Solution of Assignment # 03

(a) Show that the multiple integral of the joint density equals 1.

(b) $\int_0^2 f(x,y)\,dy = 12x^2/7 + 6x/7$

(c) $\int_0^1 \int_0^x f(x,y)\,dy\,dx = \int_0^1 (6x^3/7 + 3x^3/14)\,dx = 15/56.$

Detail (a)

First of all, observe that f is a non-negative function. Also, we have that

$$\iint_{\mathbb{R}^2} f\,dxdy = \int_0^1 \int_0^2 \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dydx = \frac{6}{7}\int_0^1 x^2y + \frac{xy^2}{4}\Big|_0^2 dx$$
$$= \frac{6}{7}\int_0^1 2x^2 + x dx = \frac{6}{7}\left(\frac{2x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1 = \frac{6}{7}\cdot\frac{7}{6} = 1$$

Since it is a non-negative function that integrates to one, it is a valid density function.

Detail (b)

For $x \in (0,1)$ we have that

$$f_X(x) = \int_0^2 f dy = \frac{6}{7}\int_0^2 x^2 + \frac{xy}{2} dy = \frac{6}{7}\left(x^2y + \frac{xy^2}{4}\right)\Big|_0^2 = \boxed{\frac{6}{7}(2x^2 + x)}$$

otherwise it is equal to zero.

Detail(c)

In order to calculate $P(X > Y)$, we have to integrate joint PDF over appropriate region - in the rectangle $(0,1)\times(0,2)$ where $x > y$. We have that

$$P(X > Y) = \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dydx = \int_0^1 \frac{6}{7}\left(x^2y + \frac{xy^2}{4}\right)\Big|_0^x dx$$
$$= \int_0^1 \frac{6}{7}\left(x^3 + \frac{x^3}{4}\right) dx = \frac{6}{7}\cdot\int_0^1 \frac{5}{4}x^3 dx = \boxed{\frac{15}{56}}$$
