Representations and Exploration for Deep Reinforcement Learning using Singular Value Decomposition

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Motivation

Current RL systems face challenges in environments with one or more of the following:

- Complex Observations (often multimodal)
- Partial Observability
- Sparse Reward Signals

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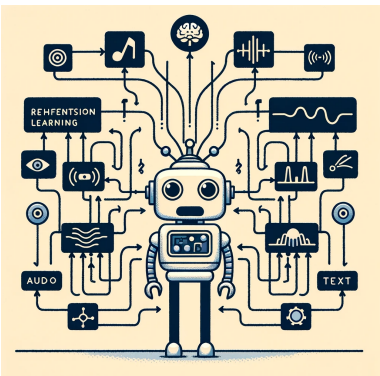

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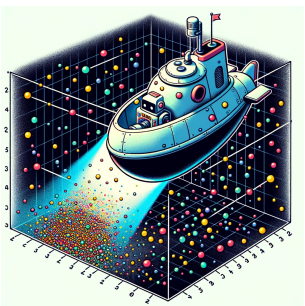

Figure: RL agent exploring a good representation space

Previous Approaches

Representation Learning:

Similar to an AutoEncoder, they try to derive useful representations by reconstructing observations.

Issues

- Can become impractical in complex observation scenarios.
- Prone to representation collapse when done in the latent space.

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Exploration:

Count-based exploration bonuses can lead an agent to discover useful regions

Issues

- Exact counts can be impractical in rich-observation settings.
- Decoupled from the agent's state representation, leading to suboptimal exploration.

Proposed solution

Auxiliary loss to drive **representation learning**.

$$\mathcal{L}_{\text{diag}}\left(\mathbf{\Sigma}(\theta)\right) \coloneqq \frac{1}{k}\sum_{i=1}^{k}\mathbf{\Sigma}(\theta)_{[i,i]},$$

$$\mathcal{L}_{\text{off}}\left(\mathbf{\Sigma}(\theta)\right) \coloneqq \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\mathbf{\Sigma}(\theta)^2_{[i,j]}.$$

Figure: Proposed losses (to be explained)

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Proposed solution

Auxiliary loss to drive **representation learning**.

1. Preserves transition structure
2. Provides pseudo-counts for **exploration bonus**
3. Suitable for large-scale problems
4. Effective in tabular and function approximation

$$\mathcal{L}_{\text{diag}}\left(\mathbf{\Sigma}(\theta)\right) \coloneqq \frac{1}{k}\sum_{i=1}^{k}\mathbf{\Sigma}(\theta)_{[i,i]},$$

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Markov Decision Process Set Up

The MDP is defined as a tuple $(\mathcal{S}, \mathcal{A}, p, r, d_0)$ such that:

- $\mathcal{S}$ - the set of finite stets
- $\mathcal{A}$ - set of finite actions
- p - the transition function
- r - the reward function
- d_0 - the starting state

Other notations

- $P_\pi \in \mathbb{R}^{|S|\times|S|}$ - the transition matrix induced by π
- $J(\pi) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R_t\right]$ - performance of policy π
- $\gamma \in [0, 1)$ - the discount factor
- R_t - reward observed at time t

Methodology

Why SVD decomposition?

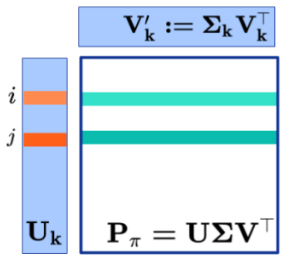
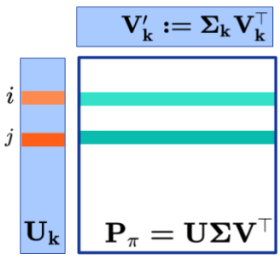


Figure: top-k SVD

Why SVD decomposition?

- Similar states will correspond to rows of U that are similar as well



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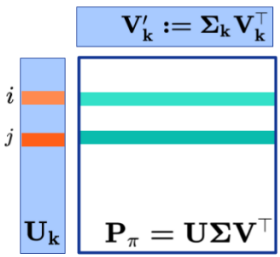

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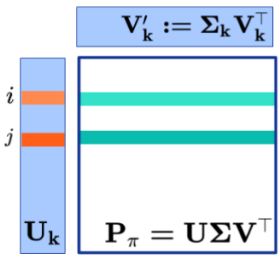

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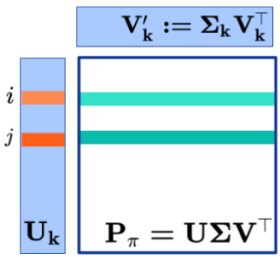
+ It generalizes the eigen decomposition of P_π
+ It always exists



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We will decompose DP_π ($DP_\pi \propto P_\pi$ when d is uniform).

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- In the practical case, it is impossible to visit every single state
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- ↑ When estimating DP_π, filling the corresponding row with 0 is a good estimation

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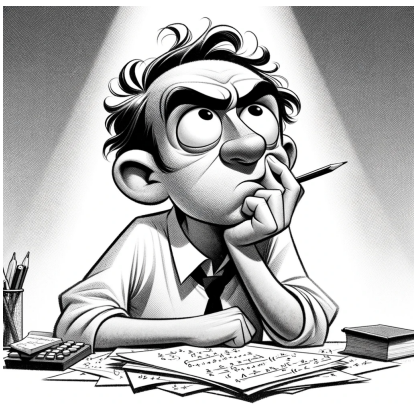
We provide a sample estimation of DP_π that we would like to decompose. However, we will **never** actually construct this.

Then how are we going to estimate DP_{π}?



Representation Learning: Estimator of DP_π

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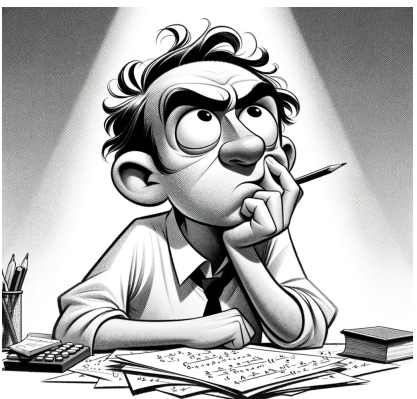
Theorem

A_n is an unbiased and a consistent estimator of DP_π, i.e.,
$\forall j, \mathbb{E}[A_{n,[i,j]}] = DP_{\pi_{[i,j]}}, \quad A_{n,[i,j]} \xrightarrow{a.s.} DP_{\pi_{[i,j]}}$.

Can the estimator and the SVD decomposition lead me to a loss function?



Representation Learning: Loss function

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Now, let $f_U : S \to \mathbb{R}^{|S|}$ and $g_V : S \to \mathbb{R}^{|S|}$:

$$\frac{1}{n} \sum_{i=1}^{n} f_U(x_i) g_V(y_i)^T = \Sigma.$$

Representation Learning: Loss function

If we want the top-k decomposition, let $\hat{U} \in \mathbb{R}^{|S| \times k}$ and $\hat{V} \in \mathbb{R}^{|S| \times k}$ be the estimates for top-k left and right singular vectors ($\Sigma(\hat{U}, \hat{V}) \in \mathbb{R}^{k \times k}$):

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$$\hat{U}^*, \hat{V}^* := \arg\min_{\hat{U}, \hat{V}} - \sum_{i=1}^{k} \Sigma(\hat{U}, \hat{V})_{[i,i]}$$
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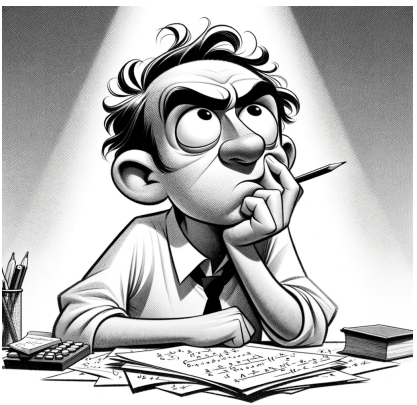
Can we extend it for Function approximation?



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We need:

- Soft constraints
- Batch normalization to constrain the weights of U and V to remain orthogonal matrices

Representation Learning: Loss with soft constraints

Let functions $f_{\theta_1} : \mathbb{R}^d \to \mathbb{R}^k$ and $g_{\theta_2} : \mathbb{R}^d \to \mathbb{R}^k$ that are parameterized by weights $\theta := [\theta_1, \theta_2]$ and take states as inputs:

$$\Sigma(\theta) := \frac{1}{n} \sum_{i=1}^{n} f_{\theta_1}(s_i) g_{\theta_2}(s_i)^\top \in \mathbb{R}^{k \times k}. \tag{1}$$

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Specifically, let

$$\mathcal{L}_{diag}(\Sigma(\theta)) := -\frac{1}{k} \sum_{i=1}^{k} \Sigma(\theta)_{i,i}, \tag{2}$$

$$\mathcal{L}_{off}(\Sigma(\theta)) := \frac{1}{k^2 - k} \sum_{i,j=1, i \neq j}^{k} \Sigma(\theta)_{i,j}^2. \tag{3}$$

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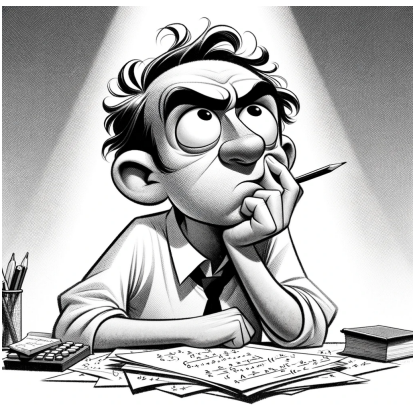
We define a loss with soft constraints by combining (2) and (3):

$$\mathcal{L}(\Sigma(\theta)) := -L_{diag}(\Sigma(\theta)) + \lambda_r L_{off}(\Sigma(\theta)),$$

How do we make it work with a mini-batch?



Representation Learning: Mini-batch Optimisation

Let us define:

$$\hat{\Sigma}(\theta) = \frac{1}{b} \sum_{i=1}^{b} f_{\theta_1}(s_i) g_{\theta_2}(s_i)^T,$$

where $b << n$ is the minibatch

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Theorem

Gradient of $L_{diag}(\hat{\Sigma}(\theta))$ is an unbiased estimator of $L_{diag}(\Sigma(\theta))$, and gradient of $L_{off}(\hat{\Sigma}(\theta))$ is in general a biased estimate of $L_{off}(\Sigma(\theta))$, i.e.,

$$\mathbb{E}\left[\frac{\partial}{\partial\theta}\mathcal{L}_{diag}(\hat{\Sigma}(\theta))\right] = \frac{\partial}{\partial\theta}\mathcal{L}_{diag}(\Sigma(\theta)),$$

$$\mathbb{E}\left[\frac{\partial}{\partial\theta}\mathcal{L}_{off}(\hat{\Sigma}(\theta))\right] \neq \frac{\partial}{\partial\theta}\mathcal{L}_{off}(\Sigma(\theta)),$$

Proof.

$$\mathbb{E}\left[\frac{\partial}{\partial\theta}\,\mathcal{L}_{\text{off}}\left(\frac{1}{b}\Sigma(\theta)\right)\right] = \mathbb{E}\left[\frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\left(\frac{1}{b}\Sigma(\theta)^2_{[i,j]}\right)\right],$$

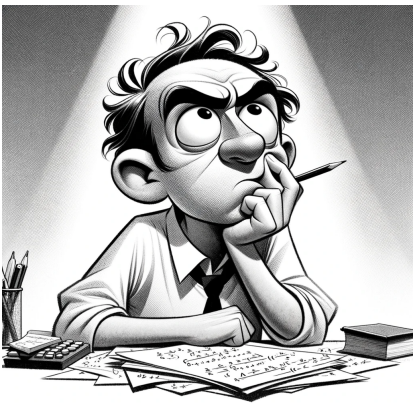
$$= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(\frac{1}{b}\Sigma(\theta)^2_{[i,j]}\right)\right],$$

$$= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(f_{\theta_1}(S)g_{\theta_2}(S')^\top_{[i,j]}\right)^2\right],$$

$$\neq \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(f_{\theta_1}(S)g_{\theta_2}(S')^\top_{[i,j]}\right)\right]^2,$$

$$= \frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\Sigma(\theta))$$

Is there a way in which we can avoid this issue?



Representation Learning: Double-sampling trick

We estimate a separate batch of data:

$$\hat{\hat{\Sigma}}(\theta) = \frac{1}{b_2} \sum_{i=1}^{b_2} f_{\theta_1}(s_i) g_{\theta_2}(s_i)^T,$$

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We define $\hat{\mathcal{L}}(\theta)$ as:

$$\hat{\mathcal{L}}_{\text{off}}(\theta) := \frac{1}{(k^2 - k)} \sum_{i,j=1, i \neq j}^{k} \hat{\Sigma}(\theta)_{[i,j]} \cdot sg\left(\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right)$$
$$+ \frac{1}{(k^2 - k)} \sum_{i,j=1, i \neq j}^{k} \hat{\hat{\Sigma}}(\theta)_{[i,j]} \cdot sg\left(\hat{\Sigma}(\theta)_{[i,j]}\right),$$

where sg is the stop gradient sign

Representation Learning: Double-sampling trick

$$\hat{\mathcal{L}}_{\text{off}}(\theta) := \frac{1}{(k^2-k)} \sum_{i,j=1, i\neq j}^{k} \hat{\Sigma}(\theta)_{[i,j]} \cdot sg\left(\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right)$$

$$+ \frac{1}{(k^2-k)} \sum_{i,j=1, i\neq j}^{k} \hat{\hat{\Sigma}}(\theta)_{[i,j]} \cdot sg\left(\hat{\Sigma}(\theta)_{[i,j]}\right),$$

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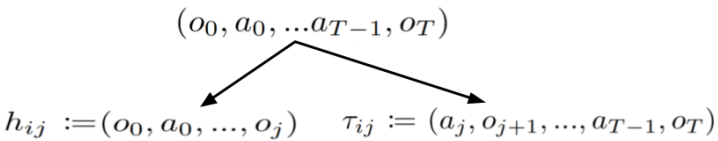
Theorem

Gradient $\frac{\partial}{\partial\theta}\hat{\mathcal{L}}_{off}(\theta)$ *is an unbiased estimator of* $\frac{\partial}{\partial\theta}\mathcal{L}_{off}(\Sigma(\theta))$*, i.e.,*

$$\mathbb{E}\left[\frac{\partial}{\partial\theta}\hat{\mathcal{L}}_{off}(\theta)\right] = \frac{\partial}{\partial\theta}\mathcal{L}_{off}(\Sigma(\theta))$$

Representation Learning: Partial Observability

Use trajectories of consecutive states as input instead of individual states, it should compensate the lack of information.



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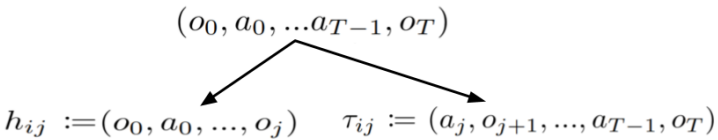

$$\Sigma(\theta) := \frac{1}{nT} \sum_{i=1}^{n} \sum_{j=0}^{T-1} f_{\theta_1}(h_{ij}) g_{\theta_2}(\tau_{ij})^\top$$

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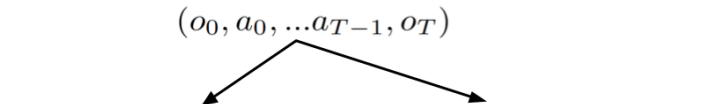

$$\Sigma(\theta) := \frac{1}{nT} \sum_{i=1}^{n} \sum_{j=0}^{T-1} f_{\theta_1}(h_{ij}) g_{\theta_2}(\tau_{ij})^\top$$

- After a state-processing network, an LSTM is used to produce the representation of a trajectory.

Representation Learning: Partial Observability

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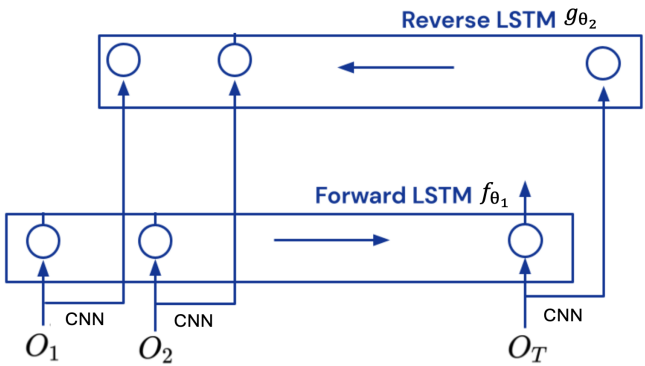

Exploration Bonus: Estimation of state visitation frequency

Let the decomposition of $DP_\pi = U\Sigma V^T$ and f_U the function parameterized with the left singular values U. Let $\Lambda = \Sigma^2$.

Theorem

If $P^T P_\pi$ and D are invertible, then for $\alpha_s := \left(P^T P_\pi\right)^{-1}_{[s,s]}$,

$$\|f_U(x_s)\|^2_{\Lambda^{-1}} = \frac{\alpha_s}{d(s)^2}.$$

- *The norms of the representations can be used to indicate pseudo-counts*
- *Estimating the bonus only requires computing the norm*

Exploration Bonus: Visualization

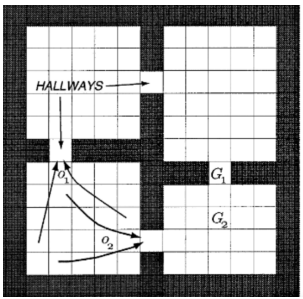

Figure: 4-room environment

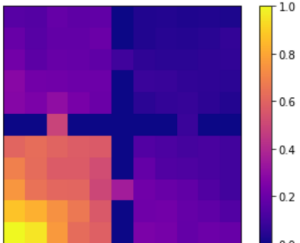

Figure: Visit frequency with a random policy

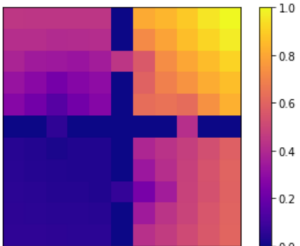

Figure: Bonus constructed using the learned representation

Criticism: Mathematics

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The authors tend to:

- remove important constraints
- mislead readers into believe their contributions are sound

.

Criticism: Flaw in Exploration Bonuses

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Reminder of Pseudo-count Theorem:

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Useless if α_s varies a lot

1. Why call it "state-dependant constant"?
2. No mathematical analyses to check the behavior of α_s

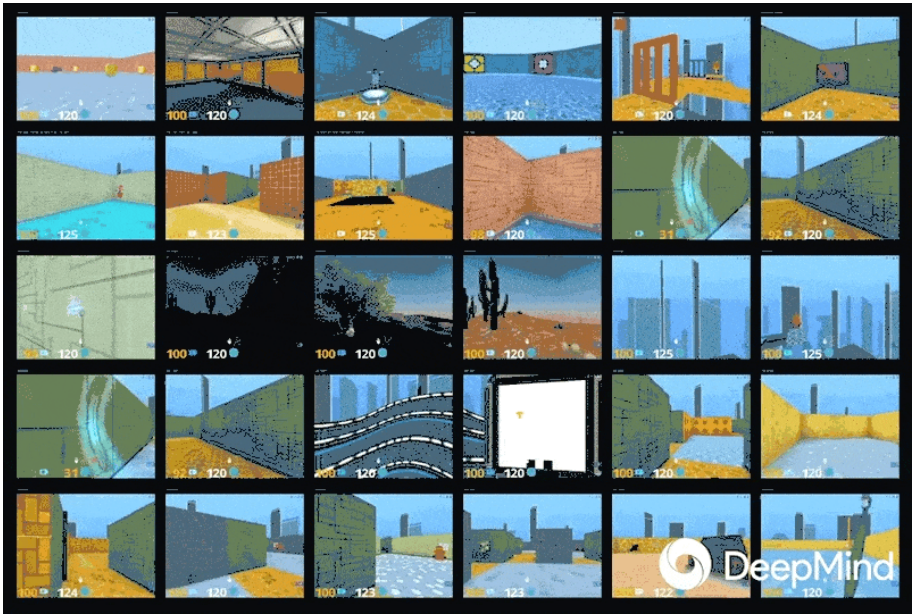
Criticism: Flaw in Function approximation setting

Using **batch-normalization is a bad way** to replace the constraint $\hat{U}^\top \hat{U} = \hat{V}^\top \hat{V} = I$ in function approximation:

1. This normalization is not w.r.t the Euclidean norm.
2. It disregards the orthogonality condition on the columns of $\hat{U}$ and $\hat{V}$.

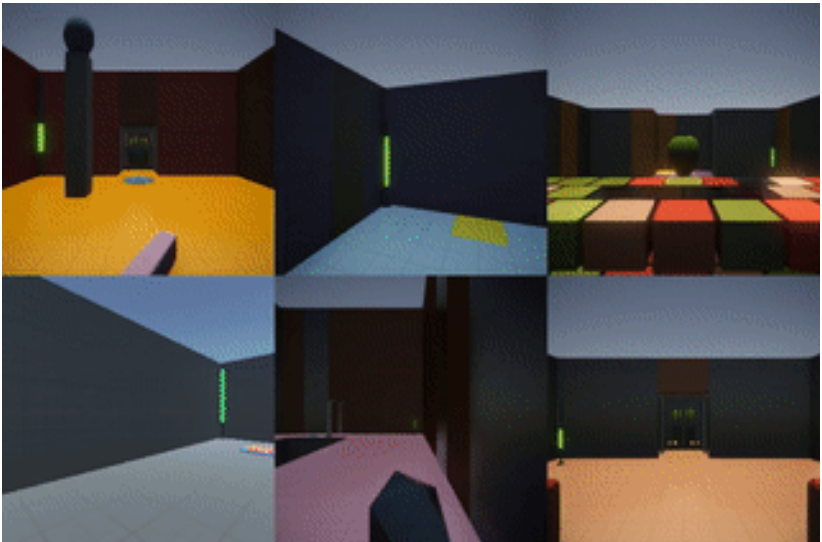
Experiments

Experiments: Environments (DM-Lab-30)



- Partial observability: Challenge for model-free approach

Experiments: Environments (DM-Hard-8)



- Partial observability
- Sparse reward: serves as exploration test bed
- Highly variable initial conditions every episode

Experiments: Baselines

Experiments: Baselines

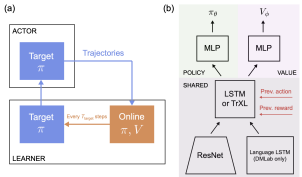
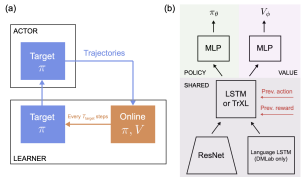


Figure: **V-MPO**: On-Policy MAP Policy Optimization (DeepMind (2019)):

- Approximate policy iteration algo.
- No exploration bonus
- No auxiliary loss (in this experiment)

Experiments: Baselines



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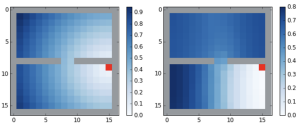

Figure: **The Laplacian in RL** (Yifan et al. (2018)). Visualization of the shaped reward defined by the L2 distance (left) and Laplacian representation (right):

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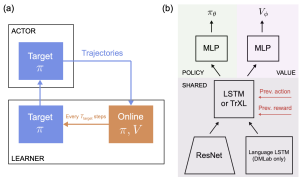
Experiments: Baselines



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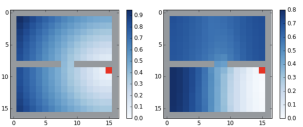

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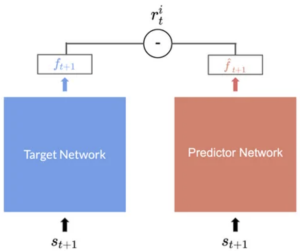

Figure: Exploration by **Random Network Distillation** (OpenAI (2018)):

- Combined with V-MPO
- Exploration bonus
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Experiments: Results

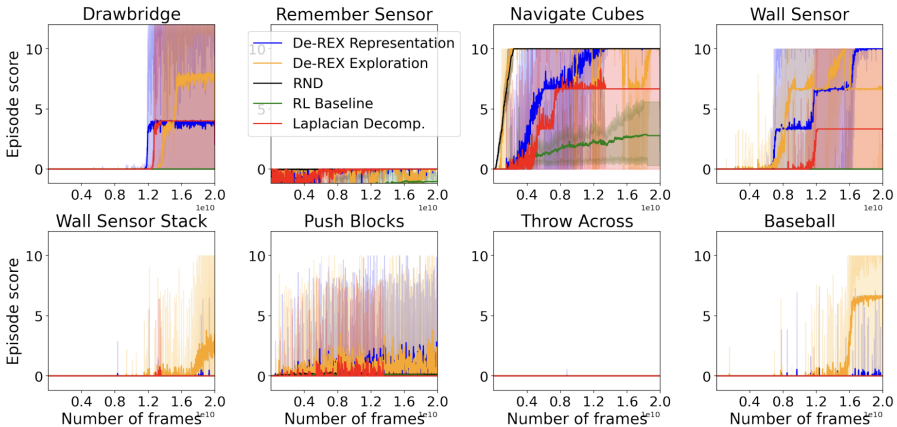

Figure: Learning curves for each task on DM-Hard-8. The shaded area corresponds to the minimum and maximum across three seeds.

Criticism: Experiments

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- The authors avoid mentioning the representation loss of the original VMPO.

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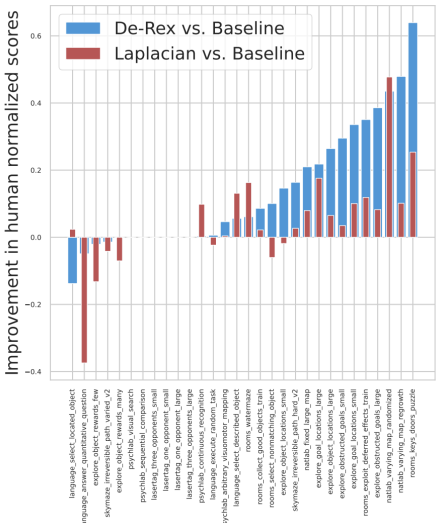
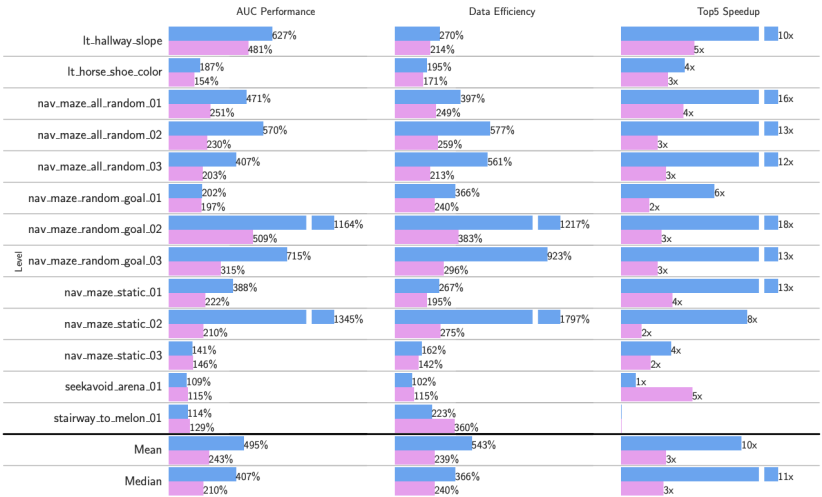
- The authors avoid mentioning the representation loss of the original VMPO.
- It is not until the appendix that it is mentioned, saying they did not use **pixel control** to "avoid confounding effects".



Appendix: Pixel control - performance boost



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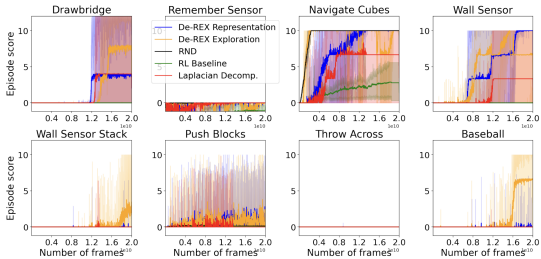

Figure: De-Rex (this paper) on DM-Hard-8

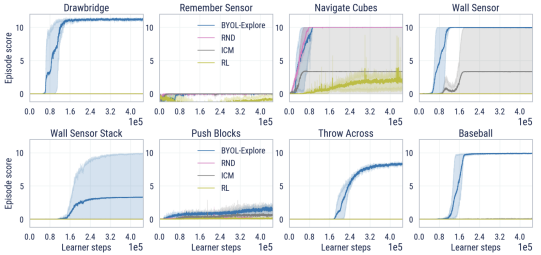

Figure: BYOL-Explore on DM-Hard-8

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Further work:

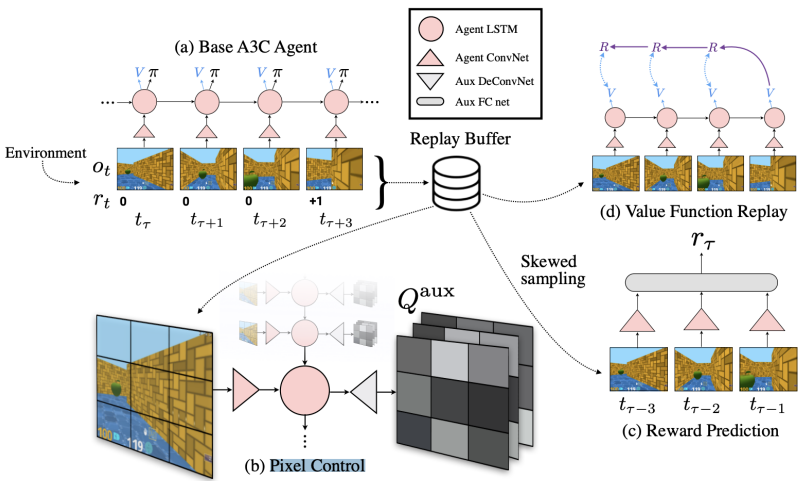
- Theoretical: Analyse the Function approximation setting
- Empirical: Minimize variance and conduct comprehensive comparisons with SoTA methods to ensure a balanced evaluation

Thank you for your attention

Appendix

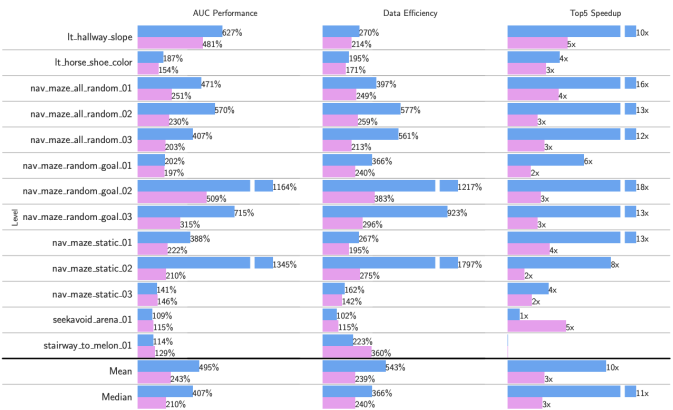
Pixel control (representation learning auxiliary loss)

Pixel Control - auxiliary policies Q^{aux} are trained to maximise change in pixel intensity of different regions of the input. The agent CNN and LSTM are used for this task along with an auxiliary deconvolution network. This auxiliary control task requires the agent to learn how to control the environment.



Pixel control - performance boost

A breakdown of the improvement over A3C for each level on Labyrinth. The values for $\mathrm{A3C} + \mathrm{RP} + \mathrm{VR}$ (reward prediction and value function replay) and UNREAL (reward prediction, value function replay and pixel control) are normalised by the A3C value. AUC Performance gives the robustness to hyperparameters (area under the robustness curve).



Proof of Theorem 1

Proof.

Let $v \in \mathbb{R}^{|S|\times 1}$ be an eigenvector of P_π and λ be its associated eigenvalue

$$P_\pi v = \lambda v.$$

By applying Neuman series, it can be seen that the eigenvectors for P_π and $(I - \gamma P_\pi)^{-1}$ are the same:

$$(I - A)^{-1} = \sum_{n=0}^{\infty} A^n$$

$$(I - \gamma P_\pi)^{-1} v = \left(\sum_{t=0}^{\infty} \gamma^t P_\pi^t \right) v = \sum_{t=0}^{\infty} (\gamma\lambda)^t v.$$

□

Proof of Theorem 1

Proof.

Further, as $0 \leq \gamma < 1$ and $|\lambda| \leq 1$,

$$(I - \gamma P_\pi)^{-1} v = \frac{1}{1 - \gamma\lambda} v$$

$$\lambda' = \frac{1}{1 - \gamma\lambda}$$

$$(I - \gamma P_\pi)^{-1} v = \lambda' v$$

Proof of Theorem 2

Proof.

$$\mathbb{E}[A_n] = \mathbb{E}\left[\frac{1}{n}X^T Y\right] = \mathbb{E}\left[\frac{1}{n}X^T E[Y|X]\right].$$

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$$\mathbb{E}[A_n] \overset{(a)}{=} \mathbb{E}\left[\frac{1}{n}X^T X P_\pi\right] = \mathbb{E}\left[\frac{1}{n}X^T X\right] P_\pi.$$

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Because of one-hot encoding, $\frac{1}{n}X^T X$ is a diagonal matrix. Therefore:

$$\mathbb{E}\left[\frac{1}{n}X^T X\right]_{[i,j]} = \mathbb{E}\left[\frac{1}{n}\sum_{j=1}^{n} x_i x_j^T\right]_{[i,j]}$$

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$$= \mathbb{E}\left[\frac{1}{n}\sum_{j=1}^{n}\{s_j = s_i \wedge s_j = s_j\}\right]$$

□

Proof of Theorem 2

Proof.

$$= \mathbb{E}\left[\frac{1}{n}\sum_{j=1}^{n}\{s_j = s_i \wedge s_j = s_j\}\right]$$

$$= \mathbb{E}[\{s_j = s_i\}]$$

$$= d(s_i).$$

Therefore, $\mathbb{E}\left[\frac{1}{n}X^T X\right] = D$, and:

$$\mathbb{E}[A_n] = DP_\pi.$$

□

Proof of Theorem 4

Proof.

We rewrite $\Sigma(\theta) := \frac{1}{n}\sum_{i=1}^{n} f_{\theta_1}(s_i) g_{\theta_2}(s_i')^\top \in \mathbb{R}^{k\times k}$ as:

$$\Sigma(\theta) := \mathbb{E}\left[f_{\theta_1}(S) g_{\theta_2}(S')^\top\right]$$

We will also use:

$$\mathcal{L}_{\text{off}}(\Sigma(\theta)) := \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k} \Sigma(\theta)_{[i,j]}^2 \tag{4}$$

$$\Sigma(\theta) := \frac{1}{b}\sum_{i=1}^{b} f_{\theta_1}(s_i) g_{\theta_2}(s_i')^\top \tag{5}$$

□

Proof.

By applying (4) and (5):

$$\begin{aligned}
\mathbb{E}\left[\frac{\partial}{\partial\theta} L_{\text{diag}}\left(\Sigma(\theta)\right)\right] &= \mathbb{E}\left[\frac{1}{k}\sum_{i=1}^{k}\frac{\partial}{\partial\theta}\Sigma(\theta)_{i,i}\right] \\
&\overset{(a)}{=} \frac{1}{k}\sum_{i=1}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\Sigma(\theta)_{i,i}\right] \\
&= \frac{1}{k}\sum_{i=1}^{k}\frac{\partial}{\partial\theta}\Sigma(\theta)_{i,i} \\
&= \frac{\partial}{\partial\theta} L_{\text{diag}}\left(\Sigma(\theta)\right),
\end{aligned}$$

(a) Is the case where the sampling distribution is fixed

□

Proof of Theorem 4

Proof.

$$
\begin{aligned}
\mathbb{E}\left[\frac{\partial}{\partial\theta}\,\mathcal{L}_{\text{off}}\left(\frac{1}{b}\Sigma(\theta)\right)\right] &= \mathbb{E}\left[\frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\left(\frac{1}{b}\Sigma(\theta)^2_{[i,j]}\right)\right], \\
&= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(\frac{1}{b}\Sigma(\theta)^2_{[i,j]}\right)\right], \\
&= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(f_{\theta_1}(S)g_{\theta_2}(S')^\top_{[i,j]}\right)^2\right], \\
&\neq \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\left(f_{\theta_1}(S)g_{\theta_2}(S')^\top_{[i,j]}\right)\right]^2, \\
&= \frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\Sigma(\theta))
\end{aligned}
$$

Proof of Theorem 5

Proof.

$$\hat{\mathcal{L}}_{\text{off}}(\theta) = \frac{1}{k^2 - k} \sum_{\substack{i,j=1 \\ i \neq j}}^{k} \hat{\Sigma}(\theta)_{[i,j]} sg\left(\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right) + \sum_{\substack{i,j=1 \\ i \neq j}}^{k} \hat{\hat{\Sigma}}(\theta)_{[i,j]} sg\left(\hat{\Sigma}(\theta)_{[i,j]}\right)$$

We note the first and the second element of the sum with *I* and *II* respectively:

$$\mathbb{E}\left[\frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\theta)\right] = \mathbb{E}\left[\frac{\partial}{\partial\theta}(\text{I})\right] + \mathbb{E}\left[\frac{\partial}{\partial\theta}(\text{II})\right]$$

□

Proof of Theorem 5

$$
\begin{aligned}
\mathbb{E}\left[\frac{\partial}{\partial\theta}(\mathsf{I})\right] &= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\mathbb{E}\left[\frac{\partial}{\partial\theta}\hat{\Sigma}(\theta)_{[i,j]}sg\left(\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right)\right], \\
&= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\mathbb{E}\left[\frac{\partial}{\partial\theta}\hat{\Sigma}(\theta)_{[i,j]}\right]\mathbb{E}\left[\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right], \\
&= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\mathbb{E}\left[\hat{\Sigma}(\theta)_{[i,j]}\right]\mathbb{E}\left[\hat{\hat{\Sigma}}(\theta)_{[i,j]}\right], \\
&= \frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\frac{\partial}{\partial\theta}\Sigma(\theta)_{[i,j]}\Sigma(\theta)_{[i,j]}, \\
&= \frac{1}{2}\frac{\partial}{\partial\theta}\left(\frac{1}{k^2-k}\sum_{\substack{i,j=1\\ i\neq j}}^{k}\Sigma(\theta)^2_{[i,j]}\right), \\
&= \frac{1}{2}\frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\Sigma(\theta))
\end{aligned}
$$

Proof of Theorem 5

Proof.

It can be observed that $\mathbb{E}\left[\frac{\partial}{\partial\theta} I)\right] = \mathbb{E}\left[\frac{\partial}{\partial\theta} II\right]$. Therefore:

$$\begin{aligned}
\mathbb{E}\left[\frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\theta)\right] &= \mathbb{E}\left[\frac{\partial}{\partial\theta}(\text{I})\right] + \mathbb{E}\left[\frac{\partial}{\partial\theta}(\text{II})\right], \\
&= 2 \cdot \mathbb{E}\left[\frac{\partial}{\partial\theta}(\text{I})\right] \\
&= \frac{\partial}{\partial\theta}\mathcal{L}_{\text{off}}(\Sigma(\theta))
\end{aligned}$$

□