Problem-1

1. word is unusual

→ Unique subsets of 5 letters (of the 7) exist

$\{u\ \_\ \_\ \_\ \_\} \rightarrow \binom{4}{4} = \frac{4!}{4!0!} = 1$

$\{u\ u\ \_\ \_\ \_\} \rightarrow \binom{4}{3} = \frac{4!}{3!1!} = 4$

$\{u\ u\ u\ \_\ \_\} \rightarrow \binom{4}{2} = \frac{4!}{2!2!} = 6$

Thus the total number of subsets of 5 letters (of the 7) are $1+4+6=11$.

→ Different strings of 5 letters from 7 letters are:-

That can be given by different arrangements of each subset

Thus,

$$= 5! \cdot \binom{4}{4} + \frac{5!}{2!}\binom{4}{3} + \frac{5!}{3!}\binom{4}{2}$$

$$= 5! + \frac{5!}{2!} \times 4 + \frac{5!}{3!} \times 6$$

$$= 120 + 240 + 120 = 480$$

2. The ways a 5 card hand can be held with 2 pairs is given by:-

Two cards are drawn of two different suit and one card out of 4 suits

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$$

$$= \frac{13!}{11!\,2!} \times \frac{4!}{2!\,2!} \times \frac{4!}{2!\,2!} \times \frac{11!}{10! \times 1!} \times \frac{4!}{1!\,3!}$$

$$= \frac{13 \times 12}{2} \times \frac{4!}{4} \times \frac{4!}{\cancel{4}} \times \frac{11}{1} \times \frac{\cancel{4}}{1}$$

$$= 123552$$

3. The violinist songs = 7

The number of couples = 16

Two Cases :-

A couple could have not listened to any song due to fight.

that means.

$$2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$$

So, that means we have

$\binom{n-1+r}{r}$, here $n = 6$

$r = 16$

$$\Rightarrow \binom{6-1+16}{16} = \binom{21}{16}$$

second Case:- Only one song was listened by the fighting couple

1 | 2 | 3 | 4 | 5 | 6 | 7

so now songs left = 15

No of couples = 6

So,

$$\binom{n-1+r}{r} = \binom{6-1+15}{15} = \binom{20}{15}$$

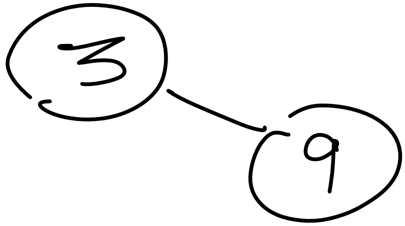
So, total ways of songs distribution are:-

$$\Rightarrow \binom{21}{16} + \binom{20}{15}$$

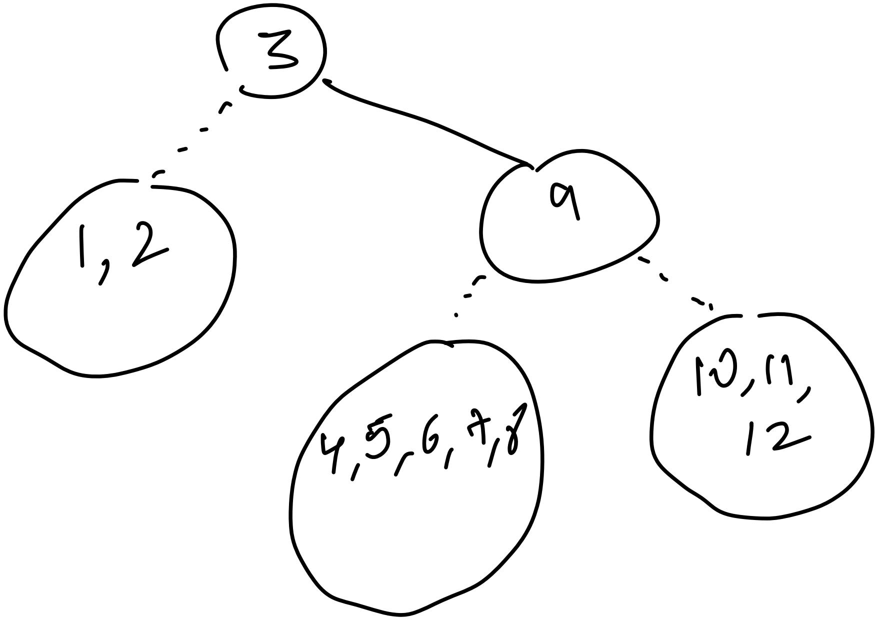
$$\Rightarrow \frac{21!}{5! \times 16!} + \frac{20!}{15! \times 5!}$$

$$\Rightarrow 35853$$

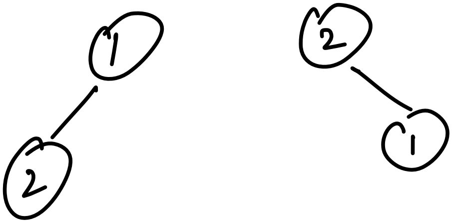
4. We have a tree with root 3 and right child 9



So, we can say

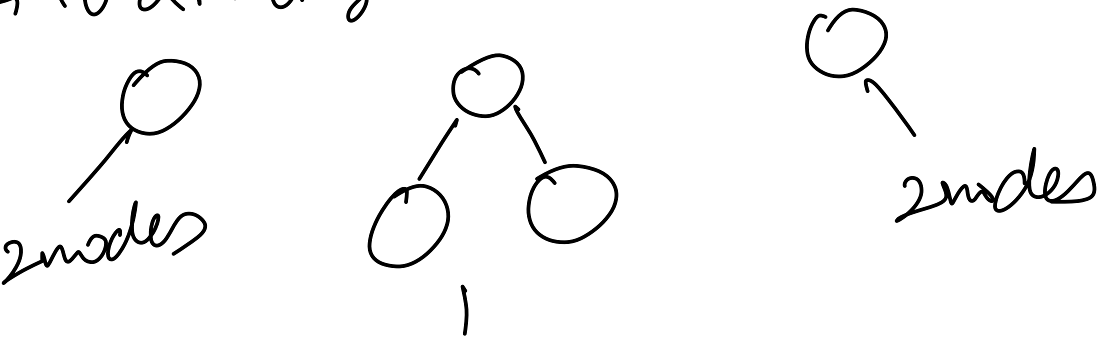


→ To arrange 2 nodes.



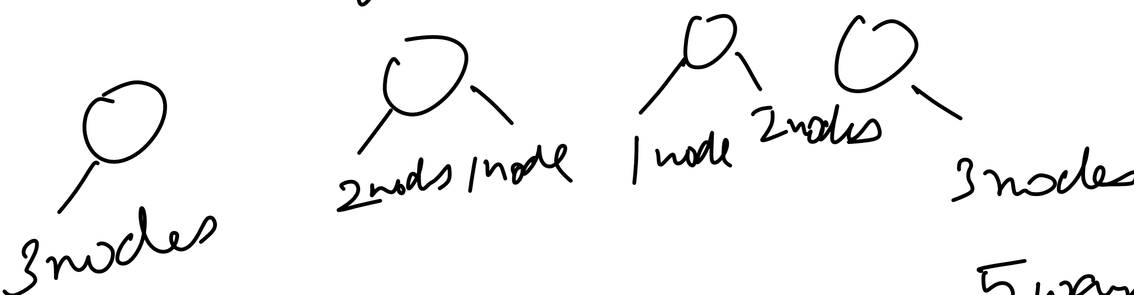
→ 2 ways.

→ To arrange 3 nodes.



Thus, 5 ways.

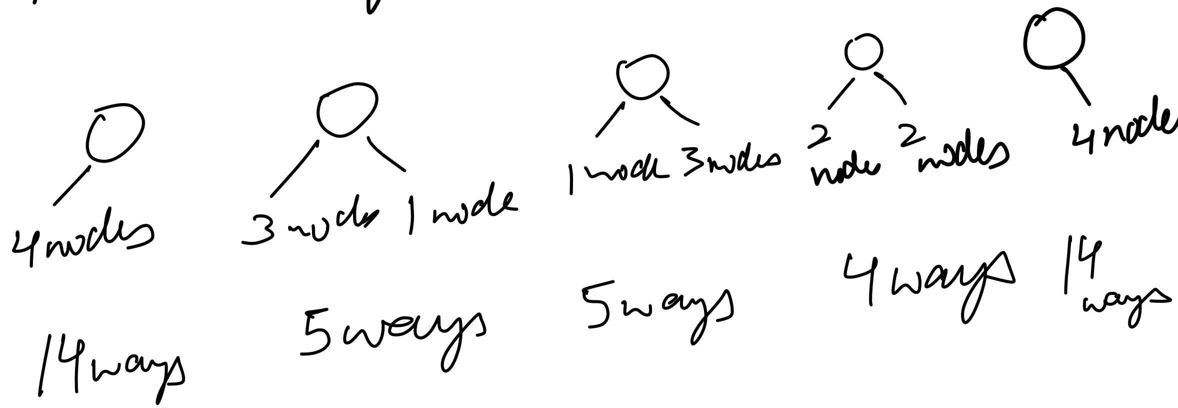
→ To arrange 4 nodes.



5ways 2 ways 2ways 5 ways

Thus, for 4 nodes there are 14 ways.

→ To arrange 5 nodes



The total ways are 42 ways

Thus, total number of BST possible $2 \times 5 \times 14 \times 42$

$= 5880$ BSTs.

5. Total Friends are 10
and no of nurses 4

Cases

4 Nurses at work

3 Nurses at work.

4 Nurses at work:

1 1 1 1

6
5 1
4 2
3 3
4 1 1
3 2 1
2 2 2
2 2 1 1
3 1 1 1

9 ways

3 Nurses at work:

1 1 1

7
6 1
5 2
4 3
5 1 1
4 2 1
3 3 1
3 2 2

8 ways

Total Ways are $9+8=17$ ways