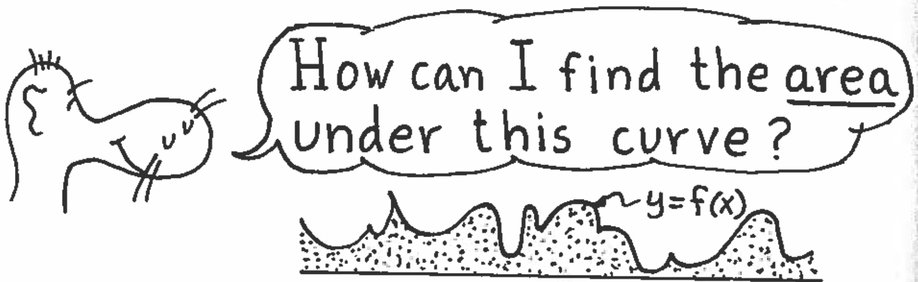
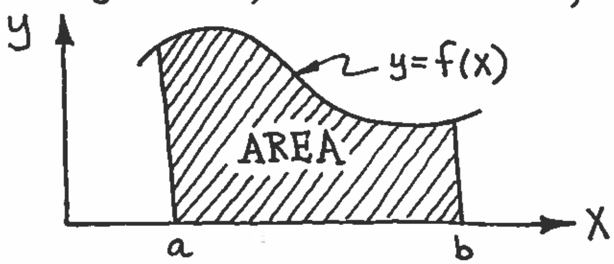
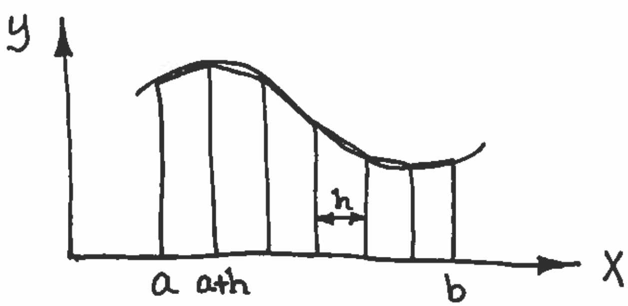


An easy way to calculate the area under a curve $y = f(x)$, from $a \le x \le b$,



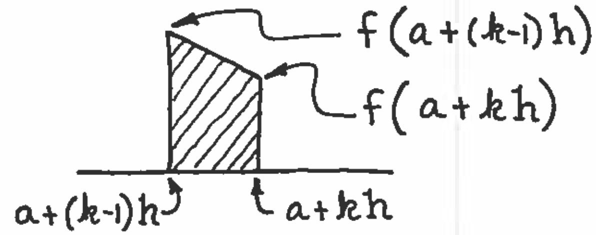
is to subdivide the area under the curve into vertical strips of equal width. The top of each strip can be approximated by a straight line as shown below:



Thus, the area under the curve is approximately the sum of the areas of n trapezoids of width h.

$$nh = b - a$$

The area under the k^{th} trapezoid



is given by:

$$\frac{h}{2}\left[f(a+(k-1)h) + f(a+kh)\right]$$

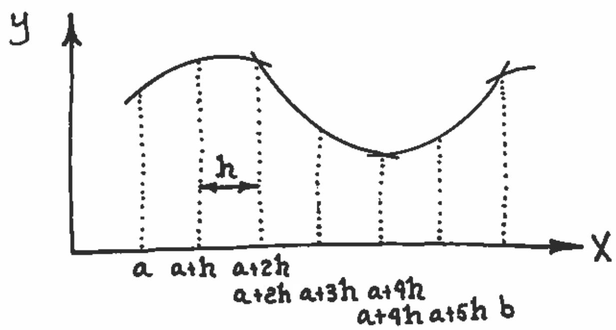
The total area under the curve from a to b is then approximately

$$A = h\left[\frac{f(a)+f(b)}{2} + f(a+h) + f(a+2h) + f(a+3h) \ldots + f(a+(n-1)h)\right]$$

Notice that at No Time did the word "Integration" pass my lips! You don't need to know calculus to find areas this way. The above procedure is obviously extremely easy to understand and to program.

Some people call the preceding formula the Trapezoidal Rule for Integration. Of course, by this name, it is much more difficult to grasp.

Another method for area estimation which is still easy but slightly more accurate than the Trapezoidal Rule involves replacing the straight line tops with parabolas:



You pick sets of 3 points off of the curve and then find the parabola that passes through them. (There is only one!) After hairy algebraic tedium, you find that the area under these parabolas is given exactly by a simple formula. If the points are evenly spaced a distance h apart, the area under the parabola from a to a+2h is given by

$$\frac{h}{3}\left[f(a)+4f(a+h)+f(a+2h)\right]$$

Similarly, the area under the parabola from a+2h to a+4h is:

$$\frac{h}{3}\left[f(a+2h)+4f(a+3h)+2f(a+4h)\right]$$

and so forth.

Thus, in toto, the area under the parabolas approximating the curve from a to b is:

$$\text{AREA}=\frac{h}{3}\Big[f(a)+4f(a+h)+2f(a+2h)+4f(a+3h)+2f(a+4h)+\ldots+2f(a+(n-2)h)+4f(a+(n-1)h)+f(b)\Big]$$

if there are n divisions. (Be sure n is even!) This formula is called <u>Simpson's One-Third Rule for Integration.</u>

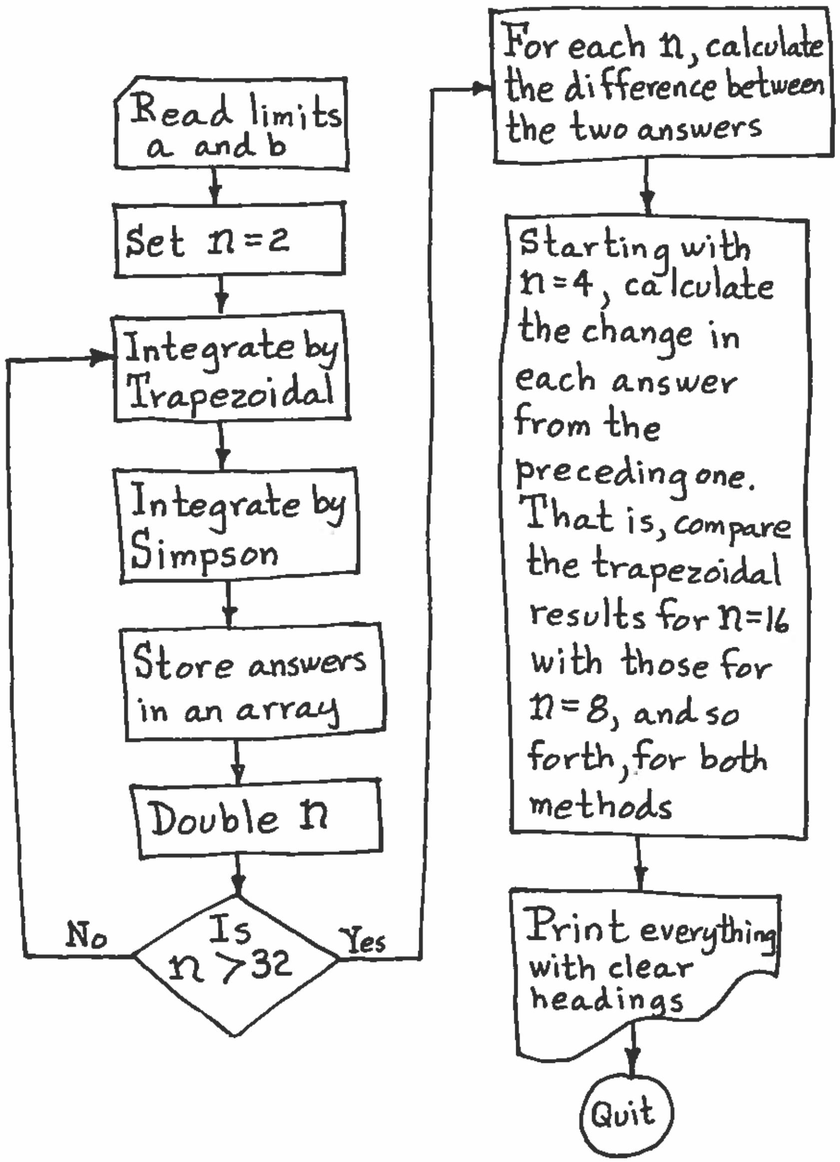
This integration stuff is easy!

It really is, so try a few problems.

First, write a numerical integration function subprogram based on the Trapezoidal Rule. Once that works, write a second function subprogram based on Simpson's Rule. These two programs should each be able to integrate another function subprogram named "FUNK," using any desired number of divisions and any given starting and ending points a & b.

Next write a mainline program. This main program is going to compare the results of the trapezoidal rule with those from Simpson's rule. It also is going to see what effect n

has on the accuracy of the results. Here's how your mainline is to work:



Now, test out your programs by seeing if you can integrate the following functions between the given limits. You can either write a separate FUNCT program for each one and run them one at a time, or else you can try using an External statement if you know what that is. I suggest running them one at a time for now.

Try these:

$$y = \frac{1}{2+X} \quad \text{from } X=0 \text{ to } X=2$$

(The answer should be 0.69315)

$$y = \frac{1}{1+X^2} \quad \text{from } X=0 \text{ to } X=1$$

(The answer should be $\frac{\pi}{4}$)

$$y = e^{-\frac{X^2}{2}} \quad \text{from } X=0 \text{ to } X=0.4$$

(I haven't the foggiest idea what the right answer is but I wouldn't be surprised if it was 0.389 something!)

$$y=(X^2-\cos X)e^{-X} \quad \text{from } X=-1 \text{ to } X=1$$

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