Integrale

Integrale nedefinite

Exemple

1. $\int dx = x + C$ — $\int 5dx = 5\int dx = 5x + C$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ — $\int x^3 dx = \frac{x^4}{4} + C$
3. $\int x\,dx = \frac{x^2}{2} + C$
4. $\int \sqrt{x}\,dx = \frac{2}{3}x\sqrt{x} + C$ — $\int \sqrt[7]{x}\,dx = \int x^{\frac{1}{7}}dx = \frac{x^{\frac{1}{7}+1}}{\frac{1}{7}+1} + C$
5. $\int \frac{dx}{x} = \ln|x| + C$
6. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$ — $\int \frac{dx}{2x-1} = \frac{1}{2}\ln|2x-1| + C$
7. $\int e^x dx = e^x + C$
8. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ — $\int 2^x\,dx = \frac{2^x}{\ln 2} + C$
9. $\int \sin x\,dx = -\cos x + C$
10. $\int \cos x\,dx = \sin x + C$
11. $\int tg\,x\,dx = -\ln|\cos x| + C$
12. $\int ctg\,x\,dx = \ln|\sin x| + C$
13. $\int \frac{dx}{\sin^2 x} = -ctg\,x + C$
14. $\int \frac{dx}{\cos^2 x} = tg\,x + C$
15. $\int (1+tg^2x)\,dx = tg\,x + C$
16. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$ — $\int \frac{dx}{x^2-4} = \frac{1}{2\cdot 2}\ln\left|\frac{x-2}{x+2}\right| + C$
17. $\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$
18. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}arctg\,\frac{x}{a} + C$ — $\int \frac{dx}{x^2+5} = \frac{1}{\sqrt{5}}arctg\,\frac{x}{\sqrt{5}} + C$
19. $\int \frac{dx}{x^2+1} = arctg\,x + C$
20. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) + C$ — $\int \frac{dx}{\sqrt{x^2+11}} = \ln(x+\sqrt{x^2+11}) + C$
21. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$ — $\int \frac{dx}{\sqrt{x^2-8}} = \ln\left|x+\sqrt{x^2-8}\right| + C$
22. $\int \frac{dx}{\sqrt{a^2-x^2}} = arcsin\,\frac{x}{a} + C$ — $\int \frac{dx}{\sqrt{4-x^2}} = arcsin\,\frac{x}{2} + C$
23. $\int \frac{dx}{\sqrt{1-x^2}} = arcsin\,x + C$
24. $\int \frac{x}{\sqrt{x^2 \pm a^2}}dx = \sqrt{x^2 \pm a^2} + C$ — $\int \frac{x}{\sqrt{x^2-4}}dx = \sqrt{x^2-4} + C$
25. $\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} + C$ — $\int \frac{x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2} + C$
26. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$
27. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$
28. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}arcsin\,\frac{x}{a} + C$

Formula Leibniz-Newton

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

$$\int_0^1 (x^2+x+1)dx = \left(\frac{x^3}{3} + \frac{x^2}{2} + x\right)\Big|_0^1 = \frac{1}{3} + \frac{1}{2} + 1 = \frac{11}{6}$$

Arie

$$A = \int_a^b f(x)dx$$

Calculaţi aria determinată de graficul funcţiei $f:\mathbb{R}\to\mathbb{R},\ f(x) = x + e^x$, axa Ox şi de dreptele de ecuaţii x=0 şi x=1.

$$A = \int_0^1 (x+e^x)dx = \left(\frac{x^2}{2} + e^x\right)\Big|_0^1 = \frac{1}{2} + e - 1 = e - \frac{1}{2}$$

Volum

$$V = \pi\int_a^b f^2(x)dx$$

Calculaţi volumul determinat de graficul funcţiei $f:[0,1]\to\mathbb{R},\ f(x) = x + e^x$ prin rotirea în jurul axei Ox.

$$V = \pi\int_0^1 (x+e^x)^2dx = \pi\int_0^1 (x^2 + 2xe^x + e^{2x})dx =$$

$$= \pi\left(\frac{x^3}{3} + 2xe^x - 2e^x + \frac{e^{2x}}{2}\right)\Big|_0^1 =$$

$$= \pi\left(\frac{1}{3} + \frac{e^2}{2}\right) - \pi\left(-2 + \frac{1}{2}\right) = \pi\left(\frac{11}{6} + \frac{e^2}{2}\right)$$