23.12.2021.

14: $\begin{cases} 2x - 3y + z = 1 \\ -4x + 6y + 2z = 3. \end{cases}$

$\bar{A} \in \mathcal{M}_{2}(5).$

$\underline{M_I}$: $A = \begin{pmatrix} 2 & -3 & 1 \\ -4 & 6 & 2 \end{pmatrix}$ $\begin{vmatrix} -3 & 1 \\ 6 & 2 \end{vmatrix} \neq 0.$

$\text{rang}(A) = 2. = \text{rang}(\bar{A}) \Rightarrow$ Sist. com. simplu ned.

$\rightarrow y, z$ - N.P.
x - N.S. $\Rightarrow \begin{cases} -3y + z = 1 - 2\alpha \\ 6y + 2z = 3 + 4\alpha \end{cases}$

$\rightarrow$ Sist. de tip Cramer. $(\Delta \neq 0) \Rightarrow$

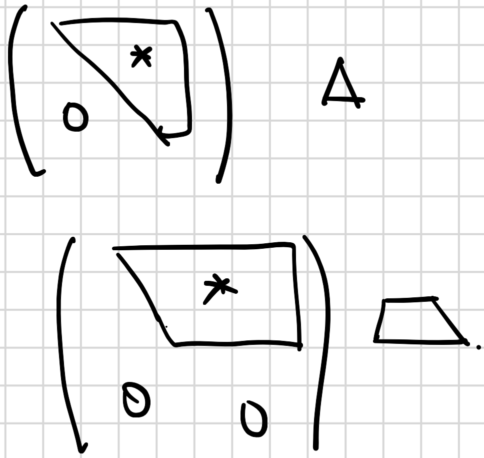
$y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}.$

$\Delta = \begin{vmatrix} -3 & 1 \\ 6 & 2 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} 1-2\alpha & 1 \\ 3+4\alpha & 2 \end{vmatrix}$

$\Delta_z = \begin{vmatrix} -3 & 1-2\alpha \\ 6 & 3+4\alpha \end{vmatrix}.$

M_{II}: Metoda eliminării lui Gauss

$$\begin{cases} 2x - 3y + z = 1 \\ -4x + 6y + 2z = 3. \end{cases}$$



$$A = \left(\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ -4 & 6 & 2 & 3 \end{array}\right)$$

$L_2 + 2L_1$

$$\sim \left(\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 0 & 0 & 4 & 5 \end{array}\right) \Rightarrow \text{Sist. dar.}$$

$$\begin{cases} 2x - 3y + z = 1 \\ 4z = 5 \end{cases} \Rightarrow z = \frac{5}{4} \Rightarrow$$

$$2x - 3y = 1 - \frac{5}{4}. \Rightarrow 2x - 3y = -\frac{1}{4}.$$

$x = \alpha$

$$y = \frac{2\alpha + \frac{1}{4}}{3} \Rightarrow \checkmark.$$

M_{III}: Scrie ec. ✓.

16)
$$\begin{cases} x+y+z+t=1 \\ x+y+z-t=0 \\ x+y-z+t=2 \end{cases}$$

Folosim M.E.G:

$$A=\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 0 \\ 1 & 1 & -1 & 1 & 2 \end{array}\right) \begin{array}{l} L_2-L_1 \\ L_3-L_1 \\ \sim \end{array}$$

forma trapezoidala

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 & -1 \\ 0 & 0 & -2 & 0 & 1 \end{array}\right) \Rightarrow$$

Sis. devine:

$$\begin{aligned} x+y+z+t&=1 \\ -2t&=-1 \\ -2z&=1. \end{aligned} \Rightarrow$$

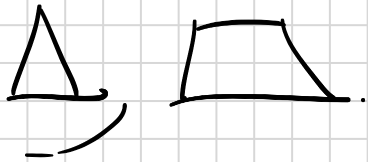
$$t=\frac{1}{2}; \quad z=-\frac{1}{2}. \Rightarrow$$

$$x+y=1. \Rightarrow$$

$$S=\left\{\left(\alpha, 1-\alpha, -\frac{1}{2}, \frac{1}{2}\right) \,\middle|\, \alpha\in\mathbb{R}\right\}.$$

□

$$\begin{pmatrix} x & k & \alpha \\ \cancel{y} & \alpha' & \beta \\ \cancel{z} & \cancel{\gamma} & \gamma' \\ t & \cancel{\gamma} & \delta \\ u & \cancel{\delta} & \delta \\ v & e & \theta \end{pmatrix} \sim \longrightarrow$$



$L_2 - \frac{y}{x} L_1$

(26.) Enunțul îl aveți.

$$\begin{cases} \beta x + \alpha y = \gamma \\ \gamma x + \alpha z = \beta \\ \gamma y + \beta z = \alpha \end{cases}$$

are sol unică $\Leftrightarrow$ $\alpha\beta\gamma \neq 0$.

$\Updownarrow$

Sist e echiv

$$\underbrace{\begin{pmatrix} \beta & \alpha & 0 \\ \gamma & 0 & \alpha \\ 0 & \gamma & \beta \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta \\ \alpha \end{pmatrix}$$

El are sol unică $\Leftrightarrow$ $\det(A) \neq 0$.

$$\begin{vmatrix} \beta & \alpha & 0 \\ \gamma & 0 & \alpha \\ 0 & \gamma & \beta \end{vmatrix} = -2\alpha\beta\gamma \Rightarrow \det(A) \neq 0 \Leftrightarrow \alpha\beta\gamma \neq 0$$

Pt rezolvare sx:

$$x=\frac{\Delta_x}{\Delta}, \quad y=\frac{\Delta_y}{\Delta}, \quad z=\frac{\Delta_z}{\Delta}$$

Δe cal

$$\Delta = -2\alpha\beta\gamma.$$

$$\Delta_x = \begin{vmatrix} \gamma & \alpha & 0 \\ \beta & 0 & \alpha \\ \alpha & \gamma & \beta \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} \beta & \gamma & 0 \\ \gamma & \beta & \alpha \\ 0 & \alpha & \beta \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} \beta & \alpha & \gamma \\ \gamma & 0 & \beta \\ 0 & \gamma & \alpha \end{vmatrix}$$

97

a).

$$S: \begin{cases} 2x - y + z + 2t = 1 \\ 2x + 2y + 4z + 2t = \alpha \\ 3x - 2y + z + 3t = 1. \end{cases}$$

Care e ideea?

S e comp $\Leftrightarrow$ $\operatorname{rg} A = \operatorname{rg} \bar{A}$.

$$\operatorname{rg}(A) = \begin{pmatrix} 2 & -1 & 1 & 2 \\ 2 & 2 & 4 & 2 \\ 3 & -2 & 1 & 3 \end{pmatrix} \in \mathcal{M}_{3,4}(\mathbb{R})$$

$= ($ Gauss $)$ ... x_n ... $\Rightarrow x_4$ e Cauchy!

S: .... $\Rightarrow$ S e Cramer!

$$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 2 & 2 & 4 & 2 \\ 3 & -2 & 1 & 3 \end{pmatrix} \overset{L_2 - L_1}{\sim}$$

$$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 0 & 3 & 3 & 0 \\ 3 & -2 & 1 & 3 \end{pmatrix} \overset{C_3 + C_2}{\sim}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 6 & 0 \\ 3 & -2 & -1 & 0 \end{pmatrix} \sim$$

$$9 \begin{vmatrix} -1 & 0 \\ 1 & 6 \end{vmatrix} \neq 0.$$

Cum $\downarrow^{Rez}$ $\begin{vmatrix} 2 & -1 & 0 \\ 0 & 3 & 6 \\ 7 & -2 & -1 \end{vmatrix} = 0 \Rightarrow$

$rg\, A = 2. \Rightarrow$ Sist. e com $\Leftarrow$

$\underline{rg\, \bar{A} = 2}$

$$\bar{A} = \begin{pmatrix} \boxed{\begin{matrix} 2 & -1 \\ 2 & 2 \end{matrix}} & 1 & 2 & 1 \\ & 4 & 2 & \alpha \\ 3 \quad -2 & 1 & 3 & 1 \end{pmatrix} \in \mathcal{M}_{3,5}(\mathbb{R})$$

$\neq 0.$

$\underline{M_I}$: Bordare (. . . . . .).

$\underline{M_{II}}$: $rg\, \bar{A} = 2 \Leftarrow \begin{vmatrix} 2 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -2 & 1 \end{vmatrix} = 0.$

b) $\underline{\text{Îmbinații}}$:

$$\begin{cases} x - 3y = -2 \\ x + 2y = 3 \\ 3x - y = \alpha \\ 2x + y = \beta. \end{cases}$$

$rg\, A = 2.$

$\Downarrow$ trebuie ca $rg\, \bar{A} = 2.$

Cum $\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} \neq 0 \Rightarrow$

$$\operatorname{rg} \bar{A} = 2 \leftarrow \begin{vmatrix} 1 & -3 & -2 \\ 1 & 2 & 3 \\ 3 & -1 & \alpha \end{vmatrix} = 0 \quad \wedge$$

$$\begin{vmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 2 & 1 & \beta \end{vmatrix} = 0.$$