Lab 1

Monday, February 21, 2022

Formula lui Taylor

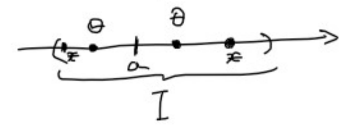
I = interval deschis, $C^k(I) = \{ f : I \to \mathbb{R} : \exists f^{(i)}, i=\overline{0,k}, f^{(k)} \text{ cont.} \}$

$\boxed{T_1}$ (formula lui Taylor cu rest Lagrange)

$f \in C^{n+1}(I)$, $a \in I \Rightarrow f(x) = \underbrace{T_n f(x)}_{\text{pol. Taylor de ord } n \text{ în jurul lui } a} + \underbrace{R_n f(x)}_{\text{restul}}$, $x \in I$,

unde $T_n f(x) = f(a) + \frac{f'(a)}{1!} \cdot (x-a) + \dots + \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$

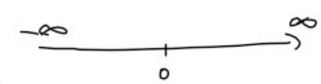
$\underbrace{R_n f(x)}_{\text{restul Lagrange}} = \frac{f^{(n+1)}(\theta_{x,a})}{(n+1)!} \cdot (x-a)^{n+1}$, unde $\theta_{x,a}$ pct. intermediar între x şi a



$a=0 \longrightarrow$ formula MacLaurin

Serii de puteri MacLaurin

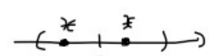
$e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \dots + \frac{1}{n!}x^n + \dots$, $x \in \underbrace{\mathbb{R}}_{(-\infty,\infty)}$



$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots + \frac{(-1)^n}{(2n+1)!} \cdot x^{2n+1} + \dots$, $x \in \mathbb{R}$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots + \frac{(-1)^n}{(2n)!} \cdot x^{2n} + \dots$, $x \in \mathbb{R}$

$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$, $|x|<1$



$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{n-1}}{n} \cdot x^n + \dots$, $|x|<1$

$(1+x)^k = 1 + \sum_{n=1}^{\infty} \underbrace{\binom{k}{n}}_{\frac{k \cdot (k-1) \cdot \ldots \cdot (k-(n-1))}{n!}} \cdot x^n$, $|x|<1$

$(k \in \mathbb{R})$

ex.: $(1+x)^{-\frac{1}{2}} = \frac{1}{\sqrt{1+x}}$

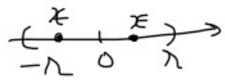
$\boxed{T_2}$ (crit. Leibniz pt. serii alternante)

$S = \sum_{k=0}^{\infty} (-1)^k a_k$, $(a_k)_{k \in \mathbb{N}}$ desc. cu lim. $= 0$ ($a_k \searrow 0$)

$S_n = \sum_{k=0}^{n} (-1)^k a_k \Rightarrow |S - S_n| \leq a_{n+1}$

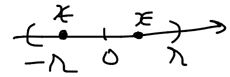
$\boxed{T_3}$ (teor. lui Abel pt. serii)

$f(x) = \sum^{\infty} a_k x^k$, $|x| < r$



$\boxed{T3}$ (teor. ...

$$f(x)=\sum_{k=0}^{\infty} a_k x^k, \quad |x|<r$$



$$\sum_{k=0}^{\infty} a_k r^k \text{ conv.} \Longrightarrow \lim_{x\nearrow r} f(x)=\sum_{k=0}^{\infty} a_k r^k$$

<u>Pb.5</u> $\ln 2 = ?$

$$\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\dots+\frac{(-1)^{n-1}}{n}x^n+\dots, \quad |x|<1$$

Seria in $x=1$: $1-\frac{1}{2}+\frac{1}{3}-\dots+\frac{(-1)^{n-1}}{n}+\dots$ este alternantă

$\boxed{T_2} \Rightarrow$ seria conv. $\overset{\boxed{T3}}{\Longrightarrow} \underbrace{\lim_{x\nearrow 1} \ln(1+x)}_{\ln 2} = 1-\frac{1}{2}+\dots+\frac{(-1)^{n-1}}{n}+\dots$

$$\Rightarrow \ln 2 = 1-\frac{1}{2}+\dots+\frac{(-1)^{n-1}}{n}+\dots$$

$$\boxed{T_2} \Rightarrow \left|\ln 2-\left(1-\frac{1}{2}+\dots+\frac{(-1)^{n-1}}{n}\right)\right| \le \frac{1}{n+1}$$

$$(?)\quad \frac{1}{n+1}\le\frac{1}{10^5} \Rightarrow \left|\ln 2-\left(1-\frac{1}{2}+\dots+\frac{(-1)^{n-1}}{n}\right)\right| \le \frac{1}{10^5}$$

<u>ex</u>: $1{,}00001 \simeq 1$ $\qquad |0{,}9999-1| \le 10^{-4}$

$0{,}9999 \simeq 1$