23.12.2022.

$f: \mathbb{R}^n \to \mathbb{R}^n$.

$[f]_e = A \Rightarrow f(x) = Ax$.

$A \sim J_A$ $\begin{bmatrix} \ddots & & \\ & \ddots & \\ & & \ddots \end{bmatrix}$.

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Jordan !

3.1.35. $S = \langle (1,2,-1) \rangle \leq \mathbb{R}^3$.

$T = \langle (1,2,1), (-2,1,-3) \rangle$.

Ecuațiile subsp. S, T ?

Sol:

Pt S: Trebuie să scriem S ca soluție unui sist. liniar.

Fie $(x,y,z) \in S \Rightarrow \begin{cases} x = \alpha \\ y = 2\alpha \\ z = -\alpha \end{cases}$ $\alpha \in \mathbb{R}$

$\Rightarrow \begin{cases} 2x = y \\ -x = z. \\ -\frac{1}{2}y = z. \end{cases} \Leftrightarrow \begin{cases} 2x - y = 0 \\ x + z = 0 \\ 2z + y = 0. \end{cases}$

V:

$$\begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0.$$

$$\operatorname{rang} A = \operatorname{rg}\begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\overset{L_3+L_1}{\sim}\begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix} \overset{C_1-C_3}{\sim}$$

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$= \det(A) = 2\begin{vmatrix} 0 \end{vmatrix} = 0. \Rightarrow \operatorname{rg} A = 2 \Rightarrow$

y, z - nu p.

$x =$ n. fn.

$\Rightarrow \begin{cases} \cancel{-y+2x=0.} \\ \cancel{} \\ \cancel{z} \end{cases}$

$$\begin{cases} 2x - y = 0 \\ x + z = 0 \\ y + 2z = 0 \end{cases} \Rightarrow \checkmark.$$

$$\mathcal{M}_{\underline{\pi}}: \quad \frac{x}{1} = \frac{y}{2} = \frac{z}{-1}.$$

b) $T = \text{span}\{(1,2,1), (-2,1,-3)\}$.

Fie $(x,y,z) \in T$.

$$\begin{cases} x = \alpha - 2\beta \\ y = 2\alpha + \beta \\ z = \alpha - 3\beta \end{cases}$$

$\alpha, \beta \in \mathbb{R}$. Ce facem?

Trebuie să eliminăm α și β.

($\exists$ o rețetă clară)

$x - z = \beta$.

$x + 2y = 5\alpha$.

$\Rightarrow$ $\alpha = \frac{x+2y}{5}$

$\beta = x - z$,

$$\begin{cases} x = \frac{x+2y}{5} - 2(x-z). \\ y = \frac{2}{5}(x+2y) + x - z \\ z = \frac{x+2y}{5} - 3(x-z). \end{cases} \Leftrightarrow$$

$$\begin{cases} 5x = x + 2y - 10x + 10z \\ 5y = 2x + 4y + 5x - 5z \\ 5z = x + 2y - 15x + 15z \end{cases} \Leftrightarrow \begin{cases} -14x + 2y + 10z = 0. \\ 7x - y - 5z = 0. \\ -14x + 2y + 10z = 0. \end{cases}$$

$$\begin{cases} 7x - y - 5z = 0. \\ \quad \downarrow \\ \quad F(x,y,z) = 0. \quad \text{Ec. imp.} \end{cases}$$

$\underline{v}$: ✓.

Ec. param: $\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t). \end{cases} \quad t \in \mathbb{R}.$

<u>3.1.37.</u>:

$S = \{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 0\},$

$T = \{(\text{—''—}) \mid x_1 = x_2 = x_3\}.$

↳ S - plan ce trece prin origine

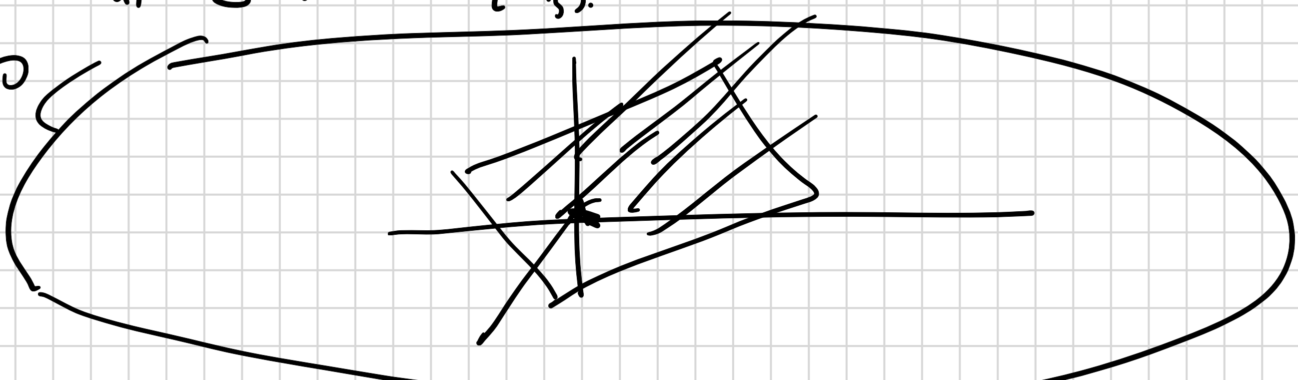
$S = \{(-\beta - \alpha, \beta, \alpha) \mid \beta, \alpha \in \mathbb{R}\}.$

$$\begin{cases} S = \langle (-1,1,0), (-1,0,1) \rangle. \\ T = \langle (1,1,1) \rangle. \end{cases} \leq \mathbb{R}^3.$$

Dem că $S \oplus T = \mathbb{R}^3$.

i) $S + T = \mathbb{R}^3$.

ii) $S \cap T = \{0_3\}$.



Cum dem că $S \oplus T = \mathbb{R}^3$.

$S = \langle (-1,1,0), (-1,0,1) \rangle$. Avem de dem (i), (ii).

$T = \langle (1,1,1) \rangle$

(i) $S + T \subseteq \mathbb{R}^3$ ✓.

$\mathbb{R}^3 \subseteq S + T$.

Fie $(x,y,z) \in \mathbb{R}^3 \Rightarrow$ Dět $\alpha, \beta, \gamma \in \mathbb{R}$:

$(x,y,z) = \alpha(-1,1,0) + \beta(-1,0,1) + \gamma(1,1,1)$.

$(x,y,z) = (-\alpha - \beta + \gamma, \alpha + \gamma, \beta + \gamma)$.

$$\underbrace{\begin{cases} x = -\alpha - \beta + \gamma \\ y = \alpha + \gamma \\ z = \beta + \gamma \end{cases}}_{\text{cramer}}$$

$$\begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \underset{\substack{C_1 + C_2 \\ C_2 + C_1}}{\sim} \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \neq 0 \Rightarrow S.S.!$$

$\Rightarrow \exists\, \alpha, \beta, \gamma \in \mathbb{R}: \ (x,y,z) \in S+T$ ✓. $\Rightarrow$

$\mathbb{R}^3 \subseteq S+T \Rightarrow \boxed{S+T = \mathbb{R}^3}$.

(ii) $S \cap T = \{0_3\}$.

Fie $x \in S \cap T \Rightarrow$

$$x = \alpha(-1,1,0) + \beta(-1,0,1) = \gamma(1,1,1).$$

$$\Updownarrow$$

$$\begin{cases} -\alpha - \beta - \gamma = 0 \\ \alpha - \gamma = 0 \\ \beta - \gamma = 0 \end{cases}$$

Unic e $\alpha = \beta = \gamma = 0$.

$\square$.

$$V = V_1 \oplus V_2 \Leftrightarrow$$

$$\forall\, x \in V \ \exists!\, \begin{matrix} x_1 \in V_1 \\ x_2 \in V_2 \end{matrix} : \ x = x_1 + x_2.$$

Exc: 3.1.38.

$$\forall A \in \mathcal{M}_2(\mathbb{R}): \ \exists!\, \begin{matrix} B \in S \\ C \in T \end{matrix} : \ A = B + C.$$

$\boxed{A = \alpha I + C}$, $tr(C) = 0$

Indicație:

$\alpha = \dfrac{tr(A)}{2}$. $\quad (tr(A) = \alpha \cdot tr(I) + 0$.

$A \in \mathcal{M}_2(\mathbb{R})$: $C = A - \frac{\operatorname{tr} A}{2} I$.

$C \in T$ ✓.

$A = A - \frac{\operatorname{tr} A}{2} I +$

$\boxed{3.1.42}$.

(i) $f : \mathbb{R}^3 \to \mathbb{R}^3$.

$f(x, y, z) = (x - y, y - z, z - x)$.

Cum gândim?

<u>Def</u>: f e lin $\Leftrightarrow$ (i) f e aditivă

(ii) f e omogenă.

(i) $f(x+y) = f(x) + f(y), \quad \forall x, y \in V$

(ii) $f(\alpha x) = \alpha f(x), \quad \forall x \in V, \ \forall \alpha \in \mathbb{K}$.

$\Updownarrow$

(iii) f e lin $\Leftrightarrow$ $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y), \ \forall x, y \in V$
$\forall \alpha, \beta \in \mathbb{K}$.

(iv) f e lin $\Leftrightarrow$ f e de forma Ax.

<u>e cont.</u>

Pt $f(x,y,z) = (x-y,\ y-z,\ z-x)$

$$f = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Pe ciorna

$f : \mathbb{R}^n \to \mathbb{R}^m$ e b_n =

$[f]_e = f(e_1) \dots f(e_n)$

pe col

$f(e_1), f(e_2), f(e_3)$ V_m

$$\begin{bmatrix} \downarrow & \downarrow & \downarrow \\ & & \end{bmatrix}, \quad f = A \cdot x$$

Cum $f = A \cdot x \Rightarrow f \in L(\mathbb{R}^n, \mathbb{R}^m)$.

② $f : \mathbb{R}^3 \to \mathbb{R}^3$

$f(x,y,z) = (x-1,\ y+2,\ z+3)$.

pe ciorna: $f(e_1) = (0, 2, 3) = f(1,0,0)$

$f(e_2) = (-1, 3, 3) = f(0,1,0)$

$f(e_3) = (-1, 2, 4)$.

$\Rightarrow$ Candidata pt poziția de mat:

$$A = \begin{pmatrix} 0 & -1 & -1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$$

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y-z, \dots \end{pmatrix} \Rightarrow f \text{ nu e liniară}$$

Știm că f nu e lin dacă arătăm că nu e liniară.

$\exists x, y \in \mathbb{R}: \quad f(x+y) \neq f(x) + f(y)$.

$f(1,1,0) \neq f(1,0,0) + f(0,1,0)$.

$(0,3,3) \neq (0,2,3) + f(-1,3,3)$ $\rightarrow f$ nu e adit..

<u>Analiză:</u>

$f = (x, y, z) + (-1, 2, 3)$.

$f = I_3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} + b_{=(-1,2,3) \neq 0}$ (f e afină).

$f = L + b$

Obs: 3 ✓ linia.
4 ✓.

(5) nu e lin

$f(\cancel{(1,0)} + f(0,1)$

$f(1,0) + f(-1,0) \neq f(0)$.

$2 \neq 0$ ✓.

(1) Det $\operatorname{Im} f$, $\operatorname{Ker} f$.

$$f = Ax \quad A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}.$$

$\operatorname{Ker} f = \{x \in \mathbb{R}^3 \mid f(x) = 0\}$. $\Rightarrow$ Rez sist

$$Ax = 0. \Leftrightarrow \begin{cases} x - y = 0 \\ y - z = 0, \\ z - x = 0. \end{cases} \Leftrightarrow$$

$\operatorname{Ker} f = \langle (1,1,1) \rangle$.

$f(x,y,z) = x(1,0,-1) + y(-1,1,0) + z(0,-1,1)$

$= \langle (1,0,-1), (-1,1,0), (0,-1,1) \rangle$.