Dynamic Asset Allocation Comparative Study

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May 8, 2023

Abstract

This report evaluates various asset allocation strategies applied to three benchmark Exchange-traded fund (ETFs), assessing their performance using multiple metrics. Our findings suggest that when selecting an asset allocation strategy, it is important to consider whether the transaction cost matters or not. If transaction costs are high, then a strategy that prioritizes low turnover can be beneficial. The closed-form risk parity method that we designed to minimize transaction costs consistently outperformed other strategies and surpassed the 1/N benchmark allocation across all assets and the market portfolio. However, if transaction costs are negligible, a strategy such as the Black-Litterman model will offer a Sharpe ratio even higher. We confirmed the robustness of our approach across various time periods, demonstrating resilience to market changes. Our asset allocation strategy, which takes turnover ratio into consideration, offers an effective, intuitive, and adaptive solution for improved portfolio optimization across diverse assets.

1 Project Introduction

In this report, we present a comprehensive analysis of various asset allocation strategies applied to three benchmark Exchange-traded fund (ETFs) - iShares US Technology, Healthcare, and Utilities - across different market conditions. The methods tested include Inverse Volatility, Max Diversification, Risk Parity, Hierarchical Risk Parity, and Black-Litterman with different priors and clustering techniques. To assess the performance of these strategies, we employed a range of metrics, including Out-of-Sample Return, Out-of-Sample Sharpe Ratio, Sortino Ratio, Maximum Drawdown, Calmar Ratio, and Turnover ratio. Our study aims to investigate the performance variations under specific conditions, explore the underlying reasons for these changes, and identify the most suitable allocation strategies.

After thorough experimentation and comparison, we found that the performance of allocation strategies is closely related to the turnover ratio. Specifically, for low transaction cost conditions, black-litterman based models delivered the best results with a relatively high turnover ratio. In contrast, for high transaction cost conditions, the closed-form solutions for risk parity consistently delivered better results on the three ETF datasets, performing well across all metrics and outperforming the benchmark 1/N allocation in every individual asset and the market portfolio. To ensure the robustness of our findings, we also tested the performance of the strategies using different training periods.

This in-depth evaluation provides valuable insights into the strengths and weaknesses of various asset allocation strategies, with a focus on the importance of considering the turnover ratio. Our study highlights the superior performance of black-litterman models under low transaction cost conditions and the robust performance of risk parity closed-form solutions under high transaction cost conditions. By understanding these results, fund managers and investors can make informed decisions in the complex financial landscape, enabling them to better navigate the real-world market and capitalize on opportunities within specific industry sectors.

2 Data

2.1 Selection of Benchmarks

We use historical data of three iShares ETFs, which represent different sectors of the US market: US Technology ETF (IYW), US Healthcare ETF (IYH), and US Utilities ETF (IDU). The choice of these three benchmarks is motivated by several factors:

- Stocks are typically the most liquid assets, and their data is readily available, making them a suitable choice for our study.
- By selecting ETFs that represent different sectors, we can capture the diverse characteristics and risks associated with various industries.
- The inclusion of different sectors in the analysis enables us to assess the performance of the portfolio optimization methods across a wide range of market conditions.
- These three sectors have relatively low correlations with each other, which allows for better diversification and risk management in the portfolio optimization process.

The data source for each ETF and its underlying stocks is Yahoo Finance.

2.2 Data Timeframe

We select data from the past 10 years, which corresponds to approximately 2500 trading days. The choice of this timeframe offers several benefits:

- It provides a sufficiently large dataset to allow for robust statistical analysis.
- The 10-year period captures different market cycles, including periods of growth, decline, and recovery.
- This timeframe enables us to assess the performance of the portfolio optimization methods over an extended period, enhancing the reliability of our findings.

As for the training period, we use four different time windows: 1 month, 3 months, 6 months, and 1 year. This allows us to evaluate the impact of varying the training period length on the performance of the portfolio optimization methods.

2.3 Rationale for Using ETFs and Underlying Stocks

The idea of using ETFs as benchmarks and their underlying stocks as individual assets in the portfolio comes from simulation studies. The main reasons for this choice are:

- The underlying stocks have a correlation with the ETFs, which allows us to study the performance of the portfolio optimization methods in a more realistic setting.
- ETFs are designed to track the performance of a specific market index or sector, making them a suitable proxy for the overall market.
- By including the underlying stocks, we can assess the ability of the portfolio optimization methods to diversify across individual assets and manage idiosyncratic risks.
- The three chosen sectors exhibit comparatively low correlations, a crucial attribute for facilitating more accurate comparisons of performance by our strategy across diverse market conditions. By demonstrating our approach's efficacy under these distinct scenarios, we can effectively establish the robustness and reliability of our strategy and outcomes.

3 Methodology

3.1 Benchmark (Computation-free)

3.1.1 (1). "1/N" with rebalancing (ew)

Simply put in $\frac{1}{d}$ in every assets(d assets in total) at each time t,

$$w_t = \begin{pmatrix} \frac{1}{d} \\ \vdots \\ \frac{1}{d} \end{pmatrix}$$

3.1.2 (2). Market Portfolio/Factor Portfolio (mkt)

Simply put all weights in the factor portfolio, we design it to be the first asset. Therefore, the weights are:

$$w_t = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

3.2 Classical Statistical Approach

3.2.1 (3). Sample-based mean-variance (mv)

The analytical solution of the mean-variance portfolio comes from an optimization problem. We have the formula for the base case. From the mean and covariance matrix,$\hat{\mu} = \mathbb{E}[\mathbf{R_M}]$,

$\hat{\mathbf{\Sigma}} = cov(\mathbf{R_M})$, the analytical solution is:

$$w_t = \frac{\kappa - r}{(\hat{\mu} - r\mathbf{e})^T \hat{\mathbf{\Sigma}}^{-1} (\hat{\mu} - r\mathbf{e})} \hat{\mathbf{\Sigma}}^{-1} (\hat{\mu} - r\mathbf{e})$$

3.3 Inverse Volatility

The inverse volatility allocation method computes portfolio weights based on the inverse of the volatility of each asset. The goal of this method is to allocate more weight to assets with lower volatility, thus reducing the overall portfolio risk. The formula for calculating the inverse volatility weights is as follows:

$$w_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{d} \frac{1}{\sigma_j}}$$

where

- w_i is the weight of asset i in the portfolio
- σ_i is the volatility (standard deviation) of the returns of asset i
- d is the total number of assets

3.4 Max Diversification

The max diversification allocation method aims to optimize the portfolio by maximizing the diversification ratio, which is the ratio of the weighted sum of individual asset volatilities to the overall portfolio volatility. The objective is to find the weights that lead to the most diversified portfolio in terms of risk. The optimization problem can be formulated as:

$$\max_{w} \frac{\sum_{i=1}^{d} w_i \sigma_i}{\sqrt{w^T \Sigma w}}$$

subject to

$$\sum_{i=1}^{N} w_i = 1$$
$$0 \leq w_i \leq 1 \quad \forall i$$

where

- w_i is the weight of asset i in the portfolio
- σ_i is the volatility (standard deviation) of the returns of asset i
- Σ is the covariance matrix of the assets' returns
- d is the total number of assets

3.5 Risk Parity

Let $w = (w_1, w_2, \ldots, w_n)$ be the vector of portfolio weights, with w_i representing the weight of the i-th asset, and let Σ be the covariance matrix of the asset returns. The total portfolio risk (variance) can be expressed as: $w^T \Sigma w$.

The risk contribution of each asset i in the portfolio can be calculated as:

$$RC_i = w_i \frac{(\Sigma w)_i}{w^T \Sigma w} \tag{1}$$

Here, $(\Sigma w)_i$ is the i-th element of the product of the covariance matrix and the weight vector. The objective of risk parity is to find the weights w such that:

$$RC_1 = RC_2 = \cdots = RC_n \tag{2}$$

This is a nonlinear optimization problem that can be solved using various numerical methods, such as gradient descent or other optimization techniques.

3.5.1 Approximate Risk Parity Solution

An approximation to the risk parity solution can be obtained by using the inverse of the covariance matrix to calculate the weights. While this method might not give the exact solution, it is computationally efficient and provides a reasonable estimation of the risk parity allocation:

$$w_t = \frac{\sqrt{e^T \Sigma^{-1} e} \cdot \Sigma^{-1} e}{\sum_{i=1}^{n} \left(\sqrt{e^T \Sigma^{-1} e} \cdot \Sigma^{-1} e \right)_i} \tag{3}$$

Here, e is a vector of ones, and Σ^{-1} denotes the inverse of the covariance matrix.

3.5.2 Risk Parity Optimization Method

The risk parity optimization problem is formulated as follows:

Minimize the squared difference between risk contributions of each asset and the target risk contribution ($\frac{1}{d}$) for all assets:

$$\min_{w} \sum_{i=1}^{d} \left[w_i \frac{(\Sigma w)_i}{w^T \Sigma w} - \frac{1}{d} \right]^2 \tag{4}$$

Subject to the following constraints:

1. The sum of the portfolio weights should be equal to 1:

$$\sum_{i=1}^{d} w_i = 1 \tag{5}$$

2. Portfolio weights must be within bounds:

$$0 \leq w_i \leq 1, \quad \forall i \in \{1, 2, \ldots, d\} \tag{6}$$

3.5.3 Hierarchical Risk Parity (HRP)

[LAS+17] [Vyand] [Sahnd]

Hierarchical Risk Parity (HRP) is a portfolio optimization technique that aims to achieve a balanced risk allocation among different assets clusters in a portfolio. It is based on the concept of hierarchical clustering, which groups assets with similar characteristics, in this case return correlations, into clusters, and then allocates risk across the clusters in a uniform risk contribution fashion.

The HRP algorithm consists of the following steps:

Compute the covariance and correlation matrix of asset returns. Let R be an $n \times m$ matrix of asset returns, where n is the number of assets and m is the number of time periods. Then the pairwise correlation matrix C of asset returns is given by:

$$C_{ij} = \frac{\sum_{t=1}^{m}(R_{ti} - \bar{R}_i)(R_{tj} - \bar{R}_j)}{(m-1)s_i s_j} \tag{7}$$

where $\bar{R}_i$ and s_i are the mean and standard deviation of asset i over the time periods.

Perform hierarchical clustering on the correlation matrix (Quasi-Diagnolization) which is a way to serialize the correlation matrix where "similar" or highly correlated stocks are put together.

Compute the inverse of the correlation matrix and use it to transform the tree structure into a portfolio allocation tree.

Let D be the inverse of the correlation matrix C. Then the portfolio allocation tree is formed by recursively applying the following formula at each node:

$$w_i = \frac{\sum_{j \in \text{leaf}(i)} d_{ij}}{\sum_{j \in \text{leaves}} \sum_{k \in \text{leaf}(j)} d_{jk}} \tag{8}$$

where leaf(i) represents the set of leaf nodes under node i.

Allocate risk to the leaf nodes of the portfolio allocation tree in proportion to their inverse variance.

At the end of clustering we will have a giant cluster of assets, where we start to perform recursively from top of the dedrogram to bisect of the clusters. Now we have two separate clusters.

Hierarchical tree clustering forms a binary tree where each cluster has a left and right child cluster V_1 and V_2. For each of these sub-clusters, we calculate its variance.

$$V_{adj} = w^T V w,$$

Let V be the $n \times n$ diagonal matrix of asset variances, where V_{ii} is the variance of asset i. Then the risk allocation to each leaf node is given by:

$$w_i = \frac{1}{V_{ii}} \tag{9}$$

This step exploits the property that for a diagonal covariance matrix, the inverse-variance

allocations are the most optimal. Since we are dealing with a quasi-diagonal matrix, it makes sense to use the inverse-allocation weights to calculate the variance for this subcluster.

A weighting factor is calculated based on the new covariance matrix:

$$\alpha_1 = \frac{1 - V_1}{V_1 + V_2}; \qquad \alpha_2 = 1 - \alpha_1.$$

You might be wondering how we arrive at the above formula for the weighting factor. It comes from the classical portfolio optimisation theory where we have the following optimisation objective,

$$\min_w \frac{1}{2} w^T \sigma w \quad \text{s.t.} \quad e^T w = 1; , e = 1^T$$

where w are the portfolio weights and σ is the covariance of the portfolio. For a minimum variance portfolio, the solution to the above equation comes out to be,

$$w = \sigma^{-1} e (e^T \sigma^{-1} e)^{-1} \sigma^{-1} e.$$

And if σ is diagonal,

$$w = \sum_{i=1}^{N} \frac{1}{\sigma_{i,i}} \sum_{j=1}^{N} \sigma_{j,j}^{-1}.$$

Since we only have $N = 2$ (2 subclusters) this equation becomes,

$$w = \frac{1}{\sigma_1} \left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2} \right) = \frac{1 - V_1}{V_1 + V_2},$$

which is the same formula used by Hierarchical Risk Parity.

The weights of stocks in the left and right subclusters are then updated respectively,

$$W_1 = \alpha_1 W_1, \qquad W_2 = \alpha_2 W_2.$$

We recursively execute steps 2-5 on V_1 and V_2 until all the weights are assigned to the stocks.

The final portfolio weights are given by normalizing the risk allocations obtained earlier:

$$W = \frac{w}{\sum_{i=1}^{n} w_i} \tag{10}$$

where w is the vector of risk allocations obtained.

The numerical solution offers a flexible and effective way to solve the risk parity optimization problem, taking into account the constraints and bounds on the portfolio weights.

(Algorithm explanation credit to https://hudsonthames.org/an-introduction-to-the-hierarchical-risk-parity-algorithm/)

3.6 Black-Litterman

[pypnd] [Marnd]

The Black-Litterman model is based on the following equation:

$$w_t = \text{argmax}_w \frac{w^T \cdot \mu}{\sqrt{w^T \cdot \Sigma \cdot w}}, \tag{11}$$

where w is the market capitalization weight vector, μ is the posterior estimate of returns, and Σ is the posterior covariance matrix.

3.6.1 Prior Returns

The Black-Litterman model takes a Bayesian approach to asset allocation. Specifically, it combines a prior estimate of returns (for example, the market-implied returns) with views on certain assets, to produce a posterior estimate of expected returns. The advantages of this are:

1. You can provide views on only a subset of assets and BL will meaningfully propagate it, taking into account the covariance with other assets.

2. You can provide confidence in your views.

3. Using Black-Litterman posterior returns results in much more stable portfolios than using mean-historical return.

In our implementation, we assume that we do not have specific investor views, so we use random numbers and technical indicators instead. This approach allows us to simulate the effect of incorporating views into the model without requiring actual views from investors. Specifically, we randomly generate prior estimates of returns and use technical indicators based on historical data, including SMA, RSI and MACD with formulas as follows. We then use these inputs to calculate the equilibrium excess return, which is used to compute the optimal portfolio weights.

$$\begin{aligned}
\text{SMA}_{20} &= \frac{1}{20} \sum_{i=n-19}^{n} R_t \\
\text{RSI} &= \frac{1}{14} \sum_{i=n-13}^{n} \mathbf{1}(R_i > 0) \\
\text{MACD} &= EMA_{12}(R_t) - EMA_{26}(R_t)
\end{aligned}$$

Notice that we don't follow the original form of RSI. And $EMA_{12}(.)$ and $EMA_{26}(.)$ are the Exponential Moving Averages with periods 12 and 26.

3.6.2 Posterior Returns

To compute the posterior estimate of returns, we use the following equations:

$$\pi = \delta\omega \cdot w, \tag{12}$$

where π is the equilibrium excess return vector, δ is the risk aversion coefficient, ω is the prior covariance matrix.

$$\mu = (\Sigma^{-1} + \tau\omega^{-1})^{-1} \cdot (\Sigma^{-1} \cdot \hat{\mu} + \tau\omega^{-1} \cdot \pi), \tag{13}$$

where μ is the posterior estimate of returns, $\hat{\mu}$ is the prior estimate of returns, Σ is the posterior covariance matrix, ω is the prior covariance matrix, π is the equilibrium excess return vector, and τ is the scaling parameter.

3.6.3 Numerical Solution

To compute the optimal portfolio weights with views, we use the following equation:

$$w_t = \text{argmax}_w \left\{ \frac{w^T \cdot \mu}{\sqrt{w^T \cdot \Sigma \cdot w}} \right\}. \tag{14}$$

3.7 Clustering Models

3.7.1 Hierarchical Clustering

We perform hierarchical clustering on asset return matrix and visualize the resulting dendrogram.

In the context of clustering methods, codependence refers to the measure of similarity between two assets or variables. Here we use the Pearson correlation coefficient as our codependence metric. Given two random variables X and Y, the Pearson correlation coefficient is defined as:

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \tag{15}$$

where $cov(X,Y)$ is the covariance between X and Y, and σ_X and σ_Y are the standard deviations of X and Y, respectively. The Pearson correlation coefficient takes values between -1 and 1, with values closer to 1 indicating a strong positive correlation, values closer to -1 indicating a strong negative correlation, and values close to 0 indicating no correlation.

As for linkage, the method used to measure the distance between clusters in a hierarchical clustering algorithm, we use the Ward linkage method, which minimizes the variance of the clusters being merged.

Given two clusters A and B with means μ_A and μ_B, and n_A and n_B observations, respectively, the Ward distance between them is defined as:

$$d_{AB} = \sqrt{\frac{n_A n_B}{n_A + n_B}} \cdot \|\mu_A - \mu_B\|^2 \tag{16}$$

where $||\cdot||$ denotes the Euclidean distance.

The Ward method has the advantage of producing compact, well-separated clusters. It tends to work well when the data contains clusters of similar size and density.

3.7.2 Hierarchical Equal Risk Contributions (HERC)

[Cajnd]

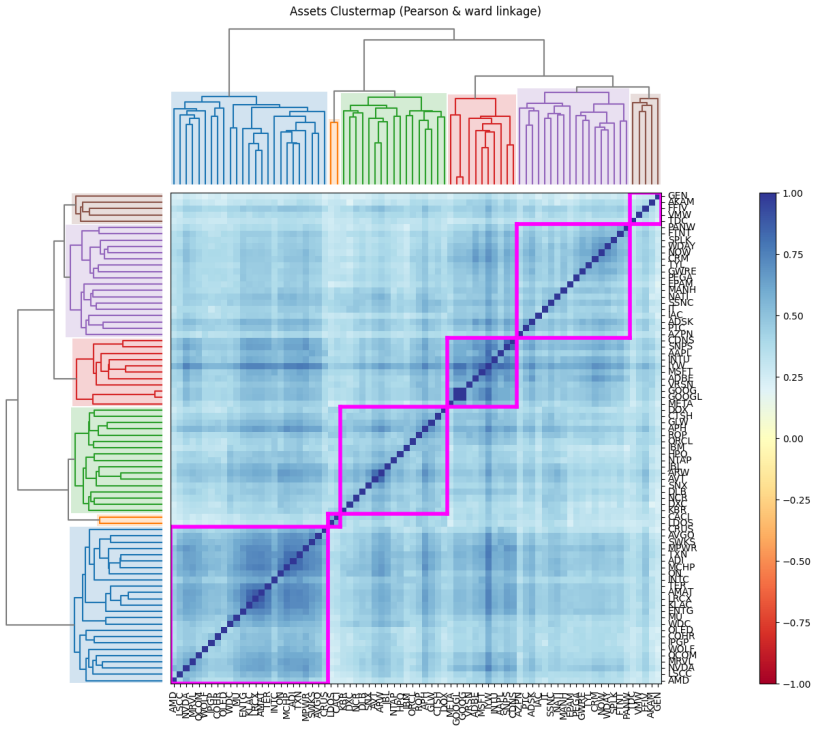

Figure 1: Asset Clustermap of Tech ETF

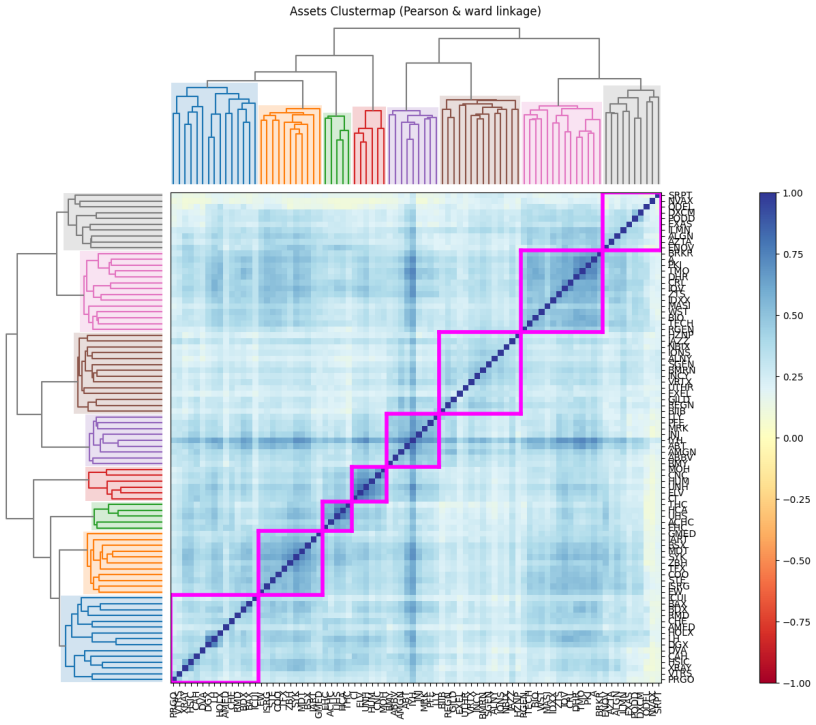

Figure 2: Asset Clustermap of Healthcare ETF

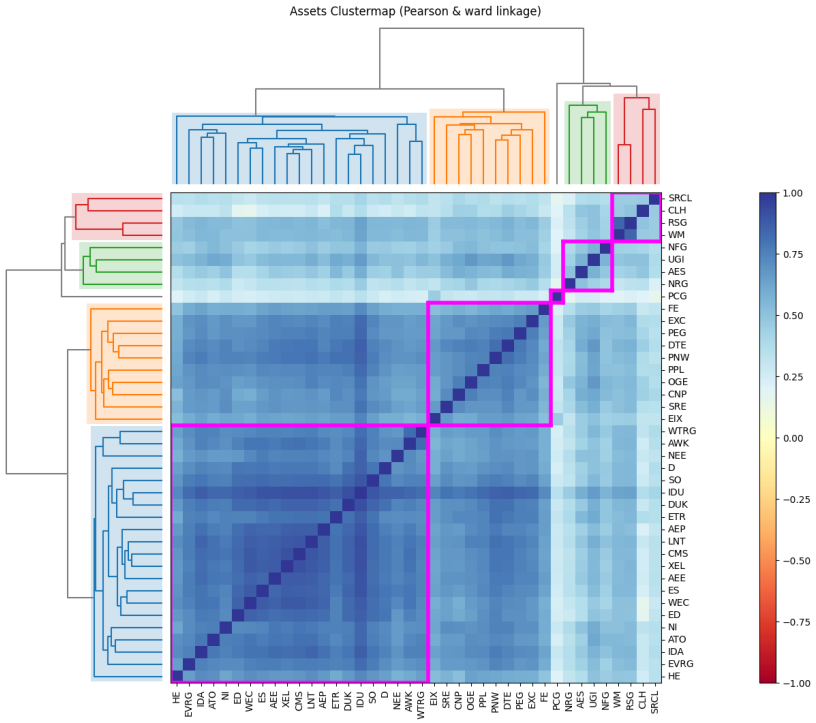

Figure 3: Asset Clustermap of Utilities ETF

The HERC portfolio optimization method is based on the idea of clustering assets into groups that exhibit similar risk characteristics, and then constructing a portfolio that achieves equal risk contributions from each group. This approach is motivated by the observation that the risk of a portfolio is often dominated by a few assets or risk factors, and that diversification across these dominant sources of risk can significantly reduce the overall risk of the portfolio.

The HERC portfolio optimization method can be formulated as follows:

$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{i,j} \\
\text{subject to} \quad & w_i \geq 0 \\
& \sum_{i=1}^{n} w_i = 1 \\
& \sum_{i \in C_k} w_i = w_{C_k} \quad \forall k = 1, 2, \ldots, K \\
& w_{C_k} = \frac{1}{\sqrt{\sum_{i \in C_k}\sum_{j \in C_k} w_i w_j \rho_{i,j}}} \quad \forall k = 1, 2, \ldots, K
\end{aligned}
\tag{17}
$$

where w is the vector of portfolio weights, ρ is the correlation matrix of asset returns, C_k is the set of assets in cluster k, and w_{C_k} is the weight assigned to cluster k.

The constraints ensure that the portfolio weights are non-negative and sum to one, and that the weights assigned to each cluster are proportional to the inverse of the square root of the total variance of the cluster.

3.7.3 Nested Clustered Optimization (NCO)

The intuition behind the NCO model is that assets within each cluster are assumed to have similar risk characteristics and therefore are more likely to co-move with each other. The model then optimizes the weights of each cluster instead of optimizing the weights of individual assets.

The NCO model has several advantages over traditional portfolio optimization models. Firstly, it reduces the estimation error by clustering assets with similar risk characteristics together. Secondly, it allows for easy interpretation of the portfolio weights, as the weights are assigned to the clusters instead of individual assets. Finally, the NCO model is computationally efficient and can handle large portfolios with many assets.

The NCO model can be formulated as follows:

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & w^T \Sigma w \\ \text{subject to} \quad & w^T \mathbf{1} = 1 \\ & w^T V w \leq \sigma_{max}^2 \\ & w_{C_k} \geq 0 \quad \forall k = 1, 2, \ldots, K \\ & w_{C_k} \leq \frac{\mu_{C_k}}{\gamma} \quad \forall k = 1, 2, \ldots, K \\ & \sum_{k=1}^{K} w_{C_k} = 1 \end{aligned} \tag{18}$$

where w is the vector of portfolio weights, Σ is the covariance matrix, V is the variance-covariance matrix of the clusters, σ_{max} is the maximum risk level allowed, C_k is the kth cluster, μ_{C_k} is the expected return of cluster C_k, and γ is the risk aversion parameter. The constraints limit the maximum risk level of the portfolio and ensure that the weights of each cluster are proportional to the expected return.

3.8 Kalman Filter

We explore the use of the Kalman Filter, a powerful linear estimation technique, in predicting stock prices and allocating assets in a portfolio by estimating the underlying states of stock prices or returns, based on observed data. It is a recursive algorithm that provides optimal estimates of the state of a system based on noisy observations. We analyze the methodology and mathematical formulas used in the implementation of this algorithm, drawing on the work of Xu & Zhang (2015)[XZ15].

In the implementation presented, the Kalman Filter is applied to stock price predictions and asset allocation using a two-step process:

(1) The algorithm estimates the underlying states of the system, such as stock prices (x_t) and their change rate ($\dot{x}_t$), based on noisy observations (z_t).

The system can be modeled using the following state-space representation:

$$\begin{bmatrix} x_{t+1} \\ \dot{x}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} w_t \quad \text{and} \quad z_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} + v_t$$

The Kalman filter comprises the following steps:

1. **Initialization**: Set the initial state estimate ($\hat{x}_{0|0}$) and covariance matrix ($P_{0|0}$).
2. **Prediction**: Predict the state and covariance matrix using the previous estimates.

$$\hat{x}_{t+1|t} = F\hat{x}_{t|t}, \quad P_{t+1|t} = FP_{t|t}F^T + \Gamma Q \Gamma^T$$

3. **Kalman Gain**: Compute the Kalman gain (K_{t+1}) to update the state estimate.

$$K_{t+1} = P_{t+1|t}H^T(HP_{t+1|t}H^T + R)^{-1}$$

4. **Update**: Update the state estimate and covariance matrix with the new observation (y_{t+1}).

$$y_{t+1} = z_{t+1} - H\hat{x}_{t+1|t}$$

$$\hat{x}_{t+1|t+1} = \hat{x}_{t+1|t} + K_{t+1}y_{t+1}$$

$$P_{t+1|t+1} = (I - K_{t+1}H)P_{t+1|t}$$

Here, F is the state transition matrix, H is the observation matrix, Q and R are the covariance matrices of the process and observation noise terms (w_t and v_t), and K_t is the Kalman gain.

Using these equations, the Kalman filter can predict stock prices in the short term, taking into account the inherent noise in the observed data.

(2) For portfolio allocation, we use the estimated states. Then, the mean-variance optimization approach is employed to allocate assets in the portfolio.

4 Performance Metrics

We will discuss six performance metrics used to evaluate the performance of a portfolio: **Out-of-Sample Return (ORtn), Out-of-Sample Sharpe Ratio (OSR), Out-of-Sample Sortino Ratio (OSSortino), Out-of-Sample Maximum Drawdown (OMaxDrawdown), Out-of-Sample Calmar Ratio (OCalmarRatio), and Out-of-Sample Turnover Ratio (OTurnoverRatio).**

4.1 Out-of-Sample Return (ORtn)

The Out-of-Sample Return is the average return of a portfolio based on the out-of-sample data. It is a measure of the portfolio's performance during the out-of-sample period, which is the period not used in the portfolio optimization process.

$$\text{Out-of-Sample Return} = \frac{1}{T}\sum_{t=1}^{T} R_t$$

Where R_t is the return of the portfolio at time t, and T is the number of periods in the out-of-sample data.

4.2 Out-of-Sample Sharpe Ratio (OSR)

The Sharpe Ratio is a widely used performance metric that evaluates the risk-adjusted return of a portfolio. It is calculated by dividing the difference between the mean return of the portfolio and the risk-free rate by the standard deviation of the portfolio's returns. A higher Sharpe Ratio indicates better risk-adjusted performance.

$$\text{Sharpe Ratio} = \frac{\bar{R} - R_f}{\sigma}$$

Where $\bar{R}$ is the mean return of the portfolio, R_f is the risk-free rate, and σ is the standard deviation of the portfolio's returns.

4.3 Out-of-Sample Sortino Ratio (OSSortino)

The Sortino Ratio is a performance metric used to evaluate the risk-adjusted return of a portfolio. It is an extension of the Sharpe Ratio, which adjusts for downside risk rather than overall risk. The Sortino Ratio is calculated by dividing the difference between the mean return of the portfolio and the risk-free rate by the downside standard deviation. A higher Sortino Ratio indicates better risk-adjusted performance.

$$\text{Sortino Ratio} = \frac{\bar{R} - R_f}{\sigma_d}$$

Where $\bar{R}$ is the mean return of the portfolio, R_f is the risk-free rate, and σ_d is the downside standard deviation.

4.4 Out-of-Sample Maximum Drawdown (OMaxDrawdown)

Maximum Drawdown is a measure of the largest decrease in the value of a portfolio from its peak to its trough. It is an important measure of risk as it quantifies the worst-case loss that could have been experienced by an investor. A lower maximum drawdown indicates better risk management.

$$\text{Max Drawdown} = \max_t \left(\max_{s \leq t} P_s - P_t \right)$$

Where P_t is the value of the portfolio at time t, and P_s is the maximum value of the portfolio up to time t.

4.5 Out-of-Sample Calmar Ratio (OCalmarRatio)

The Calmar Ratio is another measure of risk-adjusted performance. It is calculated by dividing the average annual rate of return by the maximum drawdown. A higher Calmar Ratio indicates better risk-adjusted performance.

$$\text{Calmar Ratio} = \frac{\text{Average Annual Return}}{\text{Max Drawdown}}$$

4.6 Out-of-Sample Turnover Ratio (OTurnoverRatio)

The Turnover Ratio is a measure of how frequently the assets in a portfolio are bought and sold. It is calculated by taking the average of the absolute changes in portfolio weights between rebalancing periods. A lower turnover ratio indicates a more stable portfolio, which may result in lower transaction costs.

$$\text{Turnover Ratio} = \frac{1}{N-1} \sum_{i=1}^{N-1} \sum_{j=1}^{d} \left| w_{j,i+1} - w_{j,i} \right|$$

Where d is the number of assets in the portfolio, N is the number of rebalancing periods, and $w_{j,i}$ represents the weight of asset j at the beginning of rebalancing period i.

In summary, these six performance metrics (Out-of-Sample Return, Out-of-Sample Sharpe Ratio, Out-of-Sample Sortino Ratio, Out-of-Sample Maximum Drawdown, Out-of-Sample Calmar Ratio, and Out-of-Sample Turnover Ratio) provide various ways to evaluate a portfolio's performance. Each metric offers a different perspective on risk, return, and performance, allowing investors to make more informed decisions about their investment strategies. By considering multiple metrics, investors can gain a comprehensive understanding of their portfolio's performance and risk characteristics.

5 Main Procedure

In this project, we analyze the performance of different asset allocation strategies using data from three ETFs from May 29, 2013, to April 28, 2023, which consists of a total of $T = 2,496$ days. We set the monthly net risk-free rate to be $r = 0.002$. To evaluate the effectiveness of the strategies, we adopt a rolling window approach with window length M. Specifically, we test different values of M, including $M = 21, 63, 126$, and 252, corresponding to monthly, quarterly, half-yearly, and yearly rolling. At each time step t, we use the most recent M observations to calculate the optimal allocation and form a portfolio accordingly. The allocation is held for one period, and we update our training set before repeating the process for the next time step. To assess the performance of each method, we use equal weight and market portfolio as our benchmark, and we measure their out-of-sample performance using various metrics, including Out-of-sample average return (ORtn), Out-of-sample Sharpe Ratio (OSR), Out-of-Sample Sortino Ratio (OSSortino), Out-of-Sample Maximum Drawdown (OMaxDrawdown), Out-of-Sample Calmar Ratio (OCalmarRatio), and Out-of-Sample turnover Ratio (OTurnoverRatio).

6 Experiments & Plots

In our experiments, we will begin by showing the cumulative return and turnover ratio

of our risk parity model compared with two benchmarks. Then we will compare the basic profitability performance of different methods using several main metrics. Additionally, we will select two metrics for cross-comparison of each method to evaluate their performance under different conditions.

6.1 Cumulative Return of Closed-form Risk Parity

Figure 4 is a cumulative return graph of closed-form risk parity compared with equal weight and market portfolio benchmarks, showing the performance of these investment strategies over time. As shown in the graph, risk parity consistently outperforms the equal weight benchmark and market portfolio.

The green line representing the closed-form risk parity strategy is always above the blue line representing the equal weight strategy, indicating that risk parity has better performance in terms of cumulative return. Moreover, the margin between the two lines becomes larger as the rolling basis M becomes smaller, indicating that risk parity's advantage over equal weight is more significant when it is implemented using more recent data. This implies that risk parity is more suitable for investors who prefer to take into account the latest market information and adjust their portfolio allocation accordingly.

6.2 Turnover Ratio of Closed-form Risk Parity

Figure 5 is the turnover ratio graph that shows the annualized turnover ratio of closed-form risk parity, equal weight, and market portfolio over time. The graph illustrates the percentage of the portfolio holdings that have changed over the past year due to trading activity. As can be observed, the turnover ratio of closed-form risk parity is not exceeding 0.10 in the whole period. This is due to the nature of risk parity, which is based on achieving a balanced risk contribution from each asset in the portfolio. As the market conditions change, the asset weights in the portfolio are rebalanced to ensure that the risk contribution remains equal, resulting in a low trading activity.

Furthermore, as the rolling basis M increases, the turnover ratio of closed-form risk parity even decreases, not exceeding 0.05. This is because a larger rolling basis M means that more historical data is considered in the portfolio optimization, resulting in a more stable portfolio allocation. As a result, fewer trades are needed to maintain the risk parity balance, which further reduces the turnover ratio. This is a desirable feature of the risk parity strategy, as lower turnover ratios can result in lower transaction costs and higher after-tax returns for investors.

6.3 Out-of-sample average return(ORtn) of different methods and experiments

In Figure 6, the ORtn of each method under different settings is displayed. The title of each bar chart shows the ETF dataset and rolling period M used. The first two bars (blue and orange) represent the two benchmarks, while the pink bar represents our optimal model closed-form risk parity.

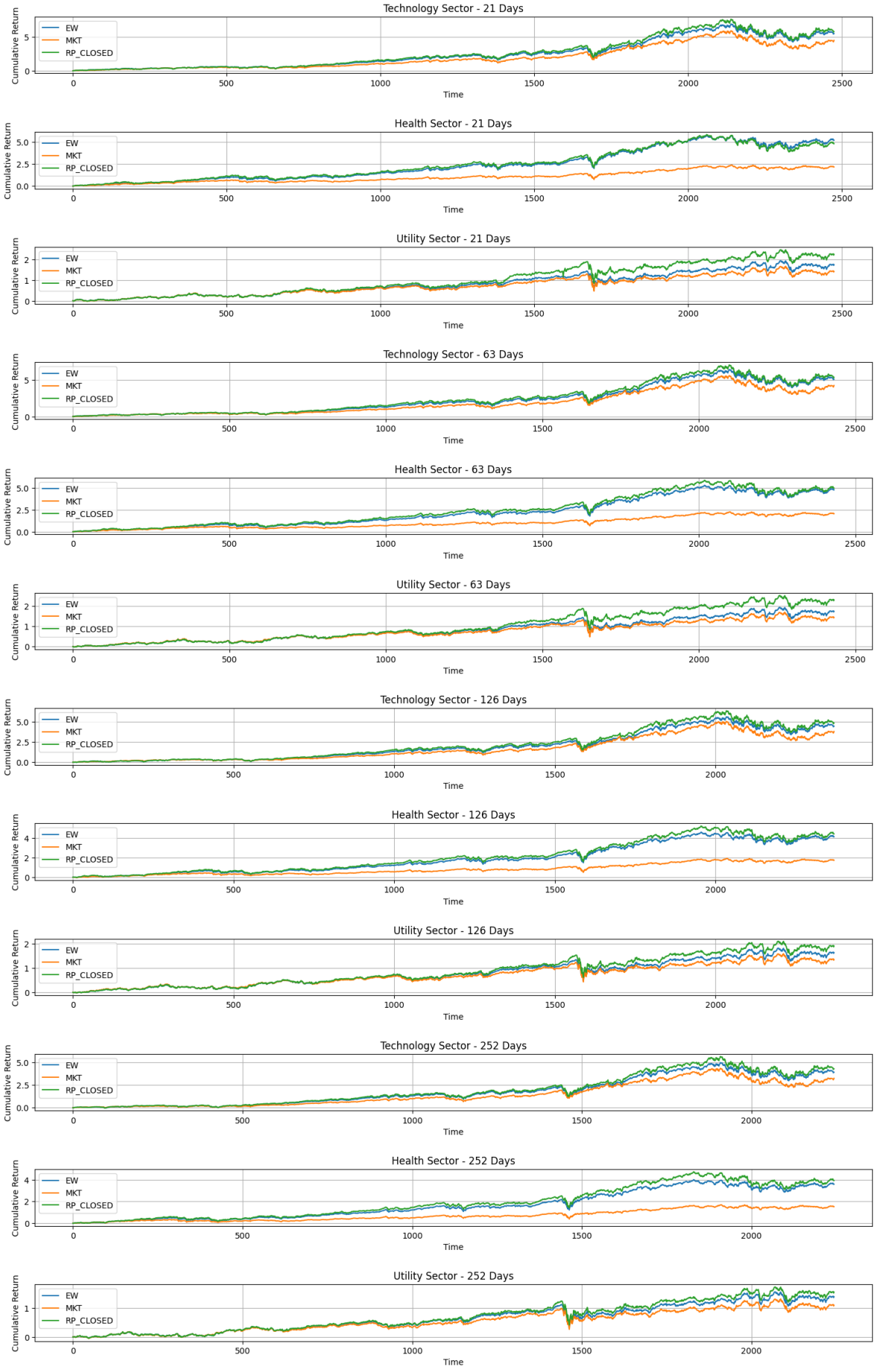

Figure 4: Cumulative Return of Closed-form Risk Parity compared with benchmarks

Figure 5: Turnover Ratio of Closed-form Risk Parity compared with benchmarks

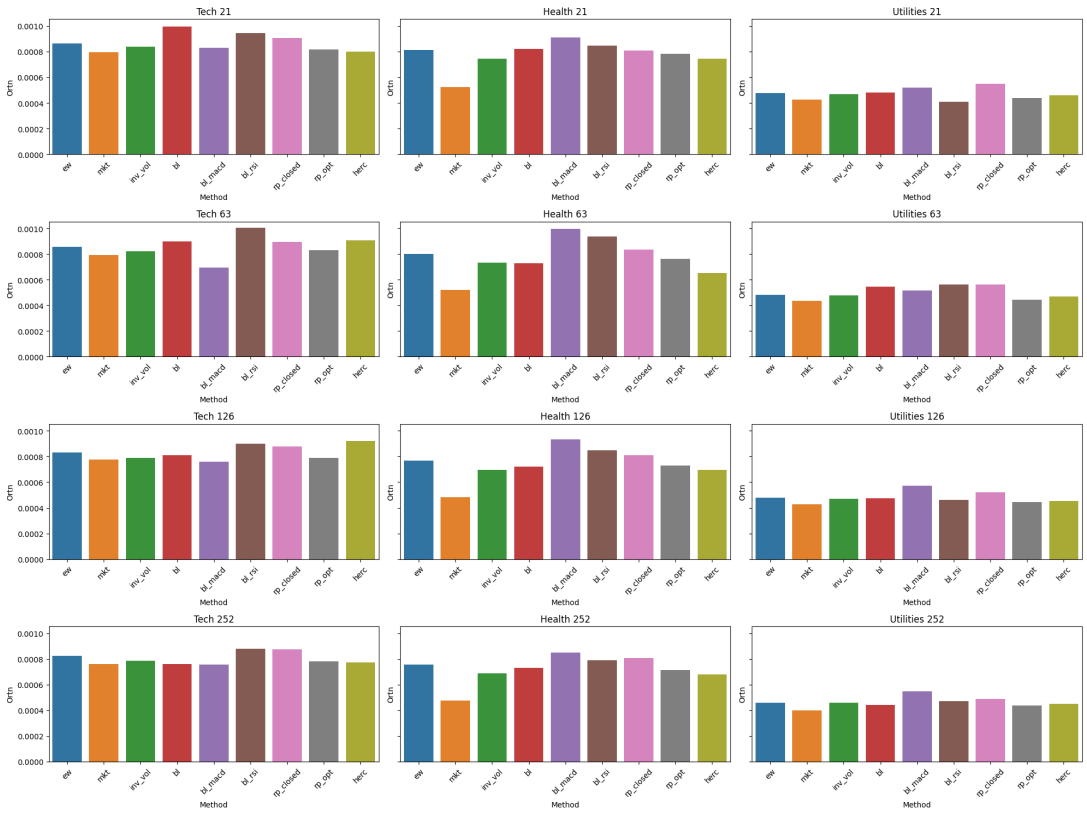

Figure 6: Out-of-sample average return(ORtn) of different methods

As can be seen from the graphs, the risk-parity model consistently outperforms the benchmarks in terms of ORtn. It is important to note that models based on black-litterman sometimes show higher returns, but they are not as robust as the risk-parity model. For instance, the black-litterman model using MACD as a weight indicator (purple) shows a high ORtn in Utilities and Health ETFs, but performs extremely poorly in Tech ETF. This observation highlights the importance of considering the specific characteristics of the underlying assets when selecting an asset allocation strategy, as a strategy that performs well in one asset class may not be as effective in another.

Therefore, risk parity is a better fit when we have no prior knowledge of the asset field we are working on and would like a robust allocation strategy. This is because the risk-parity model is based on the principle of equal risk contribution, which provides a balanced allocation regardless of the underlying asset classes or their characteristics.

6.4 Out-of-Sample Sharpe Ratio (OSR) of different methods and experiments

Figure 7 demonstrates that risk parity outperforms the benchmarks in terms of out-of-sample Sharpe Ratio. Notably, the closed-form risk parity strategy performs exceptionally well on the Utilities ETF, while risk parity with optimization exhibits better performance on the Tech and Health ETFs. Both strategies demonstrate robust performance, with an OSR on par with or higher than the benchmark.

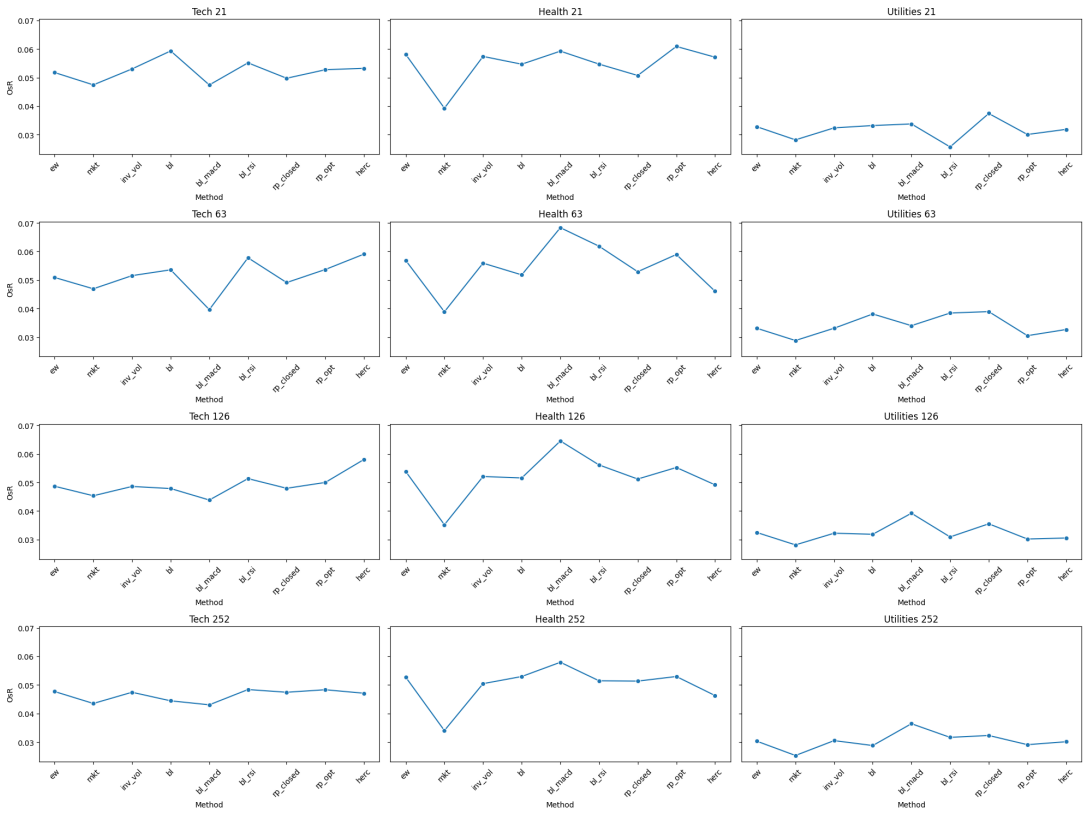

Figure 7: Out-of-Sample Sharpe Ratio (OSR) of different methods

The optimization approach used in risk parity aims to minimize the squared difference between the risk contributions of each asset and the target risk contribution, which is inversely proportional to the number of assets in the portfolio. This approach works well in situations where the risk of different assets in the portfolio is diverse and can be balanced to achieve a more robust portfolio. However, in the case of the Utilities ETF, where the assets' risk contributions are already well-balanced, the optimization approach may not add significant value, and the closed-form approach may be a better fit. On the other hand, the Tech and Health ETFs have assets with different risk contributions that need to be balanced, making the optimization approach a better fit for these ETFs.

6.5 Turnover Ratio vs OSR

After evaluating the performance of different methods in terms of OSR, we want to further investigate their efficiency in achieving a high OSR while maintaining a reasonable turnover ratio. A lower turnover ratio not only reduces transaction costs but also minimizes market impact. Thus, we aim to find a method that can achieve a higher Sharpe ratio with less frequent asset transactions, i.e., the method should be located in the lower right corner of the scatter plot.

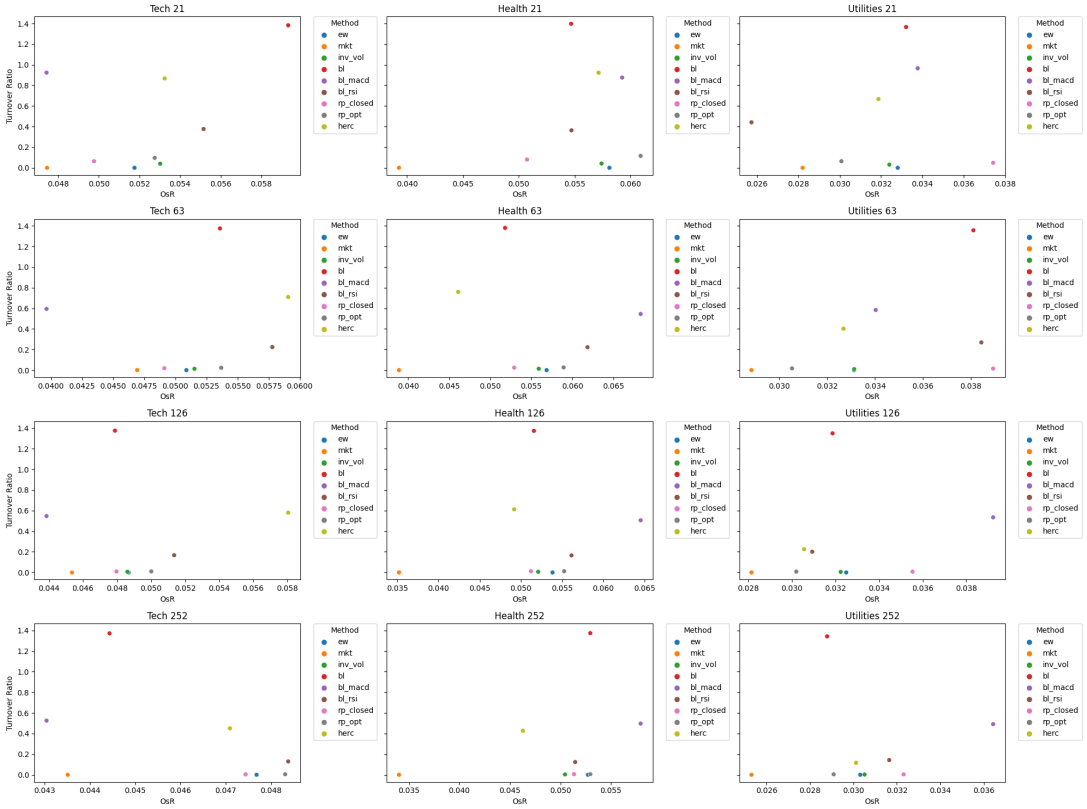

Figure 8: Turnover Ratio vs OSR of different methods

From Figure 8, we can observe that the risk parity method consistently shows a low turnover ratio, which means that it has great performance with relatively stable shares holding. In contrast, the black-litterman based models often show higher turnover ratios, making them less favorable in terms of transaction costs. Although black-litterman may sometimes demonstrate a high Sharpe ratio, this benefit may be outweighed by the costs incurred through high turnover. Therefore, in selecting an optimal model, it is important to balance both performance metrics and transaction costs, and risk parity appears to be a promising option in this regard.

6.6 Max Drawdown vs OSR

Max drawdown is a measure of the largest peak-to-trough decline in the value of a portfolio, indicating the maximum loss an investor could have experienced during a specific period. From Figure 9, a medium-ranked max drawdown for both risk parity and black-litterman methods suggests that they demonstrate a moderate ability to manage drawdowns, offering a balance between risk and return.

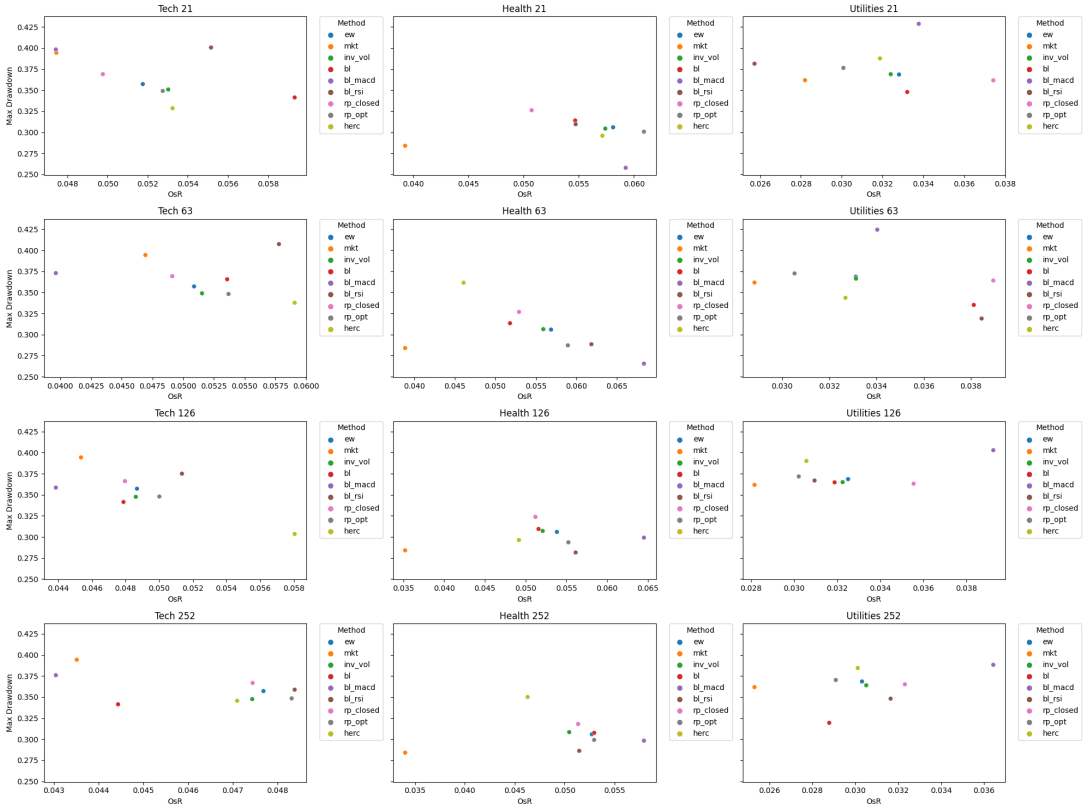

Figure 9: Max Drawdown vs OSR of different methods

6.7 Calmar Ratio vs OSR

From Figure 10, a high Calmar ratio for the risk parity and black-litterman methods indicates that these strategies demonstrate strong risk-adjusted performance. The Calmar ratio is a performance measurement that compares the average annualized return of a portfolio to its maximum drawdown, with a higher ratio reflecting better performance.

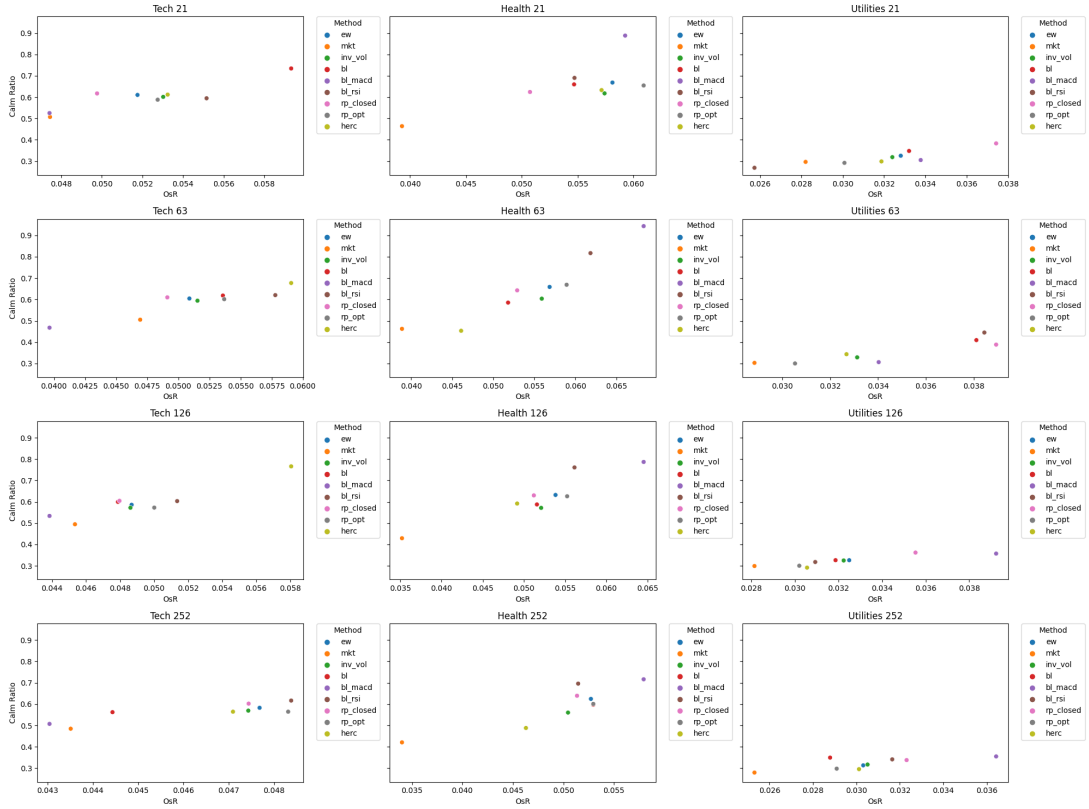

Figure 10: Calmar Ratio vs OSR of different methods

In conclusion, both the risk parity and black-litterman methods can generate relatively high returns while minimizing the impact of drawdowns, making them attractive options for investors who prioritize balancing risk and portfolio performance.

7 Other Findings

For the previous part, we analyze mostly about Risk Parity method, assuming that the turnover ratio is important. But after comparing all the results, there are several other potentially good methods, including Inverse Volatility method (method 2), and Black-Litterman based methods (methods 3-5). For this part, we would like to briefly demonstrate our findings and limitations associated with these methods.

7.1 Finding 1: Inverse Volatility method (method 2) performs similarly to Equal-Weight method (method 0) in terms of ORtn and OSR

The Inverse Volatility method (method 2) allocates more weight to low-volatility assets and less to high-volatility assets, aiming for a stable and balanced portfolio. It is somewhat like the Equal-Weight method (method 0), but it takes into account each asset's risk profile.

Both Inverse Volatility and Equal-Weight strategies aim for a balanced portfolio, but Inverse Volatility has some advantages: Firstly, Inverse Volatility considers each asset's volatility, leading to a more risk-aware allocation and potentially better diversification by focusing

on less volatile assets. Secondly, Inverse Volatility may offer slightly improved risk-adjusted performance compared to Equal-Weight, as it minimizes portfolio risk while maintaining the desired return level.

Performance metrics show that Inverse Volatility has a slightly better risk-adjusted performance (measured by the Sharpe ratio) than Equal-Weight. This suggests that Inverse Volatility might be a more appealing option for investors looking to balance risk and return while keeping a diversified portfolio. But overall, it's not that competitive among all the methods.

7.2 Finding 2: The tradeoff between high returns and high turnover ratio is significant in Black-Litterman based methods (methods 3-5)

The choice of asset allocation method depends on market structure and resources. The Black-Litterman model combines market returns with investor views, replaced in this case by technical indicators. The three methods (methods 3-5) have high returns but also high turnover ratios due to two main reasons: Firstly, with frequent updates of investor views, the model adjusts portfolio weights frequently, leading to a higher turnover ratio as it continuously incorporates new information. Secondly, it's sensitive to market signals and may cause overfitting. Methods with MACD and RSI are sensitive to short-term market fluctuations, potentially causing frequent portfolio adjustments and higher turnover ratios.

Black-Litterman methods generally have better risk-adjusted performance but higher turnover ratios, leading to frequent rebalancing and increased trading costs. Conversely, the Closed-form Risk Parity method (method 6) has a lower turnover ratio, minimizing transaction costs. In practice, the chosen method depends on factors like portfolio size, market frictions, and brokerage commissions.

7.3 Finding 3: Black-Litterman based methods (methods 3-5) show varying performance across sectors.

The performance of Black-Litterman methods varies depending on the market sector. For example, the method with MACD (method 4) performs well in Healthcare and Utilities sectors but not in Tech. However, the method with RSI (method 5) works well in Tech. This discrepancy is likely due to sector-specific features, particularly volatility:

Healthcare and utility sectors are generally less volatile than the tech sector, as they provide essential services and products less sensitive to economic cycles. MACD, a trend-following indicator, performs better in less volatile sectors with stable trends, making it more effective for healthcare and utilities. On the other hand, the RSI calculation captures recent price trends by focusing on the proportion of positive returns in a short time frame (14 days). This short-term approach may be more suitable for analyzing the tech sector compared to methods relying on longer time frames.

8 Conclusion

In choosing between the Closed-form Risk Parity and Black-Litterman methods, investors should consider their priorities and the unique features of each method.

Closed-form Risk Parity focuses on achieving equal risk contribution from all assets in the portfolio, resulting in a well-diversified and balanced allocation regardless of the underlying asset classes or their characteristics. This method consistently outperforms benchmarks in terms of out-of-sample average returns (ORtn) and out-of-sample Sharpe Ratio (OSR). Importantly, it maintains a low turnover ratio, making it a cost-efficient investment strategy across various market sectors.

Black-Litterman methods, on the other hand, combine market equilibrium returns with investor views, which are replaced by technical indicators in this case. These methods generally demonstrate better risk-adjusted performance, as indicated by higher Sharpe, Sortino, and Calmar ratios. However, they also exhibit higher turnover ratios, leading to frequent rebalancing of the portfolio and increased trading costs.

The performance of Black-Litterman methods varies across different market sectors due to the unique characteristics and market dynamics of each sector. The effectiveness of technical indicators, such as MACD and RSI, can vary across different market sectors, making these methods more sensitive to market fluctuations and potentially causing overfitting.

In summary, the choice between Closed-form Risk Parity and Black-Litterman methods depends on the investor's priorities and preferences. If the primary goal is to achieve a well-diversified and balanced portfolio with a cost-efficient strategy, the Closed-form Risk Parity method is the better choice. However, if the investor is willing to take on higher turnover ratios and trading costs for potentially better risk-adjusted performance, the Black-Litterman methods may be more suitable. Ultimately, the chosen method should align with the investor's risk tolerance, portfolio size, market frictions, and available leverage and brokerage commissions.

9 Proposal

In this analysis, we examine two investment strategies: Closed-form Risk Parity and Black-Litterman methods. Let's start by introducing these methods in simpler terms and explaining the intuition behind why they work.

Closed-form Risk Parity is a method that helps investors build a balanced and diversified portfolio. Imagine you walked into a casino and decided to play roulette. This method does something similar for your investments; it aims to divide the risk equally among all the assets in your portfolio, so no single asset dominates the risk. By spreading out the risk evenly, this method helps create a more stable investment portfolio that can potentially perform well across different market conditions.

On the other hand, the Black-Litterman method is like having a group of friends who give you advice on how to distribute tasks based on their opinions and expertise. This method combines the overall market views with the investor's views, which, in our case, are represented

by technical indicators such as MACD and RSI. These technical indicators provide signals about the current market trends and help inform the investment decisions. By incorporating these signals, the Black-Litterman method aims to achieve better risk-adjusted performance.

Now, let's discuss why these methods work. The Closed-form Risk Parity method works because it focuses on achieving a balanced portfolio where each asset contributes equally to the overall risk. This approach helps create a more stable investment portfolio that is less sensitive to the ups and downs of individual assets, making it an attractive choice for investors who want to minimize their exposure to risk.

The Black-Litterman method works because it takes into account both the overall market views and the investor's views, represented by technical indicators. These indicators can provide valuable insights into market trends, allowing the investor to make more informed decisions about their asset allocation. However, this method can sometimes result in higher turnover ratios, leading to increased trading costs.

In summary, Closed-form Risk Parity and Black-Litterman methods are two investment strategies that aim to optimize a portfolio's risk and return. Closed-form Risk Parity focuses on balancing risk evenly across all assets, making it a more stable and cost-efficient approach. In contrast, the Black-Litterman method combines market views with technical indicators to potentially achieve better risk-adjusted performance, albeit at the expense of higher trading costs. The choice between these methods depends on the investor's priorities, risk tolerance, and desired balance between risk and return.

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10 Appendix

10.1 Change in Rank for Each Sector Across Metrics

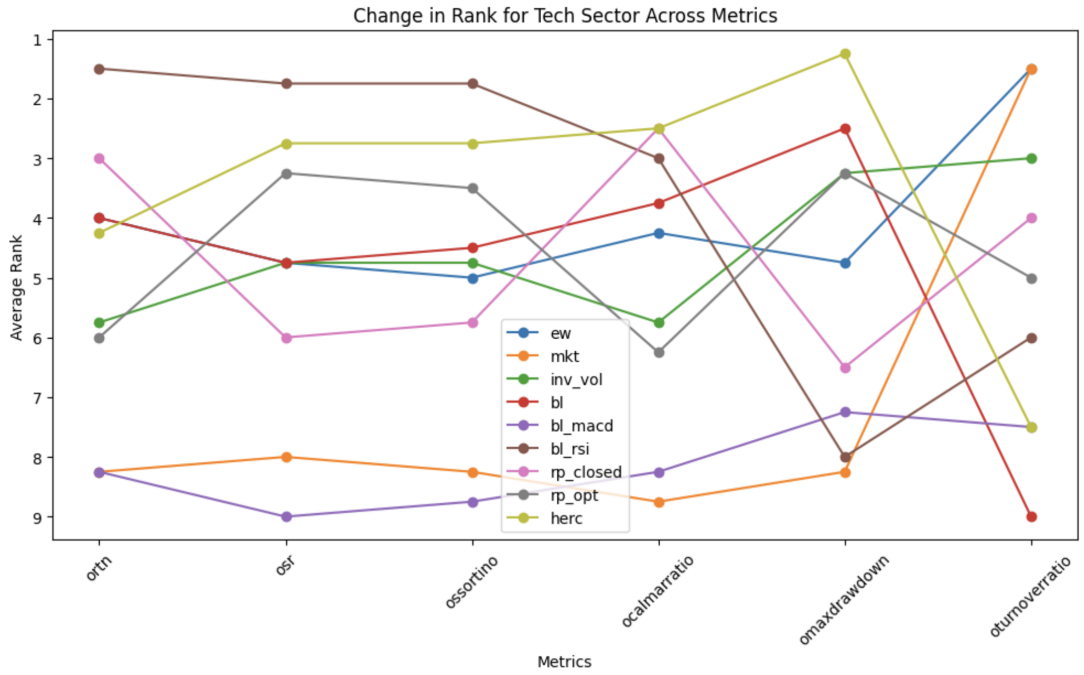

Figure 11: Change in Rank for Technology Sector Across Metrics

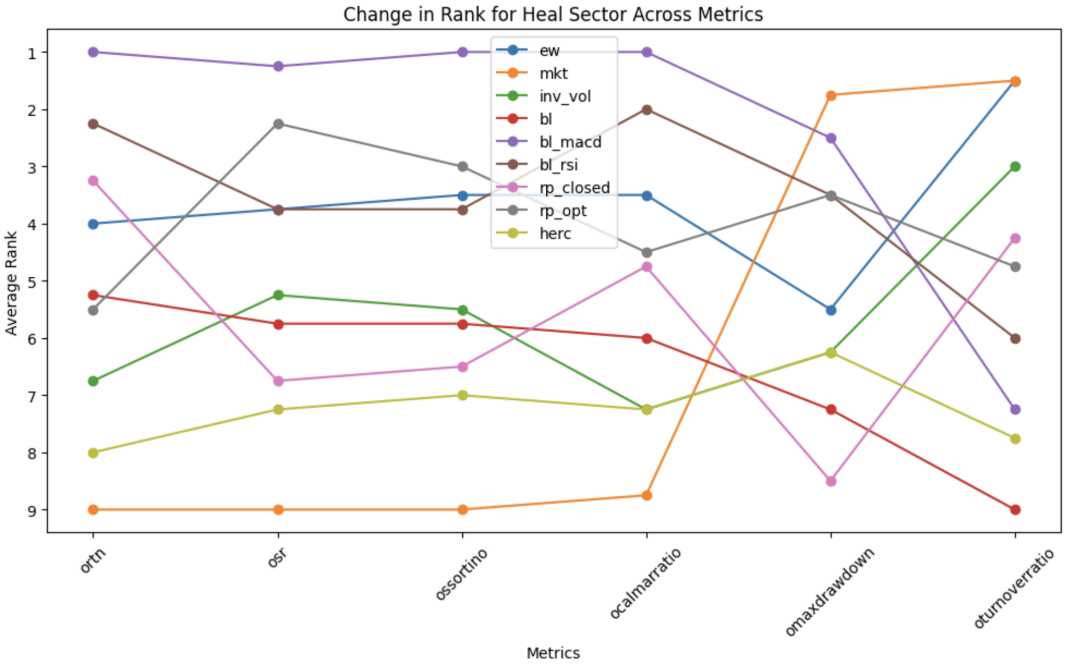

Figure 12: Change in Rank for Healthcare Sector Across Metrics

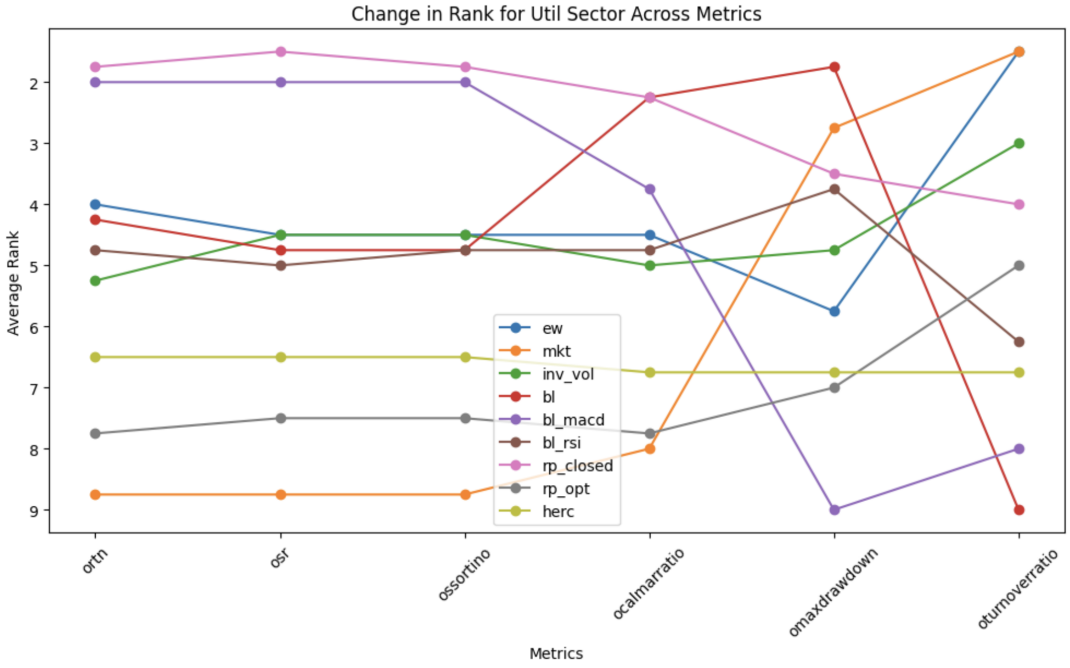

Figure 13: Change in Rank for Utilities Sector Across Metrics

10.2 Result Data frames

10.2.1 Prediction Period M = 21

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000865	0.051756	0.069463	0.357168	0.610401	0.000000
1	mkt	0.000794	0.047446	0.063896	0.394362	0.507301	0.000000
2	inverse_volatility_allocation	0.000837	0.053022	0.070843	0.350750	0.601276	0.039951
3	black_litterman	0.000994	0.059326	0.080516	0.341271	0.734353	1.383059
4	black_litterman_macd	0.000830	0.047421	0.064176	0.398216	0.525463	0.923643
5	black_litterman_rsi	0.000945	0.055158	0.074694	0.400571	0.594777	0.376269
6	rp_closed	0.000904	0.049765	0.067129	0.368949	0.617402	0.063595
7	risk_parity_optimization	0.000814	0.052750	0.070386	0.349029	0.587960	0.096411
8	herc	0.000798	0.053241	0.071752	0.328508	0.611862	0.867195

Figure 14: Metrics for Technology Sector When M = 21

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000811	0.058114	0.077598	0.305917	0.668424	0.000000
1	mkt	0.000523	0.039279	0.052000	0.284031	0.464238	0.000000
2	inverse_volatility_allocation	0.000746	0.057408	0.076077	0.304335	0.617779	0.041360
3	black_litterman	0.000822	0.054681	0.073449	0.313962	0.659721	1.397354
4	black_litterman_macd	0.000909	0.059252	0.081390	0.257839	0.888301	0.875920
5	black_litterman_rsi	0.000848	0.054717	0.074724	0.309590	0.690129	0.363478
6	rp_closed	0.000808	0.050734	0.068376	0.326151	0.624190	0.080565
7	risk_parity_optimization	0.000781	0.060908	0.081032	0.300712	0.654577	0.115425
8	herc	0.000743	0.057142	0.076086	0.295888	0.633076	0.923070

Figure 15: Metrics for Healthcare Sector When M = 21

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000476	0.032809	0.043370	0.368557	0.325592	0.000000
1	mkt	0.000426	0.028204	0.037261	0.361798	0.296806	0.000000
2	inverse_volatility_allocation	0.000467	0.032402	0.042823	0.368905	0.319138	0.030904
3	black_litterman	0.000480	0.033211	0.043777	0.347748	0.347951	1.366300
4	black_litterman_macd	0.000520	0.033775	0.044520	0.428670	0.305421	0.965603
5	black_litterman_rsi	0.000407	0.025729	0.033745	0.381376	0.269086	0.441675
6	rp_closed	0.000550	0.037426	0.049922	0.361634	0.383379	0.048134
7	risk_parity_optimization	0.000436	0.030084	0.039672	0.376374	0.292207	0.063387
8	herc	0.000460	0.031876	0.041882	0.387633	0.299033	0.667098

Figure 16: Metrics for Utilities Sector When M = 21

10.2.2 Prediction Period M = 63

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000857	0.050887	0.068297	0.357168	0.604489	0.000000
1	mkt	0.000791	0.046931	0.063230	0.394362	0.505266	0.000000
2	inverse_volatility_allocation	0.000823	0.051537	0.068834	0.348953	0.594203	0.012566
3	black_litterman	0.000897	0.053580	0.071981	0.365629	0.618408	1.375248
4	black_litterman_macd	0.000692	0.039641	0.052731	0.372951	0.467913	0.593628
5	black_litterman_rsi	0.001003	0.057791	0.079044	0.407318	0.620397	0.223502
6	rp_closed	0.000894	0.049111	0.066274	0.369283	0.609843	0.018804
7	risk_parity_optimization	0.000831	0.053687	0.071646	0.348199	0.601345	0.022562
8	herc	0.000907	0.059069	0.079635	0.337728	0.677070	0.709484

Figure 17: Metrics for Technology Sector When M = 63

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000800	0.056863	0.075908	0.305917	0.658772	0.000000
1	mkt	0.000521	0.038883	0.051495	0.284031	0.462506	0.000000
2	inverse_volatility_allocation	0.000734	0.055910	0.074045	0.306405	0.603717	0.013209
3	black_litterman	0.000728	0.051805	0.069105	0.313513	0.585374	1.380424
4	black_litterman_macd	0.000994	0.068321	0.093558	0.265533	0.943390	0.545037
5	black_litterman_rsi	0.000936	0.061842	0.084758	0.288555	0.817103	0.222355
6	rp_closed	0.000834	0.052916	0.071346	0.326906	0.642670	0.024548
7	risk_parity_optimization	0.000762	0.058937	0.078181	0.287254	0.668911	0.026372
8	herc	0.000650	0.046086	0.060072	0.361545	0.453343	0.758860

Figure 18: Metrics for Healthcare Sector When M = 63

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000482	0.033120	0.043786	0.368557	0.329344	0.000000
1	mkt	0.000435	0.028829	0.038100	0.361798	0.303067	0.000000
2	inverse_volatility_allocation	0.000478	0.033128	0.043824	0.366452	0.328840	0.009939
3	black_litterman	0.000545	0.038117	0.051091	0.335204	0.409663	1.357051
4	black_litterman_macd	0.000516	0.034028	0.045383	0.424491	0.306590	0.583333
5	black_litterman_rsi	0.000564	0.038450	0.052294	0.319099	0.445148	0.269033
6	rp_closed	0.000561	0.038944	0.051770	0.364111	0.388593	0.014915
7	risk_parity_optimization	0.000444	0.030526	0.040339	0.372556	0.300289	0.016915
8	herc	0.000469	0.032681	0.043256	0.343670	0.343618	0.402566

Figure 19: Metrics for Utilities Sector When M = 63

10.2.3 Prediction Period M = 126

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000830	0.048683	0.065298	0.357168	0.585920	0.000000
1	mkt	0.000774	0.045354	0.061072	0.394362	0.494689	0.000000
2	inverse_volatility_allocation	0.000789	0.048613	0.064877	0.347572	0.572121	0.006231
3	black_litterman	0.000812	0.047880	0.064358	0.341410	0.599487	1.375954
4	black_litterman_macd	0.000759	0.043858	0.058513	0.358539	0.533768	0.547068
5	black_litterman_rsi	0.000898	0.051357	0.069477	0.375041	0.603246	0.167266
6	rp_closed	0.000878	0.047969	0.064709	0.366161	0.604421	0.009058
7	risk_parity_optimization	0.000791	0.050012	0.066667	0.347893	0.572736	0.010146
8	herc	0.000922	0.058056	0.079345	0.303439	0.765891	0.580029

Figure 20: Metrics for Technology Sector When M = 126

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000767	0.053839	0.071759	0.305917	0.632023	0.000000
1	mkt	0.000484	0.035214	0.046543	0.284031	0.429365	0.000000
2	inverse_volatility_allocation	0.000696	0.052096	0.068870	0.307100	0.571426	0.006549
3	black_litterman	0.000721	0.051584	0.068684	0.309415	0.587477	1.374037
4	black_litterman_macd	0.000934	0.064532	0.088226	0.299169	0.786588	0.506346
5	black_litterman_rsi	0.000850	0.056141	0.076377	0.281487	0.760832	0.165825
6	rp_closed	0.000809	0.051209	0.068864	0.323793	0.629895	0.011843
7	risk_parity_optimization	0.000729	0.055249	0.073190	0.293583	0.625843	0.012002
8	herc	0.000696	0.049185	0.065355	0.296237	0.591743	0.612606

Figure 21: Metrics for Healthcare Sector When M = 126

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000477	0.032500	0.042933	0.368557	0.326342	0.000000
1	mkt	0.000430	0.028158	0.037177	0.361798	0.299236	0.000000
2	inverse_volatility_allocation	0.000471	0.032247	0.042623	0.365006	0.325143	0.005091
3	black_litterman	0.000473	0.031874	0.041823	0.364724	0.326486	1.350262
4	black_litterman_macd	0.000571	0.039236	0.052461	0.402887	0.357378	0.534997
5	black_litterman_rsi	0.000463	0.030943	0.040664	0.366908	0.317994	0.201059
6	rp_closed	0.000522	0.035539	0.047014	0.363164	0.362104	0.007450
7	risk_parity_optimization	0.000443	0.030212	0.039900	0.371745	0.300504	0.008270
8	herc	0.000452	0.030568	0.040165	0.390144	0.291647	0.224971

Figure 22: Metrics for Utilities Sector When M = 126

10.2.4 Prediction Period M = 252

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000826	0.047687	0.064003	0.357168	0.582656	0.000000
1	mkt	0.000759	0.043512	0.058616	0.394362	0.484727	0.000000
2	inverse_volatility_allocation	0.000785	0.047437	0.063391	0.347599	0.569375	0.003044
3	black_litterman	0.000762	0.044436	0.059873	0.341443	0.562181	1.372458
4	black_litterman_macd	0.000757	0.043044	0.057499	0.375956	0.507392	0.525403
5	black_litterman_rsi	0.000878	0.048385	0.065418	0.358799	0.616618	0.129534
6	rp_closed	0.000877	0.047445	0.064054	0.366796	0.602209	0.004353
7	risk_parity_optimization	0.000780	0.048318	0.064431	0.348464	0.564420	0.004837
8	herc	0.000774	0.047099	0.063029	0.345670	0.564436	0.450893

Figure 23: Metrics for Technology Sector When M = 252

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000758	0.052717	0.070275	0.305917	0.624438	0.000000
1	mkt	0.000474	0.034030	0.045008	0.284031	0.420822	0.000000
2	inverse_volatility_allocation	0.000686	0.050453	0.066750	0.308441	0.560360	0.003196
3	black_litterman	0.000729	0.052950	0.070905	0.307566	0.597239	1.374450
4	black_litterman_macd	0.000848	0.057935	0.077931	0.298295	0.716298	0.496800
5	black_litterman_rsi	0.000790	0.051456	0.069462	0.286229	0.695949	0.123531
6	rp_closed	0.000807	0.051344	0.068999	0.318081	0.639148	0.005692
7	risk_parity_optimization	0.000714	0.052952	0.070182	0.299193	0.601003	0.005669
8	herc	0.000678	0.046290	0.062001	0.350096	0.488144	0.427091

Figure 24: Metrics for Healthcare Sector When M = 252

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000458	0.030314	0.040047	0.368557	0.313260	0.000000
1	mkt	0.000401	0.025309	0.033401	0.361798	0.279371	0.000000
2	inverse_volatility_allocation	0.000458	0.030513	0.040348	0.363981	0.316958	0.002604
3	black_litterman	0.000443	0.028789	0.038408	0.319471	0.349422	1.343270
4	black_litterman_macd	0.000547	0.036431	0.048609	0.388289	0.354927	0.491347
5	black_litterman_rsi	0.000471	0.031649	0.041688	0.348089	0.341313	0.142752
6	rp_closed	0.000489	0.032311	0.042700	0.365086	0.337819	0.003785
7	risk_parity_optimization	0.000437	0.029094	0.038450	0.370241	0.297724	0.004186
8	herc	0.000451	0.030120	0.039444	0.384457	0.295515	0.115983

Figure 25: Metrics for Utilities Sector When M = 252

IEOR 4630 Asset Allocation Final Project

May 7, 2023

1 IEOR 4630 Asset Allocation Final Project

Team Members: Charles Xu(cx2280), Gechen Shen(gs3168), Yuyang Xu(yx2761), Huarui Zhang(hz2817), Jiaqi Xi(jx2504)

[77]:
```
import numpy as np
import pandas as pd
from numpy import linalg
import statsmodels.api as sm
import seaborn as sns
import matplotlib.pyplot as plt
from scipy.optimize import minimize, Bounds, LinearConstraint,␣
 ↪NonlinearConstraint, SR1
from sklearn.covariance import LedoitWolf
np.random.seed(2280)

import scipy.cluster.hierarchy as sch
from scipy.cluster.hierarchy import linkage, dendrogram, fcluster
import riskfolio as rp

import warnings
warnings.filterwarnings("ignore")

import plotly.express as px
```

1.1 Part 1: Data Processing

[5]:
```
import pandas as pd
import yfinance as yf
```

[6]:
```
US_Tech_ETF = pd.read_csv('iShares_US_Technology_ETF_holdings.csv')
US_Healthcare_ETF = pd.read_csv('iShares_US_Healthcare_ETF_holdings.csv')
US_Utilities_ETF = pd.read_csv('iShares_US_Utilities_ETF.csv')
```

[7]:
```

```

```
US_Tech_Underlying = US_Tech_ETF[US_Tech_ETF['Asset Class'] == 'Equity'].Ticker.
↪to_numpy()
US_Healthcare_ETF = US_Healthcare_ETF[US_Healthcare_ETF['Asset Class'] ==␣
↪'Equity'].Ticker.to_numpy()
US_Utilities_ETF = US_Utilities_ETF[US_Utilities_ETF['Asset Class'] ==␣
↪'Equity'].Ticker.to_numpy()
```

[8]:
```
def data_process(etf_symbol, underlying_stocks, start_date='2013-05-29',␣
↪end_date='2023-04-28'):
    etf_data = yf.download(etf_symbol, start=start_date, end=end_date)
    stocks_data = {}
    for stock in underlying_stocks:
        stocks_data[stock] = yf.download(stock, start=start_date, end=end_date)
    etf_returns = etf_data['Adj Close'].pct_change().dropna()
    etf_returns.name = etf_symbol
    stock_returns = pd.DataFrame()
    for stock, data in stocks_data.items():
        returns = data['Adj Close'].pct_change().dropna()
        stock_returns[stock] = returns
    R = pd.concat([etf_returns, stock_returns], axis=1).T
    R = R.dropna()
    return R.to_numpy()
```

[9]:
```
Tech_Returns = data_process('IYW', US_Tech_Underlying)
Healthcare_Returns = data_process('IYH', US_Healthcare_ETF)
Utilities_Returns = data_process('IDU', US_Utilities_ETF)
```

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: PerformanceWarning: DataFrame is highly fragmented.  This is usually the
result of calling `frame.insert` many times, which has poor performance.
Consider joining all columns at once using pd.concat(axis=1) instead. To get a
de-fragmented frame, use `newframe = frame.copy()`
  stock_returns[stock] = returns
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[10]:
```
tech_returns = pd.DataFrame(Tech_Returns.T)
```

[11]:
```
print(Tech_Returns.shape, Healthcare_Returns.shape, Utilities_Returns.shape)
```

```
(78, 2496) (84, 2496) (40, 2496)
```

1.1.1 – Benchmark

1.1.2 1.3.1 "1/N" with rebalancing (ew)

[12]:
```
def ew(d, M, R, para):
    w_t = np.ones((d,1))*(1/d)
    return w_t
```

1.1.3 1.3.2 Market Portfolio/Factor Portfolio (mkt)

[13]:
```
def mkt(d, M, R, para):
    w_t = np.zeros((d,1))
    w_t[0] = 1
    return w_t
```

1.1.4 1.3.3 Sample-based mean-variance (mv)

[14]:
```
def mv(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    mu_2 = mu - rf * e
    cov = np.cov(R)
    w_t = (kappa - rf)/(mu_2.T @ np.linalg.inv(cov) @ mu_2) * np.linalg.
 ↪inv(cov) @ mu_2
    return w_t
```

[15]:
```
def mean_variance_optimization(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    mu_2 = mu - rf * e
    cov = np.cov(R)
    w_t = (kappa - rf)/(mu_2.T @ np.linalg.inv(cov) @ mu_2) * np.linalg.
 ↪inv(cov) @ mu_2
    return w_t
```

[16]:
```
def risk_parity_optimization(d, M, R, para):
    cov = np.cov(R)
    initial_weights = np.ones(d) / d

    def risk_objective(w):
        portfolio_variance = w @ cov @ w.T
        risk_contributions = (w * (cov @ w.T)) / portfolio_variance
        return np.sum((risk_contributions - 1/d)**2)

    bounds = Bounds(np.zeros(d), np.ones(d))
    linear_constraint = LinearConstraint(np.ones(d), 1, 1)

    optimized_result = minimize(risk_objective, initial_weights,␣
 ↪constraints=linear_constraint, bounds=bounds)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[17]:
```
def top_bottom_performers_allocation(d, M, R, para):
    e = np.ones((d, 1))
    cumsum_returns = np.sum(R, axis=1)
    top_performers = np.argsort(cumsum_returns)[-5:]
    bottom_performers = np.argsort(cumsum_returns)[:5]
    w_t = np.zeros((d, 1))
    w_t[top_performers] = 1/5
    w_t[bottom_performers] = -1/5
    return w_t
```

1.2 Added inverse_volatility_allocation, max_diversification, black_litterman

Yuyang Xu

[18]:
```
def inverse_volatility_allocation(d, M, R, para):
    vol = np.std(R, axis=1)
    inverse_vol = 1 / vol
    w_t = (inverse_vol / np.sum(inverse_vol)).reshape(-1, 1)
    return w_t
```

[19]:
```
def max_diversification(d, M, R, para):
    cov = np.cov(R)
    initial_weights = np.ones(d) / d

    def negative_diversification_ratio(w):
        portfolio_volatility = np.sqrt(w @ cov @ w.T)
        weighted_individual_volatility = w * np.sqrt(np.diag(cov))
        diversification_ratio = weighted_individual_volatility.sum() /␣
↪portfolio_volatility
        return -diversification_ratio

    bounds = Bounds(np.zeros(d), np.ones(d))
    linear_constraint = LinearConstraint(np.ones(d), 1, 1)

    optimized_result = minimize(negative_diversification_ratio,␣
↪initial_weights, constraints=linear_constraint, bounds=bounds)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[20]:
```
def black_litterman_sma20(d, M, R, para):
    # Market capitalization weights
    market_cap = np.sum(R, axis=1).reshape(-1, 1)
    market_cap_weights = market_cap / np.sum(market_cap)

    # Prior return estimate
    # Technical indicator-based prior return estimates
    R_df = pd.DataFrame(R.T)
    prior_return = R_df.rolling(window=20).mean().iloc[-1].values.reshape(-1, 1)

    # Covariance matrix
    cov = LedoitWolf().fit(R.T).covariance_

    # Black-Litterman parameters
    tau = 0.05
    delta = 2.5

    # Compute equilibrium excess returns
    market_return = np.dot(market_cap_weights.T, prior_return)
    excess_market_return = market_return - para[2]
    omega = np.diag(np.diag(cov))
    pi = delta * np.dot(omega, market_cap_weights) * excess_market_return / tau

    # Compute posterior estimate of returns
    posterior_cov = np.linalg.inv(np.linalg.inv(tau * omega) + np.dot(np.
↪dot(market_cap_weights.T, np.linalg.inv(cov)), market_cap_weights))
```

```
    posterior_return = np.dot(posterior_cov, (np.dot(np.linalg.inv(tau *␣
↪omega), pi) + np.dot(np.dot(np.linalg.inv(cov), market_cap_weights),␣
↪market_return)))

    # Compute optimal portfolio weights
    bounds = [(0, 1) for i in range(d)]
    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
    optimized_result = minimize(lambda w: -np.dot(w.T, posterior_return) / (np.
↪sqrt(np.dot(np.dot(w.T, posterior_cov), w))), x0=np.ones(d) / d,␣
↪bounds=bounds, constraints=constraints)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[21]:
```
def black_litterman_sma50(d, M, R, para):
    # Market capitalization weights
    market_cap = np.sum(R, axis=1).reshape(-1, 1)
    market_cap_weights = market_cap / np.sum(market_cap)

    # Prior return estimate
    # Technical indicator-based prior return estimates
    R_df = pd.DataFrame(R.T)
    prior_return = R_df.rolling(window=50).mean().iloc[-1].values.reshape(-1, 1)

    # Covariance matrix
    cov = LedoitWolf().fit(R.T).covariance_

    # Black-Litterman parameters
    tau = 0.05
    delta = 2.5

    # Compute equilibrium excess returns
    market_return = np.dot(market_cap_weights.T, prior_return)
    excess_market_return = market_return - para[2]
    omega = np.diag(np.diag(cov))
    pi = delta * np.dot(omega, market_cap_weights) * excess_market_return / tau

    # Compute posterior estimate of returns
    posterior_cov = np.linalg.inv(np.linalg.inv(tau * omega) + np.dot(np.
↪dot(market_cap_weights.T, np.linalg.inv(cov)), market_cap_weights))
    posterior_return = np.dot(posterior_cov, (np.dot(np.linalg.inv(tau *␣
↪omega), pi) + np.dot(np.dot(np.linalg.inv(cov), market_cap_weights),␣
↪market_return)))

    # Compute optimal portfolio weights
    bounds = [(0, 1) for i in range(d)]
    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
```

```
    optimized_result = minimize(lambda w: -np.dot(w.T, posterior_return) / (np.
↪sqrt(np.dot(np.dot(w.T, posterior_cov), w))), x0=np.ones(d) / d,␣
↪bounds=bounds, constraints=constraints)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[22]:
```
def black_litterman_rsi(d, M, R, para):
    # Market capitalization weights
    market_cap = np.sum(R, axis=1).reshape(-1, 1)
    market_cap_weights = market_cap / np.sum(market_cap)

    # Prior return estimate
    # Technical indicator-based prior return estimates
    R_df = pd.DataFrame(R.T)
    prior_return = ((R_df > 0).rolling(window=14).sum() / 14).iloc[-1].values.
↪reshape(-1, 1)

    # Covariance matrix
    cov = LedoitWolf().fit(R.T).covariance_

    # Black-Litterman parameters
    tau = 0.05
    delta = 2.5

    # Compute equilibrium excess returns
    market_return = np.dot(market_cap_weights.T, prior_return)
    excess_market_return = market_return - para[2]
    omega = np.diag(np.diag(cov))
    pi = delta * np.dot(omega, market_cap_weights) * excess_market_return / tau

    # Compute posterior estimate of returns
    posterior_cov = np.linalg.inv(np.linalg.inv(tau * omega) + np.dot(np.
↪dot(market_cap_weights.T, np.linalg.inv(cov)), market_cap_weights))
    posterior_return = np.dot(posterior_cov, (np.dot(np.linalg.inv(tau *␣
↪omega), pi) + np.dot(np.dot(np.linalg.inv(cov), market_cap_weights),␣
↪market_return)))

    # Compute optimal portfolio weights
    bounds = [(0, 1) for i in range(d)]
    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
    optimized_result = minimize(lambda w: -np.dot(w.T, posterior_return) / (np.
↪sqrt(np.dot(np.dot(w.T, posterior_cov), w))), x0=np.ones(d) / d,␣
↪bounds=bounds, constraints=constraints)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[23]:
```
def black_litterman_macd(d, M, R, para):
    # Market capitalization weights
    market_cap = np.sum(R, axis=1).reshape(-1, 1)
    market_cap_weights = market_cap / np.sum(market_cap)

    # Prior return estimate
    # Technical indicator-based prior return estimates
    R_df = pd.DataFrame(R.T)
    prior_return = (R_df.ewm(span=12, adjust=False).mean() - R_df.ewm(span=26,␣
 ↪adjust=False).mean()).iloc[-1].values.reshape(-1, 1)

    # Covariance matrix
    cov = LedoitWolf().fit(R.T).covariance_

    # Black-Litterman parameters
    tau = 0.05
    delta = 2.5

    # Compute equilibrium excess returns
    market_return = np.dot(market_cap_weights.T, prior_return)
    excess_market_return = market_return - para[2]
    omega = np.diag(np.diag(cov))
    pi = delta * np.dot(omega, market_cap_weights) * excess_market_return / tau

    # Compute posterior estimate of returns
    posterior_cov = np.linalg.inv(np.linalg.inv(tau * omega) + np.dot(np.
 ↪dot(market_cap_weights.T, np.linalg.inv(cov)), market_cap_weights))
    posterior_return = np.dot(posterior_cov, (np.dot(np.linalg.inv(tau *␣
 ↪omega), pi) + np.dot(np.dot(np.linalg.inv(cov), market_cap_weights),␣
 ↪market_return)))

    # Compute optimal portfolio weights
    bounds = [(0, 1) for i in range(d)]
    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
    optimized_result = minimize(lambda w: -np.dot(w.T, posterior_return) / (np.
 ↪sqrt(np.dot(np.dot(w.T, posterior_cov), w))), x0=np.ones(d) / d,␣
 ↪bounds=bounds, constraints=constraints)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[24]:
```
# totally random case
def black_litterman(d, M, R, para):
    # Market capitalization weights
    market_cap = np.random.rand(d, 1)
    market_cap_weights = market_cap / np.sum(market_cap)
```

```
    # Prior return estimate and covariance matrix
    prior_return = np.random.uniform(-1, 1, size = (d, 1))
    #prior_return = R.mean(axis=1).reshape(d, 1)

    cov = LedoitWolf().fit(R.T).covariance_

    # Black-Litterman parameters
    tau = 0.05
    delta = 2.5

    # Compute equilibrium excess returns
    market_return = np.dot(market_cap_weights.T, prior_return)
    excess_market_return = market_return - para[2]
    omega = np.diag(np.diag(cov))
    pi = delta * np.dot(omega, market_cap_weights) * excess_market_return / tau

    # Compute posterior estimate of returns
    posterior_cov = np.linalg.inv(np.linalg.inv(tau * omega) + np.dot(np.
 ↪dot(market_cap_weights.T, np.linalg.inv(cov)), market_cap_weights))
    posterior_return = np.dot(posterior_cov, (np.dot(np.linalg.inv(tau *␣
 ↪omega), pi) + np.dot(np.dot(np.linalg.inv(cov), market_cap_weights),␣
 ↪market_return)))

    # Compute optimal portfolio weights
    bounds = [(0, 1) for i in range(d)]
    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
    optimized_result = minimize(lambda w: -np.dot(w.T, posterior_return) / (np.
 ↪sqrt(np.dot(np.dot(w.T, posterior_cov), w))), x0=np.ones((d, 1)) / d,␣
 ↪bounds=bounds, constraints=constraints)
    w_t = optimized_result.x.reshape(-1, 1)

    return w_t
```

[162]:
```
def rp_closed(d, M, R, para):
    covariance = np.cov(R)

    #Calculate the volatility of each asset's returns
    volatility = np.sqrt(np.diag(covariance))
#     risk_contribution = np.dot(covariance, 1/volatility) / np.dot(1/
 ↪volatility, 1/volatility)
#     w = risk_contribution / np.sum(risk_contribution)
#     #Normalizing
#     w /= np.sum(w)
    w = np.dot(covariance, 1/volatility) / np.sum(np.dot(covariance, 1/
 ↪volatility))
```

```
    d = R.shape[0]
    w_t = w.reshape(d,1)
    return w_t
```

[26]:
```
def getQuasiDiag(link):
    # Sort clustered items by distance
    link = link.astype(int)
    sortIx = pd.Series([link[-1, 0], link[-1, 1]])
    numItems = link[-1, 3]  # number of original items
    while sortIx.max() >= numItems:
        sortIx.index = range(0, sortIx.shape[0] * 2, 2)  # make space
        df0 = sortIx[sortIx >= numItems]  # find clusters
        i = df0.index
        j = df0.values - numItems
        sortIx[i] = link[j, 0]  # item 1
        df0 = pd.Series(link[j, 1], index=i + 1)
        sortIx = pd.concat([sortIx, df0], axis=0)  # item 2
        sortIx = sortIx.sort_index()  # re-sort
        sortIx.index = range(sortIx.shape[0])  # re-index
    return sortIx.tolist()

def getClusterVar(cov,cItems):
    # Compute variance per cluster
    cov_matrix = cov.loc[cItems,cItems] # matrix slice
    asset_variances = np.diag(cov_matrix)
    # calculate the vector of asset weights
    iv_weights = 1 / asset_variances
    iv_weights /= np.sum(iv_weights)

    w_ = iv_weights.reshape(-1,1)
    clusVar = np.dot(np.dot(w_.T,cov_matrix),w_)[0,0]
    return clusVar

def getRecBipart(cov, sortIx):
    # Compute HRP alloc
    w = pd.Series(1, index=sortIx)
    cItems = [sortIx]  # initialize all items in one cluster
    while len(cItems) > 0:
        cItems = [i[j:k] for i in cItems for j, k in ((0, len(i) // 2), (len(i)␣
 ↪// 2, len(i))) if len(i) > 1]  # bi-section
        for i in range(0, len(cItems), 2):  # parse in pairs
            cItems0 = cItems[i]  # cluster 1
            cItems1 = cItems[i + 1]  # cluster 2
            clusVar0 = getClusterVar(cov, cItems0)
            clusVar1 = getClusterVar(cov, cItems1)
            alpha = 1 - clusVar0 / (clusVar0 + clusVar1)
            w[cItems0] *= alpha  # weight 1
```

```
            w[cItems1] *= 1 - alpha  # weight 2
    return w
def hrp(d, M, R, para):
    d = R.shape[0]

    # Construct a hierarchical portfolio
    returns = pd.DataFrame(R.T)
    cov_matrix = returns.cov()
    corr_matrix = returns.corr()
    distance_matrix = np.sqrt((1 - corr_matrix) / 2)

    # Compute the linkage using ward method
    linkage_matrix = linkage(distance_matrix, method='ward')
    sortIx = getQuasiDiag(linkage_matrix)
    sortIx = corr_matrix.index[sortIx].tolist()
    w_t = getRecBipart(cov_matrix, sortIx).values.reshape(-1,1)
    return w_t
```

[27]:
```
def nco(d, M, R, para):
    # Building the portfolio object
    data = pd.DataFrame(R.T)
    port = rp.HCPortfolio(returns=data)
    kappa, T, rf = para[0], para[1], para[2]

    # Estimate optimal portfolio:

    model='NCO' # Could be HRP, HERC or NCO
    codependence = 'pearson' # Correlation matrix used to group assets in␣
 ↪clusters
    covariance = 'hist' # Covariance estimation technique
    obj = "MinRisk" # Posible values are "MinRisk", "Utility", "Sharpe" and␣
 ↪"ERC"
    rm = 'MV' # Risk measure used, this time will be variance
    # rf = 0 # Risk free rate
    l = 2 # Risk aversion factor, only usefull with "Utility" objective
    linkage = 'ward' # Linkage method used to build clusters
    max_k = 10 # Max number of clusters used in two difference gap statistic
    leaf_order = True # Consider optimal order of leafs in dendrogram

    w = port.optimization(model=model,
                          codependence=codependence,
                          covariance=covariance,
                          obj=obj,
                          rm=rm,
                          rf=rf,
                          l=l,
                          linkage=linkage,
```

```
                        max_k=max_k,
                        leaf_order=leaf_order)

    return w.to_numpy().reshape(d, 1)
```

[28]:
```
def herc(d, M, R, para):
    data = pd.DataFrame(R.T)
    kappa, T, rf = para[0], para[1], para[2]

    # Building the portfolio object
    port = rp.HCPortfolio(returns=data)

    # Estimate optimal portfolio:

    model = 'HERC' # Could be HRP, HERC or NCO
    codependence = 'pearson' # Correlation matrix used to group assets in␣
 ↪clusters
    rm = 'MV' # Risk measure used, this time will be variance
    linkage = 'ward' # Linkage method used to build clusters
    max_k = 10 # Max number of clusters used in two difference gap statistic
    leaf_order = True # Consider optimal order of leafs in dendrogram

    w = port.optimization(model=model,
                        codependence=codependence,
                        rm=rm,
                        rf=rf,
                        linkage=linkage,
                        max_k=max_k,
                        leaf_order=leaf_order)

    return w.to_numpy().reshape(d, 1)
```

[29]:
```
def ewma(d, M, R, para):
    lamda, rf = para[0], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    mu_2 = mu - rf * e
    cov = np.cov(R)
    w_prev = np.ones((d,1))*(1/d)
    w_t = w_prev
    for i in range(M):
        w_prev = w_t
        w_t = lamda * np.linalg.inv(cov+np.eye(d)*0.00001) @ mu_2 + (1 - lamda)␣
 ↪* w_prev
    return w_t
```

[30]:
```
from sklearn.covariance import LedoitWolf

def ewma_regularized(d, M, R, para):
    lamda, rf = para[0], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    mu_2 = mu - rf * e
    regularized_cov = LedoitWolf().fit(R.T).covariance_
    w_prev = np.ones((d,1))*(1/d)
    w_t = w_prev
    for i in range(M):
        w_prev = w_t
        w_t = lamda * np.linalg.inv(regularized_cov) @ mu_2 + (1 - lamda) *␣
 ↪w_prev
    return w_t
```

[31]:
```
def onef(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = np.array([np.mean(R, axis=1)]).T
    mu_2 = mu - rf * e
    B_hat = np.zeros((d, 1))
    res = np.zeros((d, M))
    X = sm.add_constant(R[0] - rf)
    for j in range(d):
        Y = R[j] - rf
        model = sm.OLS(Y, X).fit()
        B_hat[j, 0] = model.params[1]
        res[j, :] = model.resid
    res[0, :] = 0
    cov_R = np.var(R[0])
    sig = np.cov(res)
    cov = B_hat @ B_hat.T * cov_R + sig
    w_t = (kappa - rf)/(mu_2.T @ np.linalg.inv(cov) @ mu_2) * np.linalg.
 ↪inv(cov) @ mu_2
    return w_t
```

[32]:
```
def bs(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    cov = np.cov(R)
    mu_0 = (mu.T @ np.linalg.inv(cov) @ e) / (e.T @ np.linalg.inv(cov) @ e) * e
    a = (d + 2) / (d + 2 + (M - d - 2) * (mu - mu_0).T @ np.linalg.inv(cov) @␣
 ↪(mu - mu_0))
    mu_1 = (1 - a) * (mu - mu_0) + mu_0
    mu_2 = mu_1 - rf * e
```

```
    w_t = (kappa - rf)/(mu_2.T @ np.linalg.inv(cov) @ mu_2) * np.linalg.
↪inv(cov) @ mu_2
    return w_t
```

[33]:
```
def lw(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    mu_2 = mu - rf * e
    Ledoit = LedoitWolf().fit(R.T)
    cov = Ledoit.covariance_
    w_t = (kappa - rf)/(mu_2.T @ np.linalg.inv(cov) @ mu_2) * np.linalg.
↪inv(cov) @ mu_2
    return w_t
```

[34]:
```
def min(d, M, R, para):
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    cov = np.cov(R)
    w_t = 1/(e.T @ np.linalg.inv(cov) @ e) * np.linalg.inv(cov) @ e
    return w_t
```

[35]:
```
 def mv_c(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1)
    cov = np.cov(R)
    def objective(w_t, cov):
        return w_t.T @ cov @ w_t

    result = minimize(objective, np.ones(d) / d, args = (cov,),
        constraints=[LinearConstraint(mu-rf, kappa-rf, np.
↪inf),LinearConstraint(np.ones(d), 1, 1)], bounds=Bounds(np.zeros(d), np.inf))
    w_t = result.x.reshape(d,1)
    return w_t
```

[36]:
```
def bs_c(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    cov = np.cov(R)
    mu_0 = (mu.T @ np.linalg.inv(cov) @ e) / (e.T @ np.linalg.inv(cov) @ e) * e
    a = (d + 2) / (d + 2 + (M - d - 2) * (mu - mu_0).T @ np.linalg.inv(cov) @␣
↪(mu - mu_0))
    mu_1 = (1 - a) * (mu - mu_0) + mu_0
    def objective(w_t, cov):
        return w_t.T @ cov @ w_t
```

```
    result = minimize(objective, np.ones(d) / d, args = (cov,),
        constraints=[LinearConstraint(mu_1.reshape(d) - rf, kappa-rf, np.
↪inf),LinearConstraint(np.ones(d), 1, 1)], bounds=Bounds(np.zeros(d), np.inf))
    w_t = result.x.reshape(d,1)
    return w_t
```

[37]:
```
def lw_c(d, M, R, para):
    kappa, T, rf = para[0], para[1], para[2]
    e = np.ones((d, 1))
    mu = R.mean(axis = 1)
    mu_2 = mu - rf
    Ledoit = LedoitWolf().fit(R.T)
    cov = Ledoit.covariance_
    def objective(w_t, cov):
        return w_t.T @ cov @ w_t
    result = minimize(objective, np.ones(d) / d, args = cov,
        constraints=[LinearConstraint(mu_2, kappa-rf, np.
↪inf),LinearConstraint(np.ones(d), 1, 1)], bounds=Bounds(np.zeros(d), np.inf))
    w_t = result.x.reshape(d,1)
    return w_t
```

[38]:
```
def min_c(d, M, R, para):
    e = np.ones((d, 1))
    mu = R.mean(axis = 1).reshape(-1,1)
    cov = np.cov(R)
    def objective(w_t, cov):
        return w_t.T @ cov @ w_t
    result = minimize(objective, np.ones(d) / d, args=cov,
        constraints=[LinearConstraint(e.reshape(d), 1.0, 1.0)],␣
↪bounds=Bounds(np.zeros(d), np.ones(d)))
    w_t = result.x.reshape(d,1)
    return w_t
```

[39]:
```
def kalman_filter_single_asset(R):
    A = np.array([[1, 1], [0, 1]])
    H = np.array([[1, 0]])
    Q = np.array([[0.1, 0], [0, 0.1]])
    R_matrix = np.array([[1]])
    x = np.array([[0], [0]])
    P = np.array([[1, 0], [0, 1]])
    filtered_returns = np.zeros(R.shape[0])
    for j in range(R.shape[0]):
        y = R[j]
        x = np.dot(A, x)
        P = np.dot(np.dot(A, P), A.T) + Q
        K = np.dot(np.dot(P, H.T), np.linalg.inv(np.dot(np.dot(H, P), H.T) +␣
↪R_matrix))
```

```
        x = x + np.dot(K, (y - np.dot(H, x)))
        P = np.dot((np.eye(2) - np.dot(K, H)), P)
        filtered_returns[j] = x[0]
    return filtered_returns

def calculate_filtered_returns(R):
    d = R.shape[0]
    filtered_returns = np.zeros(R.shape)
    for i in range(d):
        filtered_returns[i] = kalman_filter_single_asset(R[i])
    return filtered_returns

def kalman_filter_allocation(d, M, R, para):
    w_t = mv(d, M, R, para)
    return w_t
```

1.3 1.4 Performance Metrics

1.3.1 1.4.1 Out-of-sample average return (ORtn): $\hat{\mu}_{OOS}$

[40]:
```
def ORtn(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w
    return np.mean(np.diag(X_t))
```

1.3.2 1.4.2 Out-of-sample Sharpe ratio OSR = $\frac{\hat{\mu}_{OOS}-r}{\hat{\sigma}_{OOS}}$

[41]:
```
def OSR(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w
    return (np.mean(np.diag(X_t)) - rf)/np.std(np.diag(X_t))
```

1.3.3 1.4.3 Out-of-sample Sortino ratio OSSortino

[42]:
```
def OSSortino(w, d, R, M, rf, target=0):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w

    # Calculate downside deviations
    downside_diff = np.minimum(X_t - target, 0)
    downside_std = np.sqrt(np.mean(np.diag(downside_diff)**2))

    # Calculate Sortino ratio
    sortino_ratio = (np.mean(np.diag(X_t)) - rf) / downside_std

    return sortino_ratio
```

1.3.4 1.4.4 Out-of-sample Max Drawdown

[43]:
```python
def OMaxDrawdown(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w

    # Calculate the cumulative returns
    cum_returns = np.cumprod(1 + np.diag(X_t))

    # Calculate the running maximum
    running_max = np.maximum.accumulate(cum_returns)

    # Calculate drawdowns and find the maximum drawdown
    drawdowns = 1 - cum_returns / running_max
    max_drawdown = np.max(drawdowns)

    return max_drawdown
```

1.3.5 1.4.4 Out-of-sample Calmar Ratio

[44]:
```python
def OCalmarRatio(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w

    # Calculate the average annual return
    avg_annual_return = np.mean(np.diag(X_t)) * 252

    # Calculate the cumulative returns
    cum_returns = np.cumprod(1 + np.diag(X_t))

    # Calculate the running maximum
    running_max = np.maximum.accumulate(cum_returns)

    # Calculate drawdowns and find the maximum drawdown
    drawdowns = 1 - cum_returns / running_max
    max_drawdown = np.max(drawdowns)

    # Calculate the Calmar Ratio
    calmar_ratio = avg_annual_return / max_drawdown

    return calmar_ratio
```

1.3.6 1.4.5 Out-of-sample Average Turnover Ratio

[45]:
```
def OTurnoverRatio(w, d, R, M, rf, rebalancing_frequency=1):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w

    # Calculate the number of periods
    n_periods = X_t.shape[1]

    # Calculate the number of rebalancing events
    n_rebalancing = int(n_periods / rebalancing_frequency)

    # Select the portfolio weights at the start of each rebalancing period
    start_indices = np.arange(0, n_periods - rebalancing_frequency,␣
 ↪rebalancing_frequency)
    end_indices = start_indices + rebalancing_frequency
    w_start = w[:, start_indices]
    w_end = w[:, end_indices]

    # Calculate the turnover for each rebalancing period
    turnovers = np.sum(np.abs(w_end - w_start), axis=0)

    # Calculate the average turnover ratio
    avg_turnover_ratio = np.mean(turnovers)

    return avg_turnover_ratio
```

1.4 1.5 Performance

[64]:
```
def get_weights(d, M, R, method, para):
    return eval(method)(d, M, R, para)

def performance(M, R, kappa, rf):
    methods_name = [ 'ew', 'mkt', \
                     "inverse_volatility_allocation",\
                    ␣
 ↪'black_litterman','black_litterman_macd','black_litterman_rsi',\
                     'rp_closed','risk_parity_optimization',\
                     'herc'
                    ]
#     methods_name = [ 'ew', 'mkt',␣
 ↪"inverse_volatility_allocation','black_litterman']
#     methods_name = [ 'ew', 'mkt',␣
 ↪'inverse_volatility_allocation','equal_risk_contribution','global_minimum_variance_portfoli
#     'kalman_filter_allocation']
#     methods_name = ['ew', 'mkt', 'min_c', 'risk_parity_optimization']
```

```
#     methods_name = ['theory', 'ew', 'mkt', 'mv', 'onef', 'bs', 'lw', 'min',␣
 ↪'mv_c', 'bs_c', 'lw_c', 'min_c']
#     kappa, T, rf, exp = para[0], para[1], para[2], para[3]
#     d, M, R, B, sigma, alpha = generate_data(exp)
    d, T = R.shape
#     R_new = calculate_filtered_returns(R)
#   21, 63, 126, 252

    para = [kappa, T, rf]
    results_df = pd.DataFrame(columns=['method', 'ortn', 'osr', 'ossortino',␣
 ↪'omaxdrawdown','ocalmarratio', 'oturnoverratio'])
    for method in methods_name:
        w = np.zeros((d, T - M))
        for i in range(0, T - M):
            R_M = R[:, i:i+M]
            if method == 'kalman_filter_allocation':
#                 print(1)
                w_t = kalman_filter_allocation(d, M, R_M[:, i:i+M], para)
#                 print(w_t)
            else:
                w_t = get_weights(d, M, R_M, method, para)
            w[:,[i]] = w_t
        ortn = ORtn(w, d, R, M, rf)
        osr = OSR(w, d, R, M, rf)
        ossortino = OSSortino(w, d, R, M, rf, kappa)
        omaxdrawdown = OMaxDrawdown(w, d, R, M, rf)
        ocalmarratio = OCalmarRatio(w, d, R, M, rf)
        oturnoverratio = OTurnoverRatio(w, d, R, M, rf)
        results_df = results_df.append({'method': method, \
                                        'ortn': ortn, \
                                        'osr': osr, \
                                        'ossortino': ossortino, \
                                        'omaxdrawdown': omaxdrawdown, \
                                        'ocalmarratio':ocalmarratio,\
                                        'oturnoverratio':oturnoverratio},␣
 ↪ignore_index=True)
        print(method, ortn, osr)
    display(results_df)
    return results_df
```

1.4.1 M = 21

[68]:
```
M = 21
kappa = 0.3/252
rf = 0.002/21
perf_tech_21 = performance(M, Tech_Returns, kappa, rf)
perf_heal_21 = performance(M, Healthcare_Returns, kappa, rf)
```

```
perf_util_21 = performance(M, Utilities_Returns, kappa, rf)
```

```
ew 0.0008651412824273943 0.05175583470479078
mkt 0.0007938890556263797 0.04744583019650232
inverse_volatility_allocation 0.000836894425268883 0.05302218711254829
black_litterman 0.0009944975374804332 0.05932558738059014
black_litterman_macd 0.0008303495329624431 0.04742116613203672
black_litterman_rsi 0.0009454374906947802 0.05515767533213805
rp_closed 0.0009039284936671028 0.04976456588948612
risk_parity_optimization 0.0008143461387101554 0.05274975353722342
herc 0.0007976258312863582 0.05324120440776111
```

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000865	0.051756	0.069463	0.357168	0.610401	0.000000
1	mkt	0.000794	0.047446	0.063896	0.394362	0.507301	0.000000
2	inverse_volatility_allocation	0.000837	0.053022	0.070843	0.350750	0.601276	0.039951
3	black_litterman	0.000994	0.059326	0.080516	0.341271	0.734353	1.383059
4	black_litterman_macd	0.000830	0.047421	0.064176	0.398216	0.525463	0.923643
5	black_litterman_rsi	0.000945	0.055158	0.074694	0.400571	0.594777	0.376269
6	rp_closed	0.000904	0.049765	0.067129	0.368949	0.617402	0.063595
7	risk_parity_optimization	0.000814	0.052750	0.070386	0.349029	0.587960	0.096411
8	herc	0.000798	0.053241	0.071752	0.328508	0.611862	0.867195

```
ew 0.0008114385415213052 0.0581135704084695
mkt 0.0005232460225117174 0.039279090350386264
inverse_volatility_allocation 0.0007460775436861689 0.05740773964877158
black_litterman 0.0008219328836775652 0.05468087582316922
black_litterman_macd 0.000908884364689351 0.05925222993927023
black_litterman_rsi 0.0008478447863525311 0.054717158728281035
rp_closed 0.0008078575513958066 0.05073397066797038
risk_parity_optimization 0.000781108071794572 0.06090793888776373
herc 0.0007433322767346053 0.05714162252135802
```

	method	ortn	osr	ossortino	omaxdrawdown
0	ew	0.000811	0.058114	0.077598	0.305917
1	mkt	0.000523	0.039279	0.052000	0.284031
2	inverse_volatility_allocation	0.000746	0.057408	0.076077	0.304335
3	black_litterman	0.000822	0.054681	0.073449	0.313962

```
4           black_litterman_macd  0.000909  0.059252   0.081390      0.257839
5            black_litterman_rsi  0.000848  0.054717   0.074724      0.309590
6                      rp_closed  0.000808  0.050734   0.068376      0.326151
7       risk_parity_optimization  0.000781  0.060908   0.081032      0.300712
8                           herc  0.000743  0.057142   0.076086      0.295888

   ocalmarratio  oturnoverratio
0      0.668424        0.000000
1      0.464238        0.000000
2      0.617779        0.041360
3      0.659721        1.397354
4      0.888301        0.875920
5      0.690129        0.363478
6      0.624190        0.080565
7      0.654577        0.115425
8      0.633076        0.923070
```

```
ew 0.0004761867940231751 0.032809456050597305
mkt 0.0004261265197495062 0.028203802089015878
inverse_volatility_allocation 0.0004671893100814236 0.032402180887596894
black_litterman 0.0004801566286648383 7 0.03321125639788524
black_litterman_macd 0.0005195425716949832 0.03377545666653067
black_litterman_rsi 0.0004072340561232192 7 0.025729247361923613
rp_closed 0.0005501697983903442 0.037426370900780506
risk_parity_optimization 0.0004364242292278452 4 0.030084186273240464
herc 0.0004599811542755385 6 0.03187554572143271
```

```
                         method      ortn       osr  ossortino  omaxdrawdown  \
0                            ew  0.000476  0.032809   0.043370      0.368557
1                           mkt  0.000426  0.028204   0.037261      0.361798
2  inverse_volatility_allocation  0.000467  0.032402   0.042823      0.368905
3                black_litterman  0.000480  0.033211   0.043777      0.347748
4           black_litterman_macd  0.000520  0.033775   0.044520      0.428670
5            black_litterman_rsi  0.000407  0.025729   0.033745      0.381376
6                      rp_closed  0.000550  0.037426   0.049922      0.361634
7       risk_parity_optimization  0.000436  0.030084   0.039672      0.376374
8                           herc  0.000460  0.031876   0.041882      0.387633

   ocalmarratio  oturnoverratio
0      0.325592        0.000000
1      0.296806        0.000000
2      0.319138        0.030904
3      0.347951        1.366300
4      0.305421        0.965603
5      0.269086        0.441675
6      0.383379        0.048134
7      0.292207        0.063387
8      0.299033        0.667098
```

1.4.2 M = 63

[70]:
```
M = 63
kappa = 0.3/252
rf = 0.002/21
perf_tech_63 = performance(M, Tech_Returns, kappa, rf)
perf_heal_63 = performance(M, Healthcare_Returns, kappa, rf)
perf_util_63 = performance(M, Utilities_Returns, kappa, rf)
```

```
ew 0.0008567618235051605 0.05088692907106489
mkt 0.0007907047366075001 0.046931272312568925
inverse_volatility_allocation 0.0008228119691783035 0.051536600037495814
black_litterman 0.0008972528460893695 0.05357977337018679
black_litterman_macd 0.0006924948113837744 0.03964121208215481
black_litterman_rsi 0.001002773086141216 0.05779143154333199
rp_closed 0.0008936698541823231 0.04911062044352832
risk_parity_optimization 0.0008309019396292729 0.05368650949655725
herc 0.0009074034137372522 0.059068737951763414
```

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000857	0.050887	0.068297	0.357168	0.604489	0.000000
1	mkt	0.000791	0.046931	0.063230	0.394362	0.505266	0.000000
2	inverse_volatility_allocation	0.000823	0.051537	0.068834	0.348953	0.594203	0.012566
3	black_litterman	0.000897	0.053580	0.071981	0.365629	0.618408	1.375248
4	black_litterman_macd	0.000692	0.039641	0.052731	0.372951	0.467913	0.593628
5	black_litterman_rsi	0.001003	0.057791	0.079044	0.407318	0.620397	0.223502
6	rp_closed	0.000894	0.049111	0.066274	0.369283	0.609843	0.018804
7	risk_parity_optimization	0.000831	0.053687	0.071646	0.348199	0.601345	0.022562
8	herc	0.000907	0.059069	0.079635	0.337728	0.677070	0.709484

```
ew 0.000799720702398694 0.056863494554335356
mkt 0.0005212946232112064 0.038883339647912324
inverse_volatility_allocation 0.0007340553613042572 0.05591025434590352
black_litterman 0.0007282633426219657 0.0518049257235896
black_litterman_macd 0.0009940507428897888 0.06832135337017604
black_litterman_rsi 0.0009356331031011949 0.06184176129805461
rp_closed 0.0008337025589605048 0.052916250112329165
risk_parity_optimization 0.0007624895962649516 0.05893709460312157
```

herc 0.0006504113589377297 0.046086062958617564

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000800	0.056863	0.075908	0.305917	0.658772	0.000000
1	mkt	0.000521	0.038883	0.051495	0.284031	0.462506	0.000000
2	inverse_volatility_allocation	0.000734	0.055910	0.074045	0.306405	0.603717	0.013209
3	black_litterman	0.000728	0.051805	0.069105	0.313513	0.585374	1.380424
4	black_litterman_macd	0.000994	0.068321	0.093558	0.265533	0.943390	0.545037
5	black_litterman_rsi	0.000936	0.061842	0.084758	0.288555	0.817103	0.222355
6	rp_closed	0.000834	0.052916	0.071346	0.326906	0.642670	0.024548
7	risk_parity_optimization	0.000762	0.058937	0.078181	0.287254	0.668911	0.026372
8	herc	0.000650	0.046086	0.060072	0.361545	0.453343	0.758860

ew 0.00048167353093291873 0.03312009418888224
mkt 0.0004351151235592921 0.028828983342894427
inverse_volatility_allocation 0.00047819027051131796 0.03312834286587052
black_litterman 0.0005449236701293961 0.03811683575271699
black_litterman_macd 0.000516446646270942 0.034028320259497626
black_litterman_rsi 0.0005636762240594835 0.038450420625107624
rp_closed 0.0005614732570416985 0.038944276594725026
risk_parity_optimization 0.0004439466500782008 0.03052588494960483
herc 0.0004686158980289957 0.03268060672164317

	method	ortn	osr	ossortino	omaxdrawdown	ocalmarratio	oturnoverratio
0	ew	0.000482	0.033120	0.043786	0.368557	0.329344	0.000000
1	mkt	0.000435	0.028829	0.038100	0.361798	0.303067	0.000000
2	inverse_volatility_allocation	0.000478	0.033128	0.043824	0.366452	0.328840	0.009939
3	black_litterman	0.000545	0.038117	0.051091	0.335204		
4	black_litterman_macd	0.000516	0.034028	0.045383	0.424491		
5	black_litterman_rsi	0.000564	0.038450	0.052294	0.319099		
6	rp_closed	0.000561	0.038944	0.051770	0.364111		
7	risk_parity_optimization	0.000444	0.030526	0.040339	0.372556		
8	herc	0.000469	0.032681	0.043256	0.343670		

```
3      0.409663        1.357051
4      0.306590        0.583333
5      0.445148        0.269033
6      0.388593        0.014915
7      0.300289        0.016915
8      0.343618        0.402566
```

1.4.3 M = 126

[71]:
```
M = 126
kappa = 0.3/252
rf = 0.002/21
perf_tech_126 = performance(M, Tech_Returns, kappa, rf)
perf_heal_126 = performance(M, Healthcare_Returns, kappa, rf)
perf_util_126 = performance(M, Utilities_Returns, kappa, rf)
```

```
ew 0.0008304443261876551 0.048682848575233896
mkt 0.0007741525741897432 0.04535369529264044
inverse_volatility_allocation 0.000789100665596077 0.048613074516168885
black_litterman 0.000812186014241688 0.04787967275912856
black_litterman_macd 0.0007594314559538519 0.04385840733850396
black_litterman_rsi 0.0008977844936171977 0.05135743273401341
rp_closed 0.0008782358139451605 0.04796871138664548
risk_parity_optimization 0.0007906787400949837 0.05001227471297807
herc 0.00092222613986985 0.05805577831210956
```

```
                         method      ortn       osr  ossortino  omaxdrawdown  \
0                            ew  0.000830  0.048683   0.065298      0.357168
1                           mkt  0.000774  0.045354   0.061072      0.394362
2  inverse_volatility_allocation  0.000789  0.048613   0.064877      0.347572
3                black_litterman  0.000812  0.047880   0.064358      0.341410
4           black_litterman_macd  0.000759  0.043858   0.058513      0.358539
5            black_litterman_rsi  0.000898  0.051357   0.069477      0.375041
6                      rp_closed  0.000878  0.047969   0.064709      0.366161
7       risk_parity_optimization  0.000791  0.050012   0.066667      0.347893
8                           herc  0.000922  0.058056   0.079345      0.303439

   ocalmarratio  oturnoverratio
0      0.585920        0.000000
1      0.494689        0.000000
2      0.572121        0.006231
3      0.599487        1.375954
4      0.533768        0.547068
5      0.603246        0.167266
6      0.604421        0.009058
7      0.572736        0.010146
8      0.765891        0.580029
```

```
ew 0.0007672483312009941 0.053838585445144964
```

mkt 0.00048394050273789497 0.035214280006448384
inverse_volatility_allocation 0.0006963686669214094 0.0520960698464392
black_litterman 0.0007213262239146956 0.051584134933553544
black_litterman_macd 0.000933819314547285 0.06453158578062747
black_litterman_rsi 0.0008498582289991166 0.05614062305295936
rp_closed 0.0008093480658153225 0.051209404810299716
risk_parity_optimization 0.0007291138000814669 0.05524862632103067
herc 0.000695620841555337 0.04918517398645623

	method	ortn	osr	ossortino	omaxdrawdown	\
0	ew	0.000767	0.053839	0.071759	0.305917	
1	mkt	0.000484	0.035214	0.046543	0.284031	
2	inverse_volatility_allocation	0.000696	0.052096	0.068870	0.307100	
3	black_litterman	0.000721	0.051584	0.068684	0.309415	
4	black_litterman_macd	0.000934	0.064532	0.088226	0.299169	
5	black_litterman_rsi	0.000850	0.056141	0.076377	0.281487	
6	rp_closed	0.000809	0.051209	0.068864	0.323793	
7	risk_parity_optimization	0.000729	0.055249	0.073190	0.293583	
8	herc	0.000696	0.049185	0.065355	0.296237	

	ocalmarratio	oturnoverratio
0	0.632023	0.000000
1	0.429365	0.000000
2	0.571426	0.006549
3	0.587477	1.374037
4	0.786588	0.506346
5	0.760832	0.165825
6	0.629895	0.011843
7	0.625843	0.012002
8	0.591743	0.612606

ew 0.00047728414979276307 0.03250048886606312
mkt 0.00042961541105506433 0.028157803580573668
inverse_volatility_allocation 0.00047094897137691934 0.03224713271409721
black_litterman 0.00047252899979166287 0.03187421444855211
black_litterman_macd 0.0005713604274422084 0.03923619580418121
black_litterman_rsi 0.00046299405722022222 0.030943104496707927
rp_closed 0.0005218383081297064 0.035539242933275865
risk_parity_optimization 0.00044329745721633778 0.03021217740502738
herc 0.00045152482374970410 0.03056814590005696

	method	ortn	osr	ossortino	omaxdrawdown	\
0	ew	0.000477	0.032500	0.042933	0.368557	
1	mkt	0.000430	0.028158	0.037177	0.361798	
2	inverse_volatility_allocation	0.000471	0.032247	0.042623	0.365006	
3	black_litterman	0.000473	0.031874	0.041823	0.364724	
4	black_litterman_macd	0.000571	0.039236	0.052461	0.402887	
5	black_litterman_rsi	0.000463	0.030943	0.040664	0.366908	
6	rp_closed	0.000522	0.035539	0.047014	0.363164	

```
7       risk_parity_optimization  0.000443  0.030212   0.039900      0.371745
8                           herc  0.000452  0.030568   0.040165      0.390144

   ocalmarratio  oturnoverratio
0      0.326342        0.000000
1      0.299236        0.000000
2      0.325143        0.005091
3      0.326486        1.350262
4      0.357378        0.534997
5      0.317994        0.201059
6      0.362104        0.007450
7      0.300504        0.008270
8      0.291647        0.224971
```

1.4.4 $\mathbf{M = 252}$

[72]:
```
M = 252
kappa = 0.3/252
rf = 0.002/21
perf_tech_252 = performance(M, Tech_Returns, kappa, rf)
perf_heal_252 = performance(M, Healthcare_Returns, kappa, rf)
perf_util_252 = performance(M, Utilities_Returns, kappa, rf)
```

```
ew 0.0008258174503199993 0.0476865312544112
mkt 0.0007585623077431988 0.043511952845754154
inverse_volatility_allocation 0.0007853733590115216 0.04743698762329803
black_litterman 0.0007617156442571624 0.04443606275922461
black_litterman_macd 0.0007569724371527667 0.043044059812833775
black_litterman_rsi 0.0008779427156403067 0.04838452642622342
rp_closed 0.0008765401818765251 0.04744466168118333
risk_parity_optimization 0.0007804771257922176 0.048318017413895506
herc 0.0007742389279623807 0.04709880812498879
```

```
                         method      ortn       osr  ossortino  omaxdrawdown  \
0                            ew  0.000826  0.047687   0.064003      0.357168
1                           mkt  0.000759  0.043512   0.058616      0.394362
2  inverse_volatility_allocation  0.000785  0.047437   0.063391      0.347599
3                 black_litterman  0.000762  0.044436   0.059873      0.341443
4            black_litterman_macd  0.000757  0.043044   0.057499      0.375956
5             black_litterman_rsi  0.000878  0.048385   0.065418      0.358799
6                       rp_closed  0.000877  0.047445   0.064054      0.366796
7       risk_parity_optimization  0.000780  0.048318   0.064431      0.348464
8                           herc  0.000774  0.047099   0.063029      0.345670

   ocalmarratio  oturnoverratio
0      0.582656        0.000000
1      0.484727        0.000000
2      0.569375        0.003044
```

```
3      0.562181        1.372458
4      0.507392        0.525403
5      0.616618        0.129534
6      0.602209        0.004353
7      0.564420        0.004837
8      0.564436        0.450893
```

```
ew 0.0007580405661401337 0.05271739059179018
mkt 0.00047431191195956705 0.034029977689829335
inverse_volatility_allocation 0.0006858655480399062 0.050453083605004855
black_litterman 0.0007289295314109642 0.05294995007600765
black_litterman_macd 0.0008478902926135107 0.057934986789955796
black_litterman_rsi 0.0007904786130345122 0.051456405968081764
rp_closed 0.0008067494648070272 0.05134414673596429
risk_parity_optimization 0.0007135549812130645 0.052951563972901856
herc 0.000678164121728128 0.04629049218255583
```

```
                         method      ortn       osr  ossortino  omaxdrawdown  \
0                            ew  0.000758  0.052717   0.070275      0.305917
1                           mkt  0.000474  0.034030   0.045008      0.284031
2  inverse_volatility_allocation  0.000686  0.050453   0.066750      0.308441
3                black_litterman  0.000729  0.052950   0.070905      0.307566
4           black_litterman_macd  0.000848  0.057935   0.077931      0.298295
5            black_litterman_rsi  0.000790  0.051456   0.069462      0.286229
6                      rp_closed  0.000807  0.051344   0.068999      0.318081
7       risk_parity_optimization  0.000714  0.052952   0.070182      0.299193
8                           herc  0.000678  0.046290   0.062001      0.350096

   ocalmarratio  oturnoverratio
0      0.624438        0.000000
1      0.420822        0.000000
2      0.560360        0.003196
3      0.597239        1.374450
4      0.716298        0.496800
5      0.695949        0.123531
6      0.639148        0.005692
7      0.601003        0.005669
8      0.488144        0.427091
```

```
ew 0.0004581504398037647 0.030314327054643264
mkt 0.000401094599122685 0.025308725511059116
inverse_volatility_allocation 0.0004578044137388481 6 0.0305128007997484
black_litterman 0.0004429761807753383 0.02878908179344527
black_litterman_macd 0.0005468811922492292 0.036430766970837226
black_litterman_rsi 0.0004714574142697991 0.03164861665579703
rp_closed 0.0004894171090073956 0.032310935208476485
risk_parity_optimization 0.0004374192544434625 4 0.029093596664197807
herc 0.0004508437612413579 6 0.030120142416548815
```

```
                         method      ortn       osr  ossortino  omaxdrawdown  \
```

0	ew	0.000458	0.030314	0.040047	0.368557
1	mkt	0.000401	0.025309	0.033401	0.361798
2	inverse_volatility_allocation	0.000458	0.030513	0.040348	0.363981
3	black_litterman	0.000443	0.028789	0.038408	0.319471
4	black_litterman_macd	0.000547	0.036431	0.048609	0.388289
5	black_litterman_rsi	0.000471	0.031649	0.041688	0.348089
6	rp_closed	0.000489	0.032311	0.042700	0.365086
7	risk_parity_optimization	0.000437	0.029094	0.038450	0.370241
8	herc	0.000451	0.030120	0.039444	0.384457

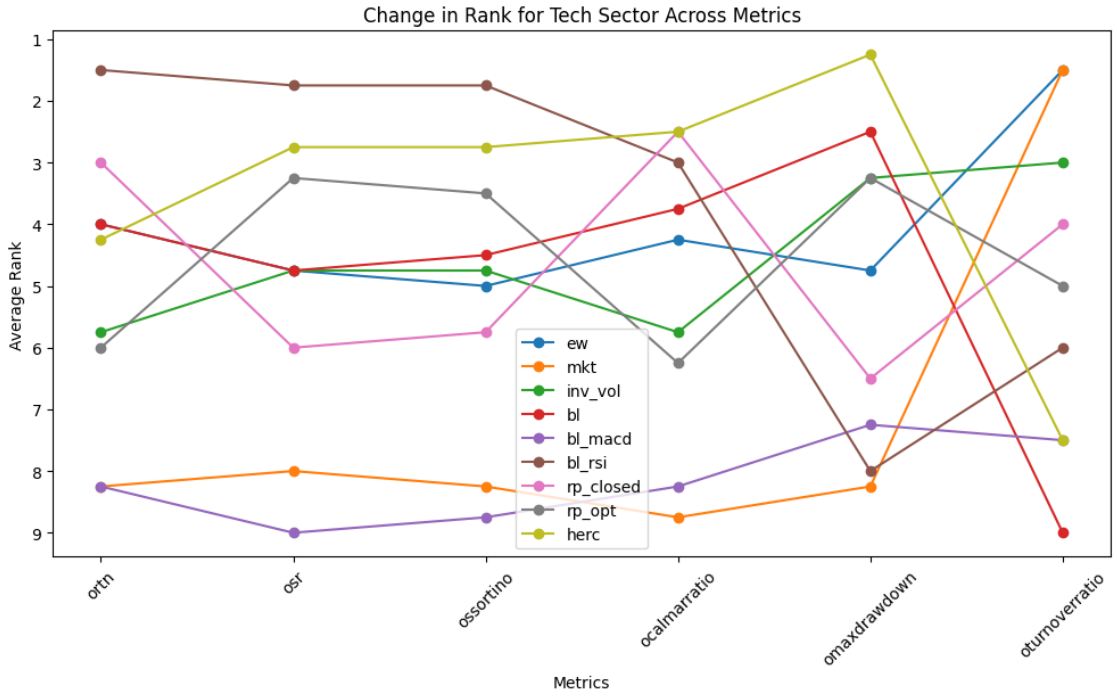
	ocalmarratio	oturnoverratio
0	0.313260	0.000000
1	0.279371	0.000000
2	0.316958	0.002604
3	0.349422	1.343270
4	0.354927	0.491347
5	0.341313	0.142752
6	0.337819	0.003785
7	0.297724	0.004186
8	0.295515	0.115983

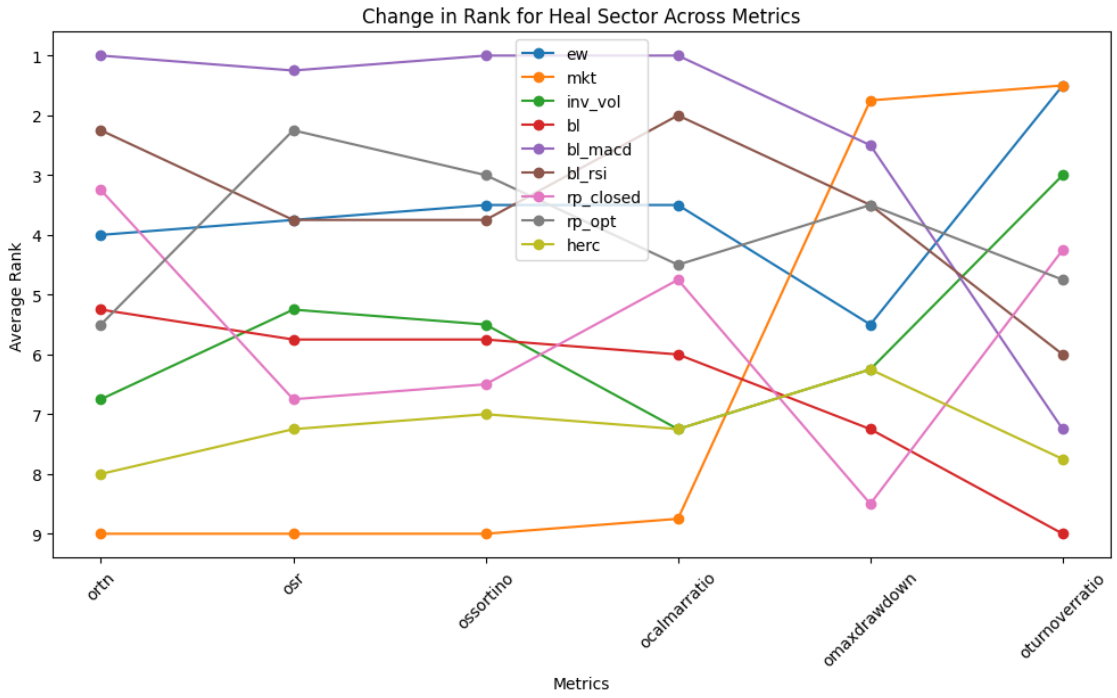
1.5 1.6 Graph Visualization

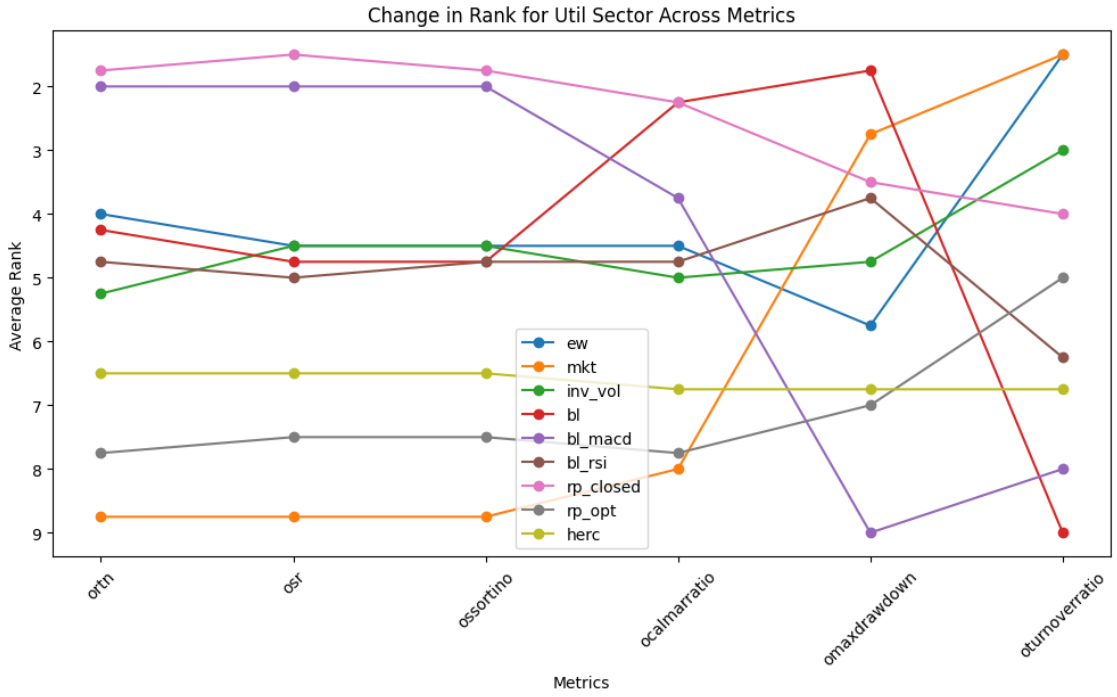
[145]:
```python
def visualize_rank_changes(dataframes, sector, metrics):
    num_dfs = len(dataframes)
    methods = dataframes[0]['method'].values

    avg_ranks = np.zeros((len(methods), len(metrics)))

    for j, metric in enumerate(metrics):
        ranks = np.zeros((num_dfs, len(methods)))

        for i, df in enumerate(dataframes):
            if metric in ['ortn', 'osr', 'ossortino', 'ocalmarratio']:
                ranks[i] = df[metric].rank(ascending=False)
            elif metric in ['omaxdrawdown', 'oturnoverratio']:
                ranks[i] = df[metric].rank(ascending=True)
        avg_ranks[:, j] = ranks.mean(axis=0)

    plt.figure(figsize=(12, 6))
    for i, method in enumerate(methods):
        plt.plot(metrics, avg_ranks[i], marker='o', label=method)
    plt.xlabel("Metrics")
    plt.ylabel(f"Average Rank")
    plt.title(f"Change in Rank for {sector.capitalize()} Sector Across Metrics")
    plt.xticks(rotation=45)
    plt.gca().invert_yaxis()  # Invert y-axis to place 1 at the top
```

```
    plt.legend()
    plt.show()
```

[146]:
```
sectors = ['tech', 'heal', 'util']
metrics = ['ortn', 'osr', 'ossortino', 'ocalmarratio', 'omaxdrawdown',␣
 ↪'oturnoverratio']
dataframes = [perf_tech_21, perf_heal_21, perf_util_21,
              perf_tech_63, perf_heal_63, perf_util_63,
              perf_tech_126, perf_heal_126, perf_util_126,
              perf_tech_252, perf_heal_252, perf_util_252]
copy_dataframes = [df.copy() for df in dataframes]
for df in copy_dataframes:
    df['method'] = df['method'].replace({
        'inverse_volatility_allocation': 'inv_vol',
        'black_litterman': 'bl',
        'black_litterman_macd': 'bl_macd',
        'black_litterman_rsi': 'bl_rsi',
        'risk_parity_optimization': 'rp_opt'
    })
titles = ['Tech 21', 'Health 21', 'Utilities 21',
          'Tech 63', 'Health 63', 'Utilities 63',
          'Tech 126', 'Health 126', 'Utilities 126',
          'Tech 252', 'Health 252', 'Utilities 252']

for sector in sectors:
    sector_dfs = [df for df, title in zip(copy_dataframes, titles) if sector.
 ↪lower() in title.lower()]
    visualize_rank_changes(sector_dfs, sector, metrics)
```


Change in Rank for Tech Sector Across Metrics
Average Rank
Metrics
ortn
osr
ossortino
ocalmarratio
omaxdrawdown
oturnoverratio
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc


Change in Rank for Heal Sector Across Metrics
Average Rank
Metrics
ortn
osr
ossortino
ocalmarratio
omaxdrawdown
oturnoverratio
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc



[154]:
```
def bar_chart_ortn_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.barplot(ax=axes[i], x='method', y='ortn', data=df)
        axes[i].set_title(title)
        axes[i].set_ylabel("Ortn")
        axes[i].set_xlabel("Method")
        axes[i].tick_params(axis='x', rotation=45)

    plt.tight_layout()
    plt.show()
```

[155]:
```
def line_graph_osr_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.lineplot(ax=axes[i], x='method', y='osr', data=df, marker='o')
        axes[i].set_title(title)
        axes[i].set_ylabel("OsR")
        axes[i].set_xlabel("Method")
        axes[i].tick_params(axis='x', rotation=45)
```

```
    plt.tight_layout()
    plt.show()
```

[156]:
```
def scatter_osr_oturnoverratio_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.scatterplot(ax=axes[i], x='osr', y='oturnoverratio', hue='method',␣
↪data=df)
        axes[i].set_title(title)
        axes[i].set_xlabel("OsR")
        axes[i].set_ylabel("Turnover Ratio")
        axes[i].legend(title='Method', bbox_to_anchor=(1.05, 1), loc=2,␣
↪borderaxespad=0.)

    plt.tight_layout()
    plt.show()
```

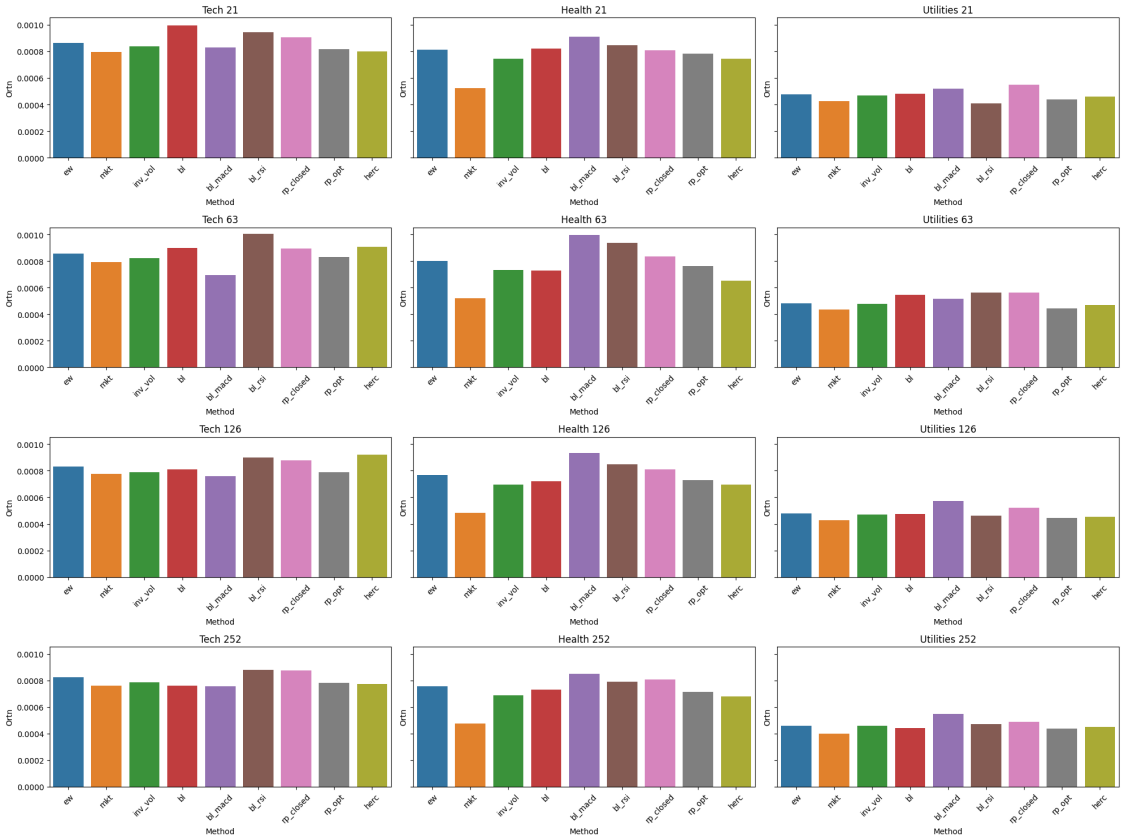
[157]:
```
def scatter_osr_ortn_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.scatterplot(ax=axes[i], x='osr', y='ortn', hue='method', data=df)
        axes[i].set_title(title)
        axes[i].set_xlabel("OsR")
        axes[i].set_ylabel("Ortn")
        axes[i].legend(title='Method', bbox_to_anchor=(1.05, 1), loc=2,␣
↪borderaxespad=0.)

    plt.tight_layout()
    plt.show()
```

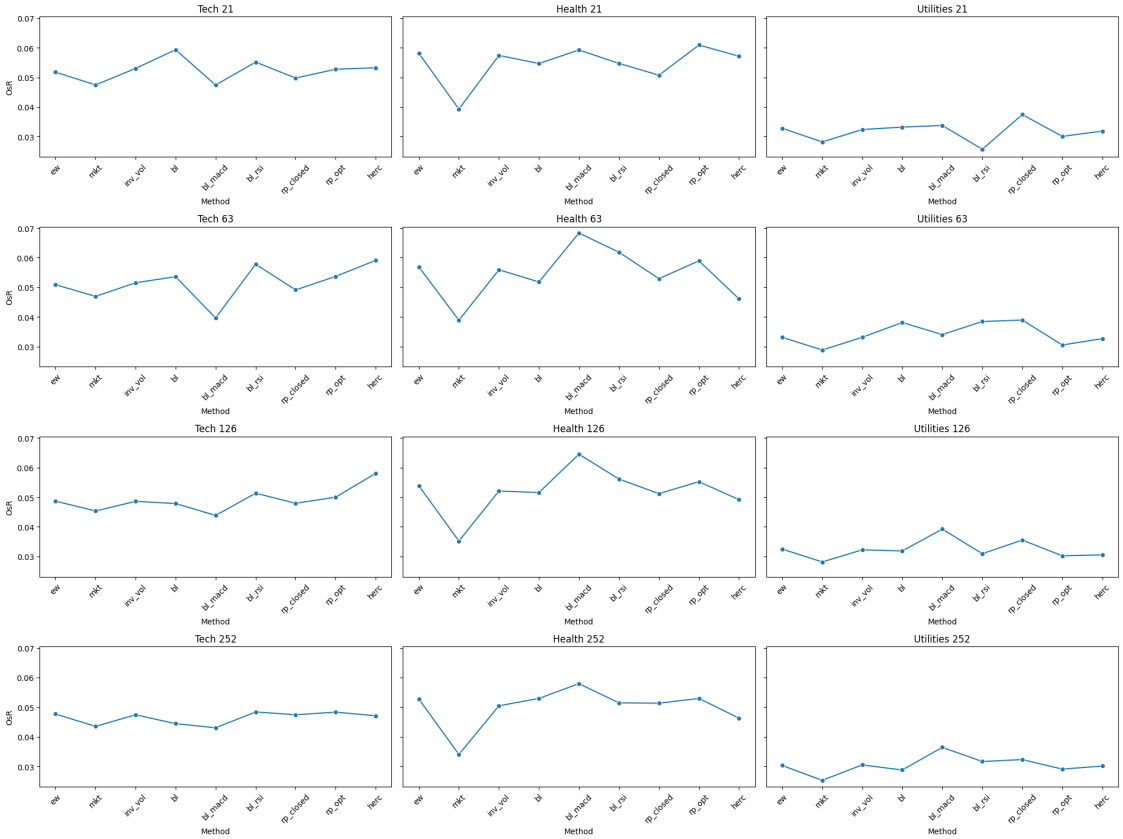
[158]:
```
def scatter_osr_maxdrawdown_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.scatterplot(ax=axes[i], x='osr', y='omaxdrawdown', hue='method',␣
↪data=df)
        axes[i].set_title(title)
        axes[i].set_xlabel("OsR")
        axes[i].set_ylabel("Max Drawdown")
        axes[i].legend(title='Method', bbox_to_anchor=(1.05, 1), loc=2,␣
↪borderaxespad=0.)
```

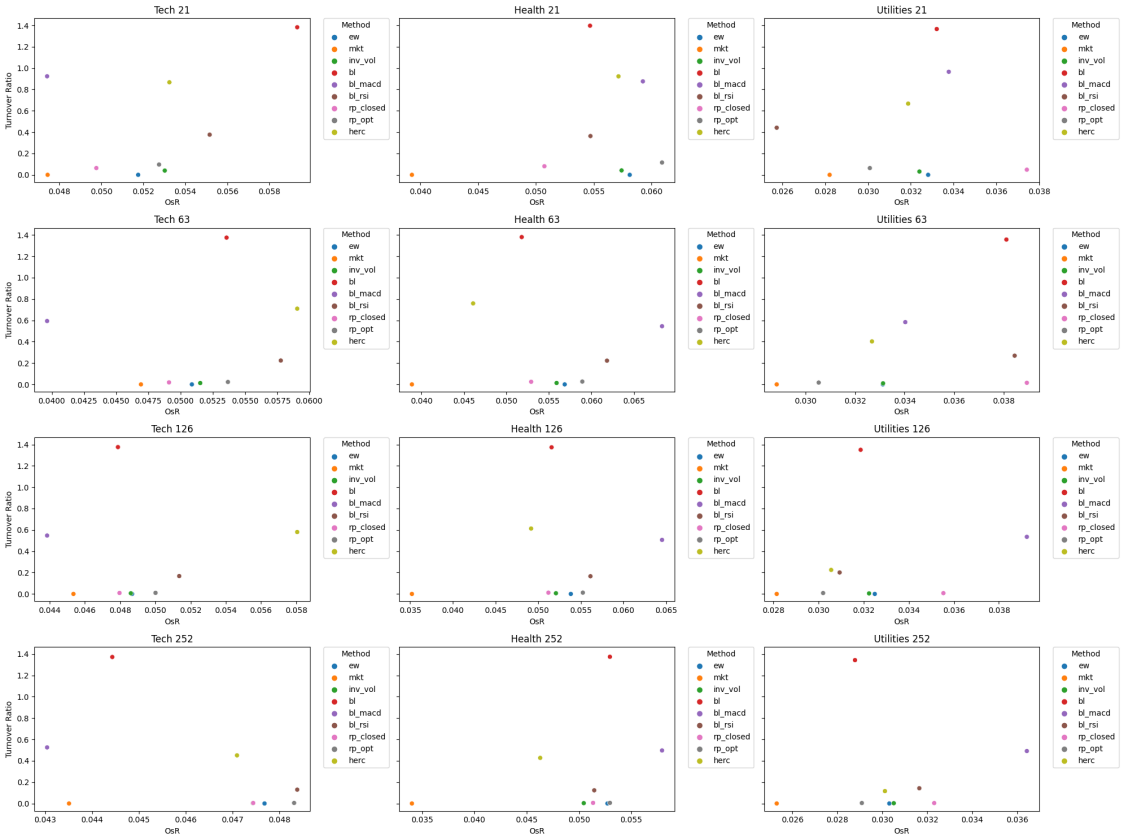
```
    plt.tight_layout()
    plt.show()
```

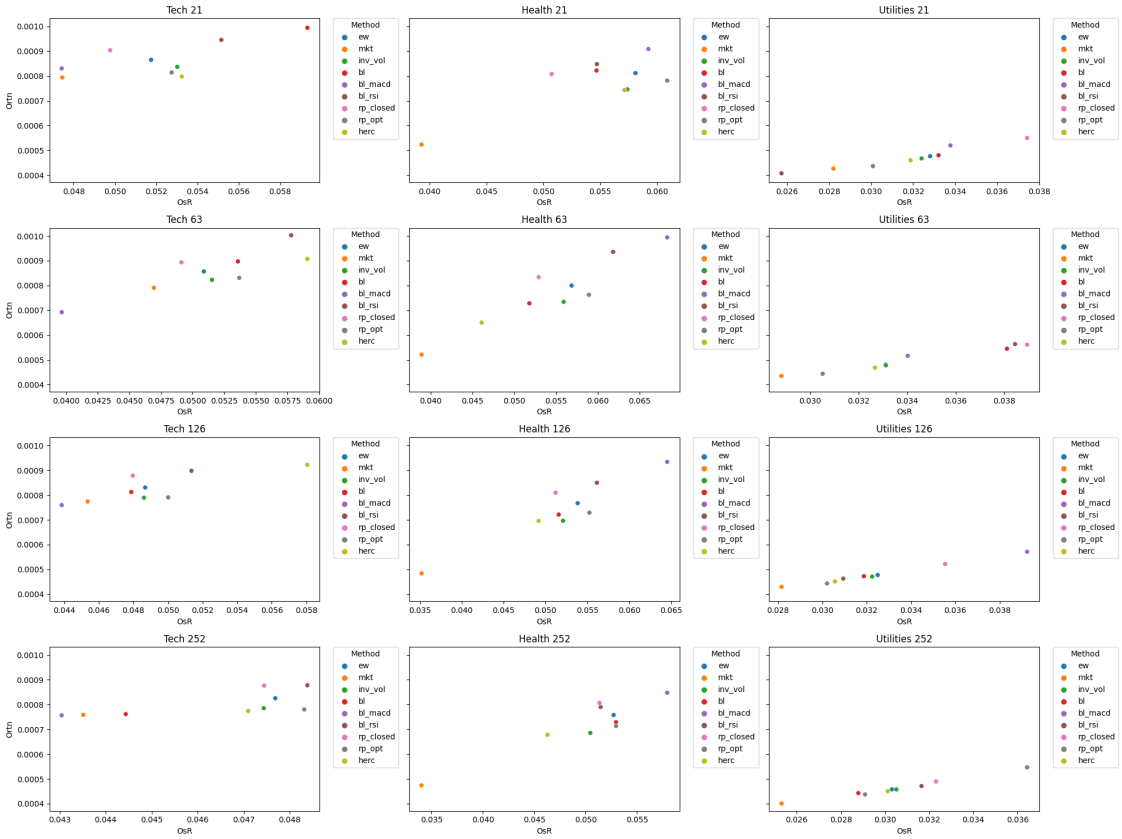
[159]:
```
def scatter_osr_ocalmarratio_multiple_dfs(dfs, titles):
    fig, axes = plt.subplots(nrows=4, ncols=3, figsize=(20, 15), sharey=True)
    axes = axes.ravel()

    for i, (title, df) in enumerate(zip(titles, dfs)):
        sns.scatterplot(ax=axes[i], x='osr', y='ocalmarratio', hue='method',␣
↪data=df)
        axes[i].set_title(title)
        axes[i].set_xlabel("OsR")
        axes[i].set_ylabel("Calm Ratio")
        axes[i].legend(title='Method', bbox_to_anchor=(1.05, 1), loc=2,␣
↪borderaxespad=0.)

    plt.tight_layout()
    plt.show()
```

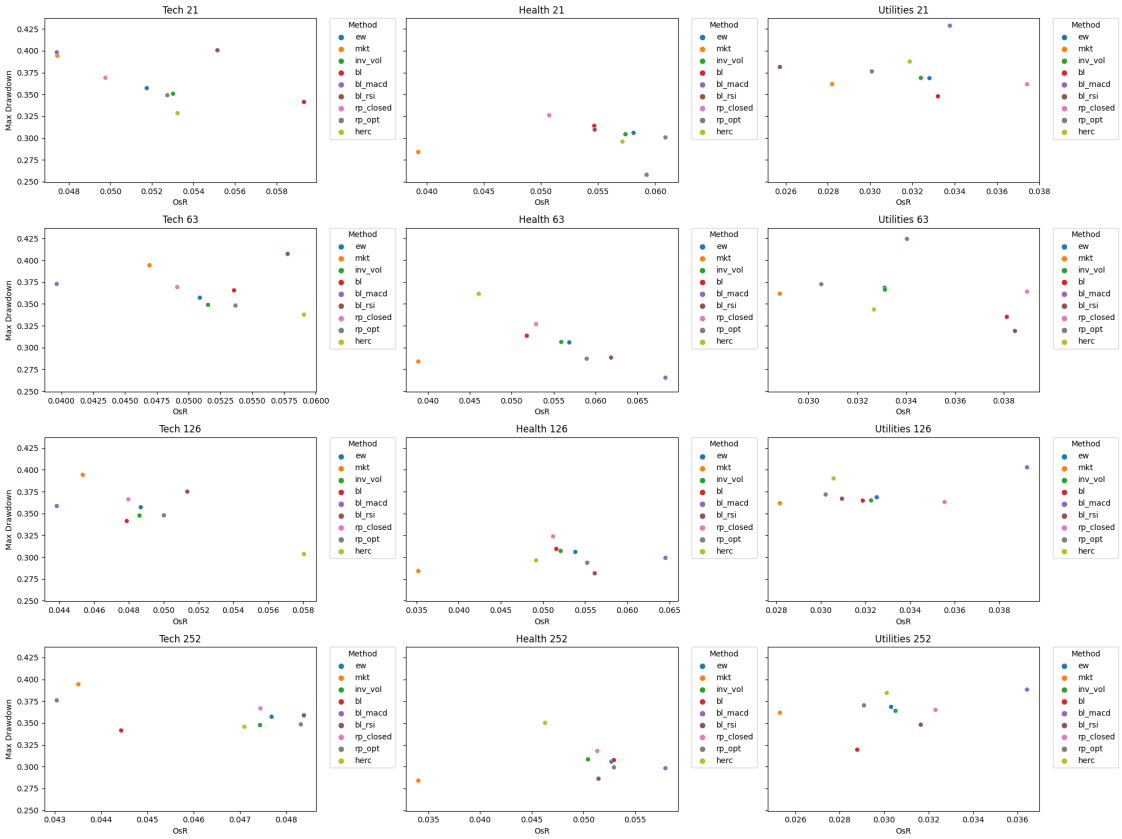
[161]:
```
bar_chart_ortn_multiple_dfs(copy_dataframes, titles)
line_graph_osr_multiple_dfs(copy_dataframes, titles)
scatter_osr_oturnoverratio_multiple_dfs(copy_dataframes, titles)
scatter_osr_ortn_multiple_dfs(copy_dataframes, titles)
scatter_osr_maxdrawdown_multiple_dfs(copy_dataframes, titles)
scatter_osr_ocalmarratio_multiple_dfs(copy_dataframes, titles)
```

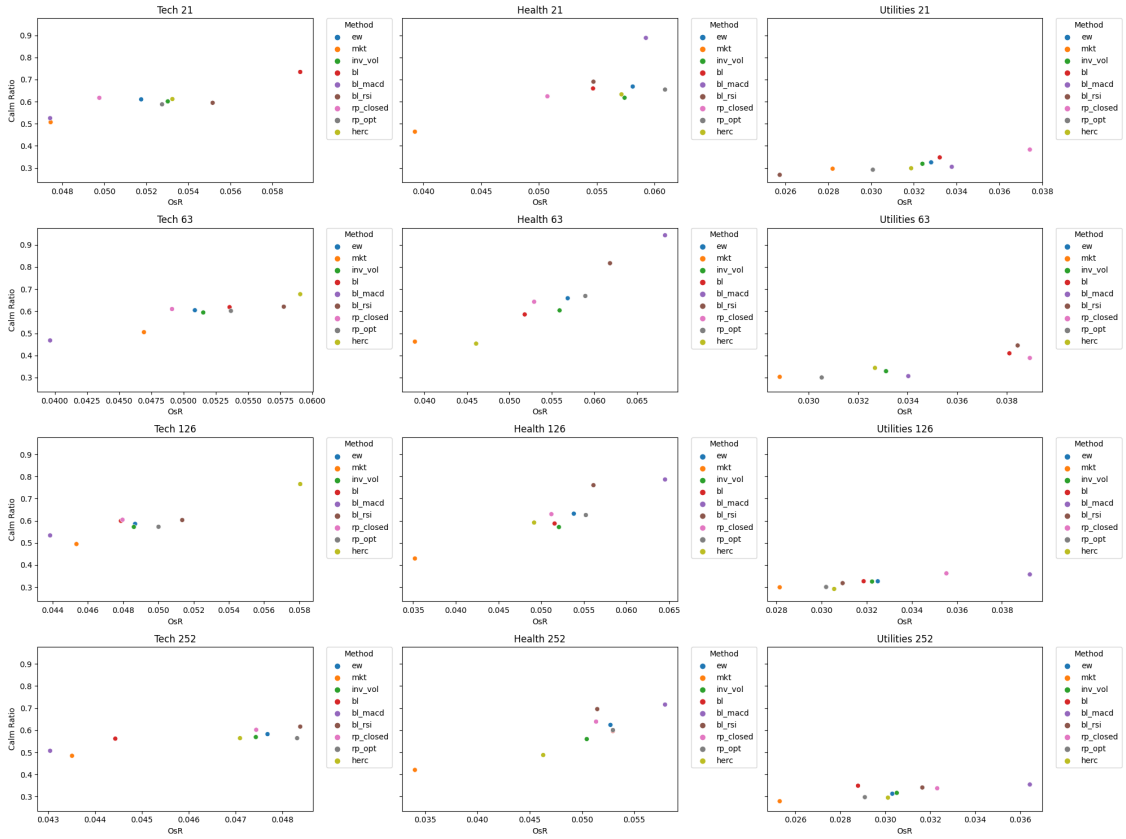

Tech 21
Health 21
Utilities 21
Tech 63
Health 63
Utilities 63
Tech 126
Health 126
Utilities 126
Tech 252
Health 252
Utilities 252
Ortn
Method
ew
mkt
inv_vol
tsl
tz_macd
tz_rsi
rp_closed
rp_spt
herc


Tech 21
Health 21
Utilities 21
Tech 63
Health 63
Utilities 63
Tech 126
Health 126
Utilities 126
Tech 252
Health 252
Utilities 252
OoR
Method
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc


Tech 21
Health 21
Utilities 21
Tech 63
Health 63
Utilities 63
Tech 126
Health 126
Utilities 126
Tech 252
Health 252
Utilities 252
Turnover Ratio
OsR
Method
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc


Tech 21
Health 21
Utilities 21
Tech 63
Health 63
Utilities 63
Tech 126
Health 126
Utilities 126
Tech 252
Health 252
Utilities 252
OstR
OsR
Method
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc


Tech 21
Health 21
Utilities 21
Tech 63
Health 63
Utilities 63
Tech 126
Health 126
Utilities 126
Tech 252
Health 252
Utilities 252
Max Drawdown
OsR
Method
ew
mkt
inv_vol
bl
bl_macd
bl_rsi
rp_closed
rp_opt
herc



1.6 1.7 Cumulative Return

[113]:
```
def OReturn(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w
    Return = np.diag(X_t)
    return Return

def getCumReturns(M, R, kappa, rf):
#     methods_name = ['ew', 'mkt', 'black_litterman_macd',␣
 ↪'black_litterman_rsi', 'rp_closed']
    methods_name = ['ew', 'mkt','rp_closed']
    d, T = R.shape

    para = [kappa, T, rf]
    method_returns = {}

    for method in methods_name:
        w = np.zeros((d, T - M))
        for i in range(0, T - M):
```

```
            R_M = R[:, i:i+M]
            w_t = get_weights(d, M, R_M, method, para)
            w[:,[i]] = w_t

        method_returns[method] = OReturn(w, d, R, M, rf)

    # Transpose the arrays in method_returns and create a DataFrame
    transposed_returns = {method: returns.T for method, returns in␣
↪method_returns.items()}
    df_method_returns = pd.DataFrame(transposed_returns)

    # Calculate and add the cumulative return columns
    for method in methods_name:
        cumulative_returns_col = f"{method}_cumulative"
        df_method_returns[cumulative_returns_col] = (1 +␣
↪df_method_returns[method]).cumprod() - 1

    display(df_method_returns)
    return df_method_returns
```

1.6.1 M=21

[114]:
```
M = 21
kappa = 0.3/252
rf = 0.002/21
cumReturn_tech_21 = getCumReturns(M, Tech_Returns, kappa, rf)
cumReturn_heal_21 = getCumReturns(M, Healthcare_Returns, kappa, rf)
cumReturn_util_21 = getCumReturns(M, Utilities_Returns, kappa, rf)
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0    -0.002317 -0.001492  -0.002345      -0.002317       -0.001492
1     0.006634  0.006655   0.005916       0.004302        0.005154
2    -0.003691  0.000539  -0.003463       0.000595        0.005696
3     0.006295  0.005528   0.006272       0.006894        0.011255
4     0.013953  0.007108   0.014048       0.020944        0.018443
…         …         …          …              …               …
2470 -0.003189 -0.003499  -0.003291       5.581860        4.378243
2471 -0.004325 -0.003950  -0.004464       5.553394        4.356999
2472 -0.027369 -0.023133  -0.031202       5.374036        4.233077
2473  0.003717  0.014434   0.006504       5.397727        4.308610
2474  0.009619  0.026123   0.004831       5.459268        4.447286

      rp_closed_cumulative
0                -0.002345
1                 0.003557
2                 0.000081
3                 0.006354
```

```
4                 0.020492
…                    …
2470              5.911971
2471              5.881119
2472              5.666412
2473              5.709769
2474              5.742182

[2475 rows x 6 columns]
```

	ew	mkt	rp_closed	ew_cumulative	mkt_cumulative	\
0	-0.004292	-0.006742	-0.004673	-0.004292	-0.006742	
1	0.012559	0.005390	0.013629	0.008213	-0.001388	
2	-0.002907	-0.001886	-0.002580	0.005282	-0.003272	
3	-0.002026	-0.002288	-0.002439	0.003245	-0.005552	
4	0.017292	0.013857	0.017799	0.020593	0.008228	
…	…	…	…	…	…	
2470	0.010644	0.007180	0.013119	5.286217	2.203004	
2471	0.002833	0.004870	-0.000430	5.304027	2.218603	
2472	-0.017544	-0.011801	-0.021262	5.193428	2.180621	
2473	-0.008526	-0.014074	-0.006960	5.140624	2.135857	
2474	0.004121	0.004650	0.002261	5.165929	2.150440	

```
      rp_closed_cumulative
0                -0.004673
1                 0.008892
2                 0.006289
3                 0.003835
4                 0.021703
…                    …
2470              4.933747
2471              4.931194
2472              4.805083
2473              4.764679
2474              4.777713

[2475 rows x 6 columns]
```

	ew	mkt	rp_closed	ew_cumulative	mkt_cumulative	\
0	0.002895	0.003821	0.002698	0.002895	0.003821	
1	-0.011170	-0.012582	-0.011562	-0.008308	-0.008809	
2	-0.000447	-0.000428	-0.000207	-0.008751	-0.009234	
3	0.000380	-0.001179	0.000312	-0.008374	-0.010401	
4	-0.000862	-0.002252	-0.001323	-0.009229	-0.012630	
…	…	…	…	…	…	
2470	0.002391	0.002102	0.002558	1.754388	1.440294	
2471	0.004563	0.004428	0.004534	1.766955	1.451100	
2472	0.000149	-0.000580	0.001144	1.767368	1.449678	
2473	-0.019023	-0.022638	-0.019878	1.714724	1.394224	

```
2474  0.012562  0.010928   0.012015       1.748827        1.420387

      rp_closed_cumulative
0                 0.002698
1                -0.008896
2                -0.009101
3                -0.008792
4                -0.010104
…                      …
2470              2.256578
2471              2.271343
2472              2.275085
2473              2.209984
2474              2.248551

[2475 rows x 6 columns]
```

1.6.2 M=63

[115]:
```
M = 63
kappa = 0.3/252
rf = 0.002/21
cumReturn_tech_63 = getCumReturns(M, Tech_Returns, kappa, rf)
cumReturn_heal_63 = getCumReturns(M, Healthcare_Returns, kappa, rf)
cumReturn_util_63 = getCumReturns(M, Utilities_Returns, kappa, rf)
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.006920  0.002737   0.008793       0.006920        0.002737
1     0.009459  0.004809   0.010206       0.016444        0.007558
2    -0.006521 -0.007114  -0.006630       0.009816        0.000391
3     0.003547  0.003647   0.003359       0.013398        0.004040
4     0.011275  0.010642   0.013283       0.024824        0.014725
…        …         …         …              …               …
2428 -0.003189 -0.003499  -0.003382       5.227861        4.167992
2429 -0.004325 -0.003950  -0.003993       5.200926        4.147578
2430 -0.027369 -0.023133  -0.031438       5.031214        4.028500
2431  0.003717  0.014434   0.006590       5.053632        4.101081
2432  0.009619  0.026123   0.007194       5.111862        4.234335

      rp_closed_cumulative
0                 0.008793
1                 0.019088
2                 0.012332
3                 0.015732
4                 0.029224
…                      …
2428              5.507717
2429              5.481730
```

```
2430              5.277956
2431              5.319328
2432              5.364790

[2433 rows x 6 columns]
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.003529  0.005024   0.004875       0.003529        0.005024
1     0.005897  0.003749   0.007169       0.009447        0.008791
2    -0.008779 -0.004788  -0.009369       0.000584        0.003961
3     0.007587  0.005677   0.010669       0.008175        0.009661
4     0.012577  0.012152   0.015229       0.020856        0.021930
…        …         …         …              …               …
2428  0.010644  0.007180   0.012735       4.912364        2.121312
2429  0.002833  0.004870   0.001386       4.929114        2.136514
2430 -0.017544 -0.011801  -0.022835       4.825092        2.099501
2431 -0.008526 -0.014074  -0.007241       4.775429        2.055879
2432  0.004121  0.004650   0.002009       4.799229        2.070089

      rp_closed_cumulative
0                 0.004875
1                 0.012079
2                 0.002597
3                 0.013293
4                 0.028724
…                    …
2428              5.154301
2429              5.162828
2430              5.022099
2431              4.978492
2432              4.990503

[2433 rows x 6 columns]
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.003543  0.003628   0.003650       0.003543        0.003628
1    -0.005015 -0.004465  -0.004976      -0.001490       -0.000853
2    -0.003053 -0.002670  -0.002875      -0.004538       -0.003521
3    -0.012385 -0.011028  -0.012404      -0.016868       -0.014510
4     0.000441  0.000433   0.000256      -0.016434       -0.014083
…        …         …         …              …               …
2428  0.002391  0.002102   0.002599       1.739472        1.453052
2429  0.004563  0.004428   0.004617       1.751972        1.463915
2430  0.000149 -0.000580   0.000566       1.752382        1.462486
2431 -0.019023 -0.022638  -0.019759       1.700024        1.406741
2432  0.012562  0.010928   0.012141       1.733941        1.433041

      rp_closed_cumulative
0                 0.003650
```

```
1                -0.001344
2                -0.004215
3                -0.016567
4                -0.016316
…                    …
2428              2.299357
2429              2.314591
2430              2.316466
2431              2.250938
2432              2.290408

[2433 rows x 6 columns]
```

1.6.3 M=126

[118]:
```
M = 126
kappa = 0.3/252
rf = 0.002/21
cumReturn_tech_126 = getCumReturns(M, Tech_Returns, kappa, rf)
cumReturn_heal_126 = getCumReturns(M, Healthcare_Returns, kappa, rf)
cumReturn_util_126 = getCumReturns(M, Utilities_Returns, kappa, rf)
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.009636  0.004911   0.010088       0.009636        0.004911
1     0.005369  0.010012   0.005122       0.015057        0.014972
2    -0.001615  0.005429  -0.002371       0.013417        0.020482
3    -0.004068 -0.004343  -0.003879       0.009295        0.016050
4    -0.003057  0.003301  -0.002359       0.006210        0.019404
…        …         …         …              …               …
2365 -0.003189 -0.003499  -0.003306       4.556288        3.735519
2366 -0.004325 -0.003950  -0.004098       4.532257        3.716814
2367 -0.027369 -0.023133  -0.031071       4.380846        3.607701
2368  0.003717  0.014434   0.006796       4.400846        3.674208
2369  0.009619  0.026123   0.007220       4.452798        3.796311

      rp_closed_cumulative
0                 0.010088
1                 0.015262
2                 0.012854
3                 0.008926
4                 0.006546
…                     …
2365              4.962426
2366              4.937994
2367              4.753494
2368              4.792594
2369              4.834415
```

```
[2370 rows x 6 columns]
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.003462 -0.003009   0.004919       0.003462       -0.003009
1     0.002602  0.000862   0.003396       0.006073       -0.002149
2     0.001092  0.000086   0.002190       0.007172       -0.002063
3     0.001584  0.000947   0.002150       0.008768       -0.001118
4    -0.004935 -0.009207  -0.005917       0.003790       -0.010314
…         …        …         …              …              …
2365  0.010644  0.007180   0.012444       4.216731        1.769318
2366  0.002833  0.004870   0.001340       4.231510        1.782805
2367 -0.017544 -0.011801  -0.023293       4.139728        1.749966
2368 -0.008526 -0.014074  -0.007040       4.095908        1.711263
2369  0.004121  0.004650   0.001952       4.116907        1.723871

      rp_closed_cumulative
0                 0.004919
1                 0.008331
2                 0.010539
3                 0.012712
4                 0.006720
…                    …
2365              4.549953
2366              4.557387
2367              4.427941
2368              4.389729
2369              4.400252
```

```
[2370 rows x 6 columns]
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0    -0.009905 -0.008282  -0.010413      -0.009905       -0.008282
1    -0.002552 -0.002818  -0.002514      -0.012432       -0.011076
2    -0.000337 -0.000524  -0.000382      -0.012765       -0.011594
3    -0.002291 -0.003875  -0.002290      -0.015026       -0.015425
4     0.002133  0.004521   0.002556      -0.012925       -0.010973
…         …        …         …              …              …
2365  0.002391  0.002102   0.002514       1.635003        1.360476
2366  0.004563  0.004428   0.004563       1.647026        1.370929
2367  0.000149 -0.000580   0.000273       1.647421        1.369554
2368 -0.019023 -0.022638  -0.019517       1.597059        1.315913
2369  0.012562  0.010928   0.012629       1.629684        1.341221

      rp_closed_cumulative
0                -0.010413
1                -0.012900
2                -0.013277
3                -0.015536
4                -0.013020
```

```
…                    …
2365              1.908541
2366              1.921814
2367              1.922611
2368              1.865570
2369              1.901760

[2370 rows x 6 columns]
```

1.6.4 M=252

[117]:
```
M = 252
kappa = 0.3/252
rf = 0.002/21
cumReturn_tech_252 = getCumReturns(M, Tech_Returns, kappa, rf)
cumReturn_heal_252 = getCumReturns(M, Healthcare_Returns, kappa, rf)
cumReturn_util_252 = getCumReturns(M, Utilities_Returns, kappa, rf)
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0    -0.006884 -0.000742  -0.009679      -0.006884       -0.000742
1    -0.001851 -0.001273  -0.002722      -0.008721       -0.002015
2     0.001521 -0.000637   0.001285      -0.007214       -0.002651
3     0.003945  0.002871   0.004115      -0.003298        0.000212
4     0.012501  0.008799   0.014893       0.009161        0.009013
…          …         …          …              …               …
2239 -0.003189 -0.003499  -0.003329       3.987254        3.167920
2240 -0.004325 -0.003950  -0.004168       3.965685        3.151457
2241 -0.027369 -0.023133  -0.030742       3.829780        3.055422
2242  0.003717  0.014434   0.006281       3.847732        3.113957
2243  0.009619  0.026123   0.007128       3.894363        3.221425

      rp_closed_cumulative
0                -0.009679
1                -0.012375
2                -0.011106
3                -0.007036
4                 0.007752
…                      …
2239              4.381194
2240              4.358766
2241              4.194025
2242              4.226649
2243              4.263906

[2244 rows x 6 columns]
```

```
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0    -0.004454  0.002480  -0.007215      -0.004454        0.002480
```

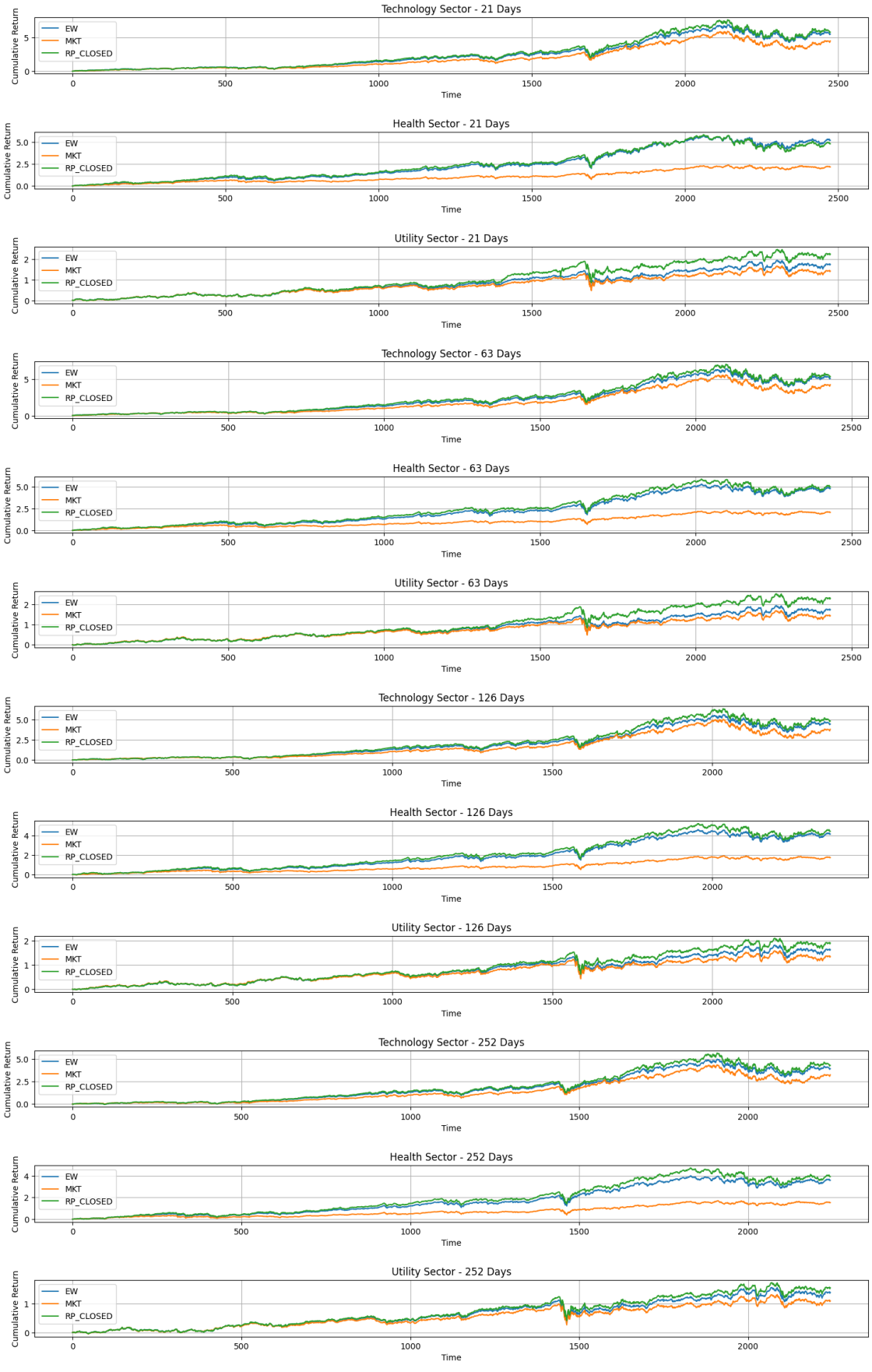
```
1    -0.002051  0.001277  -0.003099      -0.006496        0.003760
2     0.005279  0.002550   0.005256      -0.001251        0.006319
3     0.006597  0.002544   0.008363       0.005338        0.008879
4     0.009050  0.003092   0.011817       0.014436        0.011999
…        …         …         …              …               …
2239  0.010644  0.007180   0.012232       3.673288        1.563080
2240  0.002833  0.004870   0.001168       3.686527        1.575563
2241 -0.017544 -0.011801  -0.022300       3.604306        1.545170
2242 -0.008526 -0.014074  -0.007298       3.565051        1.509349
2243  0.004121  0.004650   0.002118       3.583863        1.521018

      rp_closed_cumulative
0                -0.007215
1                -0.010291
2                -0.005089
3                 0.003232
4                 0.015087
…                    …
2239              4.054334
2240              4.060238
2241              3.947394
2242              3.911285
2243              3.921686

[2244 rows x 6 columns]
            ew       mkt  rp_closed  ew_cumulative  mkt_cumulative  \
0     0.007298  0.008163   0.007378       0.007298        0.008163
1    -0.000821 -0.000931  -0.000927       0.006472        0.007224
2     0.002305  0.002236   0.002394       0.008792        0.009476
3     0.000099  0.001487  -0.000072       0.008891        0.010977
4     0.009173  0.008631   0.009443       0.018146        0.019703
…        …         …         …              …               …
2239  0.002391  0.002102   0.002500       1.384010        1.104352
2240  0.004563  0.004428   0.004571       1.394888        1.113671
2241  0.000149 -0.000580   0.000197       1.395245        1.112445
2242 -0.019023 -0.022638  -0.019462       1.349680        1.064625
2243  0.012562  0.010928   0.012392       1.379197        1.087186

      rp_closed_cumulative
0                 0.007378
1                 0.006444
2                 0.008854
3                 0.008781
4                 0.018308
…                    …
2239              1.542642
2240              1.554263
```

```
2241              1.554767
2242              1.505047
2243              1.536090

[2244 rows x 6 columns]
```

[119]:
```python
def plot_cumulative_returns(dfs, titles, figsize=(16, 24)):
    num_dfs = len(dfs)

    fig, axes = plt.subplots(nrows=num_dfs, ncols=1, figsize=figsize)
    fig.tight_layout(pad=5.0)

    for i, (df, title) in enumerate(zip(dfs, titles)):
        ax = axes[i]

        # Plot the cumulative returns for each method
        ax.plot(df['ew_cumulative'], label='EW')
        ax.plot(df['mkt_cumulative'], label='MKT')
        ax.plot(df['rp_closed_cumulative'], label='RP_CLOSED')

        # Customize the chart
        ax.set_title(title)
        ax.set_xlabel('Time')
        ax.set_ylabel('Cumulative Return')
        ax.legend(loc='upper left')
        ax.grid(True)

    # Display the charts
    plt.show()
```

[120]:
```python
dataframes = [
    cumReturn_tech_21, cumReturn_heal_21, cumReturn_util_21,
    cumReturn_tech_63, cumReturn_heal_63, cumReturn_util_63,
    cumReturn_tech_126, cumReturn_heal_126, cumReturn_util_126,
    cumReturn_tech_252, cumReturn_heal_252, cumReturn_util_252
]

titles = [
    'Technology Sector - 21 Days', 'Health Sector - 21 Days', 'Utility Sector -␣
 ↪21 Days',
    'Technology Sector - 63 Days', 'Health Sector - 63 Days', 'Utility Sector -␣
 ↪63 Days',
    'Technology Sector - 126 Days', 'Health Sector - 126 Days', 'Utility Sector␣
 ↪- 126 Days',
    'Technology Sector - 252 Days', 'Health Sector - 252 Days', 'Utility Sector␣
 ↪- 252 Days'
]
```

```
plot_cumulative_returns(dataframes, titles)
```


Technology Sector - 21 Days
Health Sector - 21 Days
Utility Sector - 21 Days
Technology Sector - 63 Days
Health Sector - 63 Days
Utility Sector - 63 Days
Technology Sector - 126 Days
Health Sector - 126 Days
Utility Sector - 126 Days
Technology Sector - 252 Days
Health Sector - 252 Days
Utility Sector - 252 Days
EW
MKT
RP_CLOSED
Cumulative Return
Time

1.7 1.8 Turnover Ratio

[121]:
```
def daily_OTurnoverRatio(w, d, R, M, rf):
    e = np.ones((d, 1))
    X_t = rf + (R[:, M:] - rf * e).T @ w

    # Calculate the number of periods
    n_periods = X_t.shape[1]

    # Calculate the turnover for each day
    turnovers = np.sum(np.abs(w[:, 1:] - w[:, :-1]), axis=0)

    return turnovers
```

[126]:
```
def getTurnover(M, R, kappa, rf):
    methods_name = ['ew', 'mkt', 'black_litterman_macd', 'black_litterman_rsi',␣
 ↪'rp_closed']
    methods_name = ['ew', 'mkt', 'rp_closed']
    d, T = R.shape

    para = [kappa, T, rf]
    method_turnovers = {}

    for method in methods_name:
        w = np.zeros((d, T - M))
        for i in range(0, T - M):
            R_M = R[:, i:i+M]
            w_t = get_weights(d, M, R_M, method, para)
            w[:,[i]] = w_t

        method_turnovers[method] = daily_OTurnoverRatio(w, d, R, M, rf)

    # Transpose the arrays in method_turnovers and create a DataFrame
    transposed_turnovers = {method: turnovers.T for method, turnovers in␣
 ↪method_turnovers.items()}
    df_method_turnovers = pd.DataFrame(transposed_turnovers)

    return df_method_turnovers
```

[127]:
```
M_values = [21, 63, 126, 252]
kappa = 0.3 / 252
rf = 0.002 / 21

# Store the DataFrames in a dictionary with keys as the DataFrame names
```

```
turnover_dfs = {}

for M in M_values:
    df_name_tech = f"turnover_tech_{M}"
    df_name_heal = f"turnover_heal_{M}"
    df_name_util = f"turnover_util_{M}"

    turnover_dfs[df_name_tech] = getTurnover(M, Tech_Returns, kappa, rf)
    turnover_dfs[df_name_heal] = getTurnover(M, Healthcare_Returns, kappa, rf)
    turnover_dfs[df_name_util] = getTurnover(M, Utilities_Returns, kappa, rf)
```

[129]:
```
def plot_daily_turnovers(dfs, titles, figsize=(16, 24)):
    num_dfs = len(dfs)

    fig, axes = plt.subplots(nrows=num_dfs, ncols=1, figsize=figsize)
    fig.tight_layout(pad=5.0)

    for i, (df, title) in enumerate(zip(dfs, titles)):
        ax = axes[i]

        # Plot the daily turnovers for each method
        ax.plot(df['ew'], label='EW')
        ax.plot(df['mkt'], label='MKT')
        ax.plot(df['rp_closed'], label='RP_CLOSED')

        # Customize the chart
        ax.set_title(title)
        ax.set_xlabel('Time')
        ax.set_ylabel('Daily Turnover Ratio')
        ax.legend(loc='upper left')
        ax.grid(True)

    # Display the charts
    plt.show()

# Create a list of turnover DataFrames
turnover_dataframes = list(turnover_dfs.values())

# Update the titles list to match the keys in the turnover_dfs dictionary
titles = [
    'Technology Sector - 21 Days', 'Health Sector - 21 Days', 'Utility Sector -␣
 ↪21 Days',
    'Technology Sector - 63 Days', 'Health Sector - 63 Days', 'Utility Sector -␣
 ↪63 Days',
    'Technology Sector - 126 Days', 'Health Sector - 126 Days', 'Utility Sector␣
 ↪- 126 Days',
```

```
    'Technology Sector - 252 Days', 'Health Sector - 252 Days', 'Utility Sector␣
 ↪- 252 Days'
]

# Call the plot_daily_turnovers function with the turnover_dataframes list and␣
 ↪the titles list
plot_daily_turnovers(turnover_dataframes, titles)
```


Technology Sector - 21 Days
Health Sector - 21 Days
Utility Sector - 21 Days
Technology Sector - 63 Days
Health Sector - 63 Days
Utility Sector - 63 Days
Technology Sector - 126 Days
Health Sector - 126 Days
Utility Sector - 126 Days
Technology Sector - 252 Days
Health Sector - 252 Days
Utility Sector - 252 Days
EW
MKT
RP_CLOSED
Daily Turnover Ratio
Time