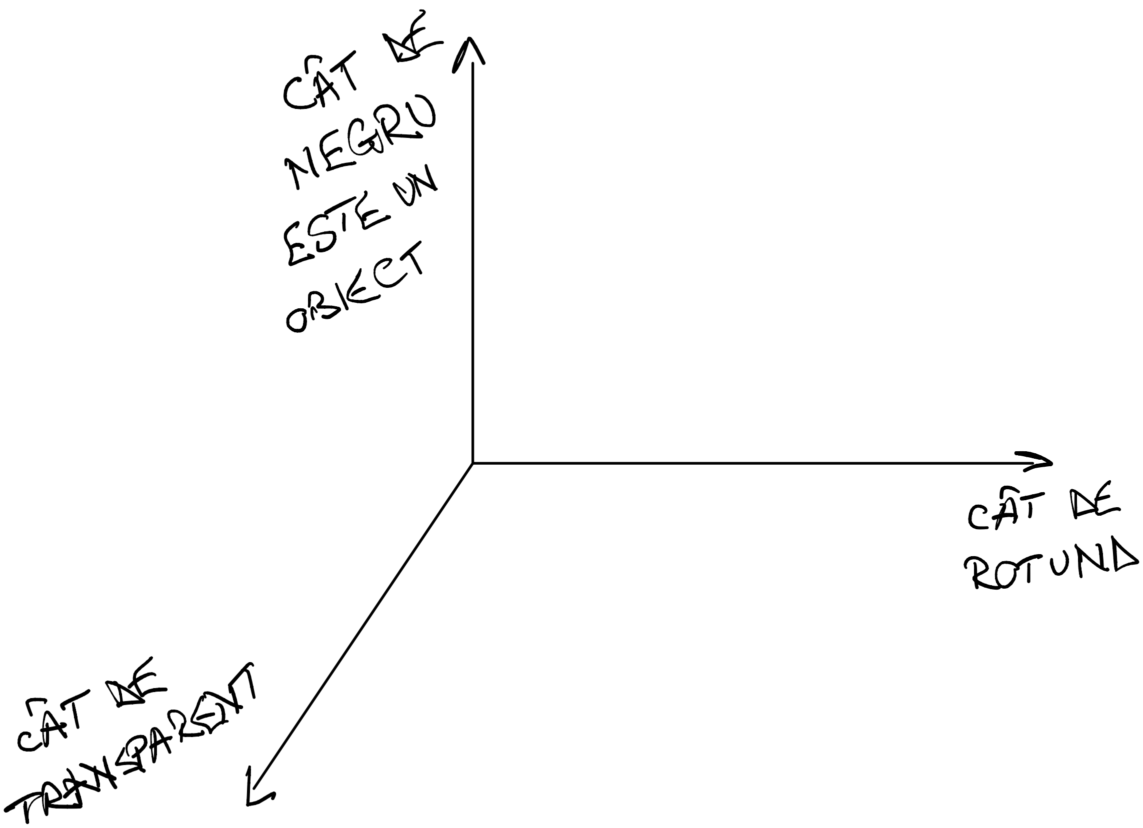
RECAPITULARE

$[x] \rightarrow$ SCALAR

$[x_1\ x_2 \dots x_n] \rightarrow$ VECTOR $\leftrightarrow$ MATRICI UNIDIMENSIONALE

$$\begin{bmatrix} [x_{11}\ x_{12} \dots x_{1n}], \\ [x_{21}\ x_{22} \dots x_{2n}] \\ \vdots \end{bmatrix} \rightarrow \text{MATRICE} \leftrightarrow \text{MATRICE BIDIMENSIONALE}$$

$\vdots \rightarrow$ TENSOR



• $\begin{cases} x_1 + x_2 = 10 \\ 3x_1 - x_2 = 2 \end{cases}$ (+)

$4x_1 = 12 \mid :4$

$x_1 = 3 \Rightarrow x_2 = 7$

$\begin{cases} x_1 + x_2 = 10 \\ 3x_1 - x_2 = 2 \end{cases} \Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

$$\boxed{X = A^{-1} \cdot Y}$$

A^{-1} = INVERSA MATRICEI A $\longrightarrow$ $\boxed{A \cdot A^{-1} = I}$

MATRICEA IDENTITATE I_n

1. Va avea 1 pe diagonala principală $\begin{bmatrix} o & & \\ & \ddots & \\ & & o \end{bmatrix}$
2. Va avea 0 în rest

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

↳ Elementul neutru la înmulțirea de matrici

$B = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}$

$B \cdot I_2 = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\cdot1+4\cdot0 & 1\cdot0+4\cdot1 \\ 3\cdot1+0\cdot0 & 3\cdot0+0\cdot1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix} = B$

$$\boxed{A \cdot I_n = A}$$

$$C = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 4 \end{bmatrix}$$

$$\boxed{C_{23} * D_{32} = \text{rezultat}}$$

$$C * D = \begin{bmatrix} 2\cdot 1 + 0\cdot 0 + 1\cdot 0 & 2\cdot 0 + 0\cdot 3 + 1\cdot 4 \\ 3\cdot 1 + 1\cdot 0 + 2\cdot 0 & 3\cdot 0 + 1\cdot 3 + 2\cdot 4 \end{bmatrix}$$

$$C * D = \begin{bmatrix} 2 & 4 \\ 3 & 11 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$C_{23} * I_{33} = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

A^{-1} = INVERSA MATRICEI A $\rightarrow$ $\boxed{A \cdot A^{-1} = A^{-1} \cdot A = I}$

- DETERMINANTUL MATRICEI A

$$\boxed{A_{11} = [x] \rightarrow \det A = x}$$

$$\boxed{A_{22} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow \det A = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}}$$

$$\rightarrow \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$A_{22} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow \det A = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

Ex.:

$$B = \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix} \rightarrow \det B = 1 \cdot 3 - 4 \cdot 5 = 3 - 20 = -17$$

$$A_{33} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow \det A = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{21} \cdot a_{32} \cdot a_{13} - a_{13} \cdot a_{22} \cdot a_{31} - a_{11} \cdot a_{23} \cdot a_{32} - a_{12} \cdot a_{21} \cdot a_{33}$$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & 4 \end{bmatrix} = 1 \cdot 1 \cdot 4 + 0 \cdot 0 \cdot 0 + 3 \cdot 2 \cdot 2 - 0 \cdot 1 \cdot 2 - 1 \cdot 0 \cdot 2 - 0 \cdot 3 \cdot 4$$
$$= 4 + 0 + 12 - 0 - 0 - 0 = 16$$

- TRANSPUSA LUI $A = A^T$ $\qquad A = (A^T)^T$

$$A_{11} = [a] \rightarrow A^T_{11} = [a]$$

$$A_{22} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot A^{*}$$

- ADJUNCTA LUI A

$$A_{22} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \to A^{T} = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$A^{*} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = (-1)^{1+1} \cdot |a_{22}| \qquad b = (-1)^{1+2} \cdot |a_{12}|$$

$$c = (-1)^{2+1} \cdot |a_{21}| \qquad d = (-1)^{2+2} \cdot |a_{11}|$$

$$A_{33} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \to A^{T} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A^{*} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad a = (-1)^{1+1} \cdot \begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{vmatrix}$$

$$f = (-1)^{2+3} \cdot \begin{vmatrix} a_{11} & a_{21} \\ a_{13} & a_{23} \end{vmatrix} \qquad g = (-1)^{3+1} \cdot \begin{vmatrix} a_{21} & a_{31} \\ a_{22} & a_{32} \end{vmatrix}$$

$$h = (-1)^{3+2} \cdot \begin{vmatrix} a_{11} & a_{31} \\ a_{12} & a_{32} \end{vmatrix} \qquad i = (-1)^{3+3} \cdot \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}$$

• INVERSA LUI A

$$A^{-1} = \frac{1}{\det A} \cdot A^{*}$$

1. Calculăm $\det A$. $\det A \neq 0$!
2. Determinăm A^{T}.
3. Calculăm A^{*} folosind A^{T}.
4. Calculăm A^{-1}.

$$\begin{cases} x_1 + x_2 = 10 \\ 3x_1 - x_2 = 2 \end{cases} \Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$$

$$A_{22} \cdot x_{21} = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 3x_1 - x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ 3x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \Leftrightarrow A \cdot x = y$$

$$A^{-1} \cdot / \; A \cdot x = y \Rightarrow A^{-1} \cdot A \cdot x = A^{-1} \cdot y$$

$$\underline{I} \cdot x = A^{-1} \cdot y$$

$$\boxed{x = A^{-1} \cdot y}$$

$$A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$$

1. $\det A = 1 \cdot (-1) - 3 \cdot 1 = -1 - 3 = -4$

2. $A^T = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$

3. $A^* = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\left.\begin{aligned} a &= (-1)^{1+1} \cdot (-1) = -1 \\ b &= (-1)^{1+2} \cdot 1 = -1 \\ c &= (-1)^{2+1} \cdot 3 = -3 \\ d &= (-1)^{2+2} \cdot 1 = 1 \end{aligned}\right| \Rightarrow A^* = \begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot A^* = \frac{1}{-4} \cdot \begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix} = \frac{1}{4} \cdot \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 \\ 3/4 & -1/4 \end{bmatrix}$$

$$X = \begin{bmatrix} 1/4 & 1/4 \\ 3/4 & -1/4 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 2 \end{bmatrix} = \begin{bmatrix} 10/4 + 2/4 \\ 30/4 - 2/4 \end{bmatrix} = \begin{bmatrix} 12/4 \\ 28/4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\downarrow$

$x_1 = 3$

$x_2 = 7$