Introduction to Statistical Learning and Application
CC1: Simple Linear Regression

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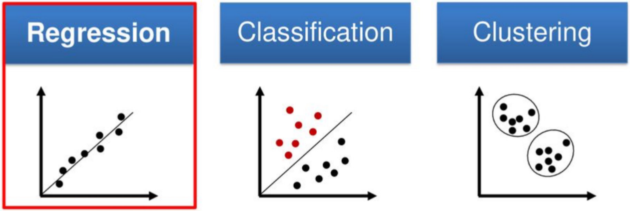
Contents

Current Section

Course Information

- This is the complementary course of "Introduction to Statistical Learning and Applications" given to students from ENSIMAG and UGA by Professor Pedro Rodrigues.
- The classes will be given on Tuesdays from 11h30 to 13h at IM2AG, room F319.
- The materials used will be made available in this page https://github.com/ISLA-Grenoble/2024-complementary.



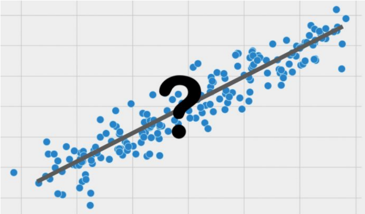
Current Section

What is Simple Linear Regression?

- Simple Linear Regression is a method used to fit the best straight line between a set of data points,
- After a graph is properly scaled, the data points must "look" like they would fit a straight line, not a parabola, or any other shape,
- The line is used as a model in order to predict a variable y from another variable x. A regression line must involve 2 variables,
- Finding the "best-fit" line is the goal of simple linear regression.

What is Simple Linear Regression?

Input, predictive or Independent varible x: this the variable whose value that is believed to influence the value of another variable,
Output, Response or Dependent variable y: this is the varaible whose value that is believed to be influenced by the value of another variable,
Best-fit Line: represents our model. It is the line that best-fits our data points. The line represents the best estimate of the y value for every given input of x,



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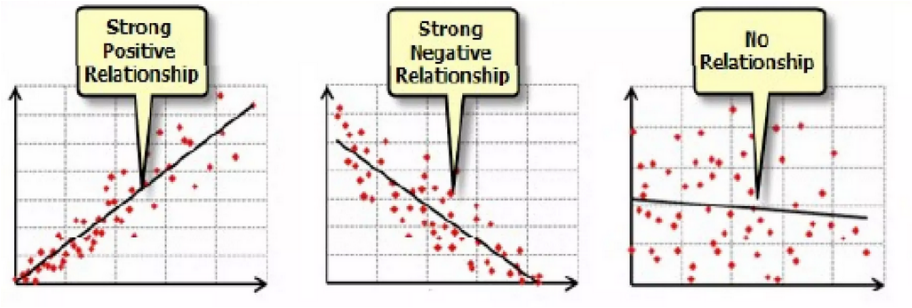
Types of Linear Regression Models



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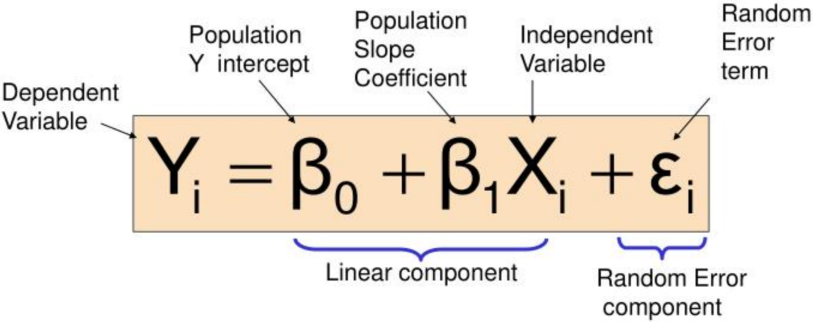
Model equation

The simple linear regression equation provides an estimate of the population regression line.



The individual random error e_i have a mean of zero.

Assumptions of Regression

Use the acronym **LINE**

- Linearity: the underlying relationship between X and Y is linear,
- Independence of errors: error values are statistically independent,
- Normality of errors: error values are normally distributed for any given value of X,
- Equal Variance (homoscedasticity)! The probability distribution of the errors has constant variance.

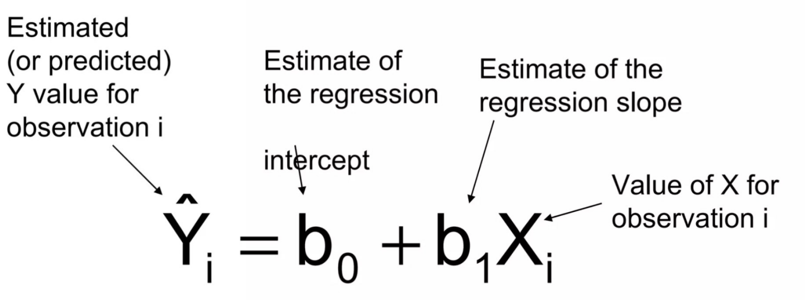
Current Section

Least Squarres Method

b_0 and b_1 are obtained by finding the values of b_0 and b_1 that minimize the sum of the squarred differences between y and $\hat{y}$

$\min \sum(y_i - \hat{y}_i)^2 = \min \sum(y_i - (b_0 + b_1 x_i))^2$

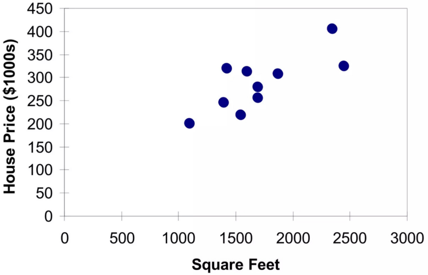
- b_0 is the estimated average value of Y when the value of X is zero,
- b_1 is the estimated change in the average value of Y as a result of a one-unit change in X.



Example

A real estate agent wishes to examine the relationship between the selling price of a home and its size measured in square feet.
A random sample of 10 houses is selected. The dependent variable Y is the house price in \$1000s. The independent variable X is the surface in squarre feet.

House Price	Square feet
245	1400
312	1600
279	1700
308	1875
199	1100
219	1550
405	2350
324	2450
319	1425
255	1700



Example

$\widehat{houseprice}$=98.24833 + 0.10977 square feet
Predict the price for a house with 2000 square feet: 98.25 + 0.1098(2000)=317.85.
The predicted price for a house with 2000 square feet is 317,850 dollars.

Current Section

Measures of variation

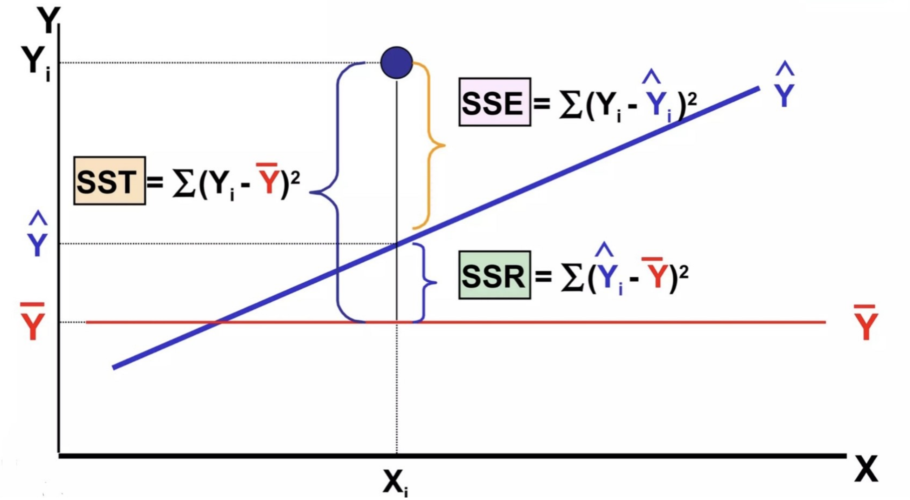
- SST= total sum of squarres
  Measures the variation of the Y_i values around their mean.
- SSR= regression sum of squares
  Explained variation attributable to the relationshiop between X and Y.
- SSE= error sum of squarres
  Variation attributable to factors other than the relationship between X and Y.

- Total variation is made up of two parts:

$$SST = SSR + SSE$$

Total Sum of Squares	Regression Sum of Squares	Error Sum of Squares
$SST = \sum (Y_i - \bar{Y})^2$	$SSR = \sum (\hat{Y}_i - \bar{Y})^2$	$SSE = \sum (Y_i - \hat{Y}_i)^2$

Measures of variation



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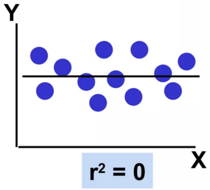
Coefficient of Determination r^2

The coefficient of determination is the portion of total variation in the dependent variable that is explained by variation in the independent variable. It is also called r-squared and is denoted as r^2

$$r^2 = \frac{SSR}{SST} = \frac{regressionsumofsquarres}{totalsumofsquares}$$



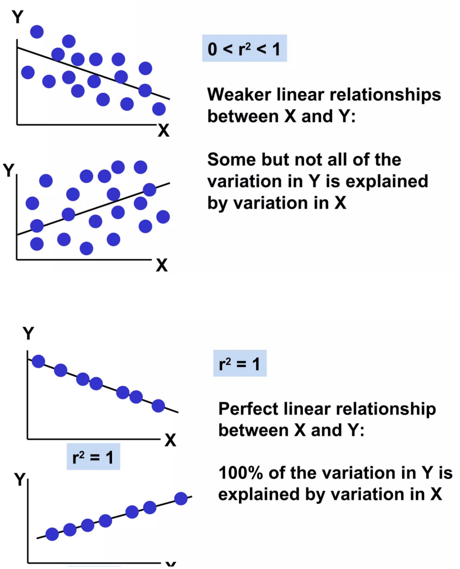
r² = 0

No linear relationship between X and Y:

The value of Y does not depend on X. (None of the variation in Y is explained by variation in X)

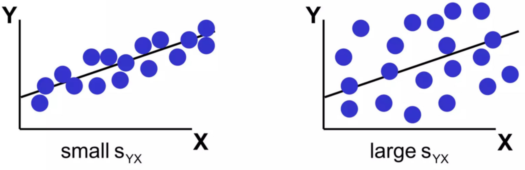
Note that r^2 values range b/w 0 and 1.

Examples of Approximate values



Standard Error of Estimate

- The standard deviation of the variation of observations around the regression line is estimated by
$$S_{YX} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-2}}$$
where SSE= error sum of squares and n=sample size.
- The magnitude of S_{YX} should always be judged relative to the size of the Y values in the sample data, i.e. S_{YX}= \$41.33k is moderately small relative to house prices in the \$200 - \$300k range.



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Residual Analysis

The residual for observation i e_i is the difference between the observed and predicted data.
Check the assumptions of regression by examining the residuals

- Examine the linearity assumption,
- Evaluate the independence assumption,
- Evaluate normal distribution assumption assumption,
- Examine for constant variance for all levels of X.

Current Section

Questions

1. Define the following: regression, regression model, dependent variable.
2. Having that SST=SSR+SSE, what is the best case scenario and the worst? Explain.
3. What does the coefficient of determination r^2 mean.
4. What does residual analysis mean?
5. Under what conditions is the linear regression model reliable?