```
> 2+2
```
$$4 \qquad (1)$$

```
> 2+2
```
$$4 \qquad (2)$$

```
> 11
```
$$11 \qquad (3)$$

```
> with(DEtools)
```
$[$ *AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM,*   **(4)**

    DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper,

    Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols,

    MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm,

    RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge,

    Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot,

    casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol,

    dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring,

    endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint,

    firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs,

    hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols,

    intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE,

    matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon,

    normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait,

    poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder,

    reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread,

    rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen,

    symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam,

    zoom $]$

```
> with(plots)
```
$[$ *animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal,* **(5)**

    conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display,

    dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal,

    interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot,

    listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot,

    pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot,

    rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve,

    sparsematrixplot, surfdata, textplot, textplot3d, tubeplot $]$

```
> ecdif:=diff(y(x),x)=2*x*(1+y^2)
```
$$ecdif := \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = 2\, x\, (y^2 + 1) \qquad (6)$$

```
> ecdif
```

$$\frac{d}{dx} y(x) = 2x\left(y^2 + 1\right) \tag{7}$$

```
> dsolve(ecdif, y(x))
```
Error, (in dsolve) y(x) and y cannot both appear in the given ODE.
```
> ecdif:=diff(y(x),x)=2*x*(1+(y(x))^2)
```
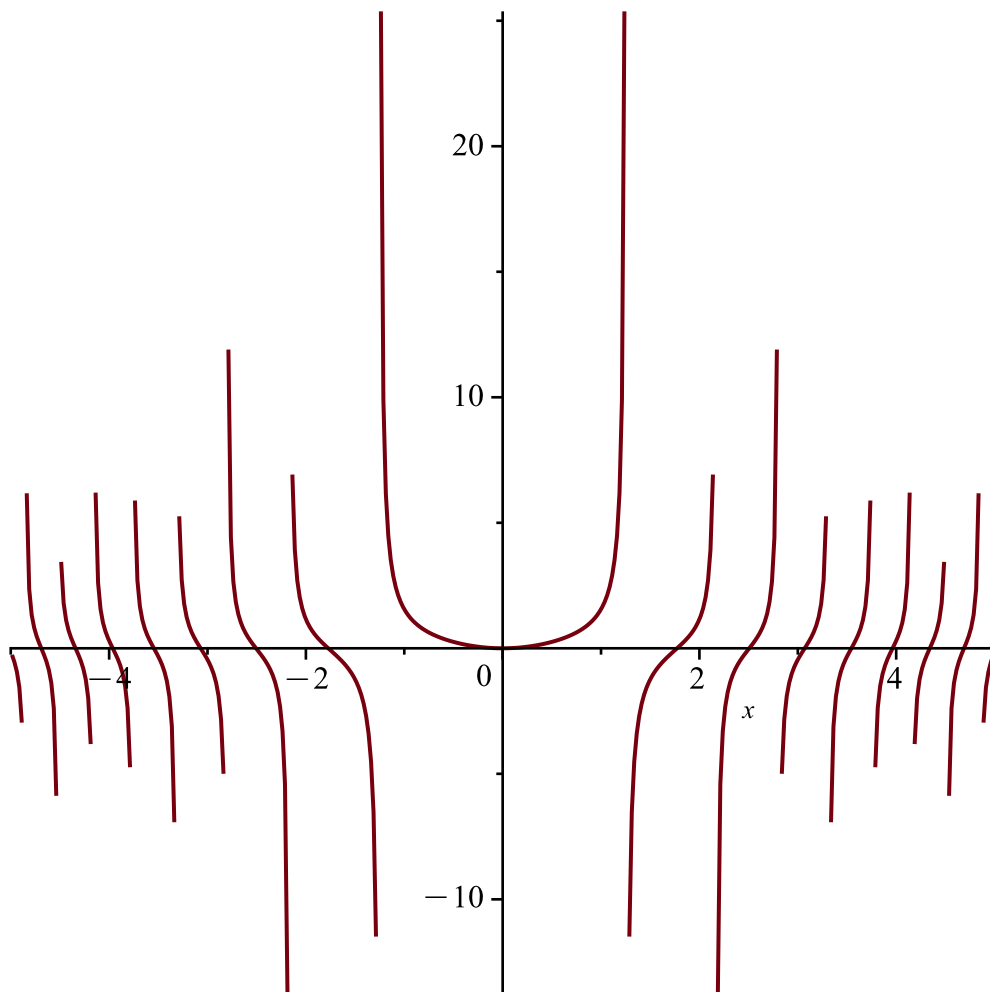
$$ecdif := \frac{d}{dx} y(x) = 2x\left(1 + y(x)^2\right) \tag{8}$$

```
> dsolve(ecdif,y(x))
```

$$y(x) = \tan\left(x^2 + 2\,c_1\right) \tag{9}$$

```
> ysol := unapply(rhs(dsolve(ecdif,y(x))),x,_C1)
```

$$ysol := (x, c_1) \mapsto \tan\left(x^2 + 2 \cdot c_1\right) \tag{10}$$

```
> plot(ysol(x, 0), x=-5..5)
```



```
1b
> ecdif:=(x^2-1)*diff(y(x),x) + 2*x*(y(x)^2)=0
```

$$ecdif := \left(x^2 - 1\right)\left(\frac{d}{dx} y(x)\right) + 2\,y(x)^2 x = 0 \tag{11}$$

```
> dsolve(ecdif,y(x))
```

$$y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \tag{12}$$

```
> ysol := unapply(rhs(dsolve(ecdif,y(x))),x,_C1)
```

$$ysol := (x, c_1) \mapsto \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \tag{13}$$

```
> plot(ysol(x, 0), x=1..infinity)
```
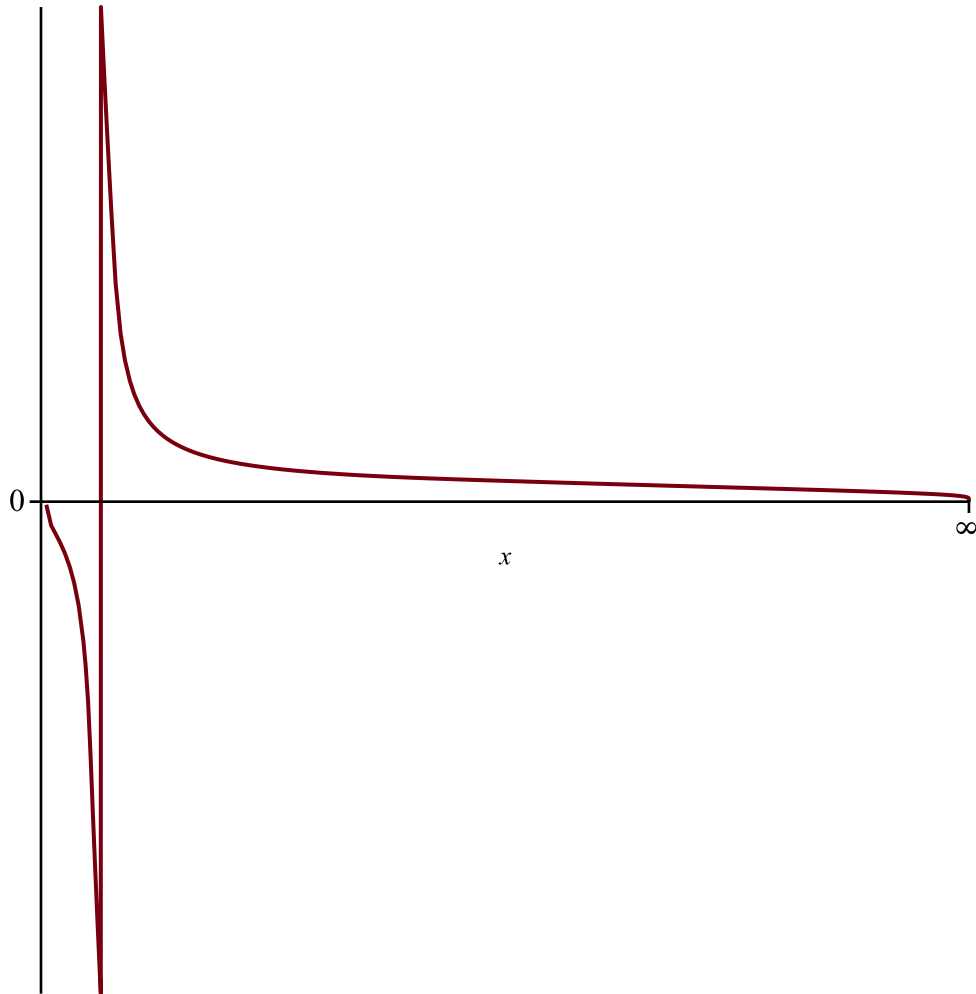

1c

```
> ecdif:=2*x^2*D(y)(x)=x^2+y(x)^2
```

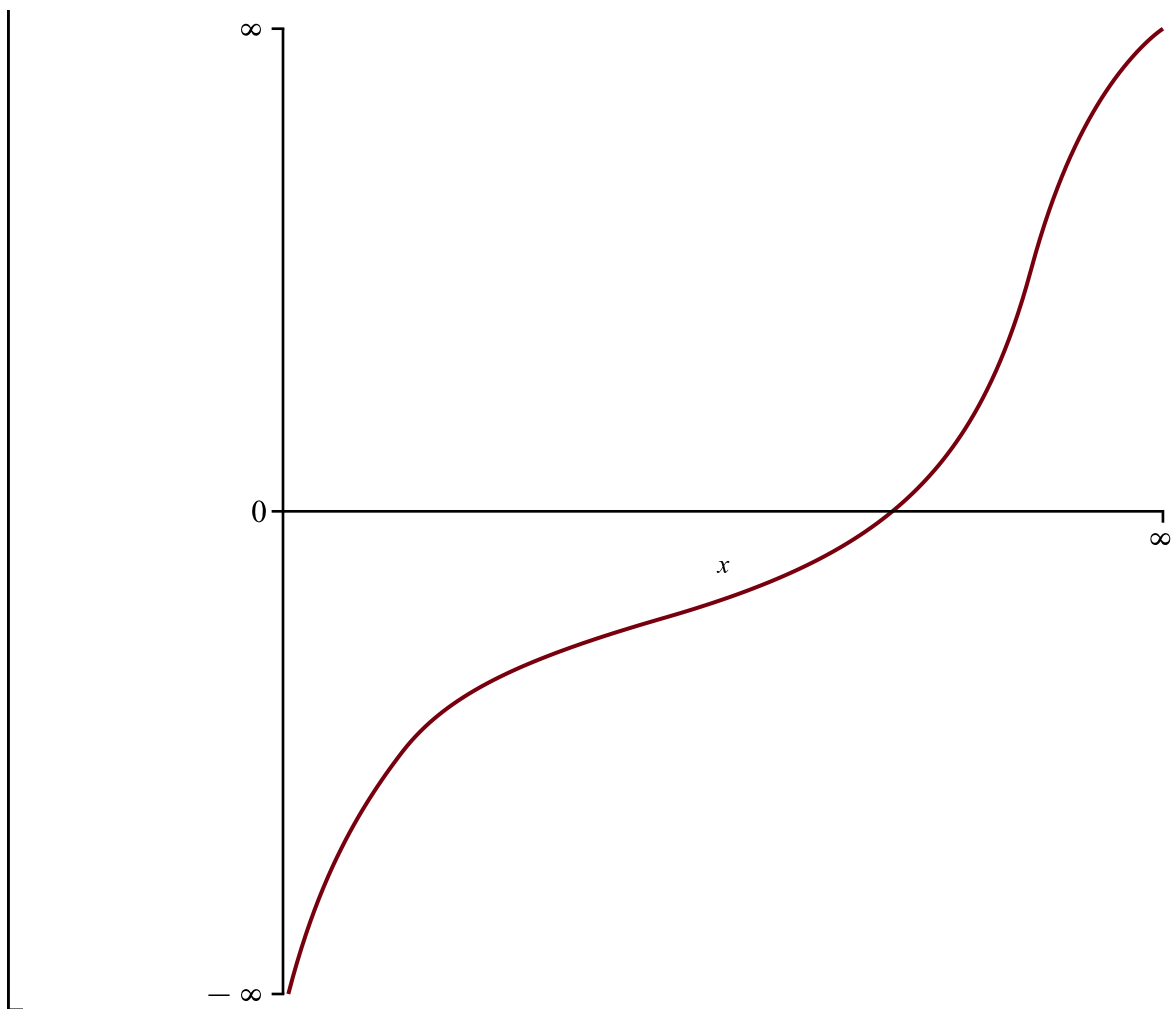$$ecdif := 2\,x^2\,\mathrm{D}(y)(x) = x^2 + y(x)^2 \tag{14}$$

```
> dsolve(ecdif,y(x))
```

$$y(x) = \frac{x\left(\ln(x) + c_1 - 2\right)}{\ln(x) + c_1} \tag{15}$$

```
> ysol:= unapply(rhs(dsolve(ecdif, y(x))), x, _C1)
```

$$ysol := (x, c_1) \mapsto \frac{x \cdot \left(\ln(x) + c_1 - 2\right)}{\ln(x) + c_1} \tag{16}$$

```
> plot(ysol(x, 0), x=1..infinity)
```

```
1d
> ecsol:=dsolve(D(y)(x)=-x/y, y(x))
Error, (in dsolve) y(x) and y cannot both appear in the given ODE.
> ecsol:=dsolve(D(y)(x)=-x/y(x),y(x),implicit)
```

$$ecsol := x^2 + y(x)^2 - c_1 = 0 \tag{17}$$

```
> ysol:=unapply(rhs(ecsol), x, _C1)
Error, invalid input: rhs expects 1 argument, but received 2
> ysol:=unapply(rhs(ecsol[0]), x, _C1)
Error, invalid subscript selector
> ecsol
```

$$y(x) = \sqrt{-x^2 + c_1}, y(x) = -\sqrt{-x^2 + c_1} \tag{18}$$

```
> ecsol[0]
Error, invalid subscript selector

> ecsol(0)
```

$$y(x)(0) = \sqrt{-x(0)^2 + c_1(0)}, y(x)(0) = -\sqrt{-x(0)^2 + c_1(0)} \tag{19}$$

```
> ysol:=unapply(subs(y(x)=y,lhs(ecsol)), x, y, _C1)
```

$$ysol := (x, y, c_1) \mapsto x^2 + y^2 - c_1 \tag{20}$$

```
> implicitplot(ysol, x=0..infinity, y=0..infinity)
```

```
> implicitplot(ysol,x=0..100,y=0..100)
```

```
> implicitplot(ysol(x, y, 2)=0, x=-5..5,y=-5..5)
```
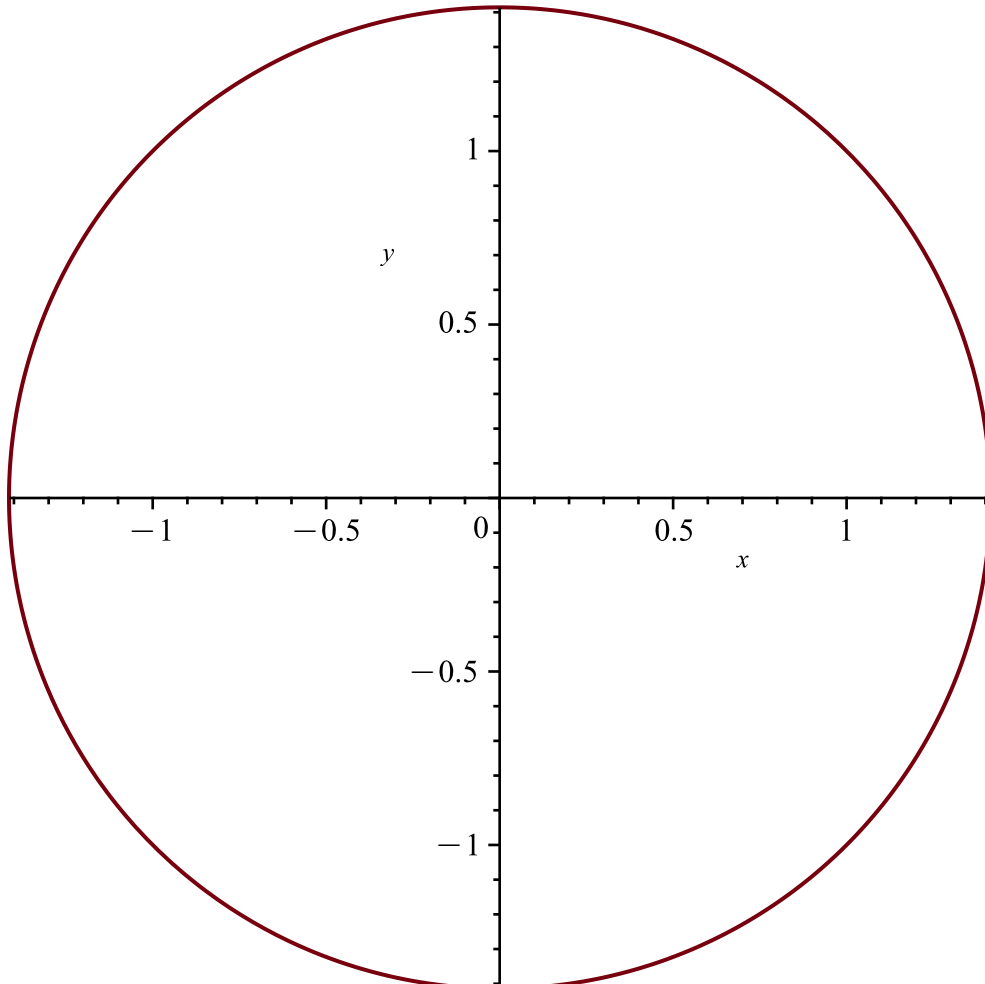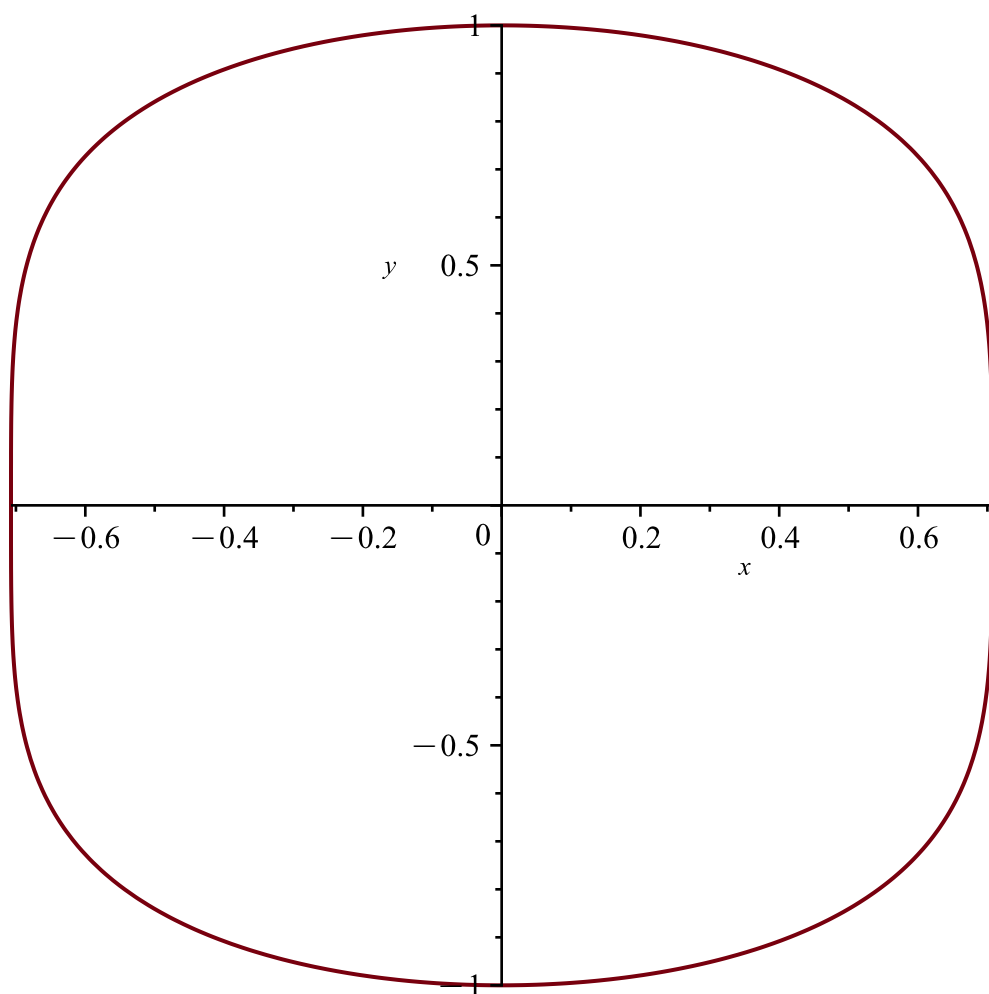


```
> implicitplot(ysol(x, y, 0), x=0..
```

1e
```
> ecsol:=dsolve(D(y)(x)=-x/(y(x)^3),y(x),implicit)
```
$$ecsol := y(x)^4 + 2\,x^2 - c_1 = 0 \tag{21}$$

```
> ysol:=unapply(subs(y(x)=y,lhs(ecsol)), x, y, _C1)
```
$$ysol := (x, y, c_1) \mapsto y^4 + 2 \cdot x^2 - c_1 \tag{22}$$

```
> implicitplot(ysol(x, y, 1), x=-10..10,y=-10..10)
```
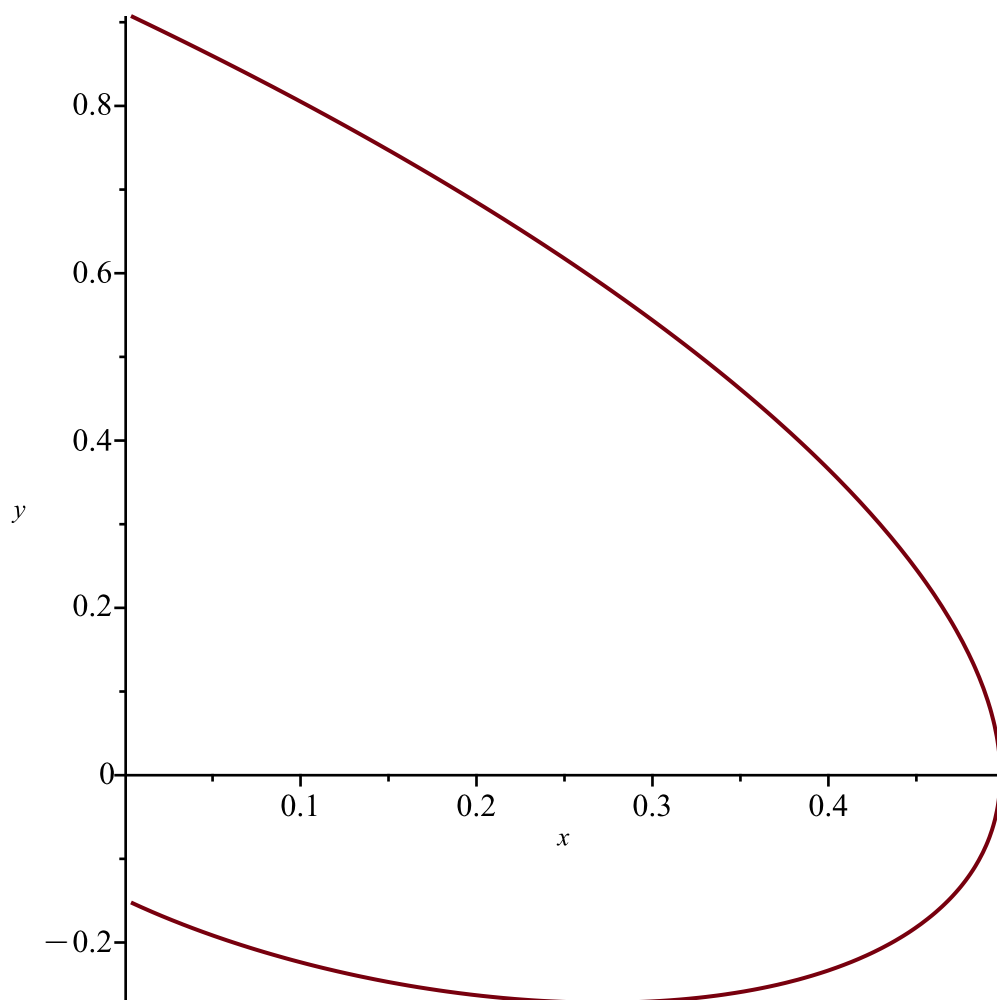
1f

```
> ecsol:=dsolve(D(y)(x)=-((x+y(x))/y(x)),y(x),implicit)
```

$$ecsol := -\frac{\ln\left(\dfrac{x^2 + x\,y(x) + y(x)^2}{x^2}\right)}{2} + \frac{\sqrt{3}\,\arctan\left(\dfrac{(2\,y(x) + x)\,\sqrt{3}}{3\,x}\right)}{3} - \ln(x) - c_1 = 0 \quad \textbf{(23)}$$

```
> ysol:=unapply(subs(y(x)=y,lhs(ecsol)),x,y,_C1)
```

$$ysol := (x, y, c_1) \mapsto -\frac{\ln\left(\dfrac{x^2 + y\cdot x + y^2}{x^2}\right)}{2} + \frac{\sqrt{3}\cdot\arctan\left(\dfrac{(2\cdot y + x)\cdot\sqrt{3}}{3\cdot x}\right)}{3} - \ln(x) - c_1 \quad \textbf{(24)}$$
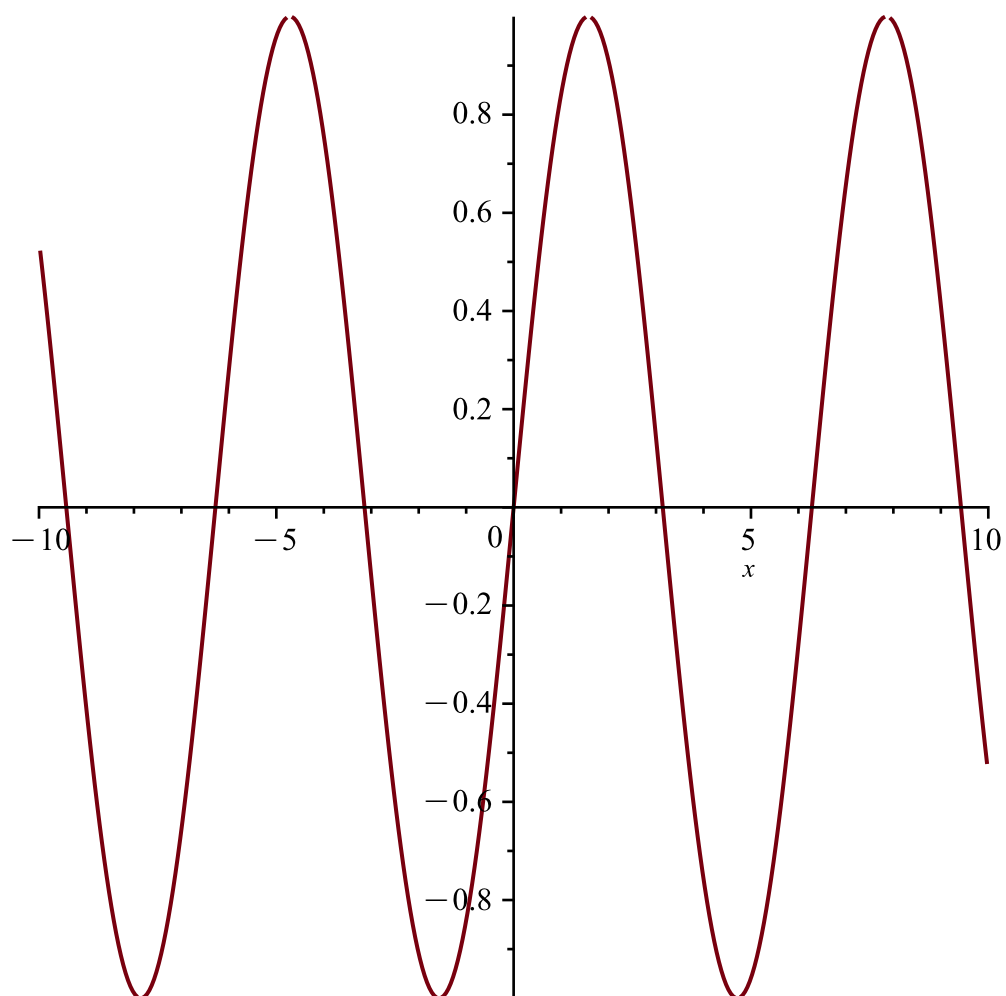
```
> implicitplot(ysol(x,y,1),x=-5..5,y=-5..5)
```

```
1g
> ecsol:=dsolve(D(y)(x)+y(x)*tan(x)=1/cos(x),y(x),implicit)
```
$$ecsol := y(x) = \left(\tan(x) + c_1 \right) \cos(x)$$ (25)

```
> ysol:=unapply(rhs(ecsol),x,_C1)
```
$$ysol := \left(x, c_1 \right) \mapsto \left(\tan(x) + c_1 \right) \cdot \cos(x)$$ (26)

```
> plot(ysol(x, 0), x=-10..10)
```

```
> ecsol:=dsolve(D(y)(x)+(2/x)*y(x)=x^3, y(x), implicit)
```
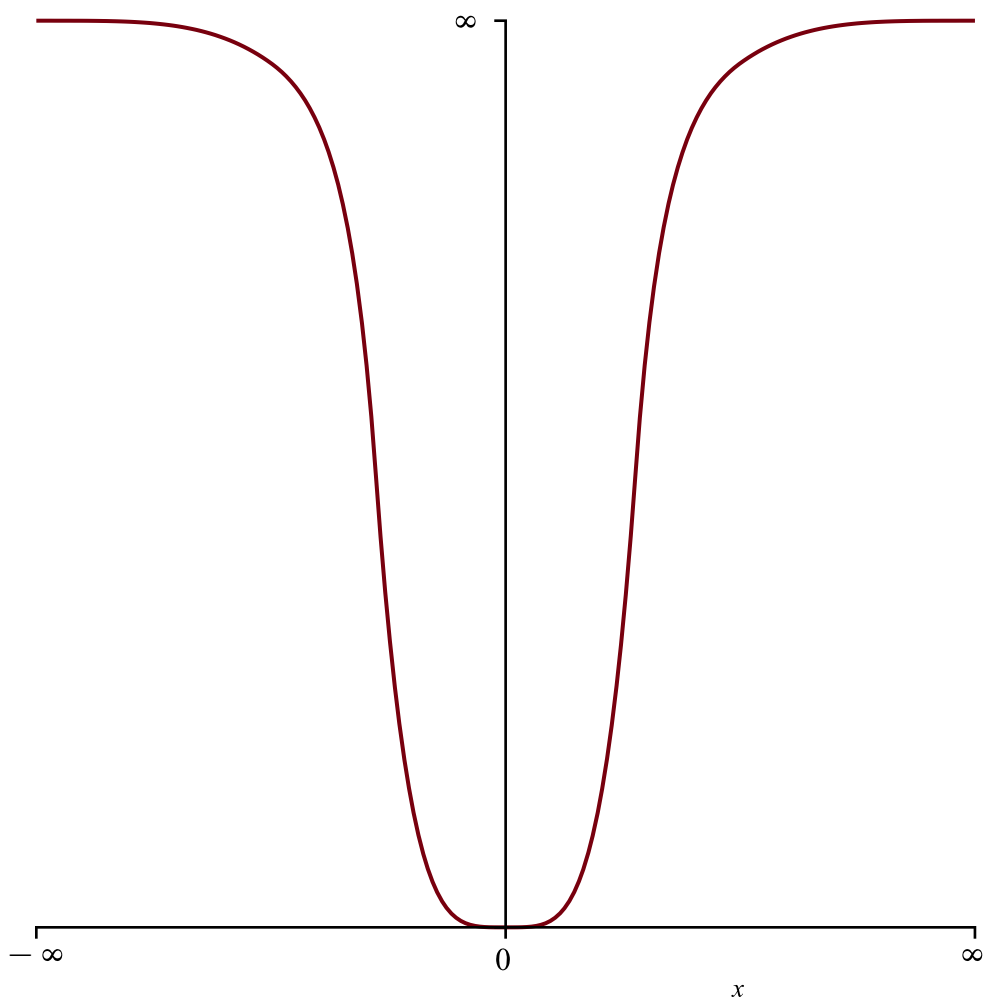
$$ecsol := y(x) = \frac{\dfrac{x^6}{6} + c_1}{x^2} \tag{27}$$

```
> ysol:=unapply(rhs(ecsol),x,_C1)
```

$$ysol := \left(x, c_1\right) \mapsto \frac{\dfrac{x^6}{6} + c_1}{x^2} \tag{28}$$
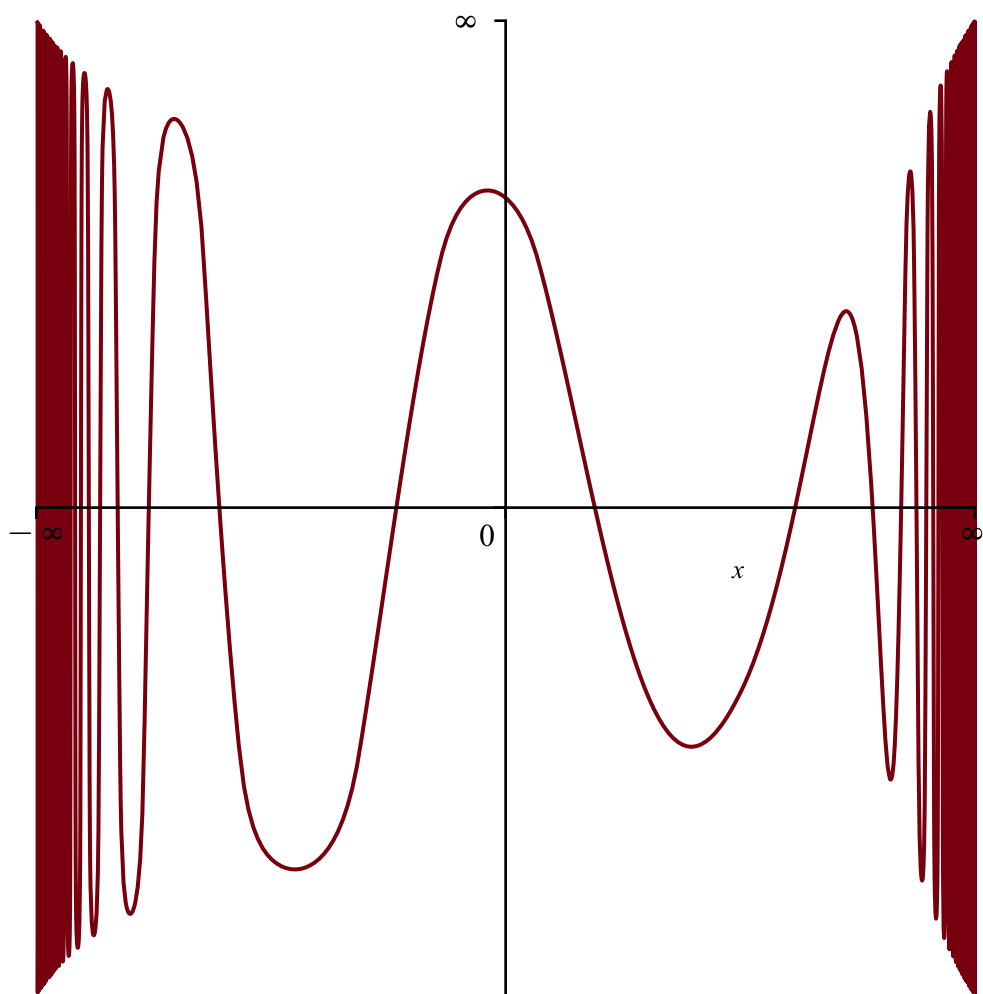
```
> plot(ysol(x, 0), x=-infinity..infinity)
```

```
> ecsol:=dsolve((D@D)(y)(x)+y(x)=sin(x)+cos(x), y(x), implicit)
```

$$ecsol := y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \frac{(1-x)\cos(x)}{2} + \frac{\sin(x)\, x}{2} \tag{29}$$

```
> ysol:=unapply(rhs(ecsol),x,_C1,_C2)
```

$$ysol := (x, c_1, c_2) \mapsto \sin(x)\cdot c_2 + \cos(x)\cdot c_1 + \frac{(1-x)\cdot\cos(x)}{2} + \frac{\sin(x)\cdot x}{2} \tag{30}$$

```
> plot(ysol(x, 5, -1), x=-infinity..infinity)
```

```
> ecsol:=dsolve((D@D)(y)(x) - y(x)=exp(2*x),y(x),implicit)
```

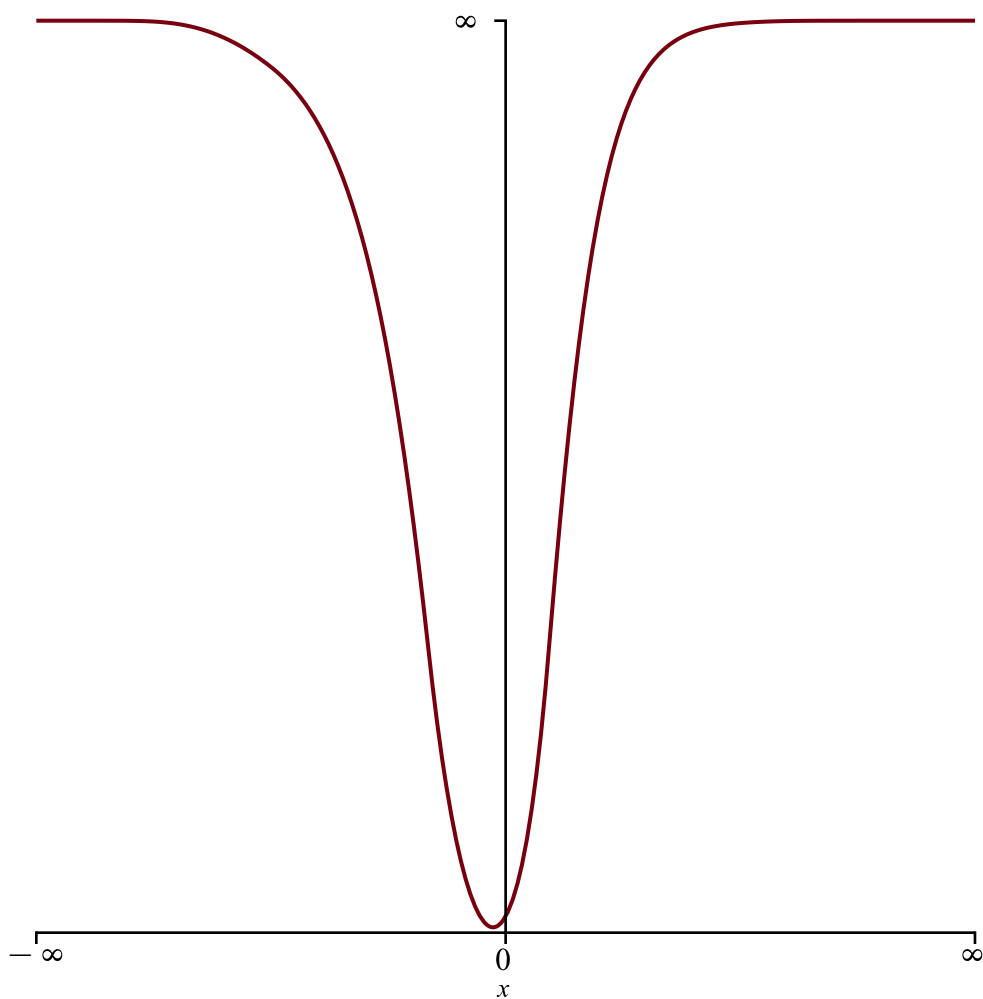$$ecsol := y(x) = e^x c_2 + e^{-x} c_1 + \frac{e^{2x}}{3}$$ (31)

```
> ysol:=unapply(rhs(ecsol),x,_C1,_C2)
```

$$ysol := \left(x, c_1, c_2\right) \mapsto e^x \cdot c_2 + e^{-x} \cdot c_1 + \frac{e^{2 \cdot x}}{3}$$ (32)
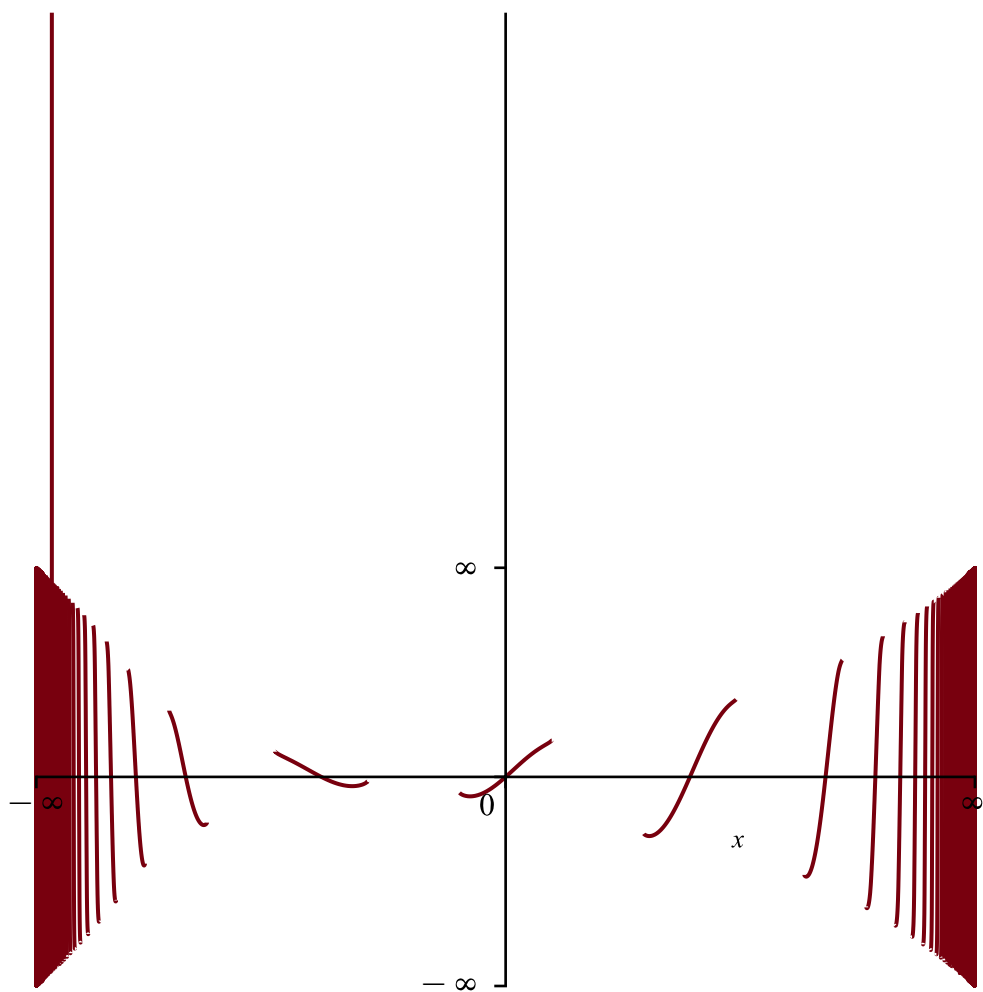
```
> plot(ysol(x, 1, 1), x=-infinity..infinity)
```

```
> ecsol:=dsolve((D@D)(y)(x)+4*y(x)=1/(cos(2*x)), y(x), implicit)
```

$$ecsol := y(x) = \sin(2\,x)\,c_2 + \cos(2\,x)\,c_1 + \frac{\sin(2\,x)\,x}{2} + \frac{\ln(\cos(2\,x))\,\cos(2\,x)}{4} \tag{33}$$

```
> ysol:=unapply(rhs(ecsol), x, _C1, _C2)
```

$$ysol := \left(x, c_1, c_2\right) \mapsto \sin(2 \cdot x) \cdot c_2 + \cos(2 \cdot x) \cdot c_1 + \frac{\sin(2 \cdot x) \cdot x}{2} + \frac{\ln(\cos(2 \cdot x)) \cdot \cos(2 \cdot x)}{4} \tag{34}$$

```
> plot(ysol(x, 0, 1), x=-infinity..infinity, numpoints=10000)
```

11

```
> ecsol:=dsolve((D@D)(y)(x)-D(y)(x)=1/(1+exp(x)),y(x),implicit)
```

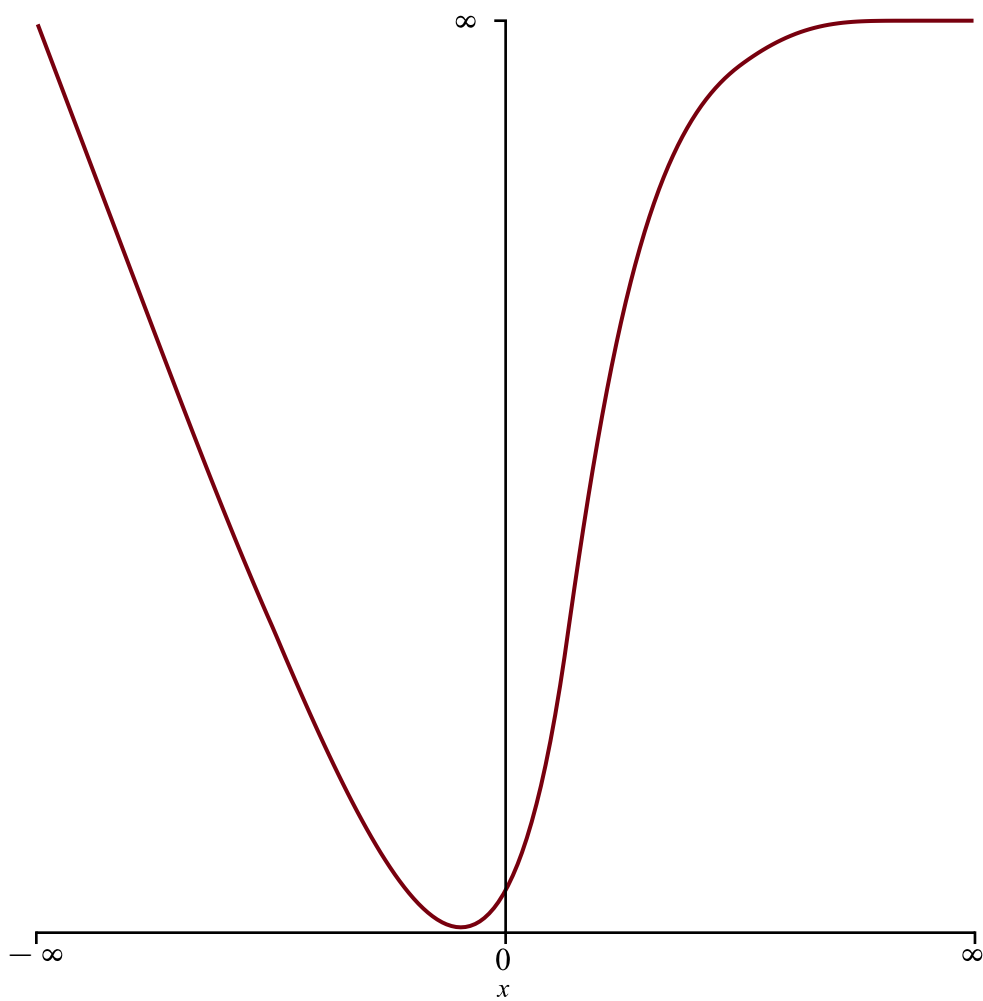$$ecsol := y(x) = -x + e^x c_1 + \ln(1 + e^x)(1 + e^x) - 1 - e^x \ln(e^x) + c_2 \qquad \textbf{(35)}$$

```
> ysol:=unapply(rhs(ecsol), x, _C1, _C2)
```

$$ysol := \left(x, c_1, c_2\right) \mapsto -x + e^x \cdot c_1 + \ln(1 + e^x) \cdot (1 + e^x) - 1 - e^x \cdot \ln(e^x) + c_2 \qquad \textbf{(36)}$$

```
> plot(ysol(x, 1, 1), x=-infinity..infinity)
```

2a

```
> ecsol:=dsolve({D(y)(x)=1+y(x)^2, y(0)=1}, y(x), implicit)
```
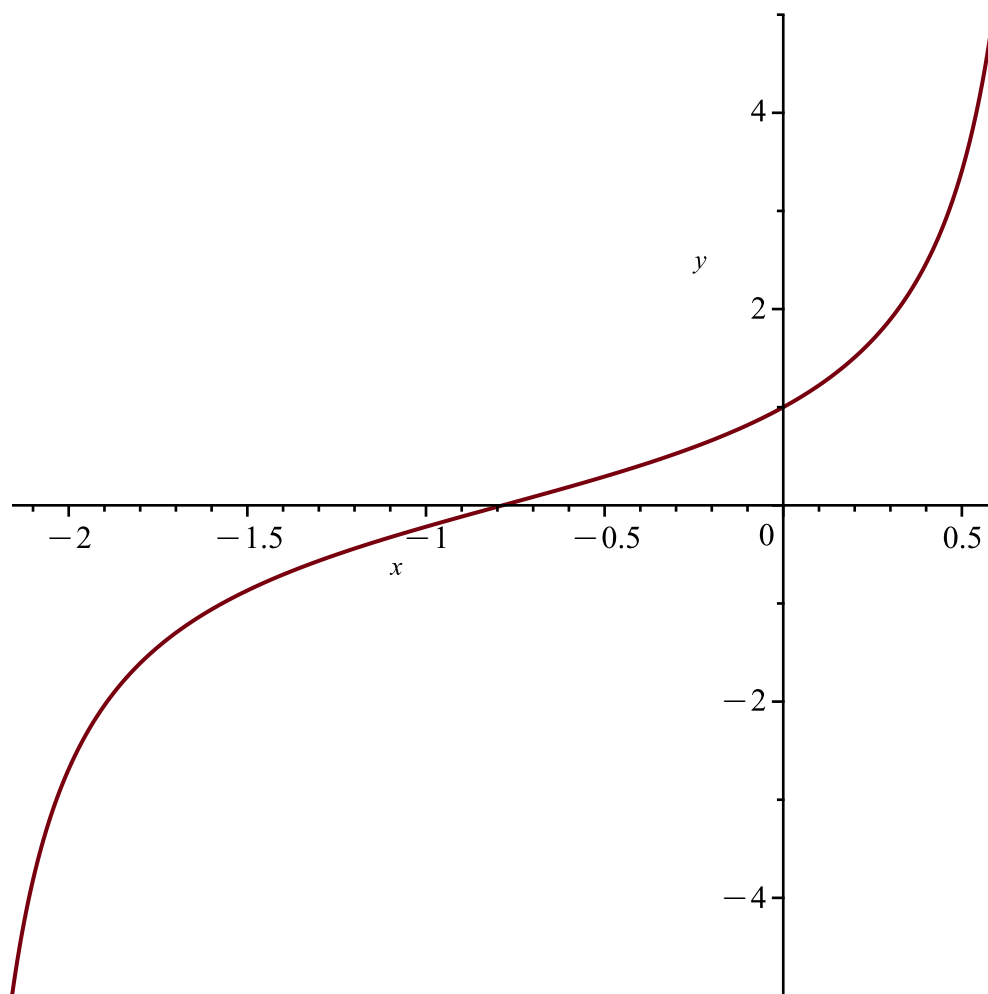
$$ecsol := x - \arctan(y(x)) + \frac{\pi}{4} = 0 \tag{37}$$

```
> ysol:=unapply(subs(y(x)=y,lhs(ecsol)), x, y)
```

$$ysol := (x, y) \mapsto x - \arctan(y) + \frac{\pi}{4} \tag{38}$$

```
> implicitplot(ysol(x, y), x=-10000..10000, y=-5..5)
```
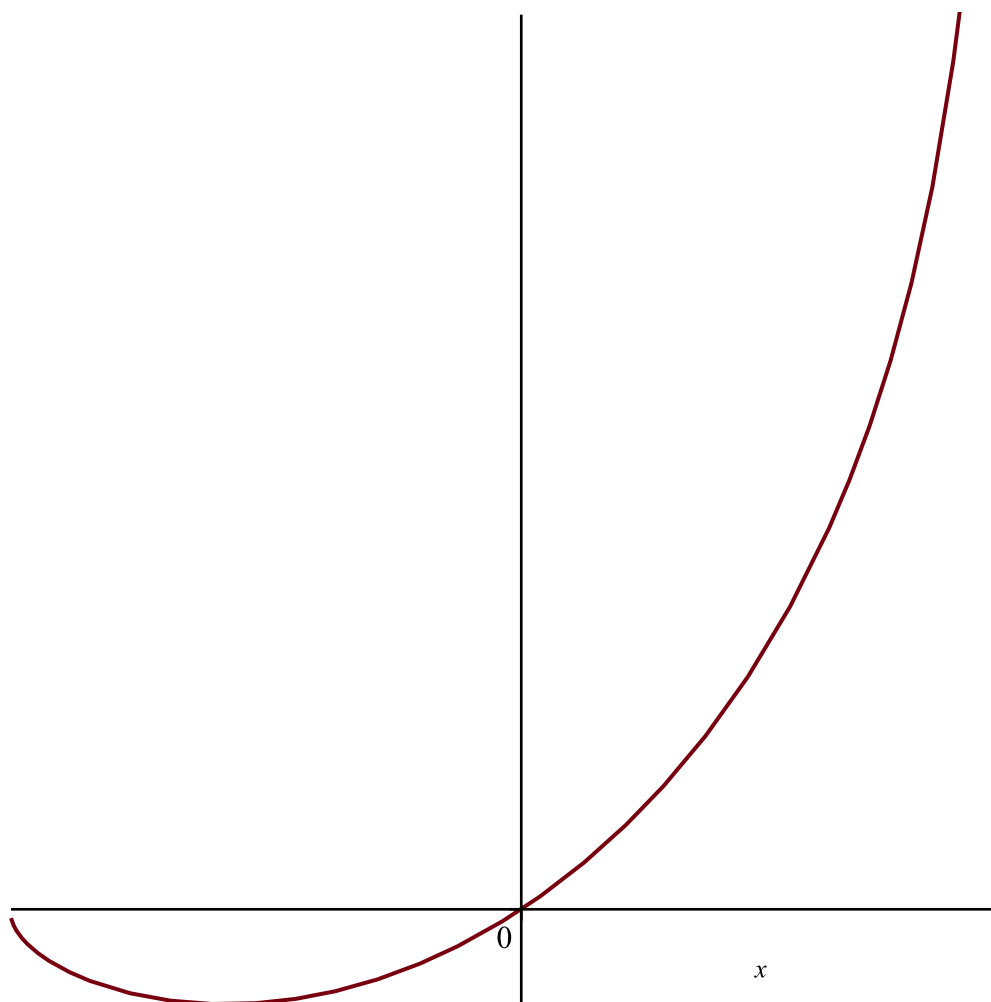
```
> ecsol:=dsolve({D(y)(x)=(1/(1-x^2))*y(x)+1+x, y(0)=0}, y(x),
  implicit)
```

$$ecsol := y(x) = \frac{\left(\dfrac{x\sqrt{-x^2+1}}{2} + \dfrac{\arcsin(x)}{2}\right)(x+1)}{\sqrt{-x^2+1}}$$

(39)

```
> ysol:=unapply(rhs(ecsol), x)
```

$$ysol := x \mapsto \frac{\left(\dfrac{x\cdot\sqrt{-x^2+1}}{2} + \dfrac{\arcsin(x)}{2}\right)\cdot(x+1)}{\sqrt{-x^2+1}}$$
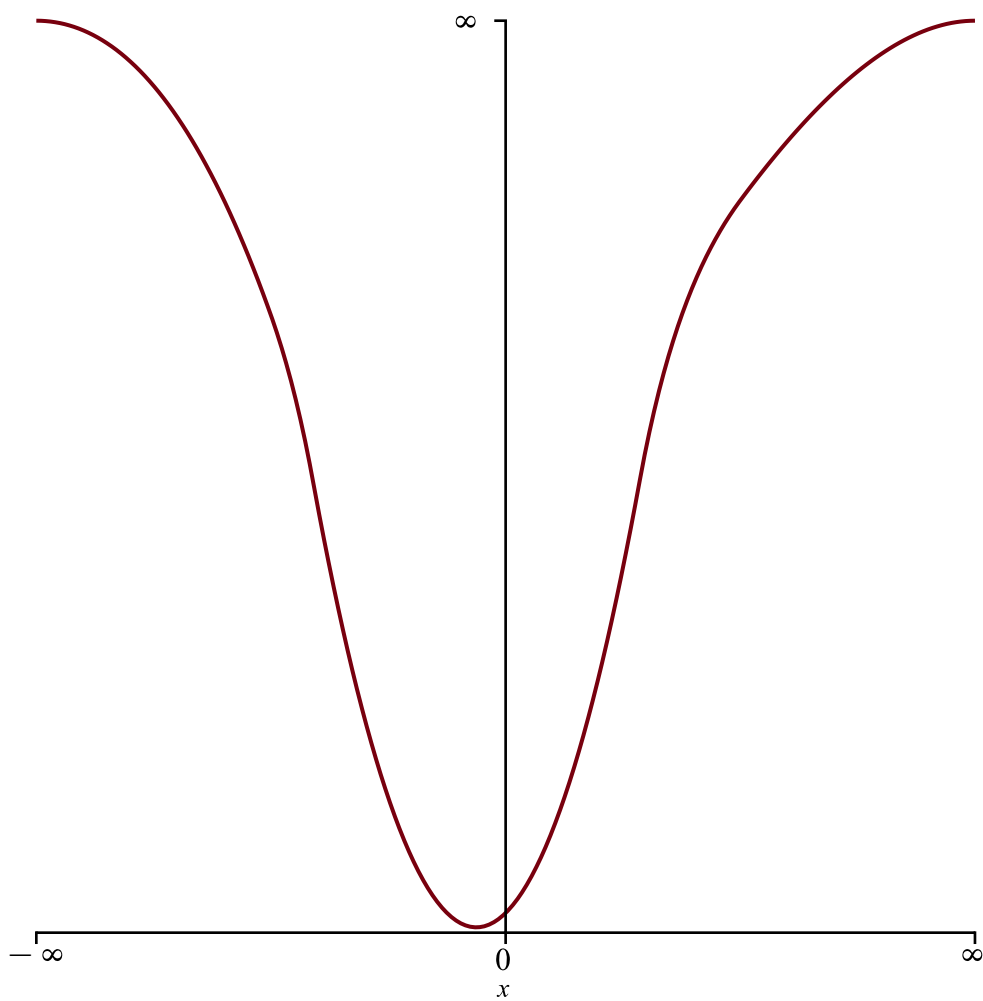
(40)

```
> plot(ysol(x), x=-infinity..infinity)
```

2c

```
> ecsol:=dsolve({D(y)(x)-2*y(x)=-x^2, y(0)=1/4}, y(x), implicit)
```

$$ecsol := y(x) = \frac{1}{2} x^2 + \frac{1}{2} x + \frac{1}{4} \tag{41}$$

```
> ysol:=unapply(rhs(ecsol), x)
```

$$ysol := x \mapsto \frac{1}{2} \cdot x^2 + \frac{1}{2} \cdot x + \frac{1}{4} \tag{42}$$
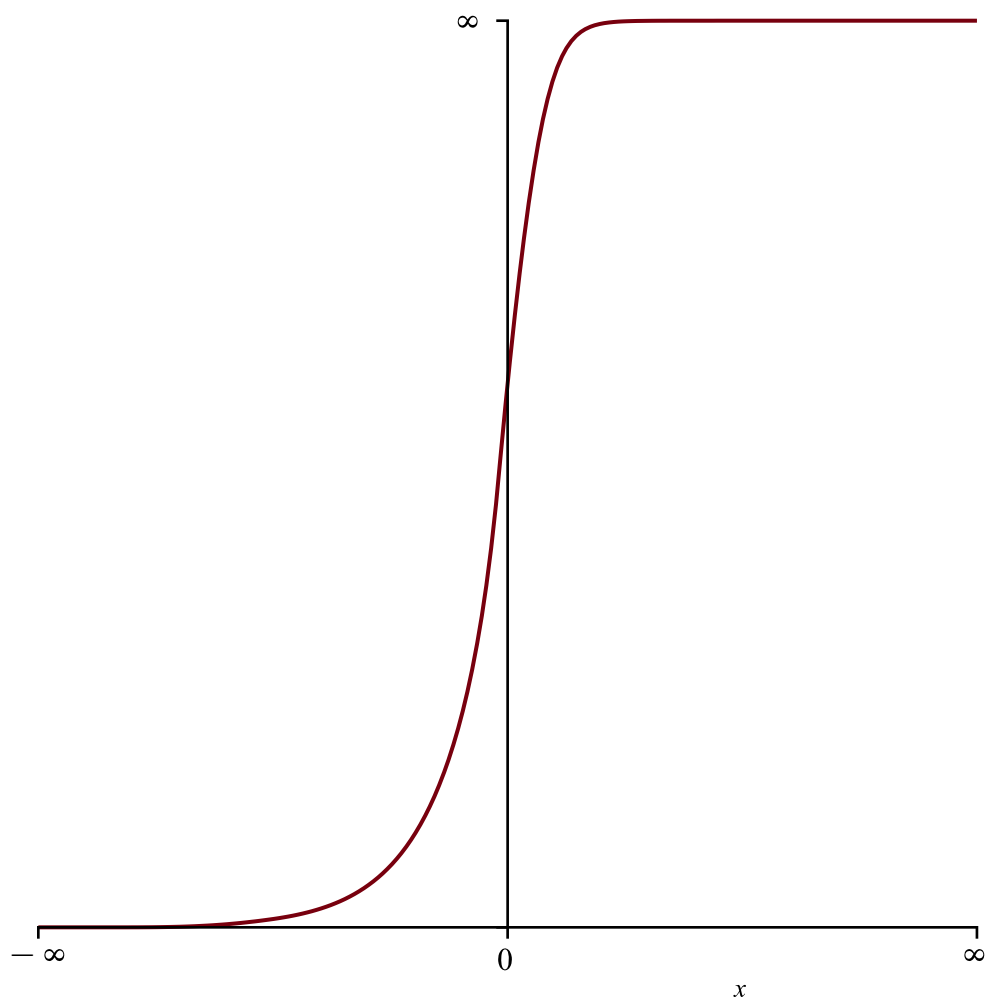
```
> plot(ysol(x), x=-infinity..infinity)
```

```
2d
> ecsol:=dsolve({(D@D)(y)(x)-5*D(y)(x)+4*y(x)=0, y(0)=5, D(y)(0)=8}
  , y(x), implicit)
```

$$ecsol := y(x) = 4\,e^{x} + e^{4\,x} \tag{43}$$

```
> ysol:=unapply(rhs(ecsol), x)
```

$$ysol := x \mapsto 4 \cdot e^{x} + e^{4 \cdot x} \tag{44}$$
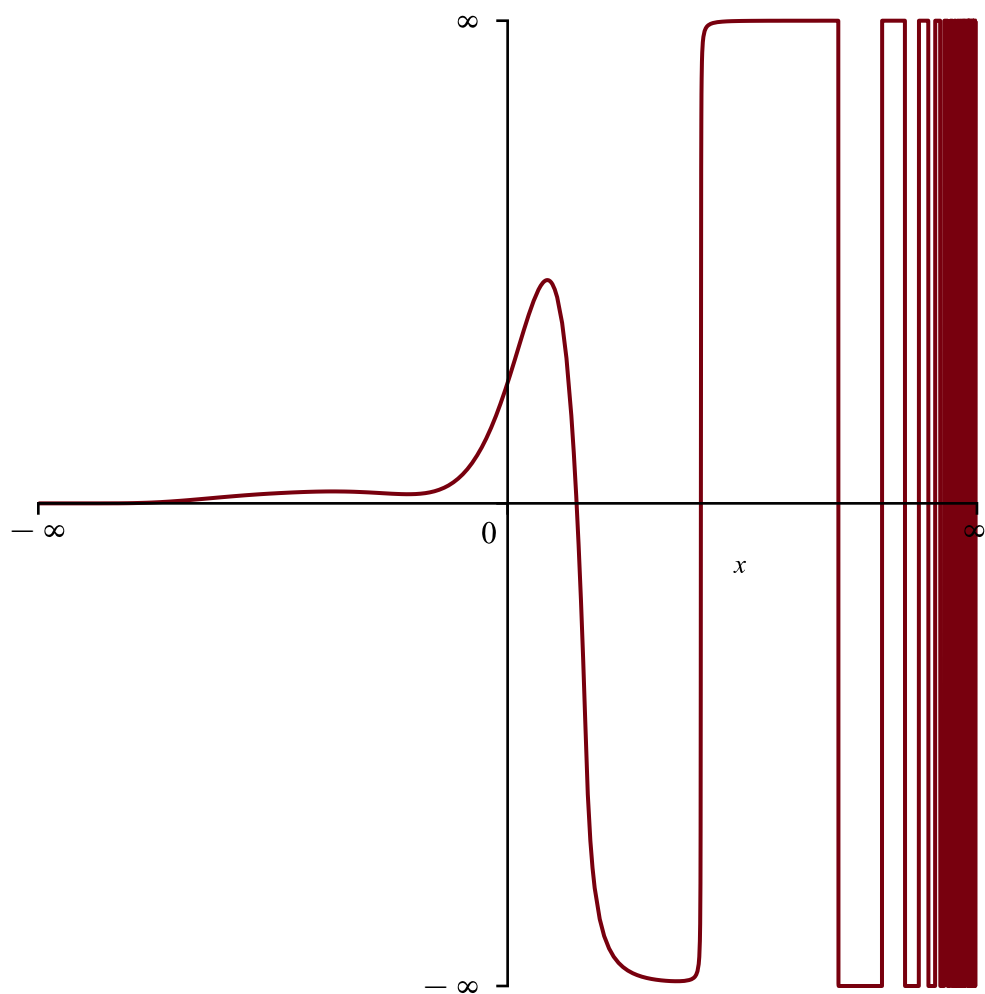
```
> plot(ysol(x),x=-infinity..infinity)
```

$2e$

```
> ecsol:=dsolve({(D@D)(y)(x)-4*D(y)(x)+5*y(x)=2*x^2*exp(x), y(0)=2,
  D(y)(0)=3}, y(x), implicit)
```

$$ecsol := y(x) = -2\,e^{2\,x}\sin(x) + e^{2\,x}\cos(x) + (x+1)^2\,e^x \tag{45}$$

```
> ysol:=unapply(rhs(ecsol), x)
```

$$ysol := x \mapsto -2\cdot e^{2\cdot x}\cdot\sin(x) + e^{2\cdot x}\cdot\cos(x) + (x+1)^2\cdot e^x \tag{46}$$

```
> plot(ysol(x),x=-infinity..infinity)
```
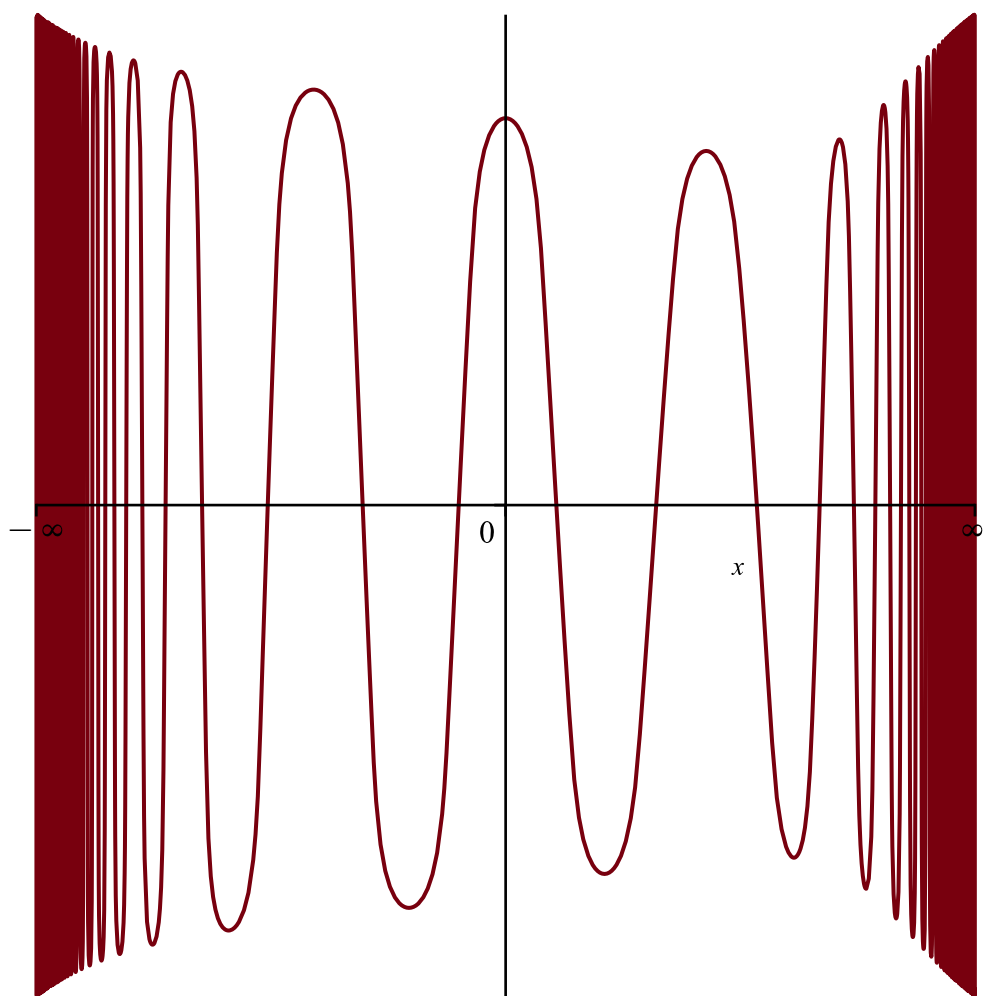
∞

0

x

− ∞

− ∞

2f

```
> ecsol:=dsolve({(D@D)(y)(x)+4*y(x)=4*(sin(2*x)+cos(2*x)), y(Pi)=D
  (y)(Pi), D(y)(Pi)=2*Pi}, y(x), implicit)
```

$$ecsol := y(x) = \frac{\sin(2\,x)}{2} + \cos(2\,x)\left(3\,\pi - \frac{1}{2}\right) + \frac{\cos(2\,x)}{2} + \sin(2\,x)\,x - \cos(2\,x)\,x \qquad \textbf{(47)}$$

```
> ysol:=unapply(rhs(ecsol), x)
```

$$ysol := x \mapsto \frac{\sin(2 \cdot x)}{2} + \cos(2 \cdot x) \cdot \left(3 \cdot \pi - \frac{1}{2}\right) + \frac{\cos(2 \cdot x)}{2} + \sin(2 \cdot x) \cdot x - \cos(2 \cdot x) \cdot x \qquad \textbf{(48)}$$

```
> plot(ysol(x),x=-infinity..infinity)
```

3a

```
> ecdiff:=(D@D)(y)(x)-(1/2)*y(x)=cos(x)
```

$$ecdiff := D^{(2)}(y)(x) - \frac{y(x)}{2} = \cos(x) \tag{49}$$

```
> DEPlot(ecdiff,y(x),x=-infinity..infinity,[[y(0)=0]],stepsize=0.1)
```

$$DEPlot\left(D^{(2)}(y)(x) - \frac{y(x)}{2} = \cos(x), y(x), x = -\infty ..\infty, [[y(0)=0]], stepsize = 0.1\right) \tag{50}$$

```
> With
```