TEMA 4

21. $A = \begin{pmatrix} 3 & 11 \\ 4 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$ $N = 26$

EXAMEN

$$\begin{pmatrix} 3 & 11 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} E & A & E \\ X & M & N \end{pmatrix} + B \pmod{26} =$$

$$= \begin{pmatrix} 3 & 11 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 4 & 0 & 4 \\ 23 & 12 & 13 \end{pmatrix} + \begin{pmatrix} 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 12+11\cdot 23 & 11\cdot 12 & 12+11\cdot 13 \\ 16+5\cdot 23 & 5\cdot 12 & 16+5\cdot 13 \end{pmatrix} +$$

$$+\begin{pmatrix} 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 265 & 132 & 155 \\ 131 & 60 & 8 \end{pmatrix} + \begin{pmatrix} 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 25 \\ 1 & 8 & 3 \end{pmatrix} + \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 9 & 32 \\ 10 & 17 & 12 \end{pmatrix} = \begin{pmatrix} 12 & 9 & 6 \\ 10 & 17 & 12 \end{pmatrix} = \begin{pmatrix} M & J & G \\ K & R & M \end{pmatrix} \Rightarrow$$

criptare

$\Rightarrow$ EXAMEN = MKJRGM

SMOGHJECHGXX

$$A = \begin{pmatrix} 3 & 11 \\ 4 & 5 \end{pmatrix}$$

$$A^{-1} = (\det A)^{-1} \cdot \underbrace{\begin{pmatrix} 5 & -11 \\ -4 & 3 \end{pmatrix}}_{A^*}$$

$$\det A = \begin{vmatrix} 3 & 11 \\ 4 & 5 \end{vmatrix} = 15 - 44 = -29 = 23$$

$$A^{-1} = 23\begin{pmatrix} 5 & -11 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 115 & -253 \\ -92 & 69 \end{pmatrix} = \begin{pmatrix} 11 & -19 \\ -14 & 17 \end{pmatrix} = \begin{pmatrix} 11 \\ 12 \end{pmatrix}$$

$$A^{-1}\left[\begin{pmatrix} S & G & H & E & H & X \\ M & G & J & C & G & X \end{pmatrix} - B\right] =$$

$$= \begin{pmatrix} 11 & 7 \\ 12 & 17 \end{pmatrix} \left[\begin{pmatrix} 18 & 14 & 10 & & & \\ 12 & 6 & 9 & 2 & 6 & 23 \end{pmatrix} - \begin{pmatrix} 9 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 11 & 7 \\ 12 & 17 \end{pmatrix} \begin{pmatrix} 11 & 7 & 3 & -3 & 3 & 16 \\ 5 & -1 & 2 & -5 & -1 & 16 \end{pmatrix} =$$

$$= \begin{pmatrix} 121+35 & 77-7 & 33+14 & -33-35 & 33-7 & 11\cdot 16+7\cdot 16 \\ 12\cdot 11+5\cdot 17 & 7\cdot 12-17 & 36+34 & -36-5\cdot 17 & 36-17 & 12\cdot 16+17\cdot 16 \end{pmatrix}$$

$$= \begin{pmatrix} 156 & 70 & 47 & -68 & 26 & 288 \\ 217 & 67 & 70 & -121 & 19 & 469 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 18 & 21 & 10 & 0 & 2 \\ 9 & 15 & 18 & 9 & 19 & 22 \end{pmatrix} =$$

$$= \begin{pmatrix} A & S & V & K & A & C \\ J & P & S & J & T & W \end{pmatrix}$$

$(\det A)^{-1} = 23^{-1} = -9 = 17$

$(26, 23)$ $x_{26} = (1,0)$ $x_{23} = (0,1)$

$26 = 23 \cdot 1 + 3$ $x_3 = x_{26} - x_{23} = (1,0) - (0,1) = (1,-1)$

$23 = 3 \cdot 7 + 2$ $x_2 = x_{23} - 7x_3 = (0,1) - (7,-7) = (-7,8)$

$3 = 2 \cdot 1 + 1$ $x_1 = x_3 - x_2 = (1,-1) - (-7,8) = (8,-9) = (8,17)$

$$A^{-1} = 17 \begin{pmatrix} 5 & -11 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 85 & -187 \\ -68 & 51 \end{pmatrix} = \begin{pmatrix} 7 & 21 \\ 10 & 25 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 21 \\ 10 & 25 \end{pmatrix} \left[\left(\begin{matrix} S & O & K & E & H & X \\ M & G & J & C & G & X \end{matrix} \right) - B \right] =$$

$$= \begin{pmatrix} 7 & 21 \\ 10 & 25 \end{pmatrix} \left[\left(\begin{matrix} 18 & 14 & 10 & 4 & 10 & 23 \\ 12 & 6 & 9 & 2 & 6 & 23 \end{matrix} \right) - \begin{pmatrix} 7 \\ 9 \end{pmatrix} \right] =$$

$$= \begin{pmatrix} 7 & 21 \\ 10 & 25 \end{pmatrix} \begin{pmatrix} 11 & 7 & 3 & -3 & 3 & 16 \\ 3 & -3 & 0 & -7 & -3 & 14 \end{pmatrix} =$$

$$= \begin{pmatrix} 77+63 & 49-63 & 21 & -21-7\cdot 21 & 21-63 & 7\cdot 16+21\cdot 14 \\ 110+75 & 70-75 & 30 & -30-7\cdot 25 & 30-75 & 160+25\cdot 14 \end{pmatrix}$$

$$= \begin{pmatrix} 140 & -14 & 21 & -168 & -42 & 406 \\ 185 & -5 & 30 & -205 & -45 & 510 \end{pmatrix} =$$

$$= \begin{pmatrix} 10 & 12 & 21 & 14 & 10 & 16 \\ 3 & 21 & 4 & 3 & 7 & 16 \end{pmatrix} =$$

$$= \begin{pmatrix} k & M & V & O & k & Q \\ D & V & E & D & H & Q \end{pmatrix} = kDMVVEODkHQQ$$