Wavelet Transform Theory and its Application in EMG Signal Processing

Xu Zhang, Yu Wang, Ray P.S. Han
Qingdao University
Qingdao, Shandong, China 266071
ray-han@pku.edu.cn

***Abstract*—Wavelet analysis is often very effective because it provides a simple approach for dealing with local aspects of a signal. The electromyogram (EMG) signals arising from muscle activities have become a useful tool for clinical diagnosis, rehabilitation medicine and sport medicine. In this paper, a time-frequency analysis based on the wavelet transform of the EMG signals is presented with a focus on 2 areas: de-noising and feature extraction.**

Keywords- EMG signal processing; wavelet transform; multi-resolution analysis; wavelet de-noising; feature extraction.

I. INTRODUCTION

The EMG signal is a biomedical signal derived from neuromuscular activities of the skeletal muscle and it provides information that is used in clinical medicine, sports medicine and biomedical areas. Surface EMG (sEMG) signals are the summation of all motor unit action potential (MUAP) within the pick-up area of electrodes and can be observed noninvasively by using electrodes affixed to the skin surface. EMG signals are non-stationary and possess highly complex time and frequency characteristics. The use of Fourier analysis to study biological signals such as EMG recordings is not the most efficient method for transient data analysis. The time-frequency analysis based on the wavelet transform is better suited to handle the non-stationary characteristics of the EMG signals.

II. INTRODUCTION TO WAVELET THEORIES

Over the past decade the wavelet method has been gaining popularity as a tool for time-frequency analysis. The starting point for both wavelet and Fourier transforms is to express the signal as a linear combination of basis functions; however, the latter has only frequency resolution and not time resolution. Wavelet transform on the other hand, possesses characteristics of multi-resolutions and thus, overcomes the shortcoming of a single resolution of the short-time Fourier transform.

A. Mother Wavelet and its Properties

Wavelets are generated from a single basic wavelet $\psi(t)$, the so-called mother wavelet by scaling and translation:

$$\psi_{a,b}(\mathrm{t}) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a,b \in R; a \neq 0\,, \tag{1}$$

in which a,b are the scale and translation factors, respectively. Hence, $1/\sqrt{a}$ represents the energy normalization across the different scales. Wavelets fulfill the conditions of admissibility and regularity. Satisfying the admissibility condition implies a band-pass spectrum described by

$$\int \frac{\left|\psi(\omega)\right|^2}{|\omega|} d\omega < +\infty\,. \tag{2}$$

Since the average value of the wavelet in time domain vanishes $\psi(t)$ must be a wave (oscillatory). Satisfying the regularity condition implies that the wavelet possesses smoothness and concentration in both time and frequency domains.

B. Discrete Wavelets Transform (DWT)

Discrete wavelets are used to overcome the redundancy of continuous wavelets transform by scaling and translating in discrete steps as follows:

$$d_{j,k} = \left\langle f, \psi_{j,k} \right\rangle, \quad \psi_{j,k} = a_0^{-\frac{j}{2}}\psi\left(a_0^{-j}t - kb_0\right), \tag{3}$$

where j,k are integers, $a_0 > 0$ is a fixed dilation step and b_0 the translation factor is dependent on a_0. Choosing $a_0 = 2$ and $b_0 = 1$ we get a dyadic sampling in both time and frequency axes and obtain the following dyadic wavelet:

$$\psi_{j,k} = 2^{-\frac{j}{2}}\psi\left(2^{-j}t - k\right). \tag{4}$$

The finite spectrum of the signals can be covered with spectra of dilated wavelets by having the stretched wavelet spectra touching each other. Additionally, the signal must have finite energy, that is:

A start-up grant provided by Qingdao University to Ray P.S. Han is gratefully acknowledged.

978-1-4244-5934-6/10/$26.00 ©2010 IEEE

$$\int \left| f(t) \right|^2 dt < \infty \,. \tag{5}$$

C. Frame Theory [1]

The energy of the wavelet coefficients must lie between 2 positive bounds in order to ensure a necessary and sufficient condition for a stable reconstruction. That is,

$$A\left\|f\right\|^2 \le \sum_{j,k} \left|\left\langle f, \psi_{j,k} \right\rangle\right|^2 \le B\left\|f\right\|^2, \;\; A > 0, B < \infty, \tag{6}$$

where $\|f\|^2$ represents the energy of the signal $f(t)$. The family of $\psi_{j,k}(t),\, j,k \in Z$ is termed as a *frame* with *frame bounds* A and B. The frame is tight when $A = B$, and the discrete wavelets behave exactly like an orthonormal basis.

Theorem: if $\psi_{j,k} \in L^2$ is a *frame* then, we have a sequence of functions $\bar{\psi}_{j,k}$ that satisfies

$$f = \sum_{j,k} \left\langle f, \psi_{j,k} \right\rangle \bar{\psi}_{j,k}, \;\; f \in L^2, \tag{7}$$

where, $\bar{\psi}_{j,k}$ is the duality of $\psi_{j,k}$. Generally, it is difficult to seek a dual frame and there are 2 ways to handle this problem: (a) setting up an orthogonal (or semi-orthogonal) condition; (b) approximating as follows:

$$f(t) \approx \frac{2}{A+B} \sum_{j,k} \left\langle f, \psi_{j,k} \right\rangle \psi_{j,k}(t). \tag{8}$$

Under normal circumstances, the $\psi_{j,k}(t)-$generated frame is confined in the Riesz Base.

D. Multi-Resolution Analysis (MRA)

To cover the spectrum down to zero, the scaling function possesses a low-pass nature of the spectrum with an admissibility condition. If a wavelet constitutes a band-pass filter and its scaling function is a low-pass filter, the wavelet can be seen as passing a signal through the filter bank. The outputs are the wavelet and scaling function transform coefficients (see Fig. 1). [2]

Following Mallat's concept of a MRA, the algorithm of a *fast wavelet transform* contains 4 operations: filtering, up-sampling, down-sampling and refactoring. Further, the 2-scale equation characterizes the inherent relationships of the basis function of the 2 adjacent scale spaces; V_j, V_{j-1} and W_j, W_{j-1}.

The Riesz basis of the subspace $\{V_j\}, j \in Z$ is a scaling function

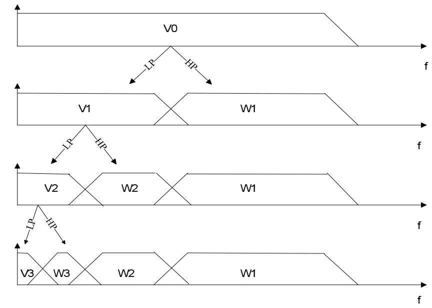


Figure 1. Splitting the signal spectrum with an iterated filter bank

$$\left\{\phi_{j,k}, k \in Z\right\}, \;\; \phi_{j,k}(t) = 2^{-\frac{j}{2}} \phi\left(2^{-j} t - k\right), \tag{9}$$

whereas, its complementary space $\{w_j\}, j \in Z$ based on the Riesz basis is

$$\left\{\psi_{j,k}, k \in Z\right\}, \;\; \psi_{j,k} = 2^{-\frac{j}{2}} \psi\left(2^{-j} t - k\right). \tag{10}$$

The space direct sum of representations is given by [3],

$$V_{j-1} = W_j \oplus V_j \,. \tag{11}$$

Note that the MRA enables a wavelet function $\psi(t)$ to be constructed using a scaling function $\phi(t)$, which in turn, can be designed by constructing the LPF $H(\omega)$ and HPF $G(\omega)$. Thus, with the design of filter banks (H, G), we can realize the MRA of the signals.

III. WAVELET ANALYSIS IN EMG SIGNAL PROCESSING

EMG signals require a reliably accurate method for each of the following 4 steps: detection, decomposition, processing and classification. To achieve this, we apply the techniques of the wavelet transformation to EMG signal analysis.

A. Wavelet Denoising

sEMG signals are recorded by electrodes affixed to the skin surface and they capture the biological signals of the activities of the neuromuscular system. Muscles emit a weak electrical signal with an amplitude of about 0.1~5.0 mv. Hence, they require a highly sensitive measurement system but this invariably leads to decreased anti-jamming capability. Other problems encountered in EMG detection are interference and noise. The energy of the sEMG power is mainly concentrated in the 0~1000Hz range, but raw EMG signals often carry a low frequency (near DC) and high-frequency interference signals. Truly useful EMG signals are between the 10~500Hz range, in particular, the 50~150Hz range [4]. It is generally believed that the noise arises from the high-frequency signals, which are assumed to obey the Gaussian distribution [5]. The following equation represents a simple model of the EMG signal,

$$f(\mathrm{t}) = s(t) + n(t), \tag{12}$$

where $s(t)$, $n(t)$ denotes EMG signals and White Gaussian Noise $N(0,\sigma^2)$, respectively. To do a multi-resolution signal analysis for $f(t)$, we note that the wavelet transform coefficients of $s(t)$ at all scales exhibit a greater value change in the odd position. Further, the wavelet coefficients of $n(t)$ at all scales are uniform but as the scale increases, the amplitude of the coefficients reduces. Since the noise coefficients are concentrated in small-scales and the EMG signals in large-scales, wavelet de-noising involves adopting an accurate method to estimate the transform coefficients of the raw signals, with the noise-generated spectral component removed. The classical wavelet de-noising is based on the assignment of a threshold value which can be divided into the soft threshold and hard threshold (see Fig. 2).

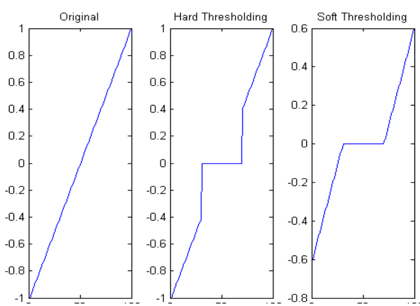
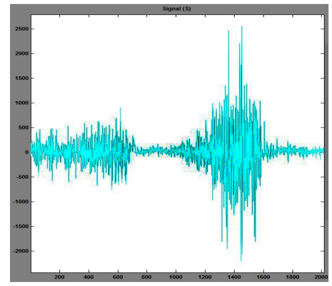


Figure 2. The difference between hard and soft thresholding

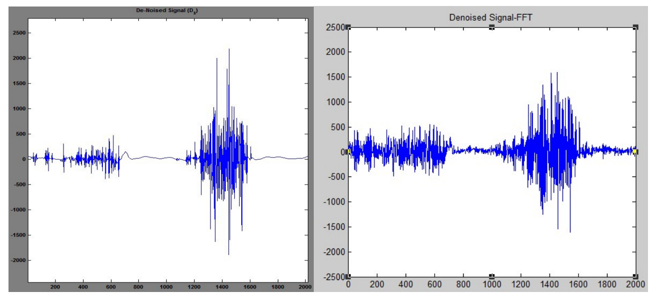
Wavelet de-noising can be divided into the following steps:

- Make multi-scale decomposition of the raw EMG signals, to observe the signal wavelet coefficients;
- Estimate the noise and choose the threshold, make the threshold analysis to wavelet coefficients and get new coefficients;
- Reconstruct the EMG signals by the revised wavelet coefficient.

To illustrate the effectiveness of the wavelet de-noising method, we performed a comparative analysis of EMG signals processed by the fast Fourier transform (FFT) and the wavelet transform. We employed a Butterworth low-pass filter of order 10 with a cutoff frequency of 500Hz to filter out the high-frequency noise. Also, we selected the Sym2 wavelet to decompose the original EMG signals into 5 levels, and then, achieved a soft threshold de-noising arithmetic based on the minmax rule in MATLAB7. Fig. 3 depicts a comparison of the 2 results with the raw EMG signals obtained from the recorded biceps signals with the right elbow bent.



(a) Raw EMG signals



De-noising EMG signal (b) with WT and (c) with FFT

Figure 3. A comparison of the two de-noising methods

From the comparison of the results, we see that the peak and mutation part of the raw signals are well preserved using the wavelet transform de-noising method, including preserving the maximum signal character. However, the FFT de-noising method cannot differentiate the high-frequency part of the useful and interferential signals, and thus, the result is clearly inferior to the wavelet transform de-noising. [6]

Li and Luo [7] have proposed a novel de-noising method based on the "airspace correlation" method to eliminate noise in EMG signals by adopting noise energies at different EMG decomposition levels. Their experimental results show the edge features of the EMG signals are better retained with a superior feature extraction capability.

B. Feature Extraction and Classification of EMG Signals

Due to the large amount of EMG signals, it is impractical to feed the time sequence directly into a classifier. Thus, they should be mapped into a smaller dimension vector or a feature vector. The scale and frequency in the wavelet analysis are interrelated: a low-scale demonstrates rapidly changing details of a high frequency signal and a high-scale illustrates slowly changing coarse features with a low frequency [3]. The MRA features of the wavelet transform offer information pertaining to the time-frequency variation of the signals that act as a mathematical microscope.

As a generalization of DWT, a wavelet packet transform allows the ''best'' adapted analysis of a signal in a timescale domain. The classification methods of the EMG signals are based on the definition of a feature space and a distance measure. The characteristics of the EMG signals are highly dependent on the level and duration of muscle contractions, the static or dynamic muscle states, fatigue, etc. The objective is to extract effective features from the EMG signals to improve the classification accuracy. The feature space depends on the wavelet parameterization and is selected in accordance to an optimized criterion.

IV. CONSIDERATIONS OF EMG SIGNAL APPLICATIONS

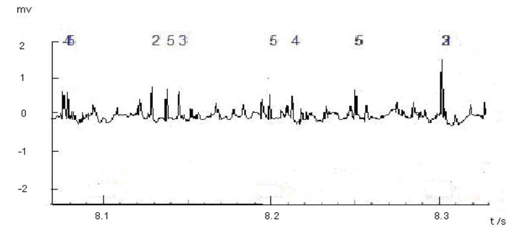
There are many applications of processed EMG signals. For example, they been widely applied to functional electrical stimulation, fatigue analysis associated with muscle contraction and clinical diagnosis, the control of powered prosthetic limbs, etc. EMG signal classification results are vastly different among the various applications. Further, in signal processing, the choice of the basis wavelet function is important. Some of the popularly used wavelets are Daubechies wavelets, 3 spline wavelet, Mexico-Colombian hat wavelet and Morlet wavelet. Since these basis functions have different properties they should be selected based on specific application requirements. Here are 4 considerations of EMG signal applications.

A. MUAP Detection

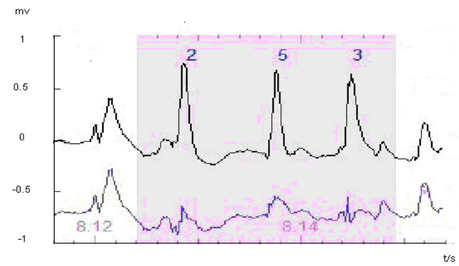
Chen and Yang [8] (among several others) have proposed a novel approach for the detection and classification of MUAP from multi-channel EMG signals. They employed the MRA of the wavelet transform to extract time-frequency characteristics of MUAP and entered feature vectors into the neural network to a separate MUAP from the EMG signals. Based on our lab experiments using 6 MUAP templates and the monophasic action potential, we obtained reliable decompositions with good accuracy. Fig. 4(a) depicts the EMG signals under a mild muscle contraction with template numbers of identified spikes given at the top. Fig. 4(b) shows an amplified segment of Fig. 4(a) with the residual signals that remain after the templates of the identified MUAP deleted. The shaded area indicates the interval shown in the close-up board in Fig. 4(c). Observe that the reconstruction is overlaid over the signals as shown.

B. Muscle Fatigue Analysis

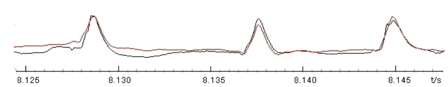
Yan and Zeng [9] used the wavelet transform with the Mexican hat wavelet function matching the M-wave shape to analyze the sEMG feature change for muscle fatigue. From the scale and the modulus maxima of the 2 half-waves in the wavelet transform coefficients relationship they define the muscle fatigue index based on the wavelet transform scale to achieve a quantitative description of the fatigue state. Jiang and Wang [4] suggested using the wavelet transform and neural networks to classify the normal and fatigue sEMG signals. They employed the Daubechies4 wavelet as the basis function to obtain wavelet decomposition of the sEMG signals. To extract the fatigue EMG signal features with varying scaling functions, the wavelet scale factor selection of 5~8 that included the main frequency range of EMG signals (50~150Hz) is used. They performed a classification of EMG signals with the BP neural network. The results showed that a direct classification of the EMG signals only yields an accuracy rate of 55%, while a classification after wavelet decomposition produces an accuracy of 87.5%. Thus, it is obvious that a wavelet decomposition method can greatly improve the EMG signal recognition accuracy.



(a) EMG signals during muscle contraction



(b) Amplified segment(up) and residual signal



(c) close-up board.

Figure 4. Results of the MUAP extraction

C. Movement Recognition

Xie and Huang [10] presented a sEMG signal classification method based on wavelet packet transformation, which decomposes the original EMG signals into 4 levels using the symmlet5 wavelet. The energies from different frequency bands are selected as robust feature vectors and 4 types of forearm movement are identified through a learning vector quantization neural network. Cai and Wang [11] extracted the maximum value of wavelet coefficient as the feature vector for pattern recognition with a neural network classifier to quantify 4 kinds of movement pattern; exhibition boxing, fist motion, forearm pronation and forearm supination. Jiang and Wang [12] made a multi-scale decomposition to the sEMG recorded from an upper extremity muscle, and used the wavelet coefficients variance of the sEMG to construct feature space. This method can be used to identify a variety of hand movements.

Here, we designed an experiment to recognize three kinds of movement patterns—tiptoe, toes upwarp, raising the forefoot and record their sEMG signals of the calf muscles. The procedures are as follows:

1) EMG Signals Acquisition.

All the EMG signals are obtained using the SA9800 MYOTRAC INFINTI EMG Acquisition Instrument from the Canadian Thought Technology Ltd. The low cutoff frequency of the amplifier is 10Hz, and the high cutoff frequency is

500Hz, together with a 50 Hz notch filter and a sampling frequency of 2048 Hz.

Ten healthy volunteers participated in this experiment, each subject was asked to do three different types of foot movement, and each movement was done three times. That means each action generates 30 group sEMG signals, and each group has a 2-second acquisition time.

2) Feature Extraction

The original sEMG signal was decomposed into 5 levels with the db4 wavelet. We obtained the wavelet coefficients after decomposition:

$$coeff = [ca_5, cd_5, cd_4, cd_3, cd_2, cd_1] \tag{13}$$

Note that each scale occupies a frequency band, and the quadratic sum of all the coefficients represents the signal energy in the scale. Thus, selecting the energies in different frequency bands as feature vectors, we get a 6-dimensional feature vector.

3) Classification Using the BP Artificial Neural Network.

The 6-dimensional feature vector is first normalized. Then, the training signals of each movement from the 15 groups are selected. These 45 sample sets are employed to train the BP neural network via gradient descent with a variable learning rate. The rest of the 45 groups are used as test samples for the pattern recognition (see results in Table 1).

TABLE 1. CLASSIFICATION RESULTS FOR THREE TYPES OF FOOT MOVEMENT

Types of Foot Movement	Test Samples Number Identified Correctly	Correct Identification Rate
Tiptoe	15	93.33%
Toes Upwarp	13	
Raising the Forefoot	14	

The correct identification rate of 93.33% shows that wavelet transform to extract feature vectors, combined with artificial neural network classifier approach can effectively improve the accuracy of the sEMG signal recognition.

D. Emotion Recognition

This is a new application area for EMG signals. Cheng and Liu [13] extracted the minmax of the wavelet coefficients to construct feature vectors that are entered into a BP neural network classifier and nearest neighbor classifier, respectively. They successfully identified 4 kinds of emotion; joy, anger, sadness and pleasure in their work.

V. CONCLUSION

This paper provides a brief introduction of the wavelet transform in EMG signals processing and feature extraction. The method is superior to traditional Fourier methods in analyzing physical situations where the signals contain discontinuities and sharp spikes. The future of wavelets lies in the as-yet uncharted territory of applications such as emotion recognition.

ACKNOWLEDGMENT

This research is supported by Qingdao University. The authors would like to convey their appreciation to the College of Automation Engineering for the use of some equipment.

REFERENCES

[1] Xiyan Yao, “A survey of development for frame theory in wavelet analysis”, Journal of Yuncheng University, vol. 23, No. 5, Oct. 2005.

[2] C. Valens, “A really friendly guide to wavelets,” 1999, http://pagesperso-orange.fr/polyvalens/clemens/wavelets/wavelets.html.

[3] Gang Wang, “The Analysis of Surface Electromyographic Signal Based on Wavelet Transform and Multifractal Analysis,” PhD Dissertation, Shanghai Jiaotong University, 2008.

[4] Mingfeng Jiang, Hong Wang, “The classification of surface EMG signal based on wavelet transform and neural networks,” Journal of Biomedical Engineering Research, vol. 24, pp. 50–52, 2005.

[5] Yin Chen, Jiahai Liu, “Study on denoising of surface EMG signals,” Computer Era, vol. 26, No. 6, pp. 22–24, 2008.

[6] Min Zhao, “Application of FFT and Wavelet Transforms to Noise Elimination of Transformer Partial Discharge Signals,” Digital Magazine (Transformer), vol. 46, 2009, http://www.gotoread.com/s/e/?vo=8616&p=34.

[7] Zhongning Li, Zhizeng Luo, “Spatial correlation based on wavelet transform method in EMG denoising,” Acta Electronica Sinica, vo1. 35, No. 7, 2007.

[8] Xiang Chen, Jihai Yang, Zheng Liang, Fan Zheng, Xiaojin Qian, and Qinghuan Feng, “Classification technology of MUAP in NEMG,” Journal of University of Science and Technology of China, vo1. 33, No. 4, pp. 466–472, Aug. 2003.

[9] Fang Yan, Xiaoping Zeng, Qinghua He, and Yanyi Song, “Study on detection and analysis of electrically evoked SEMG signal and muscle fatigue,” Journal of Chongqing University (Natural Science Edition), vol. 28, No. 1, pp. 77–81, 2005.

[10] Hongbo Xie, Zhizhong Wang, Hai Huang, “Wavelet packet transformation feature extraction and surface EMG signal classification,” Chinese Medical Equipment Journal, vol. 24, No. 9, pp. 7–10, 2003.

[11] Liyu Cai, Zhizhong Wang, Haihong Zhang, “Surface EMG signal classification method based on wavelet transform,” Journal of Data Acquisition and Processing, vol. 15, No.2, pp. 255–258, Jun 2000.

[12] Mingwen Jiang, Rencheng Wang, Jingzhang Wang, and Dewen Jin, “Hand movement recognition based on wavelet transform of multichannel EMG,” Chinese Journal of Rehabilitation Medicine, vol. 21, pp. 22–24, 2006.

[13] Bo Cheng, Guangyuan Liu, “Emotion recognition from surface EMG signal using wavelet transform and neural nerwork,” Journal of Computer Applications, vol. 28, 2008.