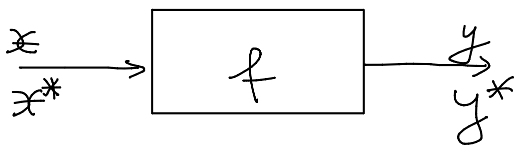
Lab 3

Monday, March 7, 2022 2:38 PM

Nr. de cond. a unei pb.



$|\Delta x| = |x - x^*|$ → er. abs. la input

$|\Delta y| = |y - y^*|$ → —"— output

$|\delta x| = \frac{|\Delta x|}{|x|}$ → er. rel. la input

$|\delta y| = \frac{|\Delta y|}{|y|}$ → —"— output

$|\delta y| \leq \text{cond} f(x)\, |\delta x|$

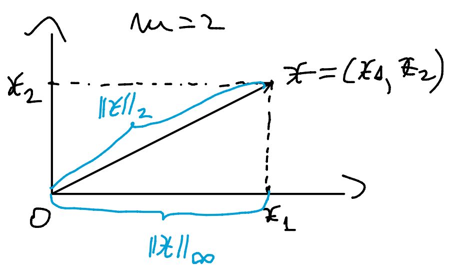
$\frac{|\delta y|}{|\delta x|} \leq (\simeq) \text{cond} f(x)$

$f : \mathbb{R}^m \to \mathbb{R}^n$

$x = [x_1, \dots, x_m]^T$

Ex. de norme: ("p-norm")

- norma Euclidiană: $\|x\|_2 = \sqrt{x_1^2 + \dots + x_m^2}$
- norma Cebîșev/supremum: $\|x\|_\infty = \max\{|x_1|, \dots, |x_m|\}$
- norma taxi/Manhattan: $\|x\|_1 = |x_1| + \dots + |x_m|$



$A \in \mathcal{M}_{n,m}(\mathbb{R}) \qquad \|A\|_p = \max_{\substack{x \in \mathbb{R}^m \\ x \neq 0_m}} \frac{\|A \cdot x\|_p}{\|x\|_p}$ → normă matriceală

2 def. nr. de cond. pt.: $f = (f_1, \dots, f_n)$, $x = (x_1, \dots, x_m)$

1) $\Gamma(x) = \left(\left| \frac{x_j \cdot \frac{\partial f_i}{\partial x_j}(x)}{f_i(x)} \right| \right)_{\substack{i=\overline{1,n} \\ j=\overline{1,m}}}$ $\qquad \text{cond}_p f(x) = \|\Gamma(x)\|_p$

2) $\frac{\partial f}{\partial x}(x) = \left(\frac{\partial f_i}{\partial x_j}(x) \right)_{\substack{i=\overline{1,n} \\ j=\overline{1,m}}}$ $\qquad \text{cond}_p f(x) = \frac{\|x\|_p \cdot \left\| \frac{\partial f}{\partial x}(x) \right\|_p}{\|f(x)\|_p}$.

- $\left.\begin{array}{l} f : I \to \mathbb{R},\ f \in C^1(I) \\ I \subseteq \mathbb{R},\ x \in I,\ f(x) \neq 0 \end{array}\right\} \Rightarrow \text{cond} f(x) = \left| \frac{x \cdot f'(x)}{f(x)} \right|$.

Ex. pt. sist. lin.: $A \cdot y = b \Rightarrow \exists!\, y$ sol. a sist.

Ex. pt. sist. lin.: $A \cdot y = b \implies \exists!\, y$ sol. a sist.

$(A \in \mathcal{M}_{n,n}(\mathbb{R}))$ $(\det A \neq 0)$

$$y = A^{-1} \cdot b$$

$$\begin{matrix} b \\ b^* \end{matrix} \longrightarrow \boxed{\; f \;} \longrightarrow \begin{matrix} y \\ y^* \end{matrix} \qquad f(b) = A^{-1} \cdot b$$

Def. 2) $\implies \operatorname{cond}_p f(b) = \dfrac{\|b\|_p \cdot \|A^{-1}\|_p}{\|A^{-1} b\|_p}$

$$\operatorname{cond}_p A := \max_{b \neq 0_n} \operatorname{cond}_p f(b) = \ldots = \|A\|_p \cdot \|A^{-1}\|_p .$$

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>> cond(A, p)
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Ex. pol.: $p(x) = x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0$

$\xi \to$ răd. nenulă și simplă $(p(\xi) = 0,\ p'(\xi) \neq 0)$

$a = [a_1, \ldots, a_n]$

$\xi = \xi(a)$ $\qquad \begin{matrix} a \\ a^* \end{matrix} \longrightarrow \boxed{\quad} \longrightarrow \begin{matrix} \xi \\ \xi^* \end{matrix}$

Temă: Def. 1) $\implies \operatorname{cond}_1 \xi(a) = \dfrac{\sum_{j=1}^{n} |a_j \cdot \xi(a)^{n-j}|}{|\xi(a) \cdot p'(\xi(a))|}$.