Derivate

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

Derivatele funcțiilor elementare

Exemple

1.
$$c' = 0$$

2.
$$x' = 1$$

$$3. \qquad (x^n)' = nx^{n-1}$$

$$4. \qquad \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

5.
$$\left(\sqrt[n]{x}\right)' = \frac{1}{\sqrt[n]{\sqrt{x^{n-1}}}}$$

$$6. \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

7.
$$(e^x)' = e^x$$

8.
$$(a^x)' = a^x \ln a$$

$$9. (lnx)' = \frac{1}{x}$$

$$10. \quad (\log_a x)' = \frac{1}{x \ln a}$$

11.
$$(\sin x)' = \cos x$$

12.
$$(\cos x)' = -\sin x$$

13.
$$(tg x)' = \frac{1}{\cos^2 x} = 1 + tg^2 x$$

$$(ctg x)' = -\frac{1}{\sin^2 x}$$

15.
$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

16.
$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

17.
$$(arctg \ x)' = \frac{1}{x^2 + 1}$$

18.
$$(arcctg x)' = -\frac{1}{x^2 + 1}$$

19.
$$\left(\sqrt{x^2 + a^2}\right)' = \frac{x}{\sqrt{x^2 + a^2}}$$

20.
$$\left(\sqrt{x^2 - a^2}\right)' = \frac{x}{\sqrt{x^2 - a^2}}$$

21.
$$\left(\sqrt{a^2 - x^2}\right)' = \frac{-x}{\sqrt{a^2 - x^2}}$$

Reguli de derivare

$$(f \pm g)' = f' \pm g'$$

$$(cf)' = cf'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$

Ecuația tangentei la graficul funcției f în punctul $(x_0, f(x_0)) \in G_f$ $y - f(x_0) = f'(x_0) \cdot (x - x_0)$

$$(x^5)' = 5x^4$$

9' = 0

$$\left(\sqrt[3]{x}\right)' = \frac{1}{\sqrt[3]{x^2}} sau\left(\sqrt[3]{x}\right)' = \left(x^{\frac{1}{3}}\right)' = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{\frac{-2}{3}}$$

$$(7^x)' = 7^x \ln 7$$

$$(\log_2 x)' = \frac{1}{x \ln 2}$$

$$\left(\sqrt{x^2+1}\right)' = \frac{x}{\sqrt{x^2+1}}$$

$$\left(\sqrt{x^2+1}\right)' = \frac{x}{\sqrt{x^2+1}}$$

$$\left(\sqrt{x^2 - 4}\right)' = \frac{x}{\sqrt{x^2 - 4}}$$

$$\left(\sqrt{7-x^2}\right)' = \frac{-x}{\sqrt{7-x^2}}$$

Exemple

$$(x + \sin x)' = x' + (\sin x)' = 1 + \cos x$$

$$(5x^2)' = 5(x^2)' = 10x$$

$$(xe^x)' = 3(x') - 10x$$

 $(xe^x)' = x'e^x + x(e^x)' = e^x + xe^x$

$$\left(\frac{e^x}{x}\right)' = \frac{(e^x)'x - e^xx'}{x^2} = \frac{xe^x - e^x}{x^2}$$

$$\left(\frac{1}{x^3}\right)' = -\frac{(x^3)'}{x^6} = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

$$sau\left(\frac{1}{x^3}\right)' = (x^{-3})' = -3x^{-4}$$

Scrieți ecuația tangentei la graficul funcției

$$f: \mathbb{R} \to \mathbb{R}$$
, $f(x) = x + e^x$ în punctul de abscisă 1.

$$y - f(1) = f'(1) \cdot (x - 1)$$

$$f'(x) = 1 + e^x \to f'(1) = 1 + e$$

$$y - (1 + e) = (1 + e) \cdot (x - 1) \rightarrow y = (1 + e)x$$