# EE5327 Optimization

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Q. Maximize

$$w = 11x - z$$

with constraints

$$10x + y - z \leq 1$$
$$2x + 2y + z \leq 2$$
$$x, y, z \geq 0$$

Then, the maximum value of w is equal to ......

## Solution

Adding slack variables $s_1 \geq 0, s_2 \geq 0$

$10x + y - z + s_1 = 1$
$2x - 2y + z + s_2 = 2$
Objective function $f(x) = 11x + 0y - z + 0s_1 + 0s_2$

Initial Simplex Table

|   | $C_j$ | 11 | 0 | -1 | 0 | 0 |  |  |
|---|---|---|---|---|---|---|---|---|
|   |   | x | y | z | $s_1$ | $s_2$ | RHS | $\theta$ |
| 0 | $s_1$ | **10** | 1 | -1 | 1 | 0 | 1 | $\frac{1}{10} \rightarrow$ |
| 0 | $s_2$ | 2 | -2 | 1 | 0 | 1 | 2 | 1 |
|   | $C_j - w_j$ | **11** $\uparrow$ | 0 | -1 | 0 | 0 | $w_{RHS} = 0$ |  |

x - Entering Variable

$s_1$ - Leaving variable

10 - pivot

## Solution

$w_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$w_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$w_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$w_4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$w_5 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$w_{RHS} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\theta = \dfrac{RHSValue}{\text{Corresponding value in columns of x}}$

First iteration

$R_1 \rightarrow \frac{R_1}{10}$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 11R_1$

|     | $C_j$ | 11 | 0 | -1 | 0 | 0 |     |     |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|     |     | x | y | z | $s_1$ | $s_2$ | RHS | $\theta$ |
| 11 | x | 1 | $\frac{1}{10}$ | $\frac{-1}{10}$ | $\frac{1}{10}$ | 0 | $\frac{1}{10}$ | - |
| 0 | $s_2$ | 0 | $\frac{-11}{5}$ | $\frac{6}{5}$ | $\frac{-1}{5}$ | 1 | $\frac{9}{5}$ | $\frac{3}{2} \rightarrow$ |
|     | $C_j - w_j$ | **11** ↑ | 0 | -1 | 0 | 0 | $w_{RHS} = \frac{11}{10}$ |     |

x - Entering Variable

$s_1$ - Leaving variable

$\frac{6}{5}$- Pivot

Second iteration :
$R_2 \to R_2 x\frac{5}{6}$
$R_1 \to R_1 + \frac{R_2}{10}$
$R_3 \to R_3 - \frac{R_2}{10}$

|    | $C_j$ | 11 | 0 | -1 | 0 | 0 |    |
|----|-------|-----|-----|-----|-----|-----|-----|
|    |       | x | y | z | $s_1$ | $s_2$ | RHS |
| 11 | x | 1 | $\frac{-1}{12}$ | 0 | $\frac{1}{10}$ | $\frac{1}{12}$ | $\frac{1}{4}$ |
| $-1$ | z | 0 | $\frac{-11}{6}$ | 1 | $\frac{-1}{6}$ | $\frac{5}{6}$ | $\frac{3}{2}$ |
|    | $C_j - w_j$ | 0 | $\frac{-11}{12}$ | 0 | $\frac{-13}{12}$ | $\frac{-1}{12}$ | $w_{RHS} = \frac{5}{4}$ |

Here all $C_j - w_j$ values are either zero or negative.
So, maximum value of $w = w_{RHS} = \frac{5}{4}$
for $x = 1$ and $z = -1$.

Q. Use cvxopt to obtain a solution to problem 51.

$$\min_x \ c^T x$$

*subject to $Ax \leq b$*

$$c = \begin{bmatrix} -11 \\ 0 \\ 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -2 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

# Solution

## Code:

```
from cvxopt import matrix, solvers

A = matrix([ [10.0, 1.0,-1.0], [2.0, -2.0, 1.0], [-1.0, 0.0, 0.0],
             [0.0, -1.0, 0.0], [0.0, 0.0, -1.0] ])
b = matrix([ 1.0, 2.0, 0.0, 0.0, 0.0])
c = matrix([ -11.0, 0.0, 1.0 ])
sol = solvers.lp(c, A.T, b)
print(sol['x'])
print("cost function=")
print(-1 * np.dot(np.reshape(c,(1,3)),sol['x']))
```

```
 Optimal solution found.
 [ 2.50e-01]
 [-3.88e-08]
 [ 1.50e+00]

 cost function=
 [[1.25000003]]
```