① a) $a \equiv b \pmod{n}$

$$a = qn + b$$
$$a - b = qn$$
$$b - a = (-q)n$$
$$b = (-q)n + a$$
$$\boxed{b \equiv a \pmod{n}}$$

b) $a \equiv b \pmod{n}$ $\qquad$ $b \equiv c \pmod{n}$

$\quad a = qn + b$ $\qquad\qquad$ $b = Kn + c$

$\qquad\qquad\qquad\qquad\qquad$ $c = b - Kn$

$$a - c = qn + b - b + Kn$$
$$a - c = (q + K)n$$
$$a = (q + K)n + c$$
$$\boxed{a \equiv c \pmod{n}}$$

② a) $1234 \mod 4321$ $\qquad\qquad\qquad$ $1 = -1082 \times 1234$

$$4321 = 1234 \times 3 + 619 \qquad\qquad \boxed{3{,}239}$$
$$1234 = 619 \times 1 + 615$$
$$619 = 615 \times 1 + 4$$
$$615 = 4 \times 153 + 3$$
$$4 = 3 \times 1 + 1$$
$$1 = 4 - 3$$
$$1 = 4 - 615 + 4 \times 153$$
$$1 = 4 \times 154 - 615$$
$$1 = (619 - 615) \times 154 - 615$$
$$1 = 619 \times 154 - 615 \times 154 - 615$$
$$1 = 619 \times 154 - 615 \times 155$$
$$1 = 619 \times 154 - (1234 - 619) \times 155$$
$$1 = 619 \times 154 - 1234 \times 155 + 619 \times 155$$
$$1 = 619 \times 309 - 1234 \times 155$$
$$1 = (4321 - 1234 \cdot 3) \times 309 - 1234 \times 155$$
$$1 = 4321 \times 309 - 927 \times 1234 - 1234 \times 155$$

b) 24140 mod 40902

$40902 = 24140 \times 1 + 16762$

$24140 = 16762 \times 1 + 7378$

$16762 = 7378 \times 2 + 2006$

$7378 = 2006 \times 3 + 1360$

$2006 = 1360 \times 1 + 646$

$1360 = 646 \times 2 + 68$

$646 = 68 \times 9 + 34$

$68 = 34 \times 2 + 0$

$\boxed{No \quad Solution}$

c) 550 mod 1769

$1769 = 550 \times 3 + 119$

$550 = 119 \times 4 + 74$

$119 = 74 \times 1 + 45$

$74 = 45 \times 1 + 29$

$45 = 29 \times 1 + 16$

$29 = 16 \times 1 + 13$

$16 = 13 \times 1 + 3$

$13 = 3 \times 4 + 1$

$\boxed{550}$

$1 = 13 - 3 \times 4$

$1 = 13 - (16 - 13) \times 4$

$1 = 13 - 16 \times 4 + 13 \times 4$

$\boxed{1 = 13 \times 5 - 16 \times 4}$

$1 = (29 - 16) \times 5 - 16 \times 4$

$1 = 29 \times 5 - 16 \times 5 - 16 \times 4$

$1 = 29 \times 5 - 9 \times 16$

$1 = 29 \times 5 - 9 \times (45 - 29)$

$1 = 29 \times 5 - 9 \times 45 + 9 \times 29$

$1 = -9 \times 45 + 14 \times 29$

$1 = -9 \times 45 + 14 \times (74 - 45)$

$1 = -9 \times 45 + 14 \times 74 - 14 \times 45$

$1 = -23 \times 45 + 14 \times 74$

$1 = -23 \times (119 - 74) + 14 \times 74$

$1 = -23 \times 119 + 23 \times 74 + 14 \times 74$

$1 = -23 \times 119 + 37 \times 74$

$1 = -23 \times 119 + 37 \times (550 - 119 \times 4)$

$1 = -23 \times 119 + 37 \times 550 - 148 \times 119$

$1 = 37 \times 550 - 171 \times 119$

$1 = 37 \times 550 - 171 \times (1769 - 550 \times 3)$

$1 = 37 \times 550 - 171 \times 1769 + 513 \times 550$

③ a) reducable

b) irreducable

c) reducable

④ a) $f = gq + r$

$x^3 - x + 1 = x^2 + 1(x) + 1 =$

$$\boxed{1}$$

$$\begin{array}{r} x \\ x^2+1 \overline{\smash{)}\, x^3 + x + 1} \\ \underline{x^3 + x} \\ 1 \end{array}$$

b) $x^5 + 1$

$x^5 + x^4 + x^3 + 2x^2 + 2x + 1 = (x^3 + x^2 + x + 1)(x^2) + (-x^3 + 2x + 1)$

$$\begin{array}{r} x^2 + \\ x^3 + x^2 + x + 1 \overline{\smash{)}\, x^5 + x^4 + x^3 + 2x^2 + 2x + 1} \\ \underline{x^5 + x^4 + x^3 + x^2} \\ x^2 + 2x + 1 \end{array}$$

$x^3 + x^2 + x + 1 = (x^2 + 2x + 1)(x + 2) + (2x - 2)$

$$\begin{array}{r} x + 2 \\ x^2 + 2x + 1 \overline{\smash{)}\, x^3 + x^2 + x + 1} \\ \underline{x^3 + 2x^2 + x} \\ 2x^2 + 1 \\ 2x^2 + x + 2 \\ \hline 2x + 2 \end{array}$$

$x^2 + 2x + 1 = (2x + 2)(2x + 2) + 0$

?

$$\boxed{2x + 2}$$

$$\begin{array}{r} 2x + 2 \\ 2x + 2 \overline{\smash{)}\, x^2 + 2x + 1} \\ \underline{x^2 + x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

(5) $H(P) = \left(\frac{1}{4}\log_2 \frac{1}{4} + \frac{1}{4}\log_2 \frac{1}{4} + \frac{1}{2}\log_2 \frac{1}{2}\right) = 1.5$

$H(K) = 1.5$

$P_c(1) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$

$P_c(2) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{4}$

$P_c(3) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + 0\left(\frac{1}{4}\right) = \frac{1}{8}$

$P_c(4) = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + 0 + 0 = \frac{1}{8}$

$H(c) = -\left(\frac{1}{2}\log_2 \frac{1}{2} + \frac{1}{4}\log_2 \frac{1}{4} + \frac{1}{8}\log_2 \frac{1}{8} + \frac{1}{8}\log_2 \frac{1}{8}\right) = 1.75$

$H(K|c) = H(K) + H(P) - H(c)$

$H(K|c) = 1.5 + 1.5 - 1.75 = \boxed{1.25}$