

EE5327 Optimization

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Question 51

Q. Maximize

$$w = 11x - z$$

with constraints

$$10x + y - z \leq 1$$

$$2x + 2y + z \leq 2$$

$$x, y, z \geq 0$$

Then, the maximum value of w is equal to

Solution

Adding slack variables $s_1 \geq 0, s_2 \geq 0$

$$10x + y - z + s_1 = 1$$

$$2x - 2y + z + s_2 = 2$$

$$\text{Objective function } f(x) = 11x + 0y - z + 0s_1 + 0s_2$$

Initial Simplex Table

	C_j	11	0	-1	0	0		
		x	y	z	s_1	s_2	RHS	θ
0	s_1	10	1	-1	1	0	1	$\frac{1}{10} \rightarrow$
0	s_2	2	-2	1	0	1	2	1
	$C_j - w_j$	11 \uparrow	0	-1	0	0	$w_{RHS} = 0$	

x - Entering Variable

s_1 - Leaving variable

10 - pivot

Solution

$$w_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 2 \end{pmatrix}$$

$$w_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$w_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$w_4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$w_5 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$w_{RHS} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\theta = \frac{RHSValue}{\text{Corresponding value in columns of } x}$$

First iteration

$$R_1 \rightarrow \frac{R_1}{10}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 11R_1$$

	C_j	11	0	-1	0	0		
		x	y	z	s_1	s_2	RHS	θ
11	x	1	$\frac{1}{10}$	$\frac{-1}{10}$	$\frac{1}{10}$	0	$\frac{1}{10}$	-
0	s_2	0	$\frac{-11}{5}$	$\frac{6}{5}$	$\frac{-1}{5}$	1	$\frac{9}{5}$	$\frac{3}{2} \rightarrow$
	$C_j - w_j$	11 \uparrow	0	-1	0	0	$w_{RHS} = \frac{11}{10}$	

x - Entering Variable

s_1 - Leaving variable

$\frac{6}{5}$ - Pivot

Second iteration :

$$R_2 \rightarrow R_2 \times \frac{5}{6}$$

$$R_1 \rightarrow R_1 + \frac{R_2}{10}$$

$$R_3 \rightarrow R_3 - \frac{R_2}{10}$$

	C_j	11	0	-1	0	0	
		x	y	z	s_1	s_2	RHS
11	x	1	$-\frac{1}{12}$	0	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{4}$
-1	z	0	$-\frac{11}{6}$	1	$-\frac{1}{6}$	$\frac{5}{6}$	$\frac{3}{2}$
	$C_j - w_j$	0	$-\frac{11}{12}$	0	$-\frac{13}{12}$	$-\frac{1}{12}$	$w_{RHS} = \frac{5}{4}$

Here all $C_j - w_j$ values are either zero or negative.

So, maximum value of $w = w_{RHS} = \frac{5}{4}$

for $x = \frac{1}{4}$ and $z = \frac{3}{2}$.

Question 51

Q. Use cvxopt to obtain a solution to problem 51.

$$\min_x c^T x$$

subject to $Ax \leq b$

$$c = \begin{bmatrix} -11 \\ 0 \\ 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -2 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Code:

```
from cvxopt import matrix, solvers

A = matrix([ [10.0, 1.0,-1.0], [2.0, -2.0, 1.0], [-1.0, 0.0, 0.0],
             [0.0, -1.0, 0.0], [0.0, 0.0, -1.0] ])
b = matrix([ 1.0, 2.0, 0.0, 0.0, 0.0])
c = matrix([ -11.0, 0.0, 1.0 ])
sol = solvers.lp(c, A.T, b)
print(sol['x'])
print("cost function=")
print(-1 * np.dot(np.reshape(c,(1,3)),sol['x']))
```

Optimal solution found.

[2.50e-01]

[-3.88e-08]

[1.50e+00]

cost function=

[[1.25000003]]