Seminar 10

1. Studiati existenta limitelor de functii
   a) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{1+xy}-1}$
   b) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$
   c) $\lim\limits_{(x,y)\to(\infty,\infty)} \frac{x^2+y^2}{x^4+y^4}$
   d) $\lim\limits_{(x,y)\to(0,0)} \frac{x\sin(x^2-y^2)}{x^2+y^2}$
   e) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+y^3}{xy}$
   f) $\lim\limits_{(x,y)\to(1,1)} \frac{(x-1)(y-1)}{xy-1}$
   g) $\lim\limits_{(x,y,z)\to O_3} \frac{(x+y+z)^2}{x^2+y^2+z^2}$
   h) $\lim\limits_{(x,y)\to(1,1)} \frac{\ln(1+x^2)-\ln(1+y^2)}{x^2-y^2}$

2. Se da functia $f:\mathbb{R}^2\to\mathbb{R}$

$$f(x,y)=\begin{cases} x\cos\frac{1}{y^2}+y\cos\frac{1}{x^2} & ,xy\neq 0 \\ x+y & ,xy=0 \end{cases}$$

Este functia continua in $(0,0)$? Dar in $(1,0)$?

3. Verificati daca functiile urmatoare isi ating valorile extreme si determinati aceste valori
   a) $f:(0,\infty)^2\to\mathbb{R},\quad f(x,y)=\frac{x}{y}+\frac{y}{x}$
   b) $f:B(O_2,1)\to\mathbb{R},\quad f(x,y)=\frac{1}{1+x^2+y^2}$
   c) $f:A\to\mathbb{R},\quad f(x,y)=xy(1-x-y),\quad A=\{(x,y)\in\mathbb{R}^2\big|\, x\geq 0, y\geq 0, x+y\leq 1\}$

4. Se da multimea $A=\{(x,y)\in[-1,1]^2\big|\, x\neq y\}$ si functia $f:A\to\mathbb{R}$, $f(x,y)=\frac{x^2+y^2}{(x-y)^2}$
   a) Exista limita functiei f in origine?
   b) Este multimea A compacta?
   c) Determinati valorile extreme ale lui f pe multimea A. Atinge functia f aceste valori?

Inegalitatea mediilor. Daca $n\in\mathbb{N}^\star$ si $x_1,x_2,\ldots,x_n\in[0,\infty)$ atunci

$$\sqrt[n]{x_1x_2\cdot\ldots\cdot x_n}\leq\frac{x_1+x_2+\ldots+x_n}{n}\leq\sqrt{\frac{x_1^2+x_2^2+\ldots+x_n^2}{n}}$$

Inegalitatea Cauchy-Schwarz. Daca $n\in\mathbb{N}^\star$ si $x_1,x_2,\ldots,x_n$, $y_1,y_2,\ldots,y_n\in\mathbb{R}$ atunci

$$(x_1y_1+x_2y_2+\ldots+x_ny_n)^2\leq(x_1^2+x_2^2+\ldots+x_n^2)\cdot(y_1^2+y_2^2+\ldots+y_n^2)$$

Exercitii suplimentare

1. Studiati existenta limitelor de functii
   a) $\lim\limits_{(x,y)\to(0,2)} \frac{\sin(xy)}{x}$
   b) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$
   c) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2y^2}$
   d) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^2y)}{x^2+y^2}$
   e) $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)^{x^2y^2}$
   f) $\lim\limits_{(x,y)\to(\infty,\infty)} \frac{x+y}{x^2+xy+y^2}$
   g) $\lim\limits_{(x,y,z)\to O_3} \frac{xyz}{x^2+y^2+z^2}$
   h) $\lim\limits_{(x,y,z)\to O_3} (xy+yz+zx)^2 \ln(x^2+y^2+z^2)$

2. Studiati continuitatea urmatoarelor functii in origine
   a)
$$f:\mathbb{R}^2\to\mathbb{R},\ f(x,y)=\begin{cases} \frac{x^3+y^3}{x^2+y^2} & ,(x,y)\neq(0,0) \\ 0 & ,(x,y)=(0,0) \end{cases}$$
   b)
$$f:\mathbb{R}^2\to\mathbb{R},\ f(x,y)=\begin{cases} (1+x^2y^2)^{\frac{1}{x^2+y^2}} & ,(x,y)\neq(0,0) \\ 1 & ,(x,y)=(0,0) \end{cases}$$

3. Verificati daca functiile urmatoare isi ating valorile extreme si determinati aceste valori
   a) $f:\mathbb{R}^2\to\mathbb{R},\quad f(x,y)=\frac{xy}{x^2+y^2+1}$
   b) $f:\overline{B}(O_2,1)\to\mathbb{R},\quad f(x,y)=x^2+xy+y^2$
   c) $f:A\to\mathbb{R},\quad f(x,y)=xy+\frac{1}{xy},\quad A=\{(x,y)\in\mathbb{R}^2 \,\big|\, x>0, y>0, x+y<2\}$

4. Se da multimea $A=\overline{B}(O_3,1)\setminus\{O_3\}$ si functia $f:A\to\mathbb{R}$, $f(x,y,z)=\frac{x+y+z}{\sqrt{x^2+y^2+z^2}}$
   a) Exista limita functiei f in origine?
   b) Este multimea A compacta?
   c) Determinati valorile extreme ale lui f pe multimea A. Atinge functia f aceste valori?