> restart; [1.a) > ec1:=diff(x(t),t)=x(t)+4*y(t) $ec1 := \frac{d}{dt} x(t) = x(t) + 4 y(t)$ (1)

> ec2:=diff(y(t),t)=x(t)+y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + y(t)$$
(2)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) + 4 y(t), \frac{d}{dt} y(t) = x(t) + y(t)$$
 (3)

> with (DEtools)

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot polygon, DFactor, DFactorLCLM, **(4)** DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism charpoly, equinv, eta k, eulersols, exactsol, expsols, exterior power, firint, firtest, formal sol, gen exp, generate ic, genhomosol, gensys, hamilton eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix riccati, maxdimsystems, moser reduce, muchange, mult, mutest, newton polygon, normalG2, ode int y, ode y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power equivalent, rational equivalent, ratsols, redode, reduceOrder, reduce order, regular parts, regularsp, remove RootOf, riccati system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve group, super reduce, symgen, symmetric power, symmetric product, symtest, transinv, translate, untranslate, varparam, zoom]

> with (plots)

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve,

sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

$$\frac{d}{dt} x(t) = x(t) + 4 y(t), \frac{d}{dt} y(t) = x(t) + y(t)$$
 (6)

dsolve({sist}, {x(t),y(t)})

$$\left\{ x(t) = c_1 e^{3t} + c_2 e^{-t}, y(t) = \frac{c_1 e^{3t}}{2} - \frac{c_2 e^{-t}}{2} \right\}$$
 (7)

sol:=dsolve({sist}, {x(t),y(t)})

$$sol := \left\{ x(t) = c_1 e^{3t} + c_2 e^{-t}, y(t) = \frac{c_1 e^{3t}}{2} - \frac{c_2 e^{-t}}{2} \right\}$$
 (8)

> ec1:=diff(x(t), t)=2*x(t)-y(t)

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = 2 \ x(t) - y(t)$$
 (9)

= > ec2:=diff(y(t), t)=x(t)+2*y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2y(t)$$
 (10)

| |> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = 2 x(t) - y(t), \frac{d}{dt} y(t) = x(t) + 2 y(t)$$
 (11)

> sol:=dsolve({sist}, {x(t),y(t)})

$$sol := \left\{ x(t) = e^{2t} \left(c_2 \cos(t) + c_1 \sin(t) \right), y(t) = -e^{2t} \left(\cos(t) c_1 - \sin(t) c_2 \right) \right\}$$
(12)

> ec1:=diff(x(t),t)=x(t)-y(t)+z(t)

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = x(t) - y(t) + z(t)$$
 (13)

> ec2:=diff(y(t),t)=x(t)+y(t)-z(t)

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t) + y(t) - z(t)$$
 (14)

= |> ec3:=diff(z(t),t)=-y(t)+2*z(t)

$$ec3 := \frac{d}{dt} z(t) = -y(t) + 2 z(t)$$
 (15)

> sist:=ec1,ec2,ec3

$$sist := \frac{d}{dt} x(t) = x(t) - y(t) + z(t), \frac{d}{dt} y(t) = x(t) + y(t) - z(t), \frac{d}{dt} z(t) = -y(t) + 2 z(t)$$
 (16)

> sol:=dsolve({sist}, {x(t), y(t), z(t)})
sol:=
$$\{x(t) = c_1 e^{2t} + c_2 e^t + c_3 e^t t + c_3 e^t, y(t) = e^t (c_3 t + c_2 - c_3), z(t) = c_1 e^{2t} + c_2 e^t + c_3 e^t t\}$$
 (17)

> ec1:=diff(x(t),t)=5*x(t)+3*y(t)+1 $ec1 := \frac{d}{dt} x(t) = 5 x(t) + 3 y(t) + 1$ (18)> ec2:=diff(y(t),t)=-6*x(t)-4*y(t)+exp(-t) $ec2 := \frac{d}{dt} y(t) = -6 x(t) - 4 y(t) + e^{-t}$ (19)> sist:=ec1,ec2 $sist := \frac{d}{dt} x(t) = 5 x(t) + 3 y(t) + 1, \frac{d}{dt} y(t) = -6 x(t) - 4 y(t) + e^{-t}$ (20)= > sol:=dsolve({sist}, {x(t),y(t)}) $sol := \left\{ x(t) = e^{2t} c_2 + e^{-t} c_1 - 2 - t e^{-t} - \frac{e^{-t}}{3}, y(t) = -e^{2t} c_2 - 2 e^{-t} c_1 + \frac{e^{-t}}{3} + 2 t e^{-t} + 3 \right\}$ (21) le > ec1:=diff(x(t),t)=x(t)+3*y(t)+cos(t) $ec1 := \frac{d}{dt} x(t) = x(t) + 3 y(t) + \cos(t)$ (22)> ec2:=diff(y(t),t)=x(t)-y(t)+2*t $ec2 := \frac{d}{dt} y(t) = x(t) - y(t) + 2t$ (23)> sist:=ec1,ec2 $sist := \frac{d}{dt} x(t) = x(t) + 3 y(t) + \cos(t), \frac{d}{dt} y(t) = x(t) - y(t) + 2 t$ (24)= > sol:=dsolve({sist}, {x(t),y(t)}) $sol := \left\{ x(t) = 3 e^{2t} c_2 - e^{-2t} c_1 + \frac{\sin(t)}{5} - \frac{\cos(t)}{5} - \frac{3t}{2}, y(t) = e^{2t} c_2 + e^{-2t} c_1 - \frac{\cos(t)}{5} \right\}$ (25) $+\frac{t}{2}-\frac{1}{2}$ $=^{11}$ > ec1:=diff(x(t),t)=x(t)-2*y(t)-2*z(t)+exp(-t) $ec1 := \frac{d}{dt} x(t) = x(t) - 2y(t) - 2z(t) + e^{-t}$ (26)= > ec2:=diff(y(t),t)=-2*x(t)+y(t)+2*z(t) $ec2 := \frac{d}{dt} y(t) = -2 x(t) + y(t) + 2 z(t)$ (27)> ec3:=diff(z(t),t)=2*x(t)-y(t)-3*z(t)+exp(-t) $ec3 := \frac{d}{dt} z(t) = 2 x(t) - y(t) - 3 z(t) + e^{-t}$ (28)> sist:=ec1,ec2,ec3 $sist := \frac{d}{dt} x(t) = x(t) - 2y(t) - 2z(t) + e^{-t}, \frac{d}{dt} y(t) = -2x(t) + y(t) + 2z(t), \frac{d}{dt} z(t)$ (29)

$$= 2x(t) - y(t) - 3z(t) + e^{-t}$$
> sol:=dsolve({sist}, {x(t),y(t),z(t)})
$$sol := \begin{cases} x(t) = -c_2 e^{\sqrt{3}t} - c_3 e^{-\sqrt{3}t} + e^{-t}c_1 + t e^{-t}, y(t) = c_2 e^{\sqrt{3}t} + c_3 e^{-\sqrt{3}t}, z(t) \end{cases}$$

$$= \frac{c_2 \sqrt{3} e^{\sqrt{3}t}}{2} - \frac{c_3 \sqrt{3} e^{-\sqrt{3}t}}{2} - \frac{3c_2 e^{\sqrt{3}t}}{2} - \frac{3c_3 e^{-\sqrt{3}t}}{2} + e^{-t}c_1 + t e^{-t} \end{cases}$$
(30)

 \Rightarrow ec2:=diff(y(t),t)=x(t)+y(t)

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t) + y(t)$$
 (32)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) + 4 y(t), \frac{d}{dt} y(t) = x(t) + y(t)$$
 (33)

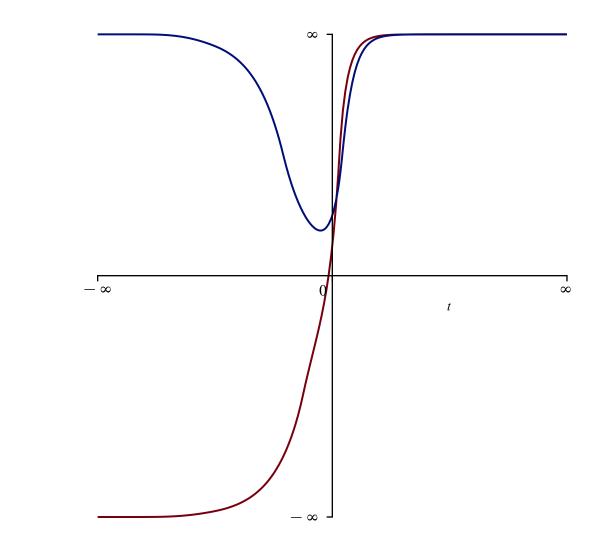
> cond_in:=x(0)=1,y(0)=2

$$cond_in := x(0) = 1, y(0) = 2$$
 (34)

> sol:=dsolve({sist,cond_in},{x(t),y(t)})

$$sol := \left\{ x(t) = \frac{5 e^{3t}}{2} - \frac{3 e^{-t}}{2}, y(t) = \frac{5 e^{3t}}{4} + \frac{3 e^{-t}}{4} \right\}$$
 (35)

> plot([unapply(rhs(sol[1]), t)(t),unapply(rhs(sol[2]), t)(t)],t=infinity..infinity)



[2b]
> ec1:=diff(x(t),t)=x(t)-y(t)+t-1
$$ec1 := \frac{d}{dt} x(t) = x(t) - y(t) + t - 1$$
(36)

> ec2:=diff(y(t),t)=-2*x(t)+4*y(t)+cos(t)

$$ec2 := \frac{d}{dt} y(t) = -2 x(t) + 4 y(t) + \cos(t)$$
 (37)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) - y(t) + t - 1, \frac{d}{dt} y(t) = -2x(t) + 4y(t) + \cos(t)$$

$$> \text{cond_in} := x(0) = 0, y(0) = 1$$

$$cond_in := x(0) = 0, y(0) = 1$$

$$> \text{sol} := \text{dsolve} \{\{sist_cond_in\}, \{x(t), y(t)\}\}$$

$$(38)$$

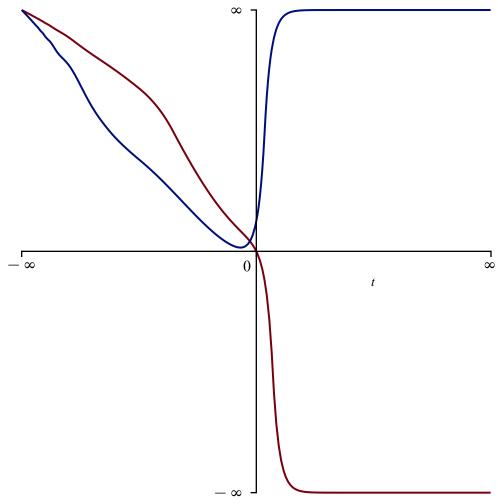
$$cond_in := x(0) = 0, y(0) = 1$$
 (39)

> sol:=dsolve({sist,cond_in},{x(t),y(t)})

$$sol := \begin{cases} x(t) = e^{\frac{\left(5 + \sqrt{17}\right)t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13}\right) + e^{-\frac{\left(-5 + \sqrt{17}\right)t}{2}} \left(\frac{33}{26} + \frac{5\sqrt{17}}{13}\right) + \frac{5\sin(t)}{26} \end{cases}$$
 (40)

$$-\frac{\cos(t)}{26} - 2t - \frac{5}{2}, y(t) = -\frac{e^{\frac{(5+\sqrt{17})t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13}\right)\sqrt{17}}{2} + \frac{e^{-\frac{(-5+\sqrt{17})t}{2}} \left(\frac{33}{26} + \frac{5\sqrt{17}}{13}\right)\sqrt{17}}{2} - \frac{3e^{\frac{(5+\sqrt{17})t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13}\right)}{2} - \frac{3\cos(t)}{13} + \frac{2\sin(t)}{13} - t - \frac{3}{2}\right\}$$

> plot([unapply(rhs(sol[1]), t)(t),unapply(rhs(sol[2]), t)(t)],t=infinity..infinity)



/ 44

$$ec2 := \frac{d}{dt} y(t) = -2 x(t) + y(t) + 1$$
 (42)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) + 2y(t) + e^{-t}, \frac{d}{dt} y(t) = -2x(t) + y(t) + 1$$
 (43)

> cond_in:=x(0)=0,y(0)=1

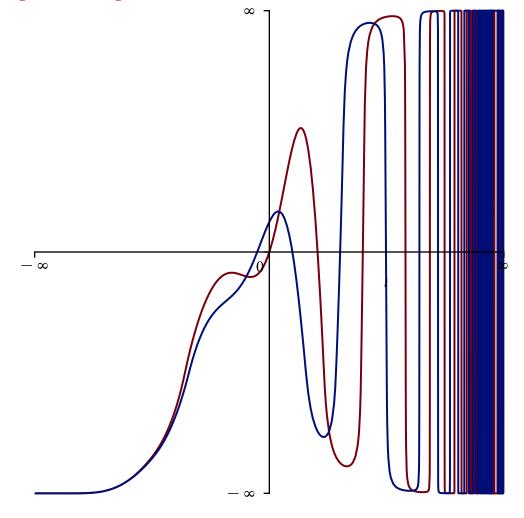
$$cond_in := x(0) = 0, y(0) = 1$$
 (44)

> sol:=dsolve({sist,cond_in},{x(t),y(t)})

$$sol := \left\{ x(t) = -\frac{3 e^{t} \cos(2 t)}{20} + \frac{29 e^{t} \sin(2 t)}{20} - \frac{e^{-t}}{4} + \frac{2}{5}, y(t) = \frac{3 e^{t} \sin(2 t)}{20} + \frac{29 e^{t} \cos(2 t)}{20} - \frac{1}{5} - \frac{e^{-t}}{4} \right\}$$

$$(45)$$

> plot([unapply(rhs(sol[1]), t)(t),unapply(rhs(sol[2]), t)(t)],t=-infinity..infinity)



> ec2:=diff(y(t),t)=2*x(t)-2*y(t)-5*z(t)+3*t
$$ec2 := \frac{d}{dt} y(t) = 2 x(t) - 2 y(t) - 5 z(t) + 3 t$$
(47)

 \rightarrow ec3:=diff(z(t),t)=-2*x(t)+3*y(t)+6*z(t)+3

$$ec3 := \frac{d}{dt} z(t) = -2 x(t) + 3 y(t) + 6 z(t) + 3$$
 (48)

> sist:=ec1,ec2,ec3

$$sist := \frac{d}{dt} x(t) = -x(t) + 3 y(t) + 3 z(t) + 27 t^2, \frac{d}{dt} y(t) = 2 x(t) - 2 y(t) - 5 z(t) + 3 t, \frac{d}{dt} (49)$$

$$z(t) = -2x(t) + 3y(t) + 6z(t) + 3$$

> cond_in:=x(0)=50,y(0)=-30,z(0)=26

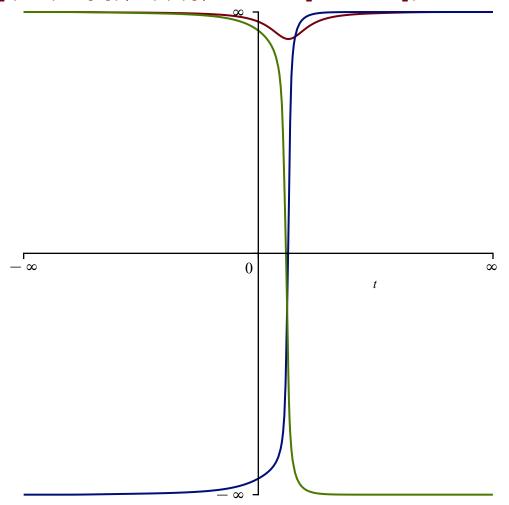
$$cond\ in:=x(0)=50,y(0)=-30,z(0)=26$$
(50)

> sol:=dsolve({sist,cond_in},{x(t),y(t),z(t)})

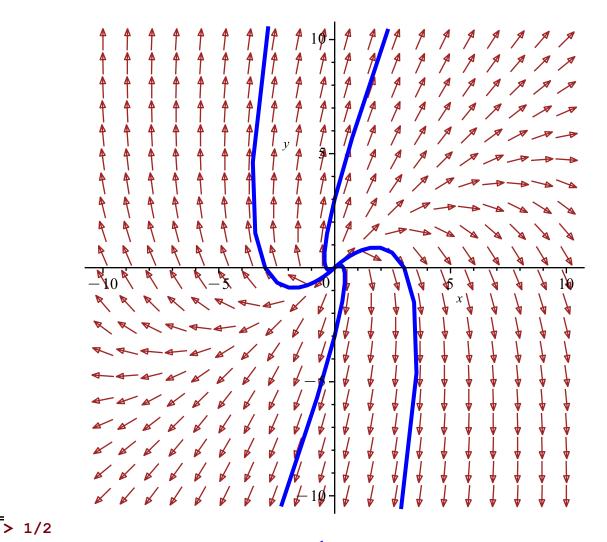
$$sol := \{x(t) = 3 e^{t} + 2 e^{-t} + 27 t^{2} - 63 t + 45, y(t) = e^{3t} + 2 e^{t} - 18 t^{2} + 24 t - 32 - e^{-t}, z(t)$$

$$= -e^{3t} - 27 t + 26 + 18 t^{2} + e^{-t}\}$$
(51)

> plot([unapply(rhs(sol[1]), t)(t),unapply(rhs(sol[2]), t)(t), unapply(rhs(sol[3]), t)(t)],t=-infinity..infinity)



```
> ec1:=diff(x(t),t)=x(t)+y(t)
                                   ec1 := \frac{d}{dt} x(t) = x(t) + y(t)
                                                                                                       (52)
> ec2:=diff(y(t),t)=-2*x(t)+4*y(t)
                                ec2 := \frac{d}{dt} y(t) = -2 x(t) + 4 y(t)
                                                                                                       (53)
> sist:=ec1,ec2
                    sist := \frac{d}{dt} x(t) = x(t) + y(t), \frac{d}{dt} y(t) = -2 x(t) + 4 y(t)
                                                                                                       (54)
> sol1:=dsolve(\{sist, x(0)=3, y(0)=0\}, \{x(t), y(t)\})
                      sol1 := \{x(t) = -3 e^{3t} + 6 e^{2t}, y(t) = -6 e^{3t} + 6 e^{2t}\}\
                                                                                                       (55)
> sol2:=dsolve({sist,x(0)=0,y(0)=3},{x(t),y(t)})
                         sol2 := \{x(t) = 3 e^{3t} - 3 e^{2t}, y(t) = 6 e^{3t} - 3 e^{2t}\}
                                                                                                       (56)
> sol3:=dsolve({sist,x(0)=-3,y(0)=0},{x(t),y(t)})
                         sol3 := \{x(t) = 3 e^{3t} - 6 e^{2t}, y(t) = 6 e^{3t} - 6 e^{2t}\}
                                                                                                       (57)
> sol4:=dsolve({sist,x(0)=0,y(0)=-3},{x(t),y(t)})
                      sol 4 := \{x(t) = -3 e^{3t} + 3 e^{2t}, y(t) = -6 e^{3t} + 3 e^{2t}\}\
                                                                                                       (58)
> lim1:=limit(sol1[1],t=infinity),limit(sol1[2],t=infinity)
                              \lim 1 := \lim_{t \to \infty} x(t) = -\infty, \lim_{t \to \infty} y(t) = -\infty
                                                                                                       (59)
> lim2:=limit(sol2[1],t=infinity),limit(sol2[2],t=infinity)
                                \lim 2 := \lim_{t \to \infty} x(t) = \infty, \lim_{t \to \infty} y(t) = \infty
                                                                                                       (60)
> lim3:=limit(sol3[1],t=infinity),limit(sol3[2],t=infinity)
                                \lim 3 := \lim_{t \to \infty} x(t) = \infty, \lim_{t \to \infty} y(t) = \infty
                                                                                                       (61)
> lim4:=limit(sol4[1],t=infinity),limit(sol4[2],t=infinity)
                              \lim 4 := \lim_{t \to \infty} x(t) = -\infty, \lim_{t \to \infty} y(t) = -\infty
                                                                                                       (62)
3c
> DEplot([sist], [x(t),y(t)], t=-4...4, x=-10...10, y=-10...10, [[x(0)=3,y
   (0) = 0], \quad [\mathbf{x}(0) = 0, \mathbf{y}(0) = 3], \quad [\mathbf{x}(0) = -3, \mathbf{y}(0) = 0], \quad [\mathbf{x}(0) = 0, \mathbf{y}(0) = -3]],
   arrows=medium,linecolor=blue);
```



$$\frac{1}{2} \tag{63}$$

$$=^{4a}$$
> ec1:=diff(x(t),t)=y(t)
$$ec1 := \frac{d}{dt} x(t) = y(t)$$
(64)

> ec2:=diff(y(t),t)=-x(t)-2*y(t)

$$ec2 := \frac{d}{dt} y(t) = -x(t) - 2 y(t)$$
 (65)

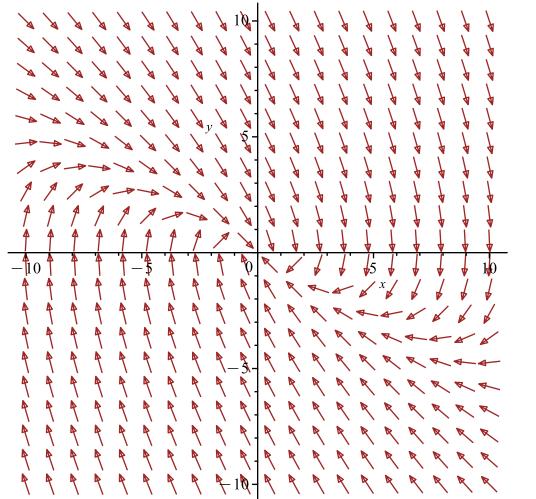
> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = y(t), \frac{d}{dt} y(t) = -x(t) - 2y(t)$$
 (66)

> sol:=dsolve({sist}, {x(t),y(t)})

$$sol := \left\{ x(t) = e^{-t} \left(c_2 t + c_1 \right), y(t) = -e^{-t} \left(c_2 t + c_1 - c_2 \right) \right\}$$
 (67)

> lim:=limit(sol[1], t=infinity), limit(sol[2], t=infinity) $\lim := \lim_{t \to \infty} x(t) = 0, \lim_{t \to \infty} y(t) = 0$ (68) DEplot([sist],[x(t),y(t)],t=-100..100,x=-10..10,y=-10..10,arrows=
 medium,linecolor=blue)



-50

> ec1:=diff(x(t), t)=2*x(t)+y(t)

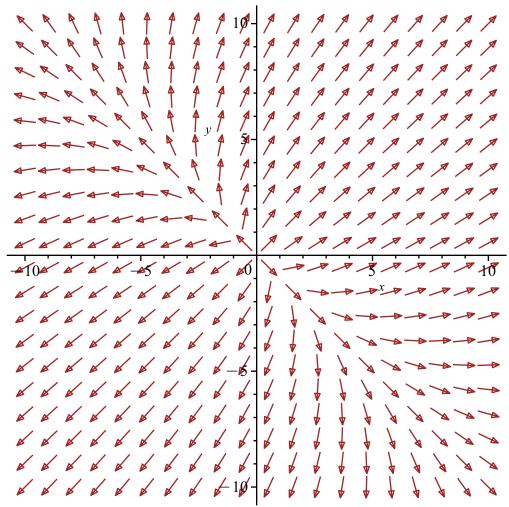
$$ec1 := \frac{d}{dt} x(t) = 2 x(t) + y(t)$$
 (69)

> ec2:=diff(y(t),t)=x(t)+2*y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2y(t)$$
 (70)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = 2 x(t) + y(t), \frac{d}{dt} y(t) = x(t) + 2 y(t)$$
 (71)



5b

> ec1:=diff(x(t),t)=-x(t)-y(t)

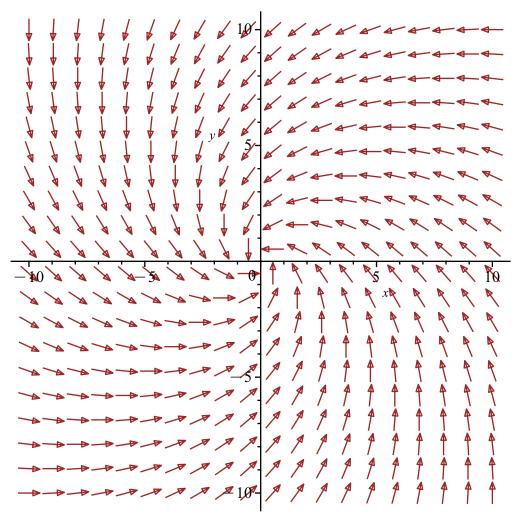
$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = -x(t) - y(t) \tag{72}$$

> ec2:=diff(y(t),t)=x(t)-y(t)

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t) - y(t) \tag{73}$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = -x(t) - y(t), \frac{d}{dt} y(t) = x(t) - y(t)$$
 (74)



5c

> ec1:=diff(x(t),t)=y(t)

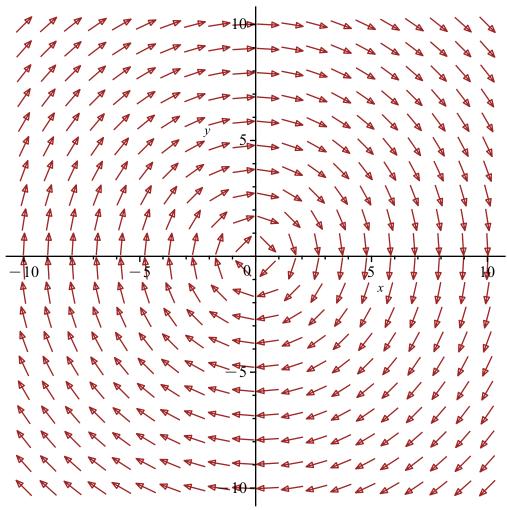
$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = y(t) \tag{75}$$

> ec2:=diff(y(t),t)=-x(t)

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = -x(t) \tag{76}$$

> sist:=ec1,ec2

$$sist := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = y(t), \ \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = -x(t)$$



5d

> ec1:=diff(x(t),t)=-2*x(t)

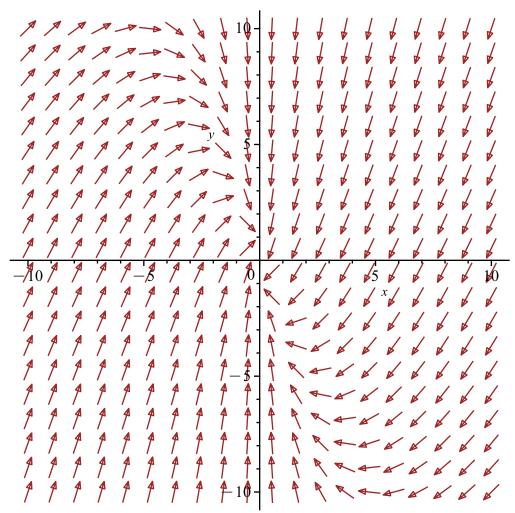
$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = -2 \ x(t) \tag{78}$$

> ec2:=diff(y(t),t)=-4*x(t)-2*y(t)

$$ec2 := \frac{d}{dt} y(t) = -4 x(t) - 2 y(t)$$
 (79)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = -2 x(t), \frac{d}{dt} y(t) = -4 x(t) - 2 y(t)$$
 (80)



5e

> ec1:=diff(x(t),t)=x(t)-4*y(t)

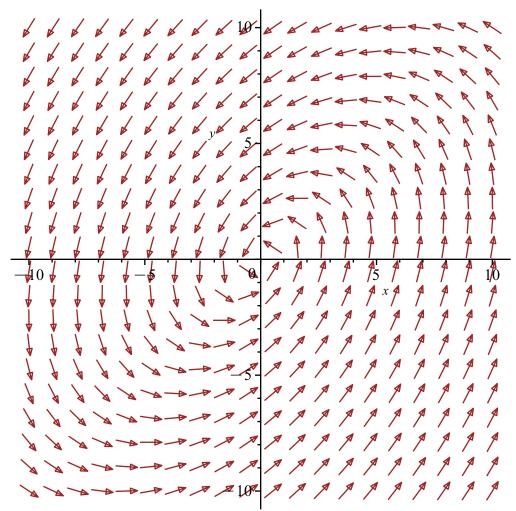
$$ec1 := \frac{d}{dt} x(t) = x(t) - 4 y(t)$$
 (81)

> ec2:=diff(y(t),t)=5*x(t)-3*y(t)

$$ec2 := \frac{d}{dt} y(t) = 5 x(t) - 3 y(t)$$
 (82)

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) - 4 y(t), \frac{d}{dt} y(t) = 5 x(t) - 3 y(t)$$
 (83)



5f

> ec1:=diff(x(t),t)=3*x(t)-y(t)

$$ec1 := \frac{d}{dt} x(t) = 3 x(t) - y(t)$$
 (84)

> ec2:=diff(y(t),t)=y(t)

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = y(t) \tag{85}$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = 3 x(t) - y(t), \frac{d}{dt} y(t) = y(t)$$
 (86)

