ITMO UNIVERSITY

Saint Petersburg, Russia

Специализированные технологии машинного обучения / Advanced Machine learning Technologies

Lecture 2 – Reinforcement Learning

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn a *function* to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

Cat

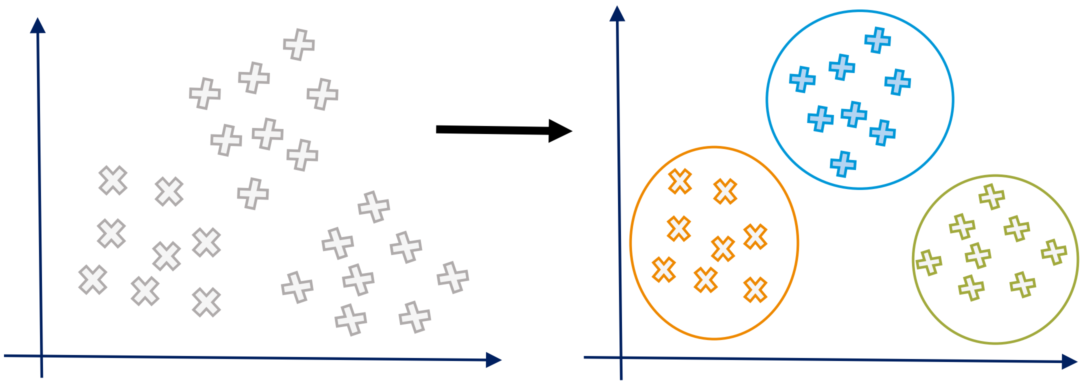
Classification

Unsupervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

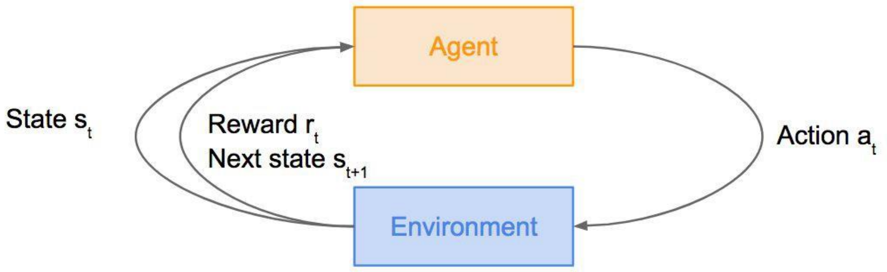


density estimation

Reinforcement Learning

Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

Goal: Learn how to take actions in order to maximize reward



Reinforcement Learning

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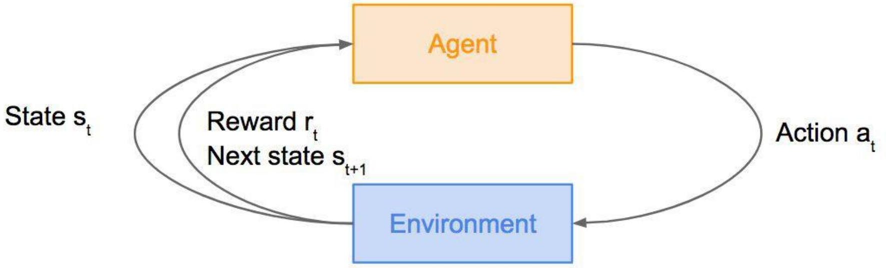

A person learns how to walk not by looking through a huge number of teaching examples, but by trying new things and making mistakes, so that, by getting both positive and negative experience

Overview

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- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients

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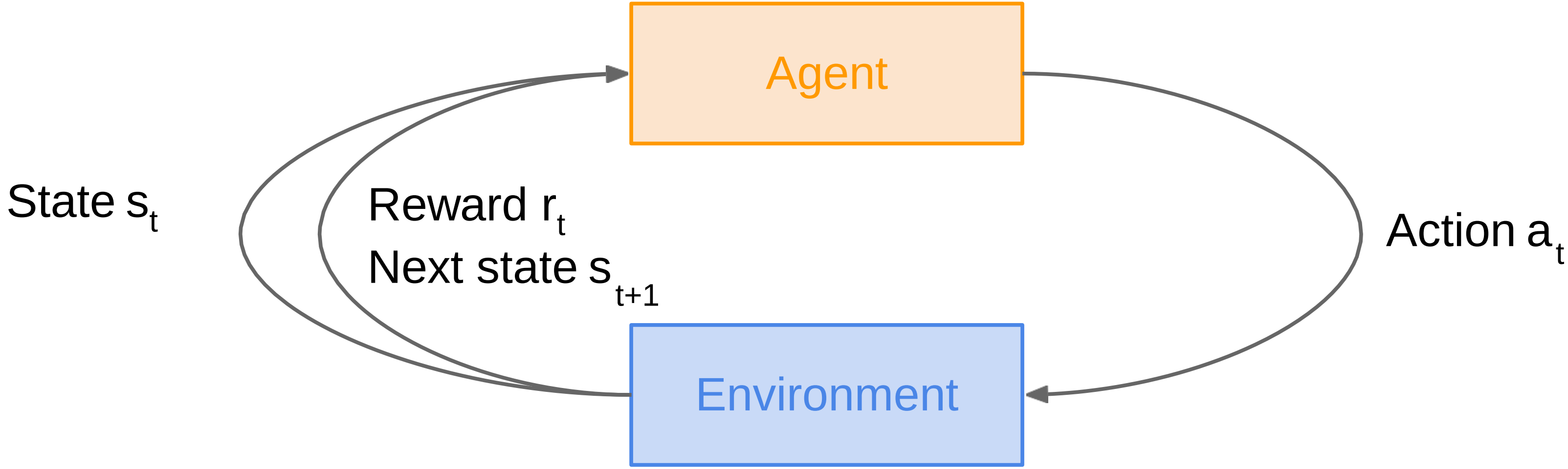
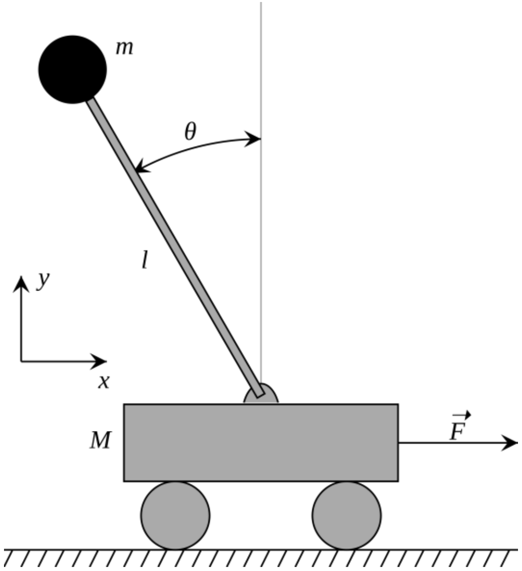
**partly based on CS231n Stanford RL lecture*

Reinforcement Learning



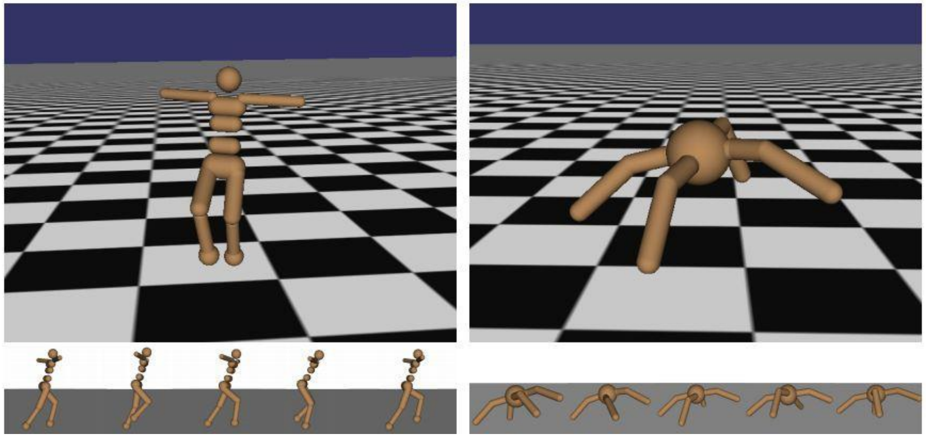
Cart-Pole Problem



Objective: Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity
Action: horizontal force applied on the cart
Reward: 1 at each time step if the pole is upright

Robot motions



Objective: Make the robot move forward

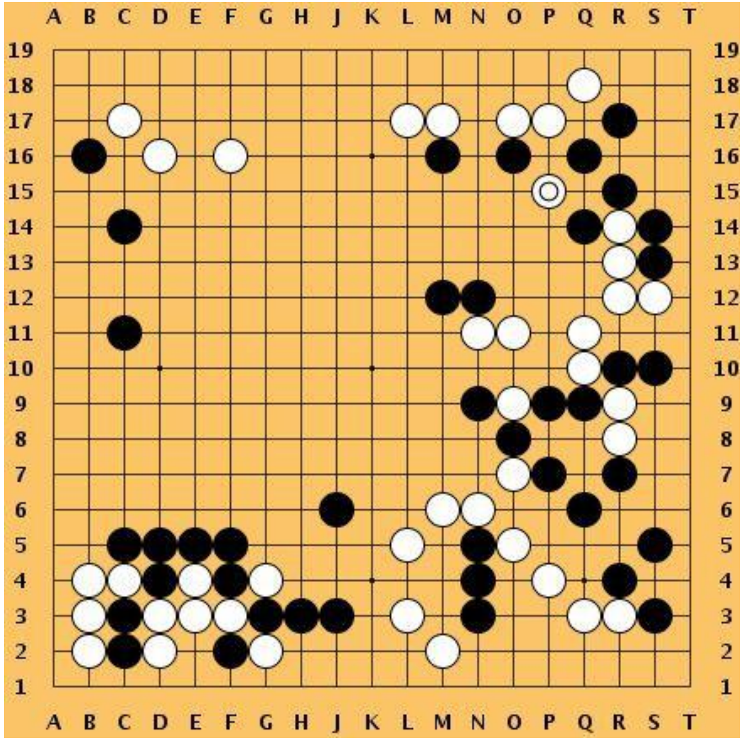
State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Atari Games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

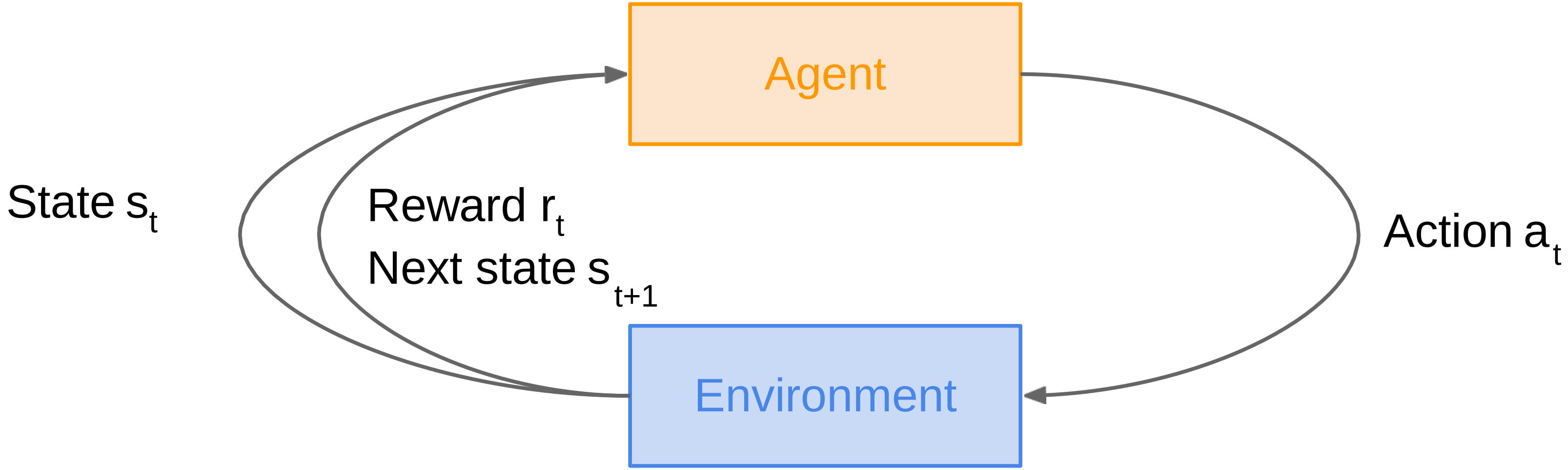
Go



Objective: Win the game!

State: Position of all pieces
Action: Where to put the next piece down
Reward: 1 if win at the end of the game, 0 otherwise

How can we mathematically formalize the RL problem?



Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Process is defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor – the less it is the less objective depends on further rewards

Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(. \mid s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(. \mid s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

- A policy π is a function from S to A that specifies what action to take in each state
- **Objective**: find policy π^* that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$

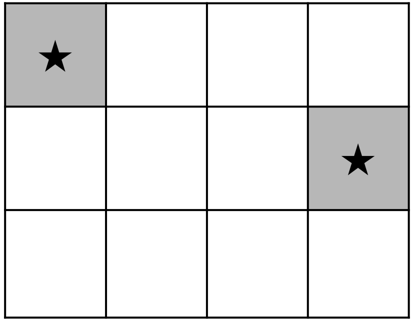
A simple MDP: Grid World

actions = {

1. right
2. left
3. up
4. down

}

states



Set a negative “reward” for each transition (e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

A simple MDP: Grid World

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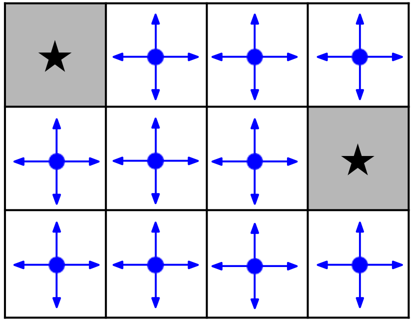
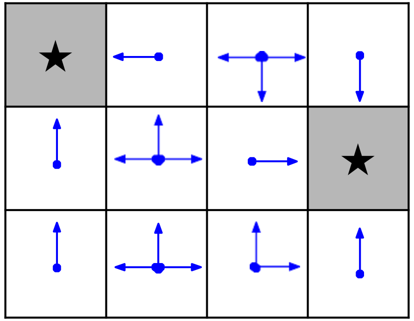


Random Policy



Optimal Policy

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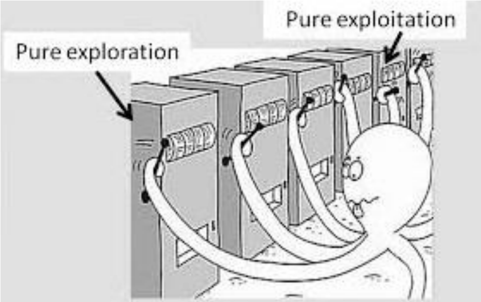
“Multi-armed bandits”

Assume that we have the following setup:

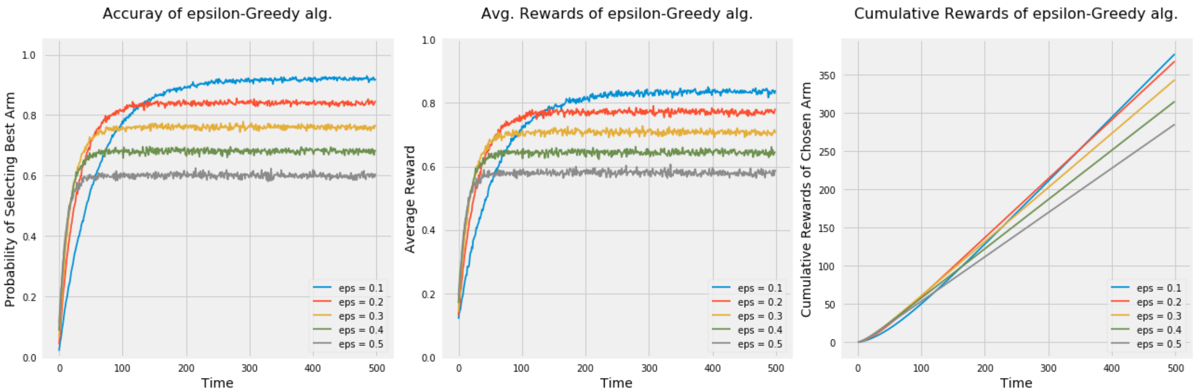
- Several automates with different reward probability (give money)
- Push the arms to earn more money (Our goal is to maximize **reward** with t→ ∞)
- No different states for agent – only actions with reward;
- How to choose the best one? (say, «the best policy”)



Greedy algorithm: on every step we choose the arm with the highest reward.

Exploration algorithm: on every step choose another arm in a hope that it will give even more reward.

ε-greedy algorithm: with probability **(1-ε)** we perform in greedy way and with **ε we explore the system.**



The optimal policy π*

We want to find optimal policy π* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?
Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) s_0, a_0, r_0, s_1, a_1, r_1, …

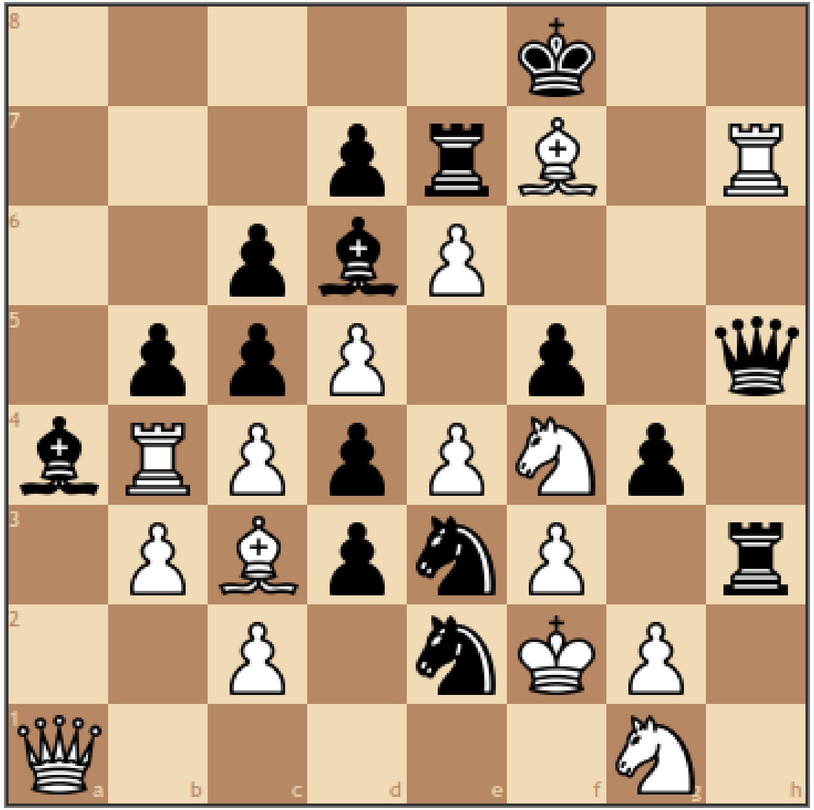
How to estimate how good the current state is?

Chess

Reward is known only in the end of a game….

How good the current state is?

How to choose the next step?



Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) s_0, a_0, r_0, s_1, a_1, r_1, …

How good is a state?

The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s'\sim\mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of

$$r + \gamma Q^*(s',a')$$

The optimal policy π* corresponds to taking the best action in any state as specified by Q*

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s,a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s',a')|s,a\right]$$

Q_i will converge to Q^* as i -> infinity

What's the problem with this?
Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s,a\right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a\right]$

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot);s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i))\nabla_{\theta_i} Q(s,a;\theta_i)\right]$$

Case Study: Playing Atari Games

Objective: Complete the game with the highest score

[Mnih et al. NIPS Workshop 2013; Nature 2015]

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Q-network Architecture

[Mnih et al. NIPS Workshop 2013; Nature 2015]

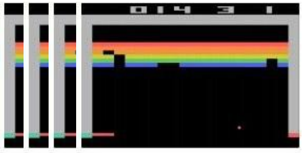
$Q(s, a; \theta)$:
neural network
with weights θ

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

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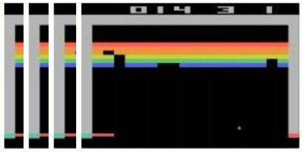

Input: state s_t

Current state s_t: 84x84x4 stack of last 4 frames
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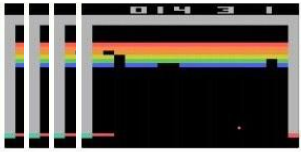
Familiar conv layers, FC layer



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32 4x4 conv, stride 2

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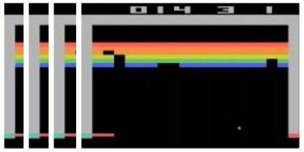
Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t,a_4)$



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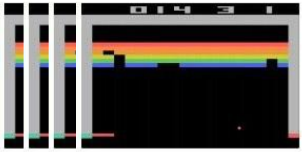

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Number of actions between 4-20 depending on Atari game

Current state s_t: 84x84x4 stack of last 4 frames
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Q-network Architecture

[Mnih et al. NIPS Workshop 2013; Nature 2015]

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$Q(s, a; \theta)$: neural network with weights θ

A single feedforward pass to compute Q-values for all actions from the current state => efficient!

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4

Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t,a_4)$

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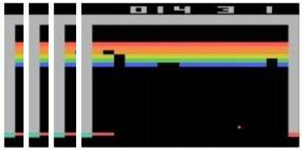

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Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

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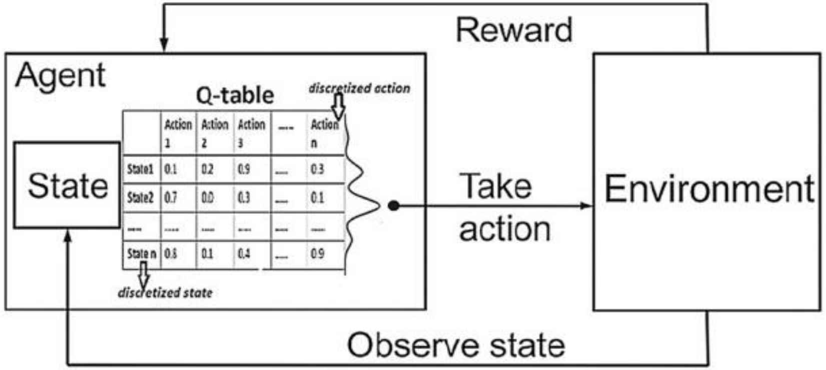
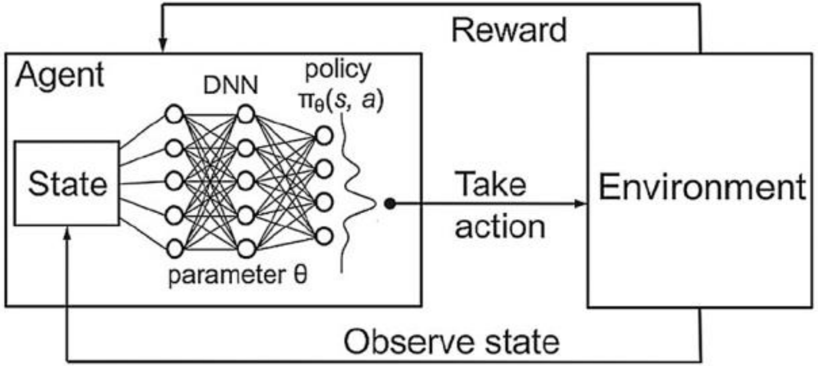
Q-learning and Deep Q-learning



a Q-learning



b Deep Q-learning

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Each transition can also contribute to multiple weight updates => greater data efficiency

Putting it together: Deep Q-Learning with Experience Replay

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Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
    Initialise state s_t
    for $t = 1, T$ **do**
        With probability ϵ select a random action a_t
        otherwise select $a_t = \max_a Q^*(s_t, a; \theta)$
        Execute action a_t and observe reward r_t and state s_{t+1}
        Store transition (s_t, a_t, r_t, s_{t+1}) in $\mathcal{D}$
        Set $s_{t+1} = s_t$
        Sample random minibatch of transitions (s_t, a_t, r_t, s_{t+1}) from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } s_{t+1} \\ r_j + \gamma \max_{a'} Q(s_{t+1}, a'; \theta) & \text{for non-terminal } s_{t+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(s_t, a_j; \theta))^2$
    end for
end for

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Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N

Initialize replay memory, Q-network

Initialize action-value function Q with random weights
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        Perform a gradient descent step on $(y_j - Q(s_t, a_j; \theta))^2$
    end for
end for

Play M episodes (full games)

Putting it together: Deep Q-Learning with Experience Replay

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  end for
end for

Initialize state (starting game screen pixels) at the beginning of each episode

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

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    Perform a gradient descent step on $(y_j - Q(s_t, a_j; \theta))^2$
  end for
end for

For each timestep t
of the game

Putting it together: Deep Q-Learning with Experience Replay

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Initialize replay memory $\mathcal{D}$ to capacity N
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        With probability ϵ select a random action a_t
        otherwise select $a_t = \max_a Q^*(s_t, a; \theta)$
        Execute action a_t and observe reward r_t and state s_{t+1}
        Store transition (s_t, a_t, r_t, s_{t+1}) in $\mathcal{D}$
        Set $s_{t+1} = s_t$
        Sample random minibatch of transitions (s_t, a_t, r_t, s_{t+1}) from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } s_{t+1} \\ r_j + \gamma \max_{a'} Q(s_{t+1}, a'; \theta) & \text{for non-terminal } s_{t+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(s_t, a_j; \theta))^2$
    end for
end for

With small probability, select a random action (explore), otherwise select greedy action from current policy

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
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end for

Take the action (a_t), and observe the reward r_t and next state s_{t+1}

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Store transition in replay memory

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end for

Experience Replay:
Sample a random minibatch of transitions from replay memory and perform a gradient descent step

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Environments for experiments!

https://www.gymlibrary.dev/environments/atari/breakout/

Example!

https://www.youtube.com/watch?v=V1eYniJ0Rnk

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Policy Gradients

What is a problem with Q-learning?
The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

Policy Gradients

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The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand
Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_\theta\right]$$

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Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$$
$$= \int_{\tau} r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

Where r(τ) is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right] = \int_\tau r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

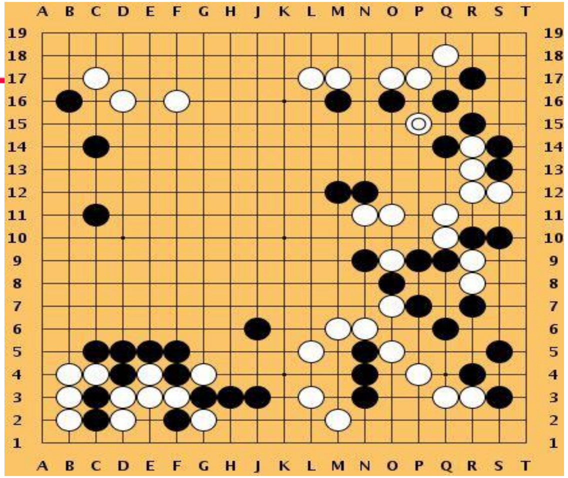
Now let's differentiate this:

$$\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)\mathrm{d}\tau$$

Intractable! Gradient of an expectation is problematic when p depends on θ

Can be estimated with Monte Carlo sampling!

More policy gradients: AlphaGo



AlphaGo - overview:

- Mix of supervised learning and reinforcement learning
- Mix of old methods (Monte Carlo Tree Search) and recent ones (deep RL)

How to beat the Go world champion:

- Featurize the board (stone color, move legality, bias, …)
- Initialize policy network with supervised training from professional go games, then continue training using policy gradient (play against itself from random previous iterations, +1 / -1 reward for winning / losing)
- Also learn value network (critic)
- Finally, combine policy and value networks in a Monte Carlo Tree Search algorithm to select actions by lookahead search

[Silver et al., Nature 2016]

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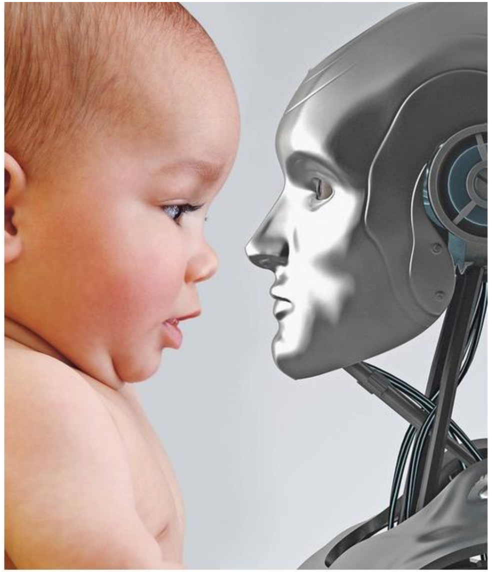
Deb Roy’s MIT experiment



90 000 hours video
140 000 hours audio

http://ai-news.ru/2018/04/kak_obuchautsya_deti_i_pochemu_iskusstvennyj_intellekt_tak_ne_mozhet.html