

```

> with(DETools)
[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, (1)
  DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper,
  Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols,
  MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm,
  RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge,
  Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot,
  casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol,
  dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring,
  endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint,
  firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs,
  hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols,
  infactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE,
  matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon,
  normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait,
  poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder,
  reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread,
  rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen,
  symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam,
  zoom]

```

```

> with(plots)
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, (2)
  conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display,
  dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal,
  interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot,
  listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot,
  pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot,
  rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve,
  sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

```

```

1.
a)
> f:=x->x^2-2*x;
                                     
$$f := x \mapsto x^2 - 2 \cdot x$$
 (3)
> ec:=diff(x(t),t)=f(x(t));
                                     
$$ec := \frac{d}{dt} x(t) = x(t)^2 - 2 x(t)$$
 (4)
> pct_ech:=solve(f(x)=0,x);
                                     
$$pct\_ech := 0, 2$$
 (5)

```

```
> D(f) (pct_ech[1])
```

-2

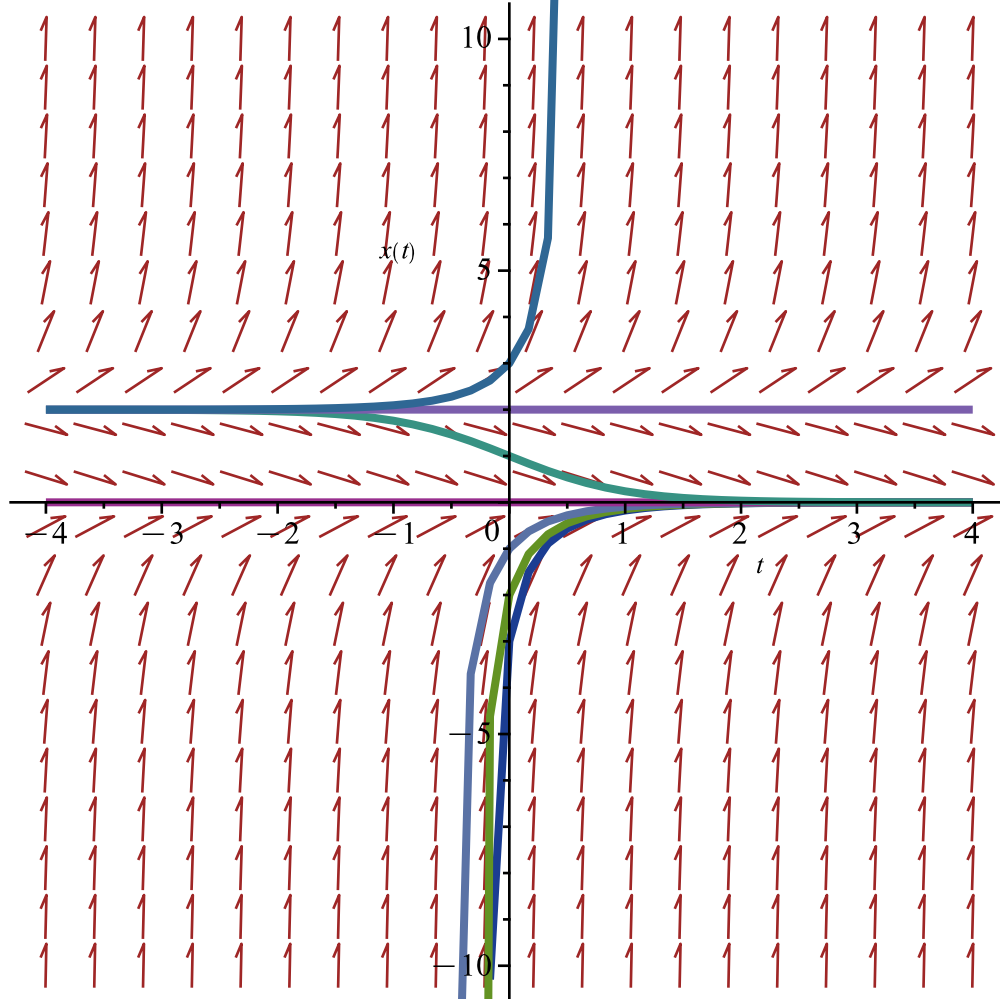
(6)

```
> D(f) (pct_ech[2])
```

2

(7)

```
> DEplot(ec, x(t), t=-4..4, x=-10..10, [[x(0)=-3], [x(0)=-2], [x(0)=-1], [x(0)=0], [x(0)=1], [x(0)=2], [x(0)=3]]);
```



b)

```
> f:=x->x*(x-1)*(x-2)
```

$f := x \mapsto x \cdot (x - 1) \cdot (x - 2)$

(8)

```
> ec:=diff(x(t),t)=f(x(t))
```

$ec := \frac{d}{dt} x(t) = x(t) (x(t) - 1) (x(t) - 2)$

(9)

```
> pct_ech:=solve(f(x)=0,x)
```

$pct_ech := 0, 1, 2$

(10)

```
> D(f) (pct_ech[1])
```

2

(11)

```
> D(f) (pct_ech[2])
```

-1

(12)

```
> D(f) (pct_ech[3])
```

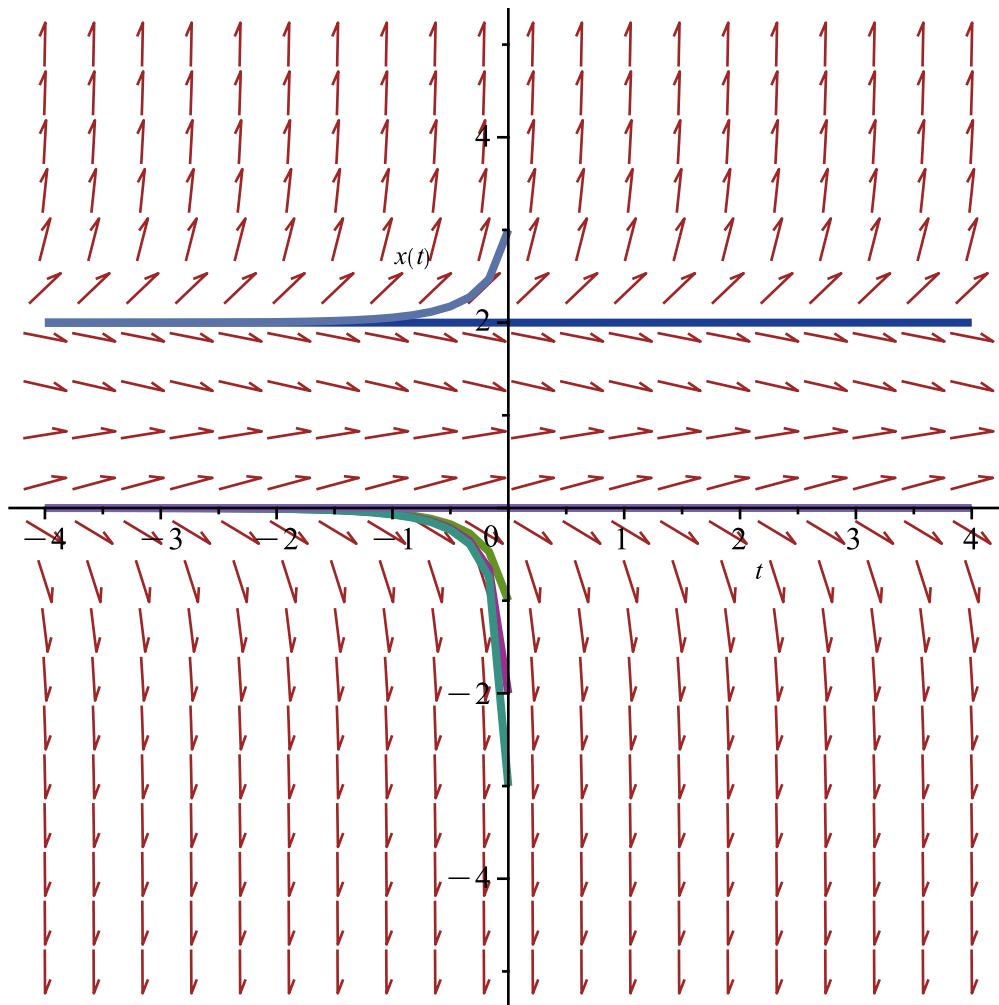
```
> DEplot(ec, x(t), t=-4..4, x=-5..5, [[x(0)=2],[x(0)=-1],[x(0)=3],
[x(0)=-2],[x(0)=-3],[x(0)=0]])
```

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of .14384102, probably
a singularity

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of .14384100, probably
a singularity

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of .58891510e-1,
probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of .32269256e-1,
probably a singularity



c)

```
> _EnvAllSolutions:=true
```

_EnvAllSolutions := true

(14)

```
> f:=x->sin(x)
```

f := x ↦ sin(x)

(15)

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = \sin(x(t)) \quad (16)$$

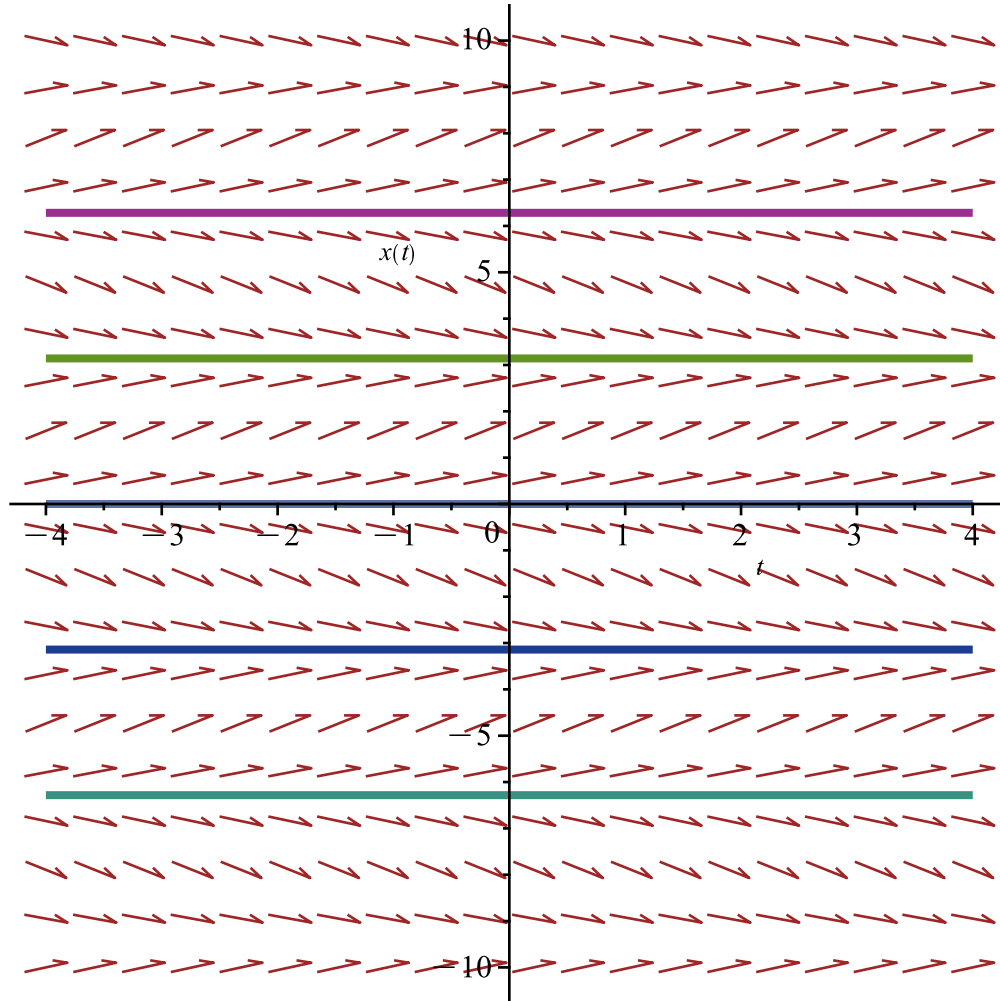
```
> pct_ech:=solve(f(x)=0,x)
```

$$pct_ech := \pi_Z3\sim \quad (17)$$

```
> D(f)(pct_ech[0])
```

$$\cos(\pi_Z3\sim_0) \quad (18)$$

```
> DEplot(ec, x(t), t=-4..4, x=-10..10, [[x(0)=-Pi],[x(0)=Pi],[x(0)=0],[x(0)=2*Pi],[x(0)=-2*Pi]])
```



```
> with(linalg)
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, (19)

mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylveste, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

2.

a)

> f1:=x->2*x+y

$$f1 := x \mapsto 2 \cdot x + y \quad (20)$$

> f1:=x->2*x+y

$$f1 := x \mapsto 2 \cdot x + y \quad (21)$$

> f2:=y->x+2*y

$$f2 := y \mapsto x + 2 \cdot y \quad (22)$$

> ec1:=diff(x(t),t)=2*x(t)+y(t)

$$ec1 := \frac{d}{dt} x(t) = 2 x(t) + y(t) \quad (23)$$

> ec2:=diff(y(t),t)=x(t)+2*y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2 y(t) \quad (24)$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = 2 x(t) + y(t), \frac{d}{dt} y(t) = x(t) + 2 y(t) \quad (25)$$

$$A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (26)$$

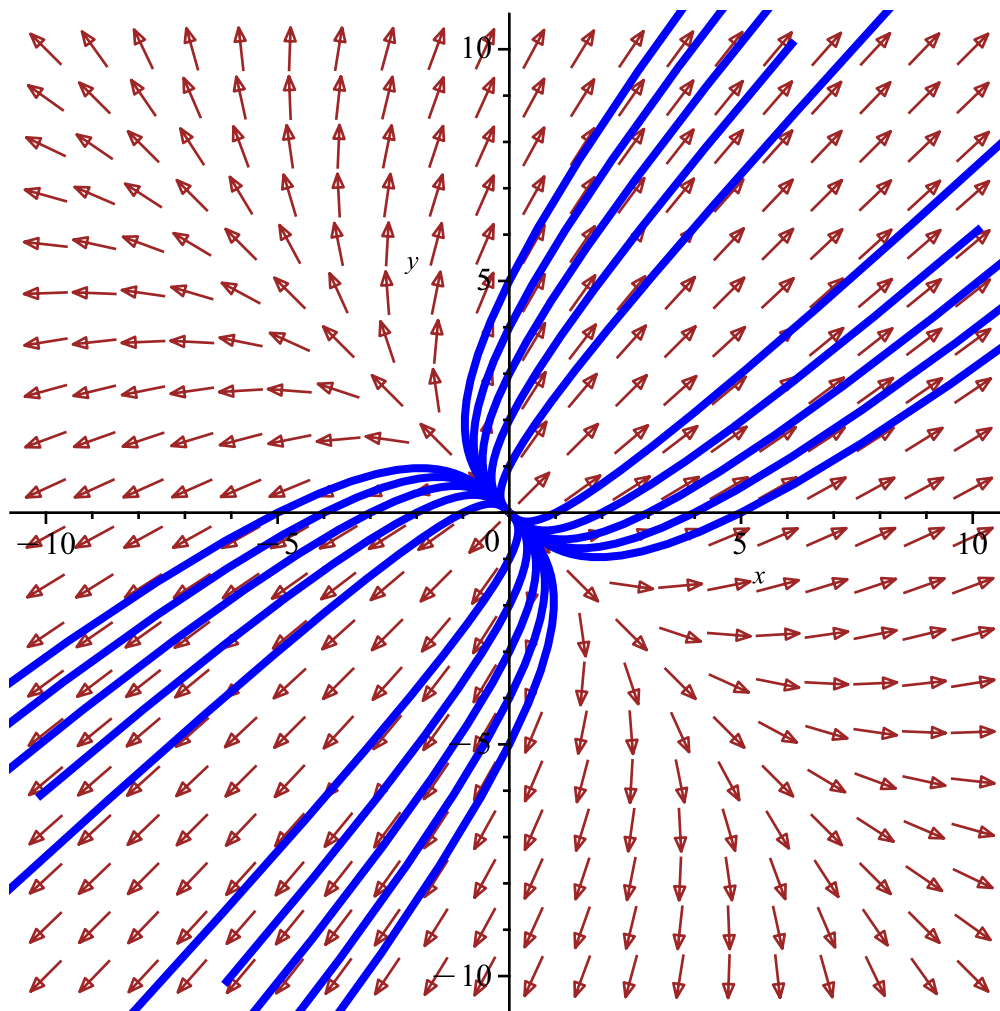
> eigenvals(A);

$$3, 1 \quad (27)$$

> cond_in2:=[x(0)=0,y(0)=i]\$i=1..5,[x(0)=-i,y(0)=0]\$i=1..5,[x(0)=0,y(0)=-i]\$i=1..5,[x(0)=i,y(0)=0]\$i=1..5;

$$cond_in2 := [x(0)=0, y(0)=1], [x(0)=0, y(0)=2], [x(0)=0, y(0)=3], [x(0)=0, y(0)=4], [x(0)=0, y(0)=5], [x(0)=-1, y(0)=0], [x(0)=-2, y(0)=0], [x(0)=-3, y(0)=0], [x(0)=-4, y(0)=0], [x(0)=-5, y(0)=0], [x(0)=0, y(0)=-1], [x(0)=0, y(0)=-2], [x(0)=0, y(0)=-3], [x(0)=0, y(0)=-4], [x(0)=0, y(0)=-5], [x(0)=1, y(0)=0], [x(0)=2, y(0)=0], [x(0)=3, y(0)=0], [x(0)=4, y(0)=0], [x(0)=5, y(0)=0] \quad (28)$$

> DEplot([sist],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[cond_in2],arrows=medium,linecolor=blue,stepsize=0.1)



b)

```
> ec1:=diff(x(t),t)=-3*x(t)+4*y(t)
```

$$ec1 := \frac{d}{dt} x(t) = -3x(t) + 4y(t) \quad (29)$$

```
> ec2:=diff(y(t),t)=-2*x(t)+3*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = -2x(t) + 3y(t) \quad (30)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -3x(t) + 4y(t), \frac{d}{dt} y(t) = -2x(t) + 3y(t) \quad (31)$$

```
> A:=matrix([[-3, -2],[4, 3]])
```

$$A := \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix} \quad (32)$$

```
> eigenvals(A)
```

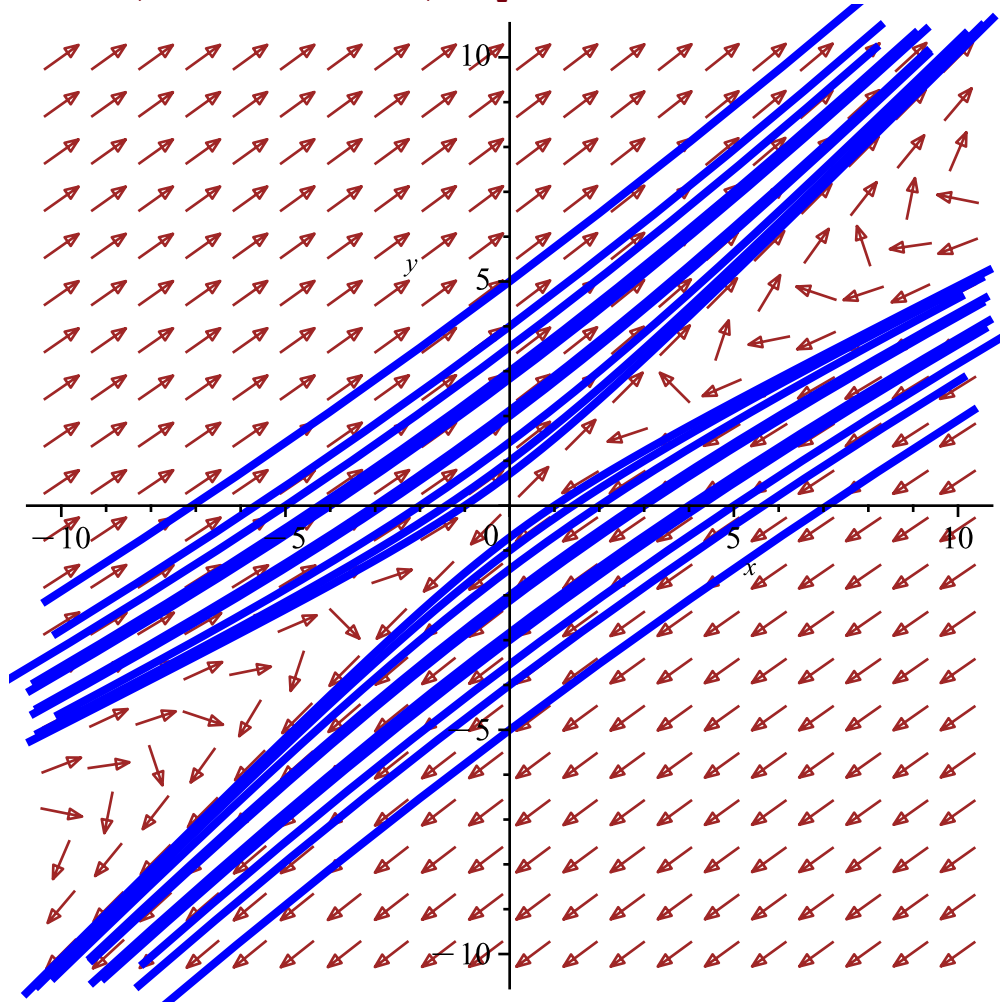
$$1, -1 \quad (33)$$

```
> cond_in2:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,
y(0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5;
```

$$cond_in2 := [x(0)=0, y(0)=1], [x(0)=0, y(0)=2], [x(0)=0, y(0)=3], [x(0)=0, y(0)=4] \quad (34)$$

=4], [x(0)=0,y(0)=5], [x(0)=-1,y(0)=0], [x(0)=-2,y(0)=0], [x(0)=-3,y(0)=0], [x(0)=-4,y(0)=0], [x(0)=-5,y(0)=0], [x(0)=0,y(0)=-1], [x(0)=0,y(0)=-2], [x(0)=0,y(0)=-3], [x(0)=0,y(0)=-4], [x(0)=0,y(0)=-5], [x(0)=1,y(0)=0], [x(0)=2,y(0)=0], [x(0)=3,y(0)=0], [x(0)=4,y(0)=0], [x(0)=5,y(0)=0]

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[cond_in2],
arrows=medium,linecolor=blue,stepsize=0.1)
```



c)

```
> ec1:=diff(x(t),t)=-x(t)-y(t)
```

$$ec1 := \frac{d}{dt} x(t) = -x(t) - y(t) \quad (35)$$

```
> ec2:=diff(y(t),t)=x(t)-3*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) - 3y(t) \quad (36)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -x(t) - y(t), \frac{d}{dt} y(t) = x(t) - 3y(t) \quad (37)$$

```
> A:=matrix([[ -1, -1], [1, -3]])
```

$$A := \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \quad (38)$$

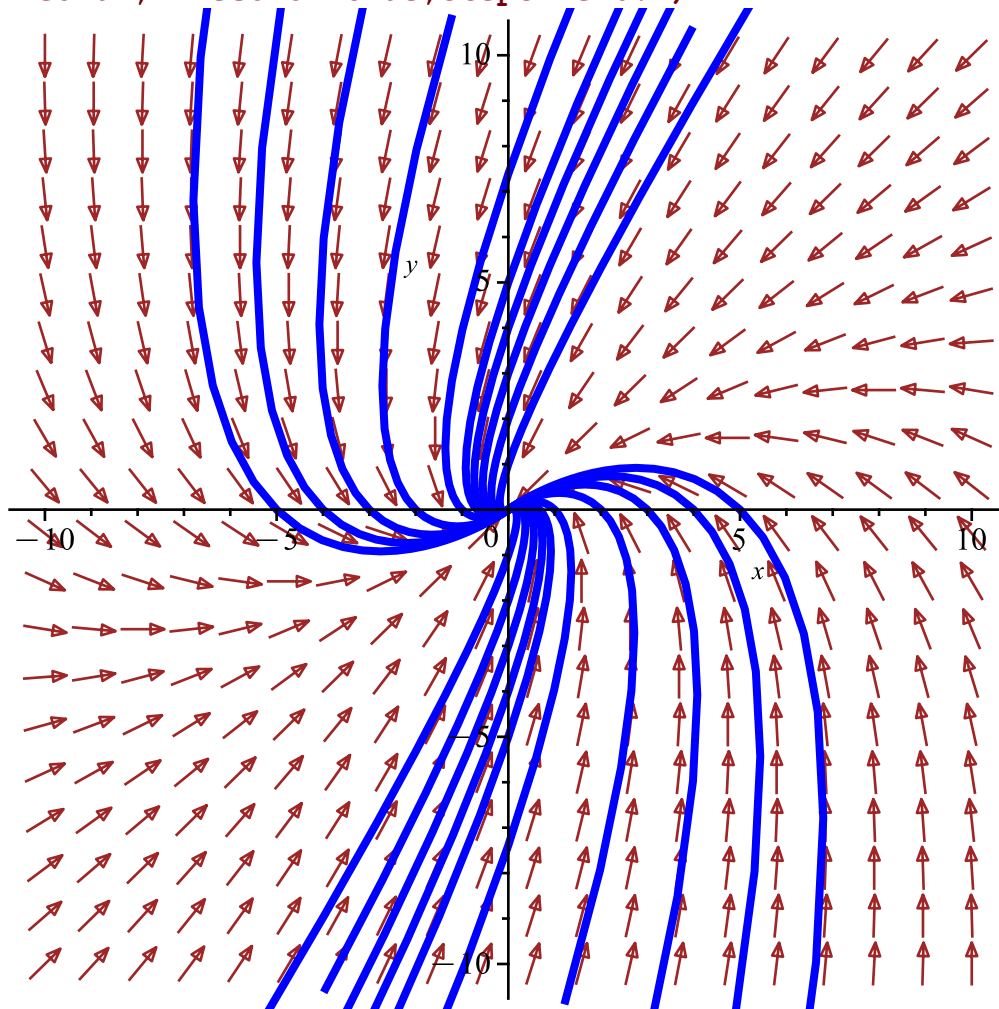
> **eigenvals (A)**

$$-2, -2 \quad (39)$$

> **cond_in2:=**[x(0)=0,y(0)=i]\$i=1..5,[x(0)=-i,y(0)=0]\$i=1..5,[x(0)=0,y(0)=-i]\$i=1..5,[x(0)=i,y(0)=0]\$i=1..5;

cond_in2 := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] (40)

> **DEplot([sist],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[cond_in2],arrows=medium,linecolor=blue,stepsize=0.1)**



d)

> **ec1:=diff(x(t),t)=-2*x(t)**

$$ec1 := \frac{d}{dt} x(t) = -2 x(t) \quad (41)$$


```
> ec2:=diff(y(t),t)=-4*x(t)-2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = -4x(t) - 2y(t) \quad (42)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -2x(t), \frac{d}{dt} y(t) = -4x(t) - 2y(t) \quad (43)$$

```
> A:=matrix([[-2, 0], [-4, -2]])
```

$$A := \begin{bmatrix} -2 & 0 \\ -4 & -2 \end{bmatrix} \quad (44)$$

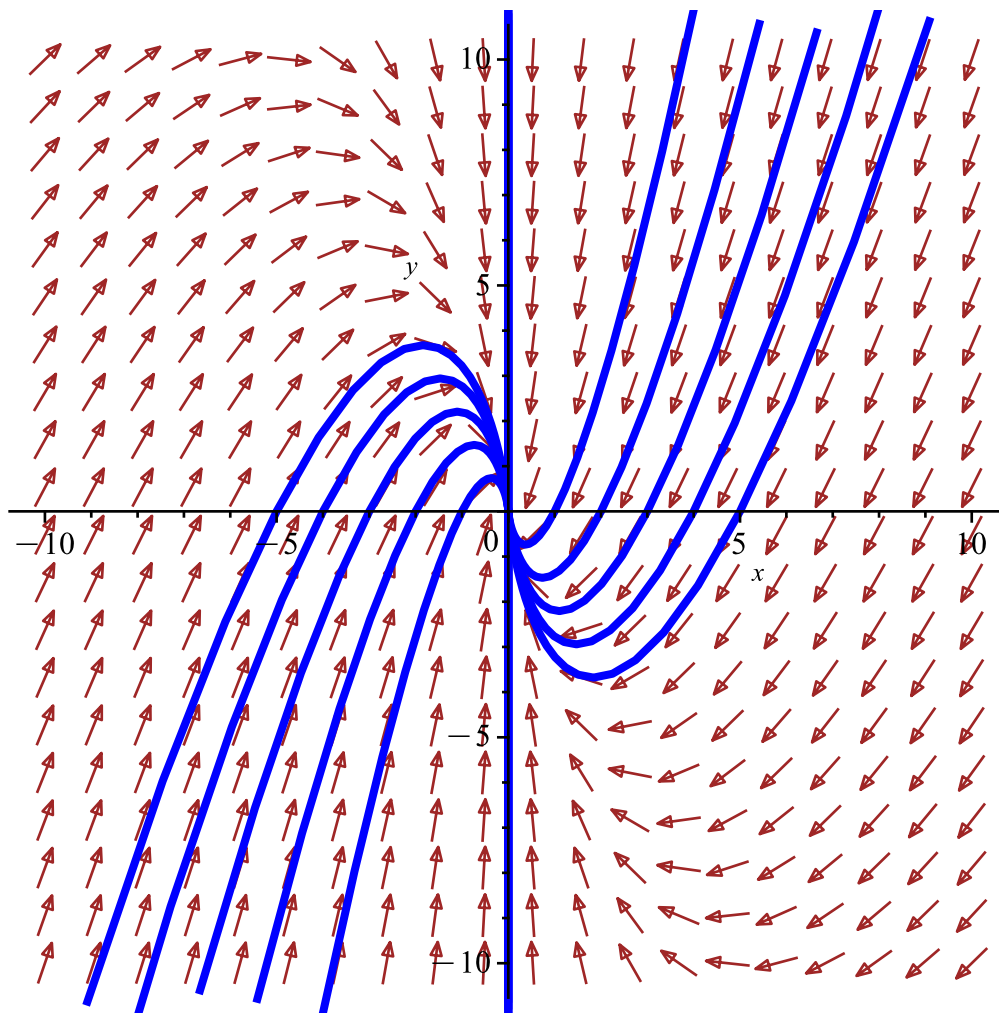
```
> eigenvals(A)
```

$$-2, -2 \quad (45)$$

```
> cond_in2:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,
y(0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5;
```

$$cond_in2 := [x(0)=0, y(0)=1], [x(0)=0, y(0)=2], [x(0)=0, y(0)=3], [x(0)=0, y(0)=4], [x(0)=0, y(0)=5], [x(0)=-1, y(0)=0], [x(0)=-2, y(0)=0], [x(0)=-3, y(0)=0], [x(0)=-4, y(0)=0], [x(0)=-5, y(0)=0], [x(0)=0, y(0)=-1], [x(0)=0, y(0)=-2], [x(0)=0, y(0)=-3], [x(0)=0, y(0)=-4], [x(0)=0, y(0)=-5], [x(0)=1, y(0)=0], [x(0)=2, y(0)=0], [x(0)=3, y(0)=0], [x(0)=4, y(0)=0], [x(0)=5, y(0)=0] \quad (46)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[cond_in2],
arrows=medium,linecolor=blue,stepsize=0.1)
```



e)

```
> ec1:=diff(x(t),t)=x(t)+4*y(t)
```

$$ec1 := \frac{d}{dt} x(t) = x(t) + 4y(t) \quad (47)$$

```
> ec2:=diff(y(t),t)=x(t)+y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) + y(t) \quad (48)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = x(t) + 4y(t), \frac{d}{dt} y(t) = x(t) + y(t) \quad (49)$$

```
> A:=matrix([[1,4],[1,1]])
```

$$A := \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \quad (50)$$

```
> eigenvals(A)
```

$$3, -1 \quad (51)$$

```
> cond_in2:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,
y(0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5;
```

$$cond_in2 := [x(0)=0,y(0)=1], [x(0)=0,y(0)=2], [x(0)=0,y(0)=3], [x(0)=0,y(0) \quad (52)$$

$=4]$, $[x(0)=0, y(0)=5]$, $[x(0)=-1, y(0)=0]$, $[x(0)=-2, y(0)=0]$, $[x(0)=-3, y(0)=0]$,
 $=0]$, $[x(0)=-4, y(0)=0]$, $[x(0)=-5, y(0)=0]$, $[x(0)=0, y(0)=-1]$, $[x(0)=0, y(0)$
 $=-2]$, $[x(0)=0, y(0)=-3]$, $[x(0)=0, y(0)=-4]$, $[x(0)=0, y(0)=-5]$, $[x(0)=1,$
 $y(0)=0]$, $[x(0)=2, y(0)=0]$, $[x(0)=3, y(0)=0]$, $[x(0)=4, y(0)=0]$, $[x(0)=5, y(0)$
 $=0]$

```

> DEplot([sist],[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,[cond_in2],
arrows=medium,linecolor=blue,stepsize=0.1)

```

