EE5327 Optimization

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Question 51

Q. Maximize

$$w = 11x - z$$

with constraints

$$\begin{aligned} 10x + y - z &\leq 1 \\ 2x + 2y + z &\leq 2 \\ x, y, z &\geq 0 \end{aligned}$$

Then, the maximum value of w is equal to ......

Solution

Adding slack variables s_1, s_2

$10x + y - z + s_1 = 1$
$2x - 2y + z + s_2 = 2$
Objective function $f(x) = 11x + 0y - z + 0s_1 + 0s_2$

Solution

Initial Simplex Table

	C_j	11	0	-1	0	0		
		x	y	z	s_1	s_2	RHS	θ
0	s_1	**10**	1	-1	1	0	1	$\mathbf{\frac{1}{10}} \rightarrow$
0	s_2	2	-2	1	0	1	2	1
	$C_j - w_j$	**11** $\uparrow$	0	-1	0	0	$w_{RHS} = 0$	

x - Entering Variable

s_1 - Leaving variable

10 - pivot

Solution

$$w_1 = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}10\\2\end{pmatrix}$$
$$w_2 = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}1\\-2\end{pmatrix}$$
$$w_3 = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}-1\\1\end{pmatrix}$$
$$w_4 = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}1\\0\end{pmatrix}$$
$$w_5 = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}0\\1\end{pmatrix}$$

$$w_{RHS} = \begin{pmatrix}0\\0\end{pmatrix} \cdot \begin{pmatrix}1\\2\end{pmatrix}$$

$$\theta = \frac{RHSValue}{\text{Corresponding value in columns of x}}$$

First iteration

$R_1 \to \frac{R_1}{10}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 11R_1$

	C_j	11	0	-1	0	0		
		x	y	z	s_1	s_2	RHS	θ
11	x	1	$\frac{1}{10}$	$\frac{-1}{10}$	$\frac{1}{10}$	0	$\frac{1}{10}$	-
0	s_2	0	$\frac{-11}{5}$	$\mathbf{\frac{6}{5}}$	$\frac{-1}{5}$	1	$\frac{9}{5}$	$\frac{3}{2} \rightarrow$
	$C_j - w_j$	**11** $\uparrow$	0	-1	0	0	$w_{RHS} = \frac{11}{10}$	

x - Entering Variable

s_1 - Leaving variable

$\frac{6}{5}$- Pivot

Second iteration :

$R_2 \longrightarrow R_2 x \frac{5}{6}$

$R_1 \longrightarrow R_1 + \frac{R_2}{10}$

$R_3 \longrightarrow R_3 - \frac{R_2}{10}$

	C_j	11	0	-1	0	0	
		x	y	z	s_1	s_2	RHS
11	x	1	$\frac{-1}{12}$	0	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{4}$
-1	z	0	$\frac{-11}{6}$	1	$\frac{-1}{6}$	$\frac{5}{6}$	$\frac{3}{2}$
	$C_j - w_j$	0	$\frac{-11}{12}$	0	$\frac{-13}{12}$	$\frac{-1}{12}$	$w_{RHS} = \frac{5}{4}$

Here all $C_j - w_j$ values are either zero or negative.

So, maximum value of w $=$ $w_{RHS} = \frac{5}{4}$

for x $=$ 1 and z $=$ -1.

Question 51

Q. Use cvxopt to obtain a solution to problem 51.

$$\min_{x} \; c^T x$$

$$\textit{subject to } Ax \preceq b$$

$$c = \begin{bmatrix} -11 \\ 0 \\ 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -2 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Code:

```
from cvxopt import matrix, solvers

A = matrix([ [10.0, 1.0,-1.0], [2.0, -2.0, 1.0], [-1.0, 0.0, 0.0],
             [0.0, -1.0, 0.0], [0.0, 0.0, -1.0] ])
b = matrix([ 1.0, 2.0, 0.0, 0.0, 0.0])
c = matrix([ -11.0, 0.0, 1.0 ])
sol = solvers.lp(c, A.T, b)
print(sol['x'])
print("cost function=")
print(-1 * np.dot(np.reshape(c,(1,3)),sol['x']))
```

```
Optimal solution found.
[ 2.50e-01]
[-3.88e-08]
[ 1.50e+00]

cost function=
[[1.25000003]]
```