QRE for State Preparation

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A key requirement for algorithms such as HHL is quantum state preparation. For example, in the HHL algorithm we need to prepare a state that encodes the vector "b" in order to solve the system $Ax = b$. The process of creating a specific initial quantum state that may encode input data or a problem instance is known as *quantum state preparation.* This project is about the implementation of an algorithm for *sparse quantum state preparation.*

INTRODUCTION

State preparation is an essential part of quantum computing and much more complex than its classical counterpart. In classical computing all possible states of a memory-block of size $O(n)$ can be obtained by setting the appropriate bits using $O(n)$ NOT-gates. The possible states of a quantum computer are superpositions of basis states and some of them can only be created with very complex circuits. In fact, it is known that using a gate library of only one-qubit-gates and CNOT-gates, some of the states of an n-qubit quantum computer can only be prepared with circuits of size $\Omega(2^n)$ [3]. However, efficient state preparation is possible for sparse states.

Definintion 1. *In contrast to general states, many (if not most) practically relevant states have the property that if S is the set of basis states of a quantum state $|S| << 2^n$ i.e only a small proportion of the basis states have nonzero coefficients. We call states with this property* sparse.

Following are some examples of sparse states:

- The proportion of basis states with nonzero coefficient among all basis states is $O(\frac{1}{\sqrt{2^n}})$ for generalized Bell states.
- $O(\frac{n}{2^n})$ for W-states and $O(\frac{1}{2^n})$ for GHZ-states.

Niels Gleinig and Torsten Hoefler [1] show how sparsity can be leveraged to make state preparation asymptotically more efficient than preparation of general states. We briefly review this algorithm in the next section.

I. AN ALGORITHM FOR SPARSE QUANTUM STATE PREPARATION

The crux of this algorithm is to choose a two basis state and then apply a specific controlled-gate G that merges them into one while not influencing change on the other elements. Repeating this eventually leads to the $|0^n\rangle$ basis state. Given the transformation G and state $|\psi\rangle$,

$$G = \begin{bmatrix} \sin(\omega) & e^{ia}\cos(\omega) \\ e^{-ia}\cos(\omega) & -\sin(\omega) \end{bmatrix}, \quad |\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix},$$

we have $G|\psi\rangle = e^{i\lambda}|0\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. This implies that

$$\begin{aligned} \alpha \sin\omega + \beta e^{ia}\cos\omega = e^{i\lambda}, &\text{ which gives} \\ \alpha e^{-ia}\cos(\omega) - \beta\sin(\omega) = 0. & \end{aligned} \tag{1}$$

Let $\alpha = e^{is}\cos\theta$ and $\beta = e^{it}\sin\theta$ where θ lies in the first quadrant. Substituting in Equation 1, we get

$$\begin{aligned} e^{is}\cos\theta\sin(\omega) + e^{it}\sin\theta e^{ia}\cos\omega = e^{i\lambda} \\ e^{is}\cos\theta e^{-ia}\cos\omega - e^{it}\sin\theta\sin\omega = 0. \end{aligned} \tag{2}$$

Further simplification yields

$$\begin{aligned} e^{is}\left(\cos\theta\sin\omega + e^{i(t-s+a)}\sin\theta\cos\omega\right) = e^{i\lambda} \\ e^{it}\left(e^{i(s-t-a)}\cos\theta\cos\omega - \sin\theta\sin\omega\right) = 0. \end{aligned} \tag{3}$$

To find a solution, without loss of generality, we may set $a = s - t$, which gives us

$$\begin{aligned} e^{is}(\cos\theta\sin\omega + \sin\theta\cos\omega) = e^{i\lambda} \\ e^{it}(\cos\theta\cos\omega - \sin\theta\sin\omega) = 0. \end{aligned} \tag{4}$$

Finally, for ω we obtain the solution

$$w = \tan^{-1}\left(\frac{|\alpha|}{|\beta|}\right). \tag{5}$$

The sparse quantum state preparation algorithm has the following specification.

Input: The input of the algorithm is the classical description of the quantum state we want to prepare. This description contains two parts: the basis states and their corresponding probabilities.

Output: The outcome of the algorithm is a circuit C having the property that applying C to the all zero state results in the required state provided as input.

The algorithm has two parts, classical and quantum.

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A. Classical Preprocessing Implementation

This part of the sparse state preparation algorithm computes all the necessary classical information needed to for the quantum algorithm. The inputs of the algorithm, i.e., both the basis states and their probabilities are provided as input to the function $Algo\_1()$.

This function identifies the two basis states that will be merged into one by providing the control and target qubit indices for multi-controlled operation G. The function returns a merged list of basis states and their probabilities which now have one less element. The function is recursively called till only one element left in the list. Along the process this function saves history of all the gates applied in different lists and passes them to the quantum part of the algorithm.

B. Quantum Algorithm Implementation

Sparse state information S, for an n qubit state is provided in the form

$$[\,[\,[basis]\,,\,probability\,]\,,\ldots]\,.$$

The function $Algo\_1()$ is used to generate history of gates corresponding to the recursive merging of basis states present in the set S till the all zero state is attained. Thus the inverse of the merging circuit computes for us the desired sparse state preparation circuit. This inversion requires only reading and applying the gates from the history lists in reverse order. The function $Sparse\_State()$ performs this job.

The implementation code can be accessed from the Github repository `https://github.com/Razeen-ud-din/Quantum-Sparse-State-Preparation`

II. SIMULATION RESULTS

We present initial results for the following simulations and quantum resource estimation using Classiq, Q# , Qiskit native routines and the sparse state preparation algorithm.

1. GHZ state preparation (Figure 3)
2. W state preparation (Figure 4)
3. Random state preparation (Figure 1)

The bump in Figure 4 for number of T gates needs to be explored further.

[1] Gleinig, Niels, and Torsten Hoefler. "An efficient algorithm for sparse quantum state preparation." 2021 58th ACM/IEEE Design Automation Conference (DAC). IEEE, 2021.

[2] Barenco, Adriano, et al. "Elementary gates for quantum computation." Physical review A 52.5 (1995): 3457.

[3] E. Knill, "Approximation by quantum circuits," 5 1995.

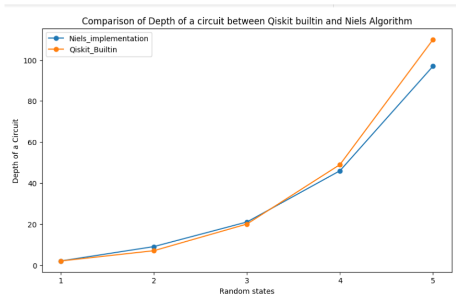

FIG. 1. Depth of circuit for state generated via a random unitary with number of qubits n on the x axis.

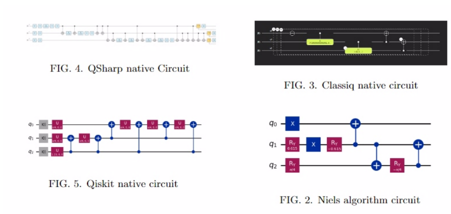

FIG. 2. W_3 state generation circuits provided by different frameworks.

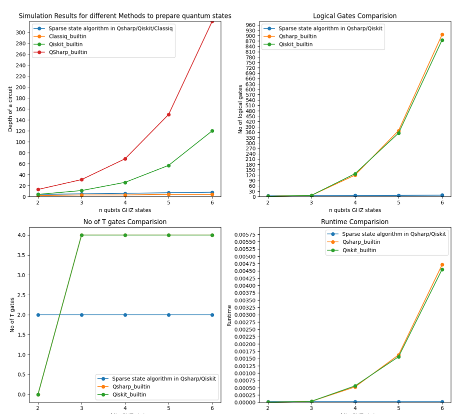

FIG. 3. Resource Estimation for GHZ states

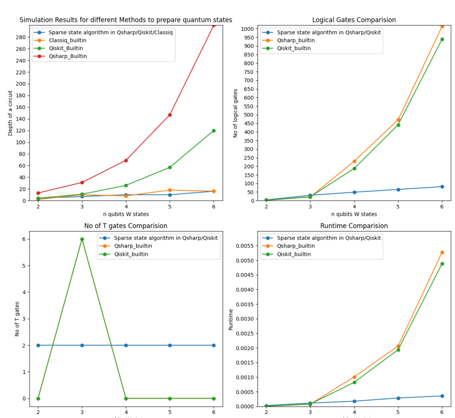

FIG. 4. Resource Estimation for W states