I. $\sum_{m=1}^{\infty} \frac{(-1)^m}{m! \cdot 2^m} \cdot (2x-8)^m; x \in \mathbb{R}$

a) Notăm $b_m = \frac{(-1)^m}{m! \cdot 2^m}$, $y = 2x-8$ $\Big\} \Rightarrow \sum_{m=1}^{\infty} b_m \cdot y^m$; $\omega = \overline{\lim_{m\to\infty}} \left|\frac{b_{m+1}}{b_m}\right| = \overline{\lim_{m\to\infty}} \left|\frac{\cancel{(-1)^{m+1}}}{(m+1)! \cdot \cancel{2^{m+1}}} \cdot \frac{m! \cancel{2^m}}{\cancel{(-1)^m}}\right| =$

$= \overline{\lim_{m\to\infty}} \frac{1}{2(m+1)} = 0 \Rightarrow R = \frac{1}{\omega} = \infty$

$\Rightarrow$ conform teoremei Abel: pt. $y \in (-\infty, \infty)$ seria e absolut convergentă

$-\infty < 2x-8 < \infty \,/ +8 \Leftrightarrow -\infty < 2x < \infty \,/ \frac{1}{2} \Rightarrow x \in \mathbb{R}; C = \mathbb{R}$

b)

$$\boxed{\sum_{m=1}^{\infty} q^m = \frac{q}{1-q} \text{ și } \sum_{m=0}^{\infty} \frac{a^m}{m!} = e^a, \forall a \in \mathbb{R}}$$

$S(x) = \sum_{m=1}^{\infty} \frac{(-1)^m}{m! \cdot 2^m} \cdot (2x-8)^m$

$S(x) = \sum_{m=1}^{\infty} \frac{\left(\frac{8-2x}{2}\right)^m}{m!} = \sum_{m=1}^{\infty} \frac{(4-x)^m}{m!}$

$a_m = S(m) + 1 = e^{4-m} \cancel{-1+1} = e^{4-m} \Rightarrow \sum_{m=0}^{\infty} a^m = \sum_{m=0}^{\infty} e^{4-m} = e^4 \sum_{m=0}^{\infty} e^{-m} = e^4 \sum_{m=0}^{\infty} \left(\frac{1}{e}\right)^m = e^4 \cdot \frac{1}{1-\frac{1}{e}} = \frac{e^5}{e-1}$

II. Determinați pc. de extrem local ale funcției $f: \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = xyz$, condiționate de $x+y+z = 12$

P1: Notăm $g: \mathbb{R}^3 \to \mathbb{R}$, $g(x,y,z) = x+y+z = 12$

$L(x,y,z,\lambda) = f(x,y,z) + \lambda \cdot g = xyz + \lambda \cdot g$

P2: $L'x() = yz + \lambda = 0$

$L'y() = xz + \lambda = 0$

$L'z() = xy + \lambda = 0$

$L'\lambda() = x+y+z-12 = 0$

$\begin{cases} (1)-(2) \Rightarrow z(y-x) = 0 < \\ (2)-(3) \Rightarrow x(z-y) = 0 < \end{cases}$

a) $\begin{cases} z = 0 \\ x = 0 \\ xy + \lambda = 0 \\ x+y+z-12 = 0 \Rightarrow y = 12 \end{cases} \Big| \Rightarrow \lambda = 0$

b) $\begin{cases} z = 0 \\ z-y = 0 \Rightarrow y = 0 \\ xy + \lambda = 0 \Rightarrow x = 12 \\ z+x+y-12 = 0 \Rightarrow x = 12 \end{cases}$

c) $\begin{cases} y-x = 0 \Rightarrow y = 0 \\ x = 0 \\ xy + \lambda = 0 \Rightarrow \lambda = 0 \\ x+y+z-12 = 0 \Rightarrow z = 12 \end{cases}$

d) $\begin{cases} y-x = 0 \\ z-y = 0 \\ xy + \lambda = 0 \Rightarrow \lambda = -16 \\ x+y+z-12 = 0 \Rightarrow 3x = 12; x = 4 \end{cases}$ $\Big| x = y = z$

Avem: $P_1(0,12,0), \lambda_1 = 0$; $P_2(12,0,0), \lambda_2 = 0$; $P_3(0,0,12), \lambda_3 = 0$; $P_4(4,4,4), \lambda_4 = -16$

P3: Aflăm natura pc. staționare:

$L''x^2() = 0; L''y^2() = 0; L''z^2() = 0$; (după formula $\boxed{L''x^2() = [L'x()]'x}$

$L''xy() = z = L''yx()$; $L''xz() = y = L''zx$; $L''yz = x = L''zy()$ (după formula $\boxed{L''xy() = [L'x()]'y}$

$$\boxed{H_L(x,y,z)=\begin{pmatrix} L''_{x^2}() & L''_{xy}() & L''_{xz}() \\ L''_{yx}() & L''_{y^2}() & L''_{yz}() \\ L''_{zx}() & L''_{zy}() & L''_{z^2}() \end{pmatrix}} = \begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix} \Rightarrow H_L(P_1)=H_L(0;12;0)=\begin{pmatrix} 0 & 0 & 12 \\ 0 & 0 & 0 \\ 12 & 0 & 0 \end{pmatrix}$$

$\Delta_1=0 \Rightarrow$ pt. a stabili natura pc. folosim diferențiala de ordin II.

$\boxed{\text{Dacă } \Delta_1,\Delta_2,\Delta_3>0 \Rightarrow \text{pc. de minim local}}$ $\boxed{\text{Dacă } \Delta_1<0,\ \Delta_2>0,\ \Delta_3<0 \Rightarrow \text{pc. de maxim local}}$

$$d^2L(P_1)=L''_{x^2}(P_1)dx^2+L''_{y^2}(P_1)dy^2+L''_{z^2}(P_1)dz^2+2L''_{xy}(P_1)dxdy+2L''_{xz}(P_1)dxdz+$$
$$+2L''_{yz}(P_1)dydz \Rightarrow d^2L(P_1)=24dxdz.$$

Pt. a stabili semnul sau natura funcționalei pătratice se diferențiază legăt.

$$dg(P_1)=0 \Leftrightarrow g'_x(P_1)dx+g'_y(P_1)dy+g'_z(P_1)dz$$

$$g(x,y,z)=x+y+z-12 \Rightarrow \left.\begin{aligned} g'_x(x,y,z)=1 \Rightarrow g'_x(P_1)=1 \\ g'_y(x,y,z)=1 \Rightarrow g'_y(P_1)=1 \\ g'_z(x,y,z)=1 \Rightarrow g'_z(P_1)=1 \end{aligned}\right\} \Rightarrow dx+dy+dz=0 \Rightarrow dz=-dx-dy$$

$$\Rightarrow d^2L(P_1)=24dx(-dx-dy)=-24dx^2-24dxdy=-24(dx^2+dxdy)=-24\Big(\underbrace{dx^2+2dx\cdot\tfrac{1}{2}dy+\tfrac{1}{4}dy^2}_{\text{formulă }(a+b)^2}-\tfrac{1}{4}dy^2\Big)$$

$$\Rightarrow d^2L(P_1)=-24\left(dx+\tfrac{1}{2}dy\right)^2+6dy^2$$

$\Rightarrow d^2L(P_1)$ nedefinită (avem și + și −) $\Rightarrow P_1(0;12;0)$ pc. șa. Analog P_2, P_3.

$$d^2L(P_4)=L''_{x^2}(P_4)dx^2+L''_{y^2}(P_4)dy^2+L''_{z^2}(P_4)dz^2+2L''_{xy}(P_4)dxdy+2L''_{xz}(P_4)dxdz+$$
$$+L''_{yz}(P_4)dydz \Rightarrow d^2L(P_4)=8dxdy+8dxdz+8dydz$$

$$dg(P_4)=0 \Leftrightarrow dx+dy+dz \Rightarrow dz=-dx-dy$$

$$\Rightarrow d^2L(P_4)=8dxdy+8dx(-dx-dy)+8dy(-dx-dy)=-8dx^2-8dxdy-8dy^2=$$
$$=-8\left(dx^2+dxdy+\tfrac{1}{4}dy^2-\tfrac{1}{4}dy^2\right)-8y^2=8\left(dx+\tfrac{1}{2}dy^2\right)-6dy^2=-8u^2-6v^2$$

$\Rightarrow d^2L(P_4)$ negativ definită $\Rightarrow P_4(4,4,4)$ pc. de maxim local.

b) $f:\mathbb{R}^2\to\mathbb{R},\ f(x,y)=\begin{cases} \dfrac{x^3y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0; & (x,y)=(0,0) \end{cases}$

$$0\le \lim_{(x,y)\to(0,0)} f(x,y)=\lim_{(x,y)\to(0,0)}\left|\frac{x^3y^2}{x^2+y^2}\right|\le \lim_{(x,y)\to(0,0)}\frac{|x^3|\cdot \cancel{y^2}}{\cancel{y^2}}=0 \Rightarrow \lim_{(x,y)\to(0,0)} f(x,y)=0=f(0,0)$$

$\Rightarrow f$ continuă în $(0,0)$

Fie $(a,b)\neq(0,0) \Rightarrow \lim\limits_{(x,y)\to(0,0)} f(x,y)=\dfrac{a^3b^2}{a^2+b^2}=f(a,b) \Rightarrow f$ continuă în $(a,b)\ \forall (a,b)\in\mathbb{R}^2\setminus\{(0,0)\}$

• $f:\mathbb{R}^2\to\mathbb{R}, f(x,y)=\begin{cases}\frac{\sqrt{|x||y|^3}}{x^2+y^2}, (x,y)\neq(0,0)\\ 0, (x;y)=(0;0)\end{cases}$; Demonstrați că $\nexists$ lim în $(0,0)$.

Fie $(x_m;y_m)=\left(\frac{1}{m};\frac{1}{m}\right)\xrightarrow[m\to\infty]{}0\left(\frac{1}{m}\neq0\right)\Rightarrow\lim_{m\to\infty}f(x_m;y_m)=\lim_{m\to\infty}\frac{\sqrt{\frac{1}{m}\cdot\frac{1}{m^3}}}{\frac{1}{m^2}+\frac{1}{m^2}}=\frac{1}{2}$ (1)

Fie $(x'_m;y'_m)=\left(\frac{2}{m};\frac{1}{m}\right)\xrightarrow[m\to\infty]{}0\Rightarrow\lim_{m\to\infty}f(x'_m;y'_m)=\frac{\sqrt{\frac{2}{m}\cdot\frac{1}{m^3}}}{\frac{3}{m^2}+\frac{1}{m^2}}=\frac{\sqrt{2}}{5}$ (2)

Din (1),(2) $\Rightarrow\nexists\lim f(x,y)\Rightarrow f$ nu e continuu în $(0,0)$.

III. 1. $y''-5y'+4y=3x+2$

P1. $y''-5y'+4y=0\Rightarrow$ ecuația caracteristică este $r^2-5r+4=0 \left\langle \begin{matrix} r=1\\ r=4\end{matrix}\right.$; cum $r_1\neq r_2$:
$y_0=C_1e^{r_1x}+C_2e^{r_2x}, C_1,C_2\in\mathbb{R}=C_1e^x+C_2e^{4x}$

P2: determinăm o soluție particulară, de forma $y_p=ax+b, a,b\in\mathbb{R}$, $y_p'=a; y_p''=0$ $\Rightarrow$

(1) $\Leftrightarrow 0-5a+4(ax+b)=3x+2\Rightarrow 4ax+4b-5a=3x+2\Rightarrow\begin{cases}4a=3\Rightarrow a=3/4\\4b-5a=2\Rightarrow b=23/16\end{cases}$

$y=y_0+y_p; y=C_1e^x+C_2e^{x}+\frac{3}{4}x+\frac{23}{16}; C_1,C_2\in\mathbb{R}$

2. $9y''-6y'+y=12xe^{-x}$

P1: $9y''-6y'+y=0\Rightarrow 9r^2-6r+1=0; r_1=r_2=r=\frac{1}{3}$
$y_0=e^{r_1x}(C_1+C_2x)=e^{\frac{x}{3}}(C_1+C_2x), C_1,C_2\in\mathbb{R}$

P2: $y_p=(ax+b)\cdot e^{-x}, a,b\in\mathbb{R}\Rightarrow y_p'=a\cdot e^{-x}+(ax+b)(-e^{-x})=e^{-x}(-ax+a-b)$.
$y_p''=-e^{-x}(-ax+a-b)+e^{-x}(-a)=e^{-x}(ax-2a+b)$

(1) $\Rightarrow 9e^{-x}(ax-2a+b)-6e^{-x}(-ax+a-b)+(ax+b)e^{-x}=12xe^{-x}$
$\begin{cases}9a+6a+a=12\Rightarrow a=3/4\\-18a+9b-6a+6b+b=0\Rightarrow -24a+16b=0\Rightarrow 16b=-24a\Rightarrow b=\frac{9}{8}\end{cases}$
$y_p=e^{-x}\left(\frac{3}{4}x+\frac{9}{8}\right)$
$y=y_0+y_p\Rightarrow y=e^{\frac{x}{3}}(C_1x+C_2)+e^{-x}\left(\frac{3}{4}x+\frac{9}{8}\right)$

3. $y''-4y'+13y=3e^{4x}\sin 5x$

P1: $r^2-4r+13=0; \Delta=-36\Rightarrow r_{1,2}=\frac{4\pm i\overbrace{\sqrt{-\Delta}}^{6}}{2}=2\pm3; r_{1,2}\in\mathbb{C}\setminus\mathbb{R}; r_{1,2}=\alpha\pm\beta, \alpha,\beta\in\mathbb{R}$

$y_0 = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x) = e^{2x}(C_1\cos 3x + C_2\sin 3x);\ C_1, C_2 \in \mathbb{R}$

P_2: $y_p = e^{4x}(a\sin 5x + b\cos 5x)$

IV. a) $I = \int_{-\infty}^{-1} \frac{(x+1)^3}{9}\cdot e^{\frac{x+1}{3}}dx;\ \Gamma(a) = \int_0^{\infty} x^{a-1}\cdot e^{-x}dx,\ a>0;$

1. $\Gamma(1)=1;$
2. $\Gamma(m)=(m-1)!,\ \forall m\in\mathbb{N}^*$
3. $\Gamma(a)=(a-1)\Gamma(a-1),\ \forall a>1$
4. $\Gamma(\frac{1}{2})=\sqrt{\pi}$
5. $\Gamma(a)$ convergentă, $\forall a>0$

$\frac{x+1}{3} = -y;\ x=-3y-1 \Rightarrow (x)'dx = (-3y-1)'dy \Rightarrow dx=-3dy$

$x\to-\infty \Rightarrow y\to\infty$ și $x\to -1 \Rightarrow y\to 0$

$I = \int_{\infty}^{0} \frac{(-3y)^3}{9}\cdot e^{-y}(-3)dy = -9\int_0^{\infty} y^3 e^{-y}dy \ \underset{a=4}{\overset{a-1=3}{=}}\ -9\Gamma(4)$

• $I = \int_1^e \frac{1}{x}(\ln x)^2(1-\ln x)^5 dx;$

$\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx$

$\beta(a,b) = \int_0^{\infty} \frac{x^{a-1}}{(1+x)^{a+b}}dx$

$I = \int_1^e \frac{1}{x}(\ln x)^2(1-\ln x)^5 dx;\ \ln x = t \Rightarrow \frac{1}{x}dx = dt;\ x=1\Rightarrow t=0;\ x=e\Rightarrow t=1;$

$I = \int_0^1 t^2(1-t)^5 dt = \beta(3,6) = \frac{\Gamma(3)\Gamma(6)}{\Gamma(3+6)} = \frac{2!5!}{8!}$

• $I = \int_{-2}^{\infty} e^{-x^2-4x+5}dx;\ y=x+2 \Rightarrow dy=dx;\ x=-2\Rightarrow y=0;\ x\to\infty \Rightarrow y\to\infty$

$I = \int_0^{\infty} e^{-(y-2)^2-4(y-2)+5}dy = \int_0^{\infty} e^{-y^2+9}dy = e^9\int_0^{\infty} e^{-y^2}dy$

$\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2};\ \int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$

Met II: $y^2 = t \Rightarrow y = t^{\frac{1}{2}};\ dy = \frac{1}{2}t^{-\frac{1}{2}}dt;\ y\to 0 \Rightarrow t\to 0;\ y\to\infty \Rightarrow t\to\infty$

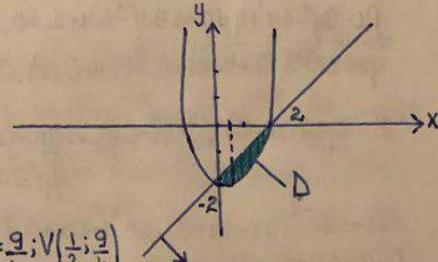
$I = e^9\int_0^{\infty} e^{-t}\frac{1}{2}t^{-\frac{1}{2}}dt = \frac{e^9}{2}\int_0^{\infty} t^{-\frac{1}{2}}e^{-t}dt \ \underset{a=\frac{1}{2}}{\overset{a-1=-\frac{1}{2}}{=}}\ \frac{e^9}{2}\Gamma(\frac{1}{2})$

b) $\Delta = \{(x,y)\in\mathbb{R} / y\le x-2;\ y\ge x^2-x-2\}$

$\iint_{\Delta}(2x-y)dxdy;\ y=x-2;\ x=0\Rightarrow y=-2 \Rightarrow (0;-2)$

$x=2 \Rightarrow y=0 \Rightarrow (2;0)$

$y\le x-2;\ (0,0) \Rightarrow 0\le -2\ (F)$



$y = x^2-x-2;\ x_V = -\frac{b}{2a} = \frac{1}{2};\ y_V = -\frac{\Delta}{4a} = f(x_V) = (\frac{1}{2})^2 - \frac{1}{2} - 2 = \frac{9}{4};\ V(\frac{1}{2};\frac{9}{4})$

$d\cap P \begin{cases} y=x-2; \\ y=x^2-x-2;\ x-2=x^2-x-2 \Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0 \end{cases}$ $\begin{matrix} x_1=0\Rightarrow y_1=-2 \\ x_2=0 \Rightarrow y_2=0 \end{matrix} \Rightarrow d\cap P = \{(0,-2);(2;0)\}$

Luăm $(0,0) \Rightarrow 0\ge -2\ (A);\ D = \{(x,y)\in\mathbb{R}^2 / 0\le x\le 2,\ x^2-x-2\le y\le x-2\}$

$I = \int_0^2\left(\int_{x^2-x-2}^{x-2}(2x-y)dy\right)dx = \int_0^2 \left(2xy - \frac{y^2}{2}\right)\Big|_{x^2-x-2}^{x-2} dx = \int_0^2 \left\{\left[2x(x-2) - \frac{(x-2)^2}{2}\right] - \left[2x(x^2-x-2) - \frac{(x^2-x-2)^2}{2}\right]\right.$

• $\Delta = \{(x,y)\in\mathbb{R}^2 / 3x-2y+6\ge 0;\ 3x+2y-6\le 0,\ y\ge 0\}$

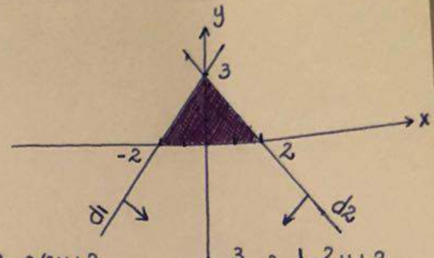
$d_1: 3x-2y+6=0$ $\quad$ $d_2: 3x+2y-6=0$

$x=0 \Rightarrow y=3 \Rightarrow (0,3)$ $\quad$ $x=0 \Rightarrow y=3 \Rightarrow (0,3)$

$y=0 \Rightarrow x=-2 \Rightarrow (-2;0)$ $\quad$ $y=0 \Rightarrow x=2 \Rightarrow (2;0)$

$3x-2y+6 \geq 0$ $\quad$ $3x+2y-6 \leq 0$

$(0;0) \Rightarrow 6 \geq 0\,(A) \Rightarrow (0,0) \in f_1$ $\quad$ $(0;0) \Rightarrow -6 \leq 0\,(A) \Rightarrow (0,0) \in f_2$

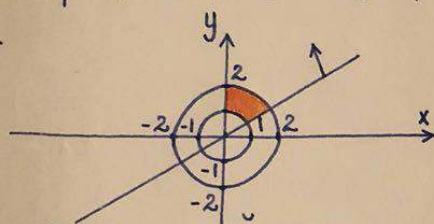


Observăm $(0,3)$ pc. comunal celor 2

$$D=\left\{(x,y)\in\mathbb{R}^2 / 0\leq y\leq 3;\ \frac{2}{3}y-2\leq x\leq -\frac{2}{3}y+2\right\};\ I=\int_0^3\left(\int_{2/3y-2}^{-2/3y+2} xy\,dx\right)dy=\int_0^3 \frac{x^2}{2}y\,\Bigg|_{\frac{2}{3}y-2}^{-\frac{2}{3}y+2}$$

• $D=\{(x,y)\in\mathbb{R}^2 / 0\leq x\leq y;\ 1\leq x^2+y^2\leq 4\}$; $C_r(a,b): (x-a)^2+(y-b)^2=r^2 \Rightarrow C_1(0,0): x^2+y^2=1;\ x^2+y^2\geq 1$

$C_2(0,0): x^2+y^2=4;\ x^2+y^2\leq 4$



S.V. $\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases} \Rightarrow D=D'=\left\{(\rho;\theta)\in\mathbb{R}^2 / 1\leq\rho\leq 2;\ \frac{\pi}{4}\leq\theta\leq\frac{\pi}{2}\right\}$

$$I=\iint_D f(x,y)\,dx\,dy=\iint_{D'} f(\rho\cos\theta;\rho\sin\theta)\left|\frac{D(x,y)}{D(\rho;\theta)}\right|d\rho\,d\theta \Rightarrow I=\iint_{D'}\sqrt{\rho^2\cos^2\theta+\rho^2\sin^2\theta}\cdot\rho\,d\rho\,d\theta$$

$$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\int_1^2\rho^2\,d\rho\right)d\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\rho^3}{3}\Bigg|_1^2 d\theta=\frac{7}{3}\theta\,\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

*1: $x^4y'=4x^3y+x^8e^{-4y/x^4}$ fol. s.v. $z=\frac{y}{x^4}$, $x>0$; $\begin{cases} y=zx^4 \\ y'=z'x^4+z4x^3 \end{cases} \Rightarrow x^4(z'x^4+4x^3z)=4x^3\cdot zx^4+x^8e^{-4z}$

$\Rightarrow x^8z'+4x^7z=4x^7z+x^8e^{-4z} \,/\cdot\frac{1}{x^8}, x\neq 0 \Rightarrow z'=e^{-4z} \Leftrightarrow \frac{dz}{dx}=e^{-4z};\ \frac{dz}{e^{-4z}}=1\,dx$

$\Rightarrow \int e^{4z}dz=\int dx;\ \frac{1}{4}e^{4z}=x+C;\ C\in\mathbb{R}\,/\cdot 4;\ e^{4z}=4x+C;\ e=4x+C, C\in\mathbb{R}$

*2: $x^2+y^2=xyy' \,/\cdot\frac{1}{xy}, x\neq 0, y\neq 0;$

$y'=\frac{x}{y}+\frac{y}{x};\ y'=f\left(\frac{y}{x}\right)\to$ ec. dif. omogenă

S.V. $\frac{y}{x}=z \Rightarrow y=xz \Rightarrow y'=z+xz'$

$\frac{dz}{dx}=\frac{1}{x}\cdot\frac{1}{z} \Leftrightarrow \int z\,dz=\int\frac{1}{x}dx$

$\frac{z^2}{2}=\ln|x|+C;\ C\in\mathbb{R}\,/\cdot 2;\ z^2=2\ln|x|+C$

$z^2=\ln x^2+C;\ \frac{y^2}{x^2}=\ln x^2+C \Rightarrow y^2=x^2(C+\ln x^2)$

*3: $y'+2xy=2x\cdot e^{-x^2}$

$\boxed{y'+P(x)y+Q(x)=0}$ ec. dif. liniară

P1: rezolvăm ec. atașată: $y'+2xy=0;\ y'=-2xy \Leftrightarrow$

$\frac{dy}{dx}=-2xy \Rightarrow \int\frac{1}{y}dy=-\int 2x\,dx \Leftrightarrow \ln|y|=-x^2+C, C\in\mathbb{R}$

P2: determinăm o sol. de forma $y=k(x)\cdot e^{-x^2}$:

$y'=k'(x)\cdot e^{-x^2}-2xe^{-x^2}\cdot k(x) \Rightarrow k'(x)\cdot e^{-x^2}-2xe^{-x^2}\cdot k(x)+$

$+2x\cdot k(x)e^{-x^2} \Rightarrow k'(x)=2x \Rightarrow k(x)=\int 2x=x^2+C$

$y=k(x)e^{-x^2} \Rightarrow y=(x^2+C)e^{-x^2}, C\in\mathbb{R}$