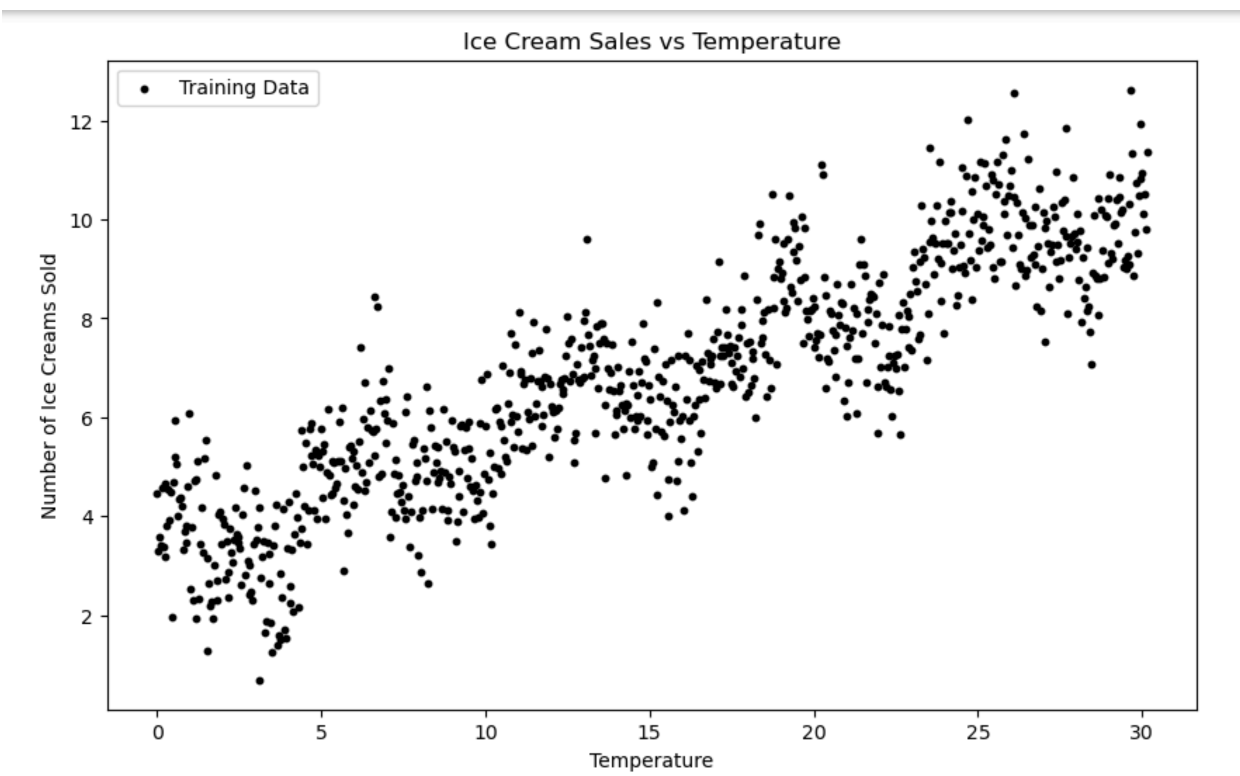
1. Introduction

- This report analyzes the relationship between temperature and ice cream sales based on data provided in `train.txt` and `test.txt`. Using various statistical and machine learning techniques, we build and evaluate models to predict ice cream sales based on temperature. The objective is to identify the best predictive model using both linear and non-linear approaches, including feature engineering and regularization techniques (Lasso and Ridge).

Data Description:

- The data is sourced from `train.txt`, and `test.txt` containing two columns: temperature (independent variable) and ice cream sales (dependent variable). We aim to find the best linear and non-linear models, including Lasso and Ridge regression, to maximize prediction accuracy on sales.

2. Initial Data Analysis and Plotting



Data plotting

- Figure above: Temperature vs Ice cream sales
- The plot suggests a positive trend between temperature and sales. This relationship will be explored through linear and non linear modeling techniques.

3. Linear Regression Model

- Model description
  - I start by fitting linear regression model $y=\beta 0+\beta 1x$ to understand baseline relationship between temperature and ice cream sales (code for it)

- Results and Interpretation
  - Coefficients:
    - Estimated $\beta 0$ (Intercept): 3.191037800253055
      Estimated $\beta 1$ (Slope): 0.23839763045936505
  - Statistical Inference:
    - The OLS summary (Table 1) reveals a statistically significant positive relationship between temperature and ice cream sales, as both the intercept ($\beta 0$) and slope ($\beta 1$) have p-values less than 0.05, indicating they are significantly different from zero.

      This table below confirms the linear relationship, supporting the hypothesis that temperature positively influences ice cream sales.

	coef	std err	t	P>|t|	[0.025	0.975]
const	3.1910	0.078	40.962	0.000	3.038	3.344
x1	0.2384	0.004	53.289	0.000	0.230	0.247

4. Feature Engineering and Model Selection with Non-linear Features

- Feature Selection Approach:
  - To improve the model beyond a simple linear fit, I included non-linear transformations of temperature, such as cos(x), log(x), cos(4x), sin(3x), sin(5x), and sin(2x)×cos(2x). Given that the feature set was relatively small, I evaluated all possible combinations of these features to identify the best subset.

- Metric Used:
  - Adjusted R^2 was employed as the metric for feature selection. Unlike standard R^2 , adjusted R^2 accounts for the number of predictors in the model, ensuring that added features improve the model's fit without simply increasing complexity.
- **Selection Process and Results:**
  - The exhaustive feature selection process revealed that the optimal feature combination is: x,cos(x),sin(3x) with an adjusted R^2 of 0.852. This combination showed the highest adjusted R^2. (screenshot with results below)

```
Best feature combination: ('x', 'cos_x', 'sin_3x')
Highest Adjusted R^2: 0.852448879534917
Model with highest Adjusted R^2 uses features: ('x', 'cos_x', 'sin_3x')
```

5. Lasso and Ridge Regression with All Features

- **Model Setup and Selection**
  - Lasso and Ridge regression models are trained with all features, using cross-validation to find the optimal alpha values for each model.
  - Code for Lasso and Ridge Regression
  - `lasso_model, ridge_model = train_lasso_ridge(x_train, y_train)`

- **Results and Interpretation**
  - **Lasso Best Alpha**: 0.019
  - **Ridge Best Alpha**: 10.0
  - **Lasso Adjusted R2R^2R2**: 0.8514
  - **Ridge Adjusted R2R^2R2**: 0.8518

Both regularized models perform similarly, with Ridge having a slightly higher adjusted R^2.

Screenshot below: Best Alpha Values and Adjusted R^2 for Lasso and Ridge

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
Lasso Best Alpha: 0.019414860526572875
Ridge Best Alpha: 10.0
Lasso Adjusted R^2: 0.8514
Ridge Adjusted R^2: 0.8518
```

6. Model Evaluation on Test Data

- **Test Data Prediction**
  - We evaluate all models using the test dataset `test.txt`.

- **Code for Model Evaluation**
  - evaluate_on_test_data(x_test, y_test, best_model, all_features[list(best_features)], model_name="Linear Regression with Best Features")
  - evaluate_on_test_data(x_test, y_test, lasso_model, all_features, model_name="Lasso Regression with All Features")
  - evaluate_on_test_data(x_test, y_test, ridge_model, all_features, model_name="Ridge Regression with All Features")

Results and Comparison

The Mean Squared Error (MSE) and R2R^2R2 values for each model on the test data are summarized below.

- Linear Regression with Best Features: MSE = 0.7831, R^2 = 0.4246
- Lasso Regression with All Features: MSE = 0.7890, R^2 = 0.4203
- Ridge Regression with All Features: MSE = 0.7797, R^2 = 0.4271

Visual Comparison of Predictions

Each figure below compares the predicted vs. actual ice cream sales for Linear Regression, Lasso, and Ridge models.

Figure 2: Linear Regression with Best Features on Test Data

```
Linear Regression with Best Features on Test Data:
Mean Squared Error (MSE): 0.7831
R^2: 0.4246
```

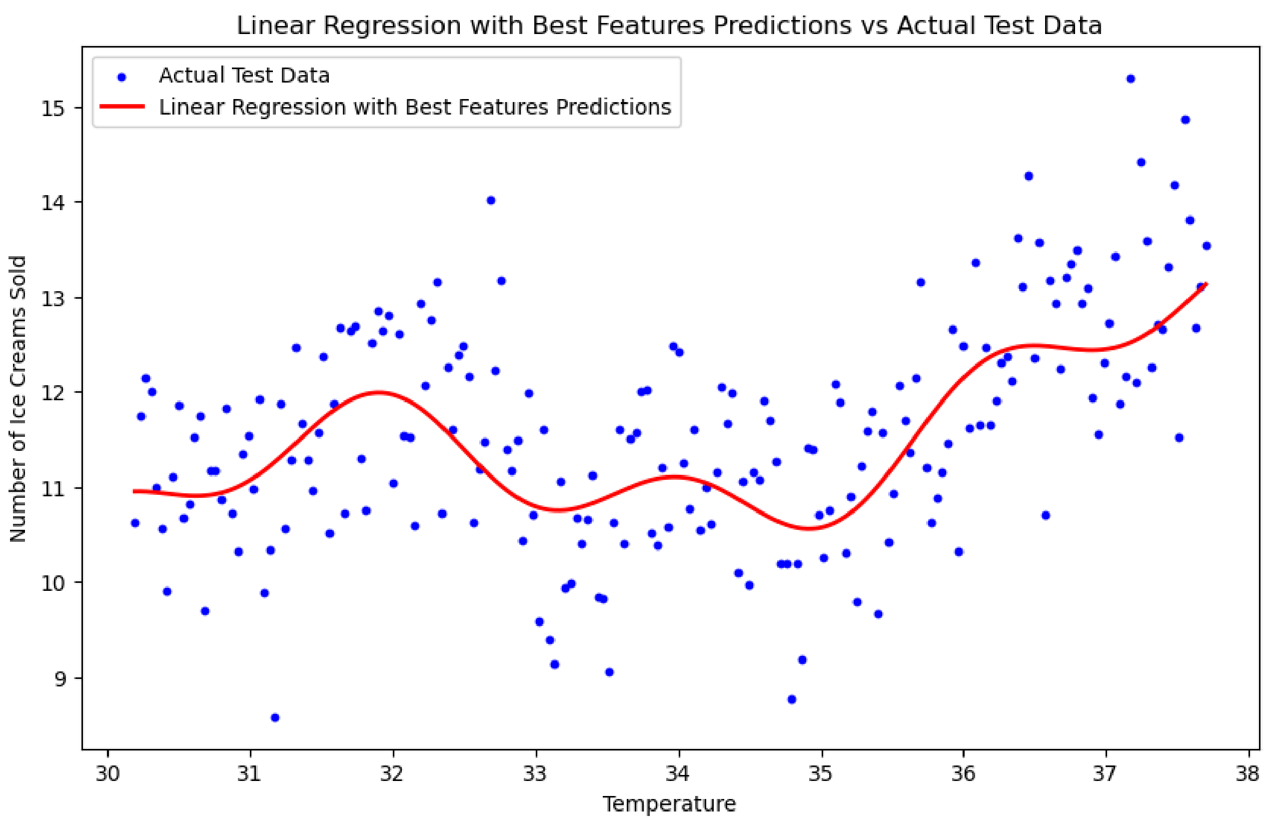


Figure 3: Lasso Regression with All Features on Test Data

```
Lasso Regression with All Features on Test Data:
Mean Squared Error (MSE): 0.7890
R^2: 0.4203
```

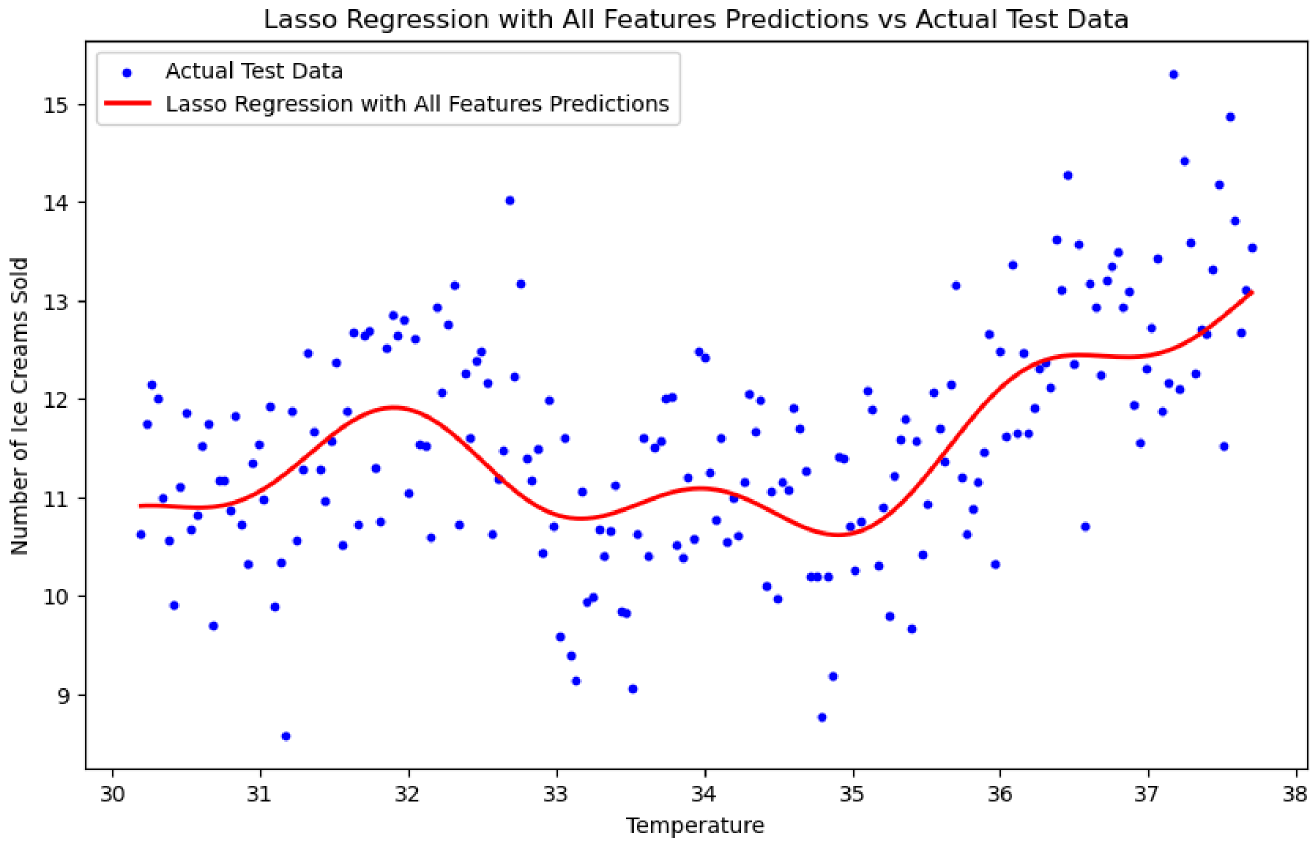
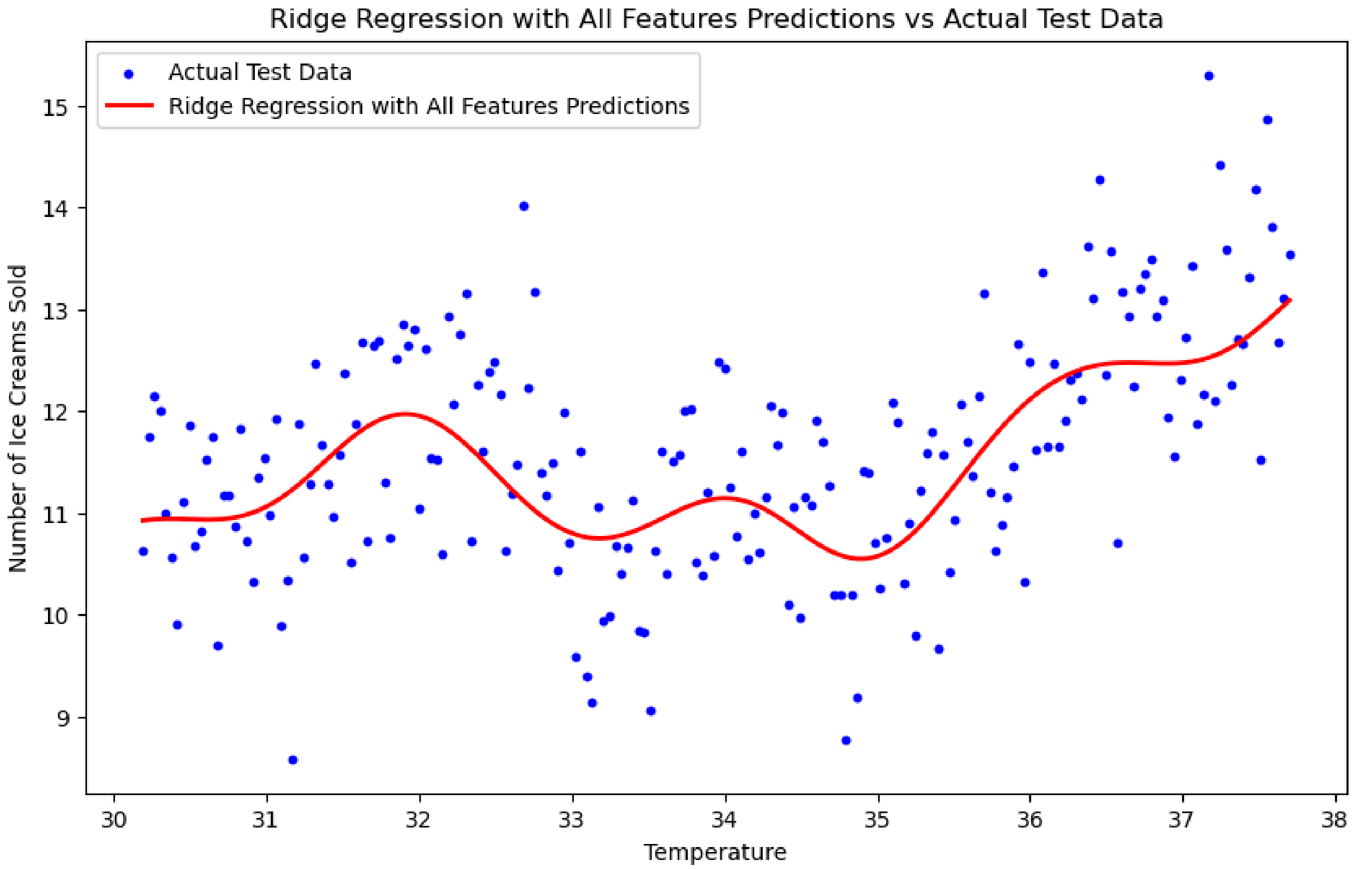


Figure 4: Ridge Regression with All Features on Test Data

```
Ridge Regression with All Features on Test Data:
Mean Squared Error (MSE): 0.7797
R^2: 0.4271
```



7. Conclusion

Summary of Findings

- The linear regression model confirmed a positive relationship between temperature and ice cream sales.
- Adding non-linear features improved the model, with the best feature combination achieving an adjusted R^2 of 0.852.
- Lasso and Ridge models achieved comparable adjusted R^2 values, but Ridge provided a slightly better generalization on test data.