

The Secant Method

Newton's method is an extremely powerful technique, but it has a major weakness: the need to know the value of the derivative of f at each approximation. Frequently, $f'(x)$ is far more difficult and needs more arithmetic operations to calculate than $f(x)$.

To circumvent the problem of the derivative evaluation in Newton's method, we introduce a slight variation. By definition,

$$f'(p_{n-1}) = \lim_{x \rightarrow p_{n-1}} \frac{f(x) - f(p_{n-1})}{x - p_{n-1}}.$$

If p_{n-2} is close to p_{n-1} , then

$$f'(p_{n-1}) \approx \frac{f(p_{n-2}) - f(p_{n-1})}{p_{n-2} - p_{n-1}} = \frac{f(p_{n-1}) - f(p_{n-2})}{p_{n-1} - p_{n-2}}.$$

Using this approximation for $f'(p_{n-1})$ in Newton's formula gives

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

The next example involves a problem considered in Example 1, where we used Newton's method with $p_0 = \pi/4$.

Example 2:

Use the Secant method to find a solution to $x = \cos x$, and compare the approximations with those given in Example 1 which applied Newton's method with the initial approximation $p_0 = \pi/4$. For the Secant method we need **two** initial approximations. Suppose we use $p_0 = 0.5$ and $p_1 = \pi/4$. Succeeding approximations are generated by the formula

Secant	
n	p_n
0	0.5
1	0.7853981635
2	0.7363841388
3	0.7390581392
4	0.7390851493
5	0.7390851332

These give the results in Table 2.5.

n	Newton p_n
0	0.7853981635
1	0.7395361337
2	0.7390851781
3	0.7390851332
4	0.7390851332

Comparing the results in Table 2.5 from the Secant method and Newton's method, we see that the Secant method approximation p_5 is accurate to the tenth decimal place, whereas Newton's method obtained this accuracy by p_3 . For this example, the convergence of the Newton's method obtained this accuracy by p_3 . For this

example, the convergence of the Secant method is much faster than functional iteration but slightly slower than Newton's method. This is generally the case.