# AstroMath Fall 2023 Workshop

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## 1 Introduction

This educational workshop aims to provide a foundational understanding of the mathematics essential for astronomy. The workshop is tailored for those interested in learning or refining their skills in astronomical calculations and understanding celestial mechanics through mathematics.

The workshop is divided into several key sections:

- **Unit Conversions:** Gain a solid understanding of converting between different units commonly used in astronomical measurements.
- **Introduction to Ratios:** Learn the fundamentals of ratios and their application in comparing celestial bodies.
- **Basic Ratios:** Delve deeper into the concept of ratios as they are applied to astronomy, such as comparing the diameters or masses of different celestial objects.
- **Distance Modulus:** Understand this important concept used for calculating astronomic distances.
- **Logarithms:** Familiarize yourself with logarithmic operations, crucial for dealing with the large numbers often encountered in astronomy.

Each section includes lessons and problems that aim to apply the learned mathematical concepts in real-world astronomical settings. By the end of this workshop, you will have a practical understanding of the mathematical techniques and principles that are critical for both amateur and professional astronomical observations and research.

## 2 Unit Conversions

Understanding unit conversions is crucial for astronomy where measurements span multiple scales. Let's start with some basic conversions involving meters.

**Lesson:** 1 meter (m) is:

- 100 centimeters (cm)
- 1,000 millimeters (mm)
- 0.001 kilometers (km)

**Problem A:** Convert 5 meters into centimeters.

**Solution A:** $5 \text{ m} \times 100 \text{ cm/m} = 500 \text{ cm}$.

**Problem B:** Convert 2.5 kilometers into meters.

**Solution B:** 2.5 km $\times$ 1000 m/km = 2500 m.

# 3 Introduction to Ratios

Before diving into the application of ratios in astronomy, let's understand the fundamental concept of ratios.

**Lesson:** A ratio is a way to show a relationship or comparison between two numbers. For example, the ratio 1:3 means for every 1 of the first number, there are 3 of the second number.

# 4 Basic Ratios

Ratios are comparisons or relationships between two things. In astronomy, we often use ratios to compare sizes, distances, and other properties of celestial objects.

**Problem 1:** The ratio of the diameter of Earth to the diameter of Jupiter is about 1:11. If the diameter of Earth is approximately 12,742 kilometers, estimate the diameter of Jupiter.

**Solution:** We can calculate this using the following ratio equation:

$$\frac{12,742 \text{ km}}{1} = \frac{x \text{ km}}{11}$$

Solving for $x$ gives $x \approx 140,162$ kilometers.

**Problem 2:** The mass ratio of the Sun to Earth is approximately $3.33 \times 10^5$. If the mass of Earth is about $5.97 \times 10^{24}$ kg, estimate the mass of the Sun.

**Solution:** Setting up the ratio equation, we have:

$$\frac{x \text{ kg}}{5.97 \times 10^{24} \text{ kg}} = 3.33 \times 10^5$$

Solving for $x$, we find $x \approx 1.989 \times 10^{30}$ kg.

# 5 Distance Modulus

The distance modulus is a way to calculate distances in astronomy using the difference between apparent magnitude and absolute magnitude. It is especially useful when parallax measurements are not available.

**Problem 3:** A certain star has an apparent magnitude of 3.4 and an absolute magnitude of -2.5. Calculate the distance to the star in parsecs.

**Solution:** The distance modulus formula is:

$$m - M = 5\log(d) - 5$$

Solving for $d$, we get:

$$d = 10^{(m-M+5)/5}$$

Substituting the given values, we find $d \approx 151.35$ parsecs.

**Problem 4:** A distant galaxy has an apparent magnitude of 14 and an absolute magnitude of -20. Calculate the distance to the galaxy in parsecs.

**Solution:** Using the distance modulus formula, we find $d \approx 10^{(14-(-20)+5)/5} = 10^{39/5} \approx 63095734$ parsecs.

## 6 Logarithms

Logarithms are used in astronomy to deal with the large numbers and exponential relationships that are often encountered. A key example is the inverse square law of light, which states that the intensity of light decreases with the square of the distance.

**Problem 5:** The brightness of a star decreases with the square of the distance. If Star A is 10 times further away than Star B and Star B is 100 times brighter than Star A, verify this relationship using logarithms.

**Solution:** We can express the brightness relationship as:

$$\frac{B_A}{B_B} = \left(\frac{D_B}{D_A}\right)^2$$

Taking the logarithm of both sides of the brightness equation, we simplify the exponential term:

$$\log\left(\frac{B_A}{B_B}\right) = 2\log\left(\frac{D_B}{D_A}\right)$$

Substituting in the given values, we find:

$$\log\left(\frac{1}{100}\right) = 2\log\left(\frac{1}{10}\right)$$

Solving these expressions, we find $\log\left(\frac{1}{100}\right) = \log\left(\frac{1}{100}\right)$, thus verifying the relationship.

**Problem 6:** If a galaxy is observed to have a brightness 1/16 that of the Milky Way, and both are similar types of galaxies, so their luminosities are similar, how much farther is the observed galaxy compared to the distance from us to the Milky Way?

**Solution:** Using the inverse square law of light, the ratio of the brightness is equal to the square of the ratio of the distances:

$$\left(\frac{D_{\mathrm{MW}}}{D_{\mathrm{G}}}\right)^2 = \frac{B_{\mathrm{MW}}}{B_{\mathrm{G}}}$$

Given that $B_{\mathrm{MW}}/B_{\mathrm{G}} = 16/1$, solving for $D_{\mathrm{G}}/D_{\mathrm{MW}}$ gives us $\sqrt{16} = 4$. So, the observed galaxy is approximately 4 times farther away than the Milky Way.

## 7 Ratios

**Problem 7:** The ratio of the diameter of Mars to the diameter of the Moon is approximately 2.15. If the diameter of the Moon is about 3,474 kilometers, estimate the diameter of Mars.

**Solution:** Using the ratio equation:

$$\frac{x \text{ km}}{3,474 \text{ km}} = 2.15$$

Solving for $x$, we find $x \approx 7,469$ kilometers.

## 8 Distance Modulus

**Problem 8:** The apparent magnitude of a star in the Andromeda Galaxy is 21, and the absolute magnitude of the star is estimated to be 1. Calculate the distance to the star in parsecs.

**Solution:** Given the apparent magnitude $m = 21$, and the absolute magnitude $M = 1$, we can find the distance $d$ to the star using the formula:

$$d = 10^{\frac{(m-M+5)}{5}}$$

Substituting the given values into the formula:

$$d = 10^{\frac{(21-1+5)}{5}}$$
$$d \approx 10^5 \text{ parsecs}$$

# 9 Logarithms

**Problem 9:** The intensity of a signal from a spacecraft decreases as the square of the distance from the Earth. If the signal is 900 times weaker when the spacecraft is at the edge of the solar system compared to when it is in low Earth orbit, approximately how many times farther is the edge of the solar system than low Earth orbit?

**Solution:** We use the inverse square law of light intensity:

$$\left(\frac{D_{\mathrm{E}}}{D_{\mathrm{S}}}\right)^2 = \frac{I_{\mathrm{E}}}{I_{\mathrm{S}}}$$

Given that $I_{\mathrm{E}}/I_{\mathrm{S}} = 900$, solving for $D_{\mathrm{S}}/D_{\mathrm{E}}$ gives us $\sqrt{900} = 30$. So, the edge of the solar system is approximately 30 times farther than low Earth orbit.

**Problem 10:** The brightness of a distant quasar is 1/10000 of a similar quasar at a distance of 100 Mpc. How much farther is the distant quasar compared to the 100 Mpc quasar?

**Solution:** Using the inverse square law of light, the ratio of the brightness is equal to the square of the ratio of the distances:

$$\left(\frac{D_1}{D_2}\right)^2 = \frac{B_1}{B_2}$$

Given that $B_1/B_2 = 1/10000$, solving for $D_2/D_1$ gives us $\sqrt{10000} = 100$. So, the distant quasar is approximately 100 times farther away than the 100 Mpc quasar.