```
> restart;
```

1.
a)

```
> ec1 := diff(x(t), t)=-k * x(t)
```

$$ec1 := \frac{d}{dt} x(t) = -k\,x(t) \tag{1}$$

```
> cond_in:=x(0)=x0
```

$$cond\_in := x(0) = x0 \tag{2}$$

```
> sol:=dsolve({ec1, cond_in}, x(t))
```

$$sol := x(t) = x0\,e^{-k\,t} \tag{3}$$

```
> with(DETools)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeometricsols*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] **(4)**

```
> with(plots)
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(5)**

b)

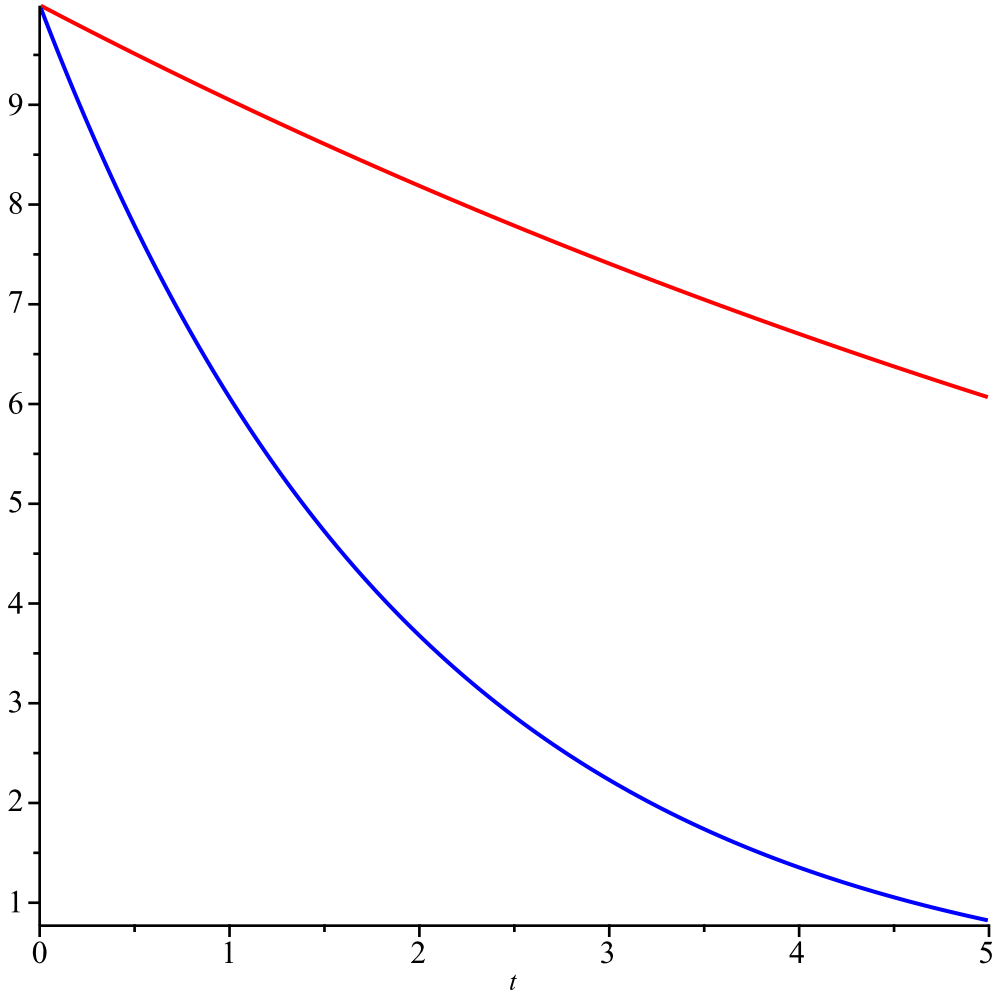
```
> sol_unapp:=unapply(rhs(sol), t, k, x0)
```

$$sol\_unapp := (t, k, x0) \mapsto x0 \cdot e^{-k \cdot t} \quad (6)$$

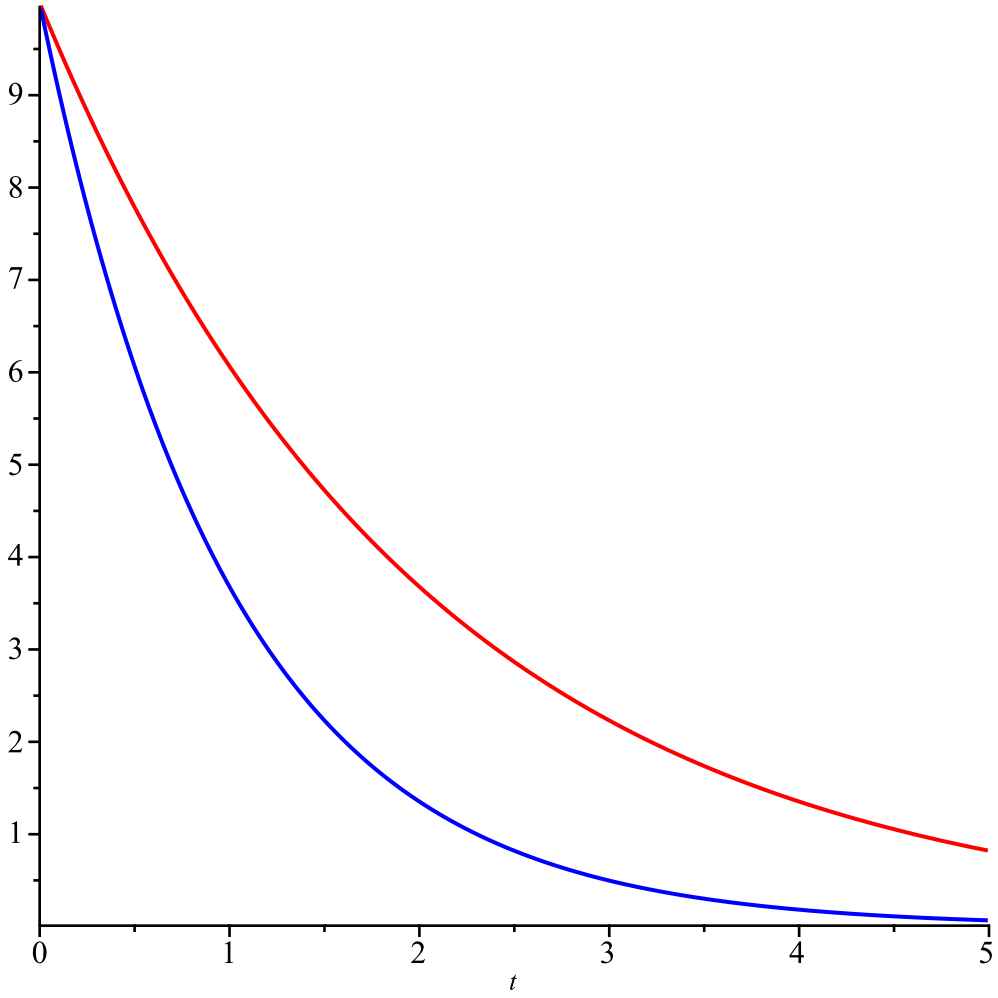
```
> x0:=100
```

$$x0 := 100 \quad (7)$$

```
> plot([sol_unapp(t, 0.1, 10), sol_unapp(t, 0.5, 10)], t = 0..5,
  color=[red, blue])
```




```
> plot([sol_unapp(t, 0.5, 10), sol_unapp(t, 1, 10)], t = 0..5,
  color=[red, blue])
```



c)

```
> ec:=sol_unapp(5730, k, x0)=x0/2
```

$$ec := x0\, e^{-5730\, k} = \frac{x0}{2} \tag{8}$$

```
> kC14:=solve(ec, k)
```

$$kC14 := \frac{\ln(2)}{5730} \tag{9}$$

d)

```
> ec:=sol_unapp(T, kC14, x0)=(20/100)*x0
```

$$ec := x0\, e^{-\frac{\ln(2)\, T}{5730}} = \frac{x0}{5} \tag{10}$$

```
> solve(ec, T)
```

$$\frac{5730 \ln(5)}{\ln(2)} \tag{11}$$

```
> evalf(solve(ec, T))
```

$$13304.64798 \tag{12}$$

e)

```
> ec1:=sol_unapp(T, kC14, x0)=(91.57/100)*x0; ec2:=sol_unapp(T,
  kC14, x0)=(93.021/100)*x0;
```

$$ec1 := x0\, e^{-\frac{\ln(2)\, T}{5730}} = 0.9157000000\, x0$$

$$ec2 := x0\, e^{-\frac{\ln(2)\, T}{5730}} = 0.9302100000\, x0 \quad \textbf{(13)}$$

```
> sol1e:=solve(ec1, T), solve(ec2, T)
```

$$sol1e := 728.0141045, 598.0495293 \quad \textbf{(14)}$$

```
> 1988-sol1e[1]
```

$$1259.985896 \quad \textbf{(15)}$$

```
> 1988-sol1e[2]
```

$$1389.950471 \quad \textbf{(16)}$$

2

a)

```
> ec1:=diff(T(t), t)=-k * (T(t)-Tm)
```

$$ec1 := \frac{d}{dt} T(t) = -k\,(T(t) - Tm) \quad \textbf{(17)}$$

```
> cond_in:=T(0)=T0
```

$$cond\_in := T(0) = T0 \quad \textbf{(18)}$$

```
> sol:=dsolve({ec1, cond_in}, T(t))
```

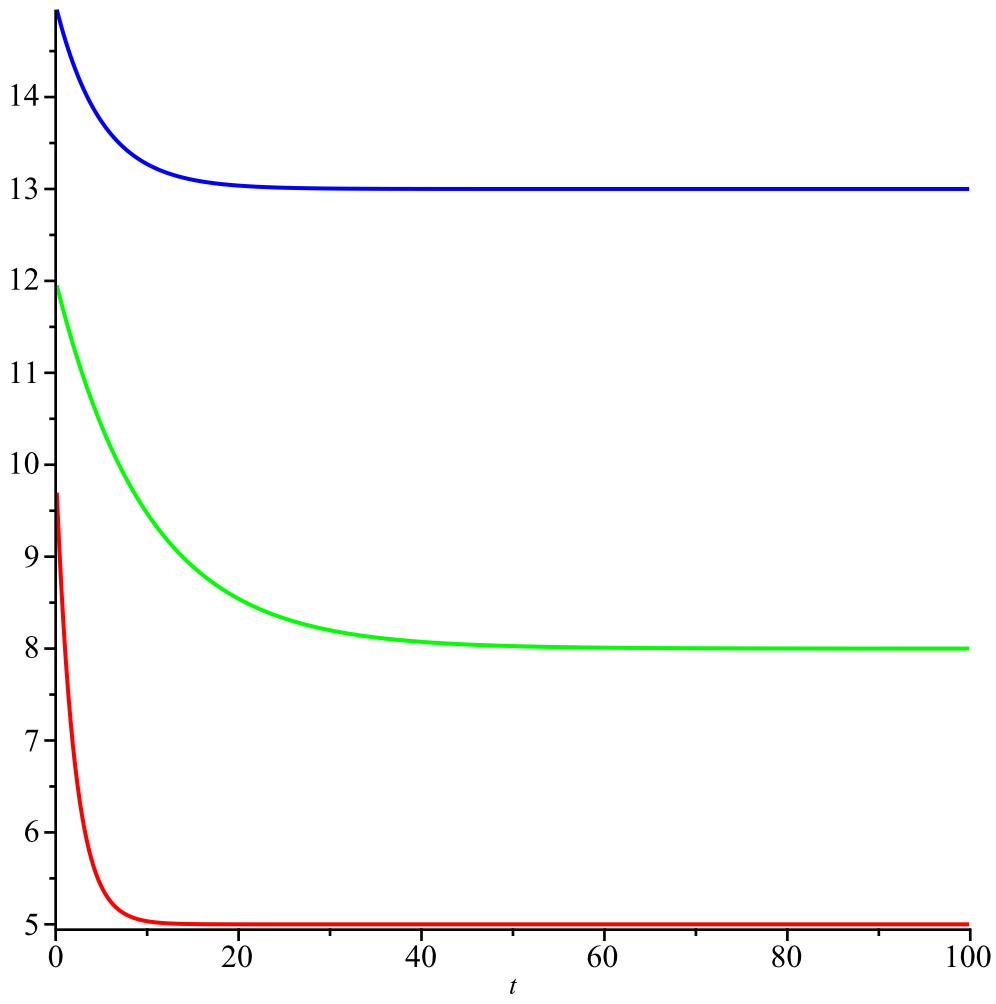
$$sol := T(t) = Tm + e^{-k\,t}\,(T0 - Tm) \quad \textbf{(19)}$$

b)

```
> sol_unapp:=unapply(rhs(sol), t, k, T0, Tm)
```

$$sol\_unapp := (t, k, T0, Tm) \mapsto Tm + e^{-k\cdot t}\cdot(T0 - Tm) \quad \textbf{(20)}$$

```
> plot([sol_unapp(t, 0.5, 10, 5), sol_unapp(t, 0.2, 15, 13),
  sol_unapp(t, 0.1, 12, 8)], t=0..100, color=[red, blue, green])
```



c)

```
> ec1:=sol_unapp(ta, k, 36, 21)=34.22
```

$$ec1 := 21 + 15\,e^{-k\,ta} = 34.22 \quad (21)$$

```
> ec2:=sol_unapp(ta+1, k, 36, 21)=34.11
```

$$ec2 := 21 + 15\,e^{-k\,(ta+1)} = 34.11 \quad (22)$$

```
> solve({ec1, ec2}, {k, ta})
```

$$\{k = 0.008355536648,\ ta = 15.11804352\} \quad (23)$$

3.

a)

```
> ec1:=diff(x(t), t)=r * x(t)
```

$$ec1 := \frac{d}{dt}x(t) = r\,x(t) \quad (24)$$

```
> cond_in:=x(0)=x0
```

$$cond\_in := x(0) = x0 \quad (25)$$

```
> sol1:=dsolve({ec1, cond_in}, x(t))
```

$$sol1 := x(t) = x0\,e^{r\,t} \quad (26)$$

```
> ec2:=diff(x(t), t)=r0 * x(t) * (1 - (x(t)/K))
```

$$ec2 := \frac{d}{dt}\,x(t) = r0\,x(t)\left(1 - \frac{x(t)}{K}\right) \tag{27}$$

```
> sol2:=dsolve({ec2, cond_in}, x(t))
```

$$sol2 := x(t) = \frac{K\,x0}{(K - x0)\,e^{-r0\,t} + x0} \tag{28}$$

b)

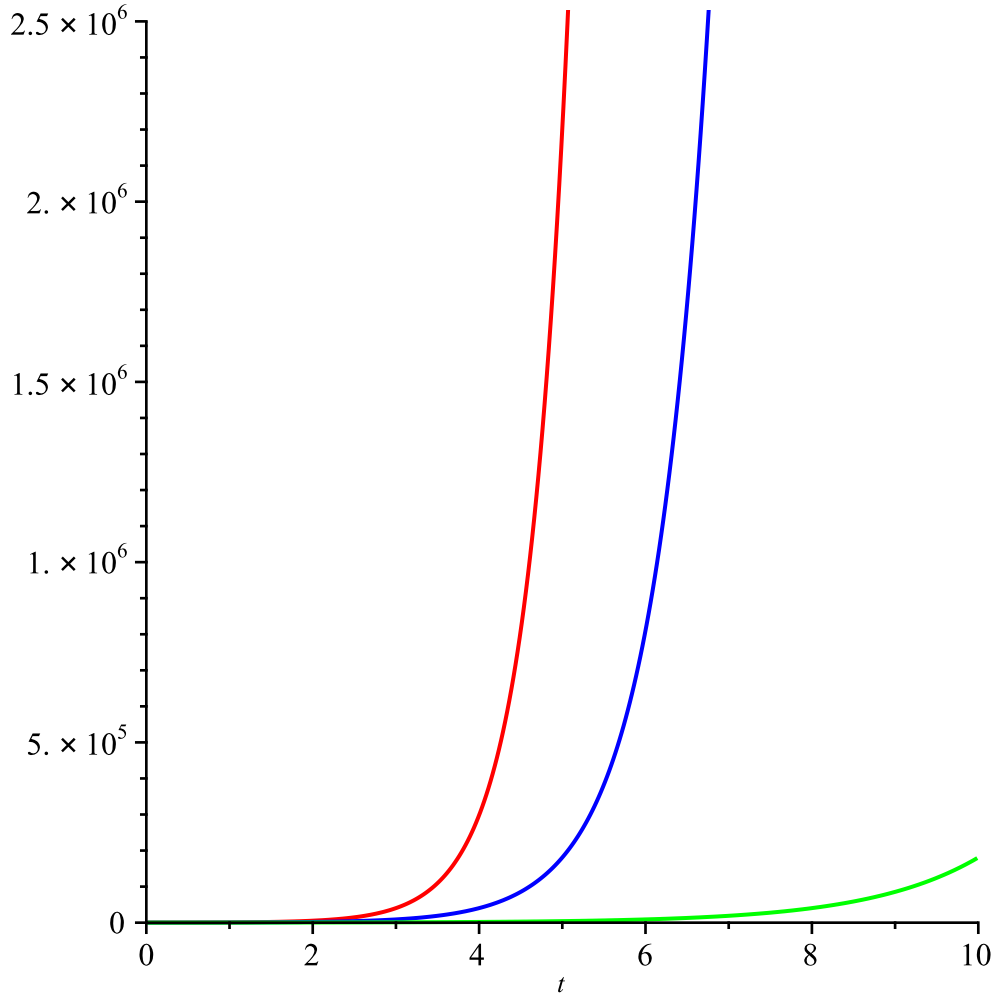
```
> sol1_unapp:=unapply(rhs(sol1), t, r, x0)
```

$$sol1\_unapp := (t, r, x0) \mapsto x0 \cdot e^{r \cdot t} \tag{29}$$

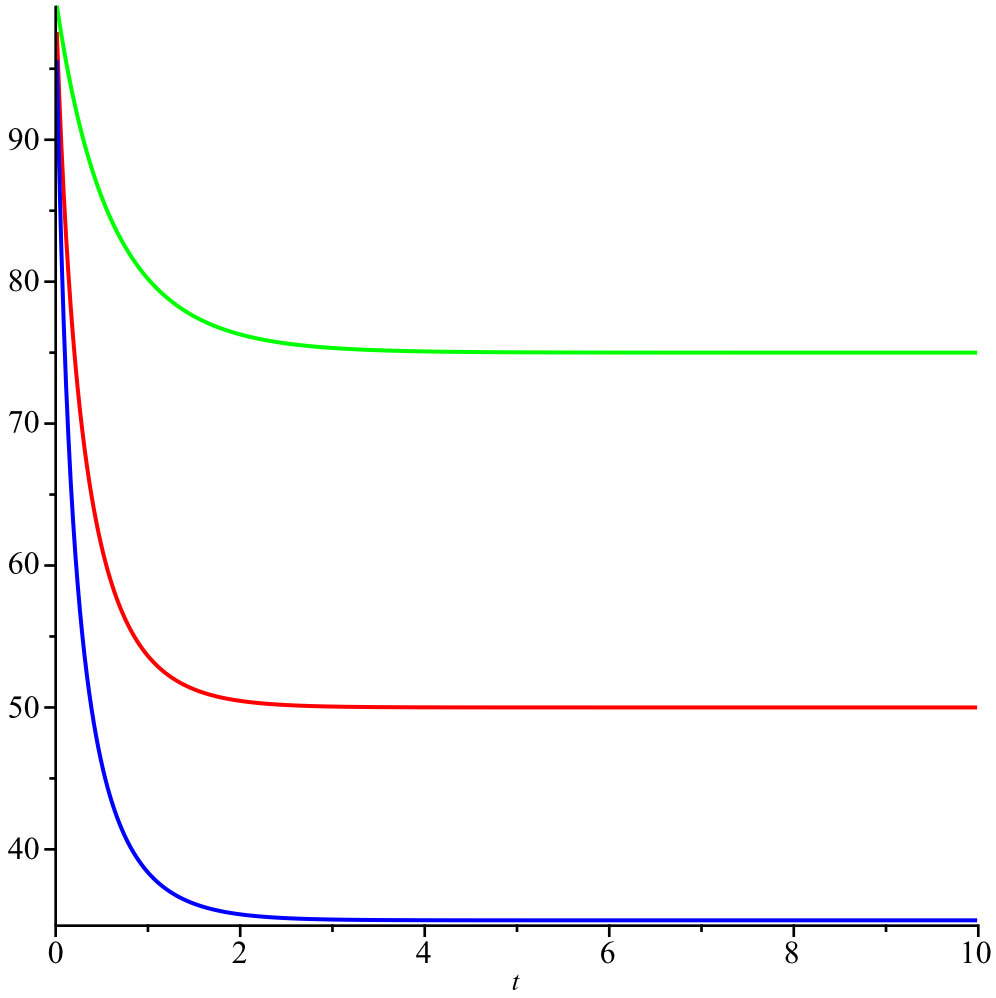
```
> sol2_unapp:=unapply(rhs(sol2), t, r0, K, x0)
```

$$sol2\_unapp := (t, r0, K, x0) \mapsto \frac{K \cdot x0}{x0 + e^{-r0 \cdot t} \cdot (K - x0)} \tag{30}$$

```
> plot([sol1_unapp(t, 2, 100), sol1_unapp(t, 1.5, 100), sol1_unapp
  (t, 0.75, 100)], t=0..10, color=[red, blue, green])
```




```
> plot([sol2_unapp(t, 2, 50, 100), sol2_unapp(t, 2, 35, 100),
  sol2_unapp(t, 1.35, 75, 100)], t=0..10, color=[red, blue, green])
```



c)

```
> ec1:=sol1_unapp(2, r, 25000)=30000
```

$$ec1 := 25000\, e^{2\,r} = 30000 \tag{31}$$

```
> solve(ec1, r)
```

$$\frac{\ln\left(\frac{6}{5}\right)}{2} \tag{32}$$

```
> sol1_unapp(5, %, 25000)
```

$$7200\sqrt{30} \tag{33}$$

```
> evalf(%)
```

$$39436.02414 \tag{34}$$

d)

```
> ec1:=sol2_unapp(2, r0, K, 20000)=40000
```

$$ec1 := \frac{20000\, K}{20000 + e^{-2\,r0}\,(K - 20000)} = 40000 \tag{35}$$

```
> ec2:=sol2_unapp(3, r0, K, 20000)=50000
```

(36)

$$ec2 := \frac{20000\,K}{20000 + e^{-3\,r0}\,(K - 20000)} = 50000 \quad \textbf{(36)}$$

```
> solve({ec1, ec2}, {r0, K})
```

$$\left\{K = \frac{200000\,RootOf\left(5\,\_Z^2 - \_Z - 1\right)}{7} + \frac{400000}{7},\ r0 = -\ln\left(RootOf\left(5\,\_Z^2 - \_Z - 1\right)\right)\right\} \quad \textbf{(37)}$$