STAT 535- FORECASTING METHODS FOR MANAGEMENT
Assignment 3
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```
radv<-read.csv("/Users/renukachintalapati/Downloads/Lydiamonthly4052.txt")
attach(radv)
fMonth<-as.factor(month)
#Augmenting fMonth to rconstruct:
radv<-data.frame(radv,fMonth)
```

Q1.

```
model1<-lm(msales~fMonth+feb44+dec44+jan45+sep45+madv+madvl1+madvl2+madvl3+madvl4+msalesl1+msalesl2+msalesl3+msalesl4+c220+s220+c348+s348+c432+s432);summary(model1)
```

OBSERVATION: We notice that many terms are not significant. Madvl2, madvl3, madvl3, msalesl3, msalesl4, c220 & s220, c348 & s348, and c432 & s432 are not significant.
Thus, we remove these terms and refit the model as follows:

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.446e+04  9.630e+03   2.540 0.012315 *
fMonth2     -1.870e+04  9.583e+03  -1.951 0.053235 .
fMonth3     -1.204e+04  9.822e+03  -1.226 0.222438
fMonth4     -2.855e+04  9.249e+03  -3.087 0.002492 **
fMonth5     -3.026e+04  7.968e+03  -3.798 0.000227 ***
fMonth6     -2.745e+04  6.878e+03  -3.991 0.000112 ***
fMonth7     -1.002e+04  6.844e+03  -1.463 0.145865
fMonth8     -1.231e+04  7.623e+03  -1.615 0.108774
fMonth9     -8.944e+03  8.584e+03  -1.042 0.299435
fMonth10    -3.883e+03  8.728e+03  -0.445 0.657209
fMonth11    -5.000e+04  9.278e+03  -5.388 3.40e-07 ***
fMonth12    -5.291e+04  7.931e+03  -6.671 7.37e-10 ***
feb44       -4.191e+04  1.594e+04  -2.628 0.009653 **
dec44       -5.188e+04  1.515e+04  -3.425 0.000832 ***
jan45        7.176e+04  1.560e+04   4.601 1.02e-05 ***
sep45       -4.263e+04  1.519e+04  -2.807 0.005801 **
madv         2.129e-01  4.720e-02   4.510 1.47e-05 ***
madvl1       1.179e-01  5.278e-02   2.233 0.027327 *
madvl2      -6.808e-03  5.464e-02  -0.125 0.901042
madvl3      -3.101e-02  5.206e-02  -0.596 0.552483
madvl4       9.280e-03  4.963e-02   0.187 0.851967
msalesl1     3.954e-01  7.991e-02   4.948 2.37e-06 ***
msalesl2     2.903e-01  8.245e-02   3.521 0.000601 ***
msalesl3     1.053e-01  8.189e-02   1.286 0.200917
msalesl4     5.409e-02  7.168e-02   0.755 0.451903
c220         1.633e+02  1.690e+03   0.097 0.923159
s220        -4.043e+03  1.668e+03  -2.424 0.016801 *
c348        -3.306e+03  1.725e+03  -1.917 0.057563 .
s348         2.160e+03  1.714e+03   1.260 0.209915
c432         6.634e+02  1.661e+03   0.399 0.690287
s432         3.585e+02  1.691e+03   0.212 0.832419
```

model2<-lm(msales~fMonth+feb44+dec44+jan45+sep45+madv+madvl1+msalesl1+msalesl2
);summary(model2)

```
Call:
lm(formula = msales ~ fMonth + feb44 + dec44 + jan45 + sep45 + 
    madv + madvl1 + msalesl1 + msalesl2)

Residuals:
   Min     1Q Median     3Q    Max 
-30671  -9075   -389   6803  41995 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.193e+04  7.941e+03   4.021 9.56e-05 ***
fMonth2     -2.111e+04  7.666e+03  -2.754 0.006685 ** 
fMonth3     -2.156e+04  6.211e+03  -3.471 0.000696 ***
fMonth4     -3.552e+04  7.130e+03  -4.981 1.88e-06 ***
fMonth5     -3.600e+04  6.774e+03  -5.314 4.27e-07 ***
fMonth6     -3.211e+04  6.354e+03  -5.054 1.37e-06 ***
fMonth7     -1.261e+04  6.052e+03  -2.084 0.039070 *  
fMonth8     -1.757e+04  6.212e+03  -2.828 0.005387 ** 
fMonth9     -1.490e+04  6.537e+03  -2.279 0.024210 *  
fMonth10    -1.199e+04  6.969e+03  -1.720 0.087697 .  
fMonth11    -5.823e+04  7.683e+03  -7.580 4.83e-12 ***
fMonth12    -5.534e+04  7.133e+03  -7.758 1.84e-12 ***
feb44       -3.748e+04  1.572e+04  -2.385 0.018470 *  
dec44       -6.185e+04  1.502e+04  -4.117 6.61e-05 ***
jan45        7.597e+04  1.553e+04   4.892 2.78e-06 ***
sep45       -5.334e+04  1.512e+04  -3.527 0.000574 ***
madv         2.603e-01  4.370e-02   5.957 2.08e-08 ***
madvl1       8.162e-02  4.850e-02   1.683 0.094698 .  
msalesl1     4.755e-01  6.776e-02   7.017 9.72e-11 ***
msalesl2     3.376e-01  6.550e-02   5.154 8.79e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 14170 on 136 degrees of freedom
Multiple R-squared:  0.8647,	Adjusted R-squared:  0.8459 
F-statistic: 45.76 on 19 and 136 DF,  p-value: < 2.2e-16
```

Not let us perform the Anova test.

anova(model1, model2)

```
> anova(model1, model2)
Analysis of Variance Table

Model 1: msales ~ fMonth + feb44 + dec44 + jan45 + sep45 + madv + madvl1 + 
    madvl2 + madvl3 + madvl4 + msalesl1 + msalesl2 + msalesl3 + 
    msalesl4 + c220 + s220 + c348 + s348 + c432 + s432
Model 2: msales ~ fMonth + feb44 + dec44 + jan45 + sep45 + madv + madvl1 + 
    msalesl1 + msalesl2
  Res.Df        RSS  Df   Sum of Sq      F  Pr(>F)  
1    125 2.3776e+10                                 
2    136 2.7298e+10 -11 -3522206072 1.6835 0.08446 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

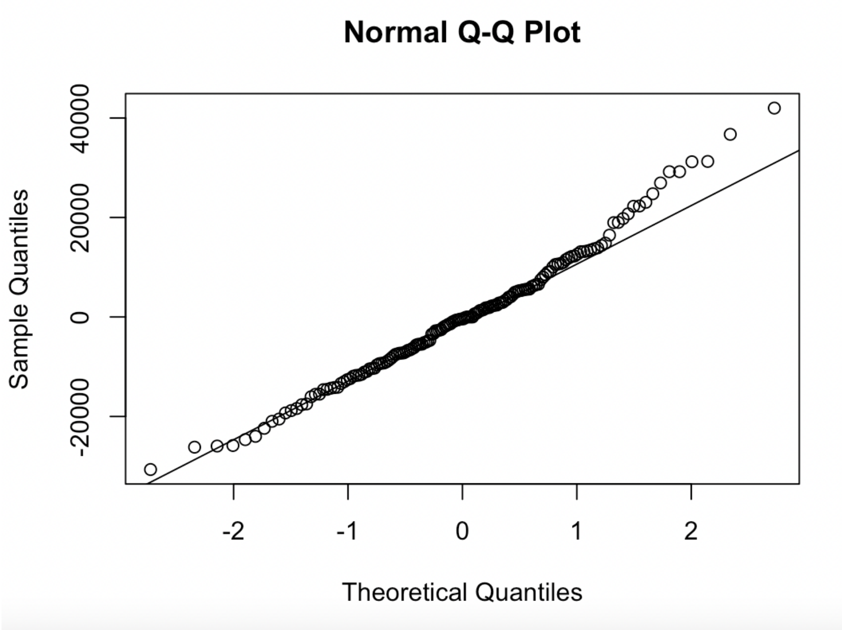
OBSERVATION: Here, we notice that the p-value is greater than 0.05. So, the terms that we have discarded are certainly insignificant. Thus, we would follow the second model for our analysis from here onwards.

#Refitting the model:
model2<-lm(msales~fMonth+feb44+dec44+jan45+sep45+madv+madvl1+msalesl1+msalesl2);summary(model2)

OBSERVATION: We notice that all the terms are significant here.

B). Let us now analyze the residuals of our chosen model.

#Normal quantile plot:
qqnorm(resid(model2))
qqline(resid(model2))



OBSERVATION: We can say that there are a few outliers notably -> 4, 73, 82 and 94. Residuals mainly at the upper tail deviate away from the normality. These are mainly 4, 73, and 94. These correspond to the dates - April 1940, January 1946, and October 1947.

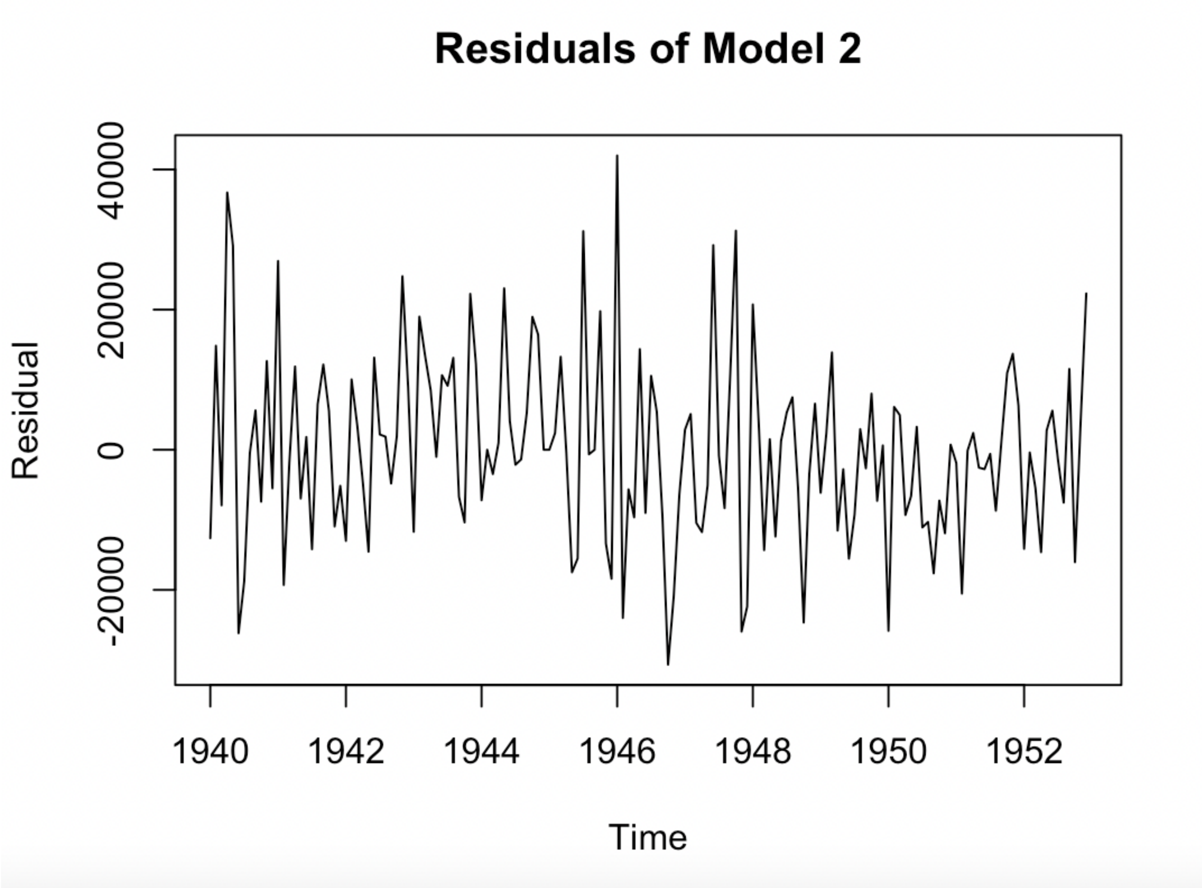
Let us now test for normality using the Shapiro- Wilk test:

#Test for normality:
shapiro.test(resid(model2))

OBSERVATION: We see that the p-value is 0.1235 which is greater than 0.05. It is thus non-significant for the rejection of normality.

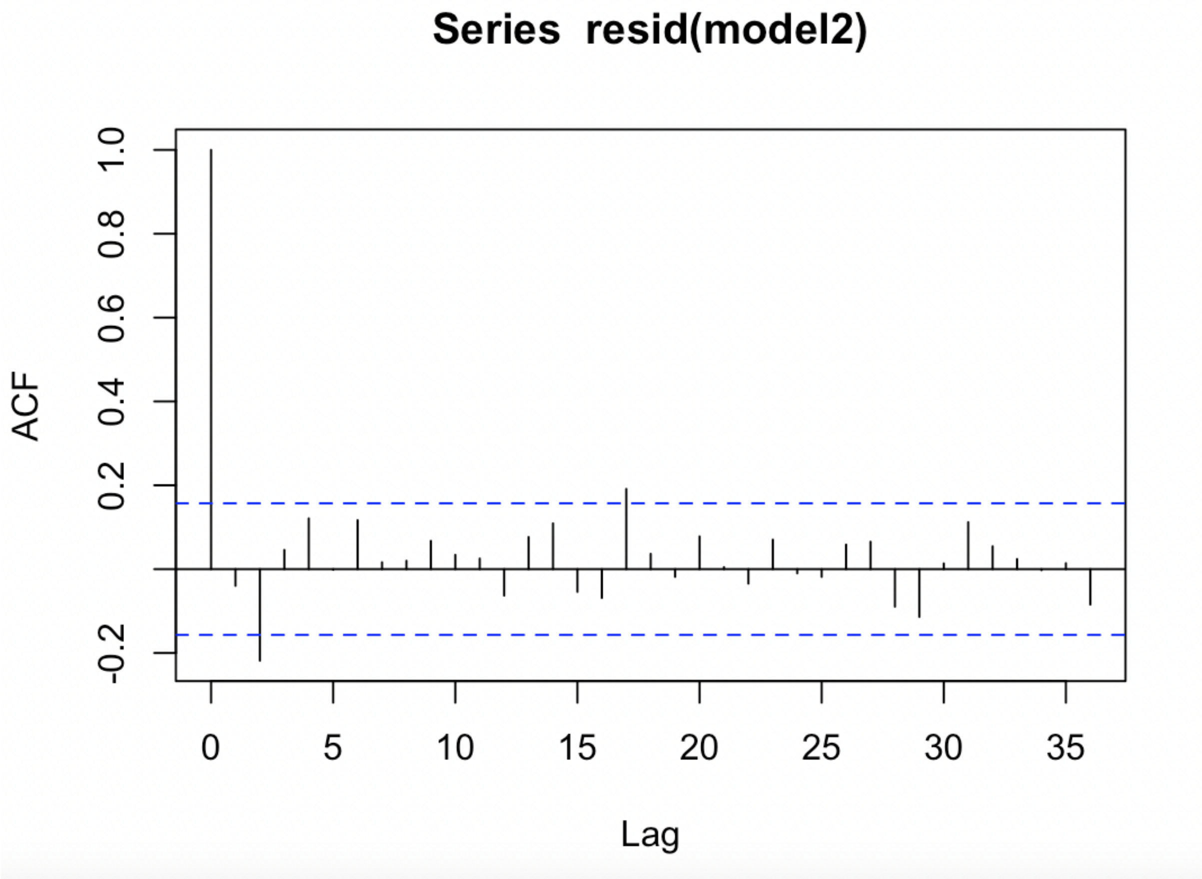
Plot of Residuals versus time:

```
plot(ts(resid(model2),start=c(1940,1),freq=12),xlab="Time",ylab="Residual",main="Residuals of Model 2")
```



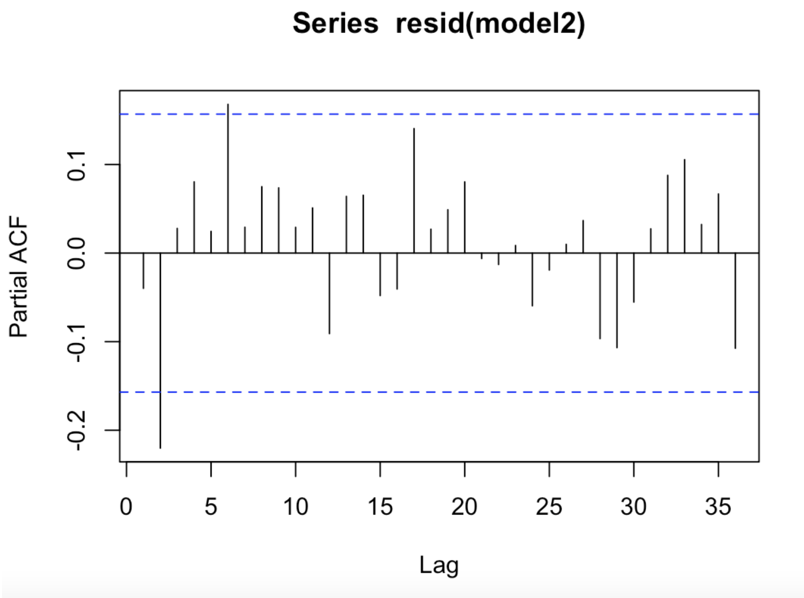
OBSERVATION: It can be said that the trend has not been captured that well by our model here. The trend part is modest and we can see some volatility at the points of outliers. The volatility is high during the first part and reduces in the later part. We can see that it reduced after 1952. The seasonal structure has also not been captured well.

```
#Autocorrelation of residual plot:
acf(resid(model2),36)
```



OBSERVATION: We observe that lag2 and lag17 are significant above. Most of the plot has been reduced to white noise above as we see that the spikes are mostly within the two blue lines.

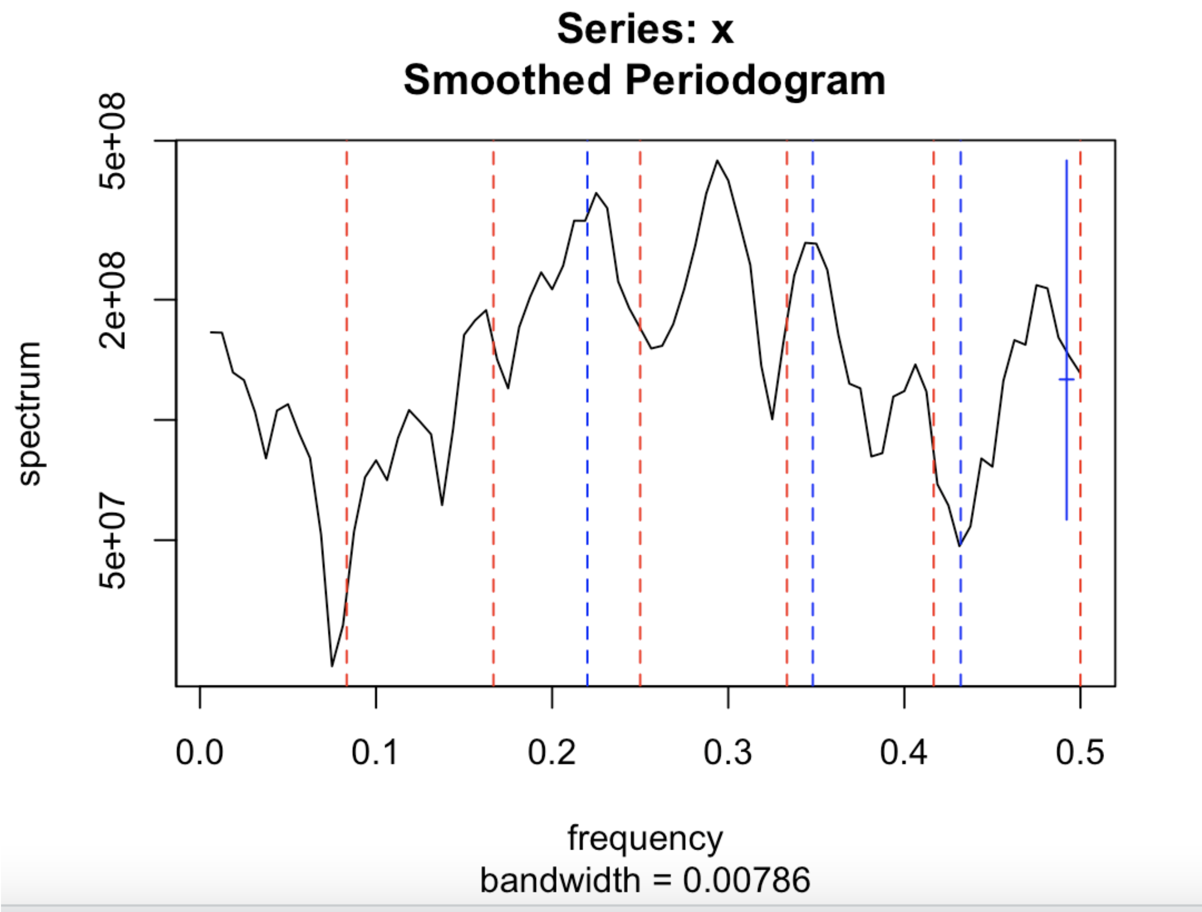
#Partial autocorrelation of residual plot:
pacf(resid(model2),36)



OBSERVATION: Here we see that lag 2 and lag 6 are significant.

#Residual spectral density:

```
spectrum(resid(model2),span=5)
abline(v=c(1/12,2/12,3/12,4/12,5/12,6/12),col="red",lty=2)
abline(v=c(0.220,0.348,0.432),col="blue",lty=2)
```



OBSERVATION: It can be noticed that the spectral plot obtained is not flat. Thus, the residuals do not conform to the structure of white noise. We can clearly see a peak at low frequency that indicates slow movement of the time series (remaining trend structure). The red lines 2/12, 3/12 and 5/12 are not prominent and thus we can say that seasonal structure is not good.
The frequencies at 0.220 and 0.348 are prominent, signifying modest calendar structure. The seasonal structure has not been captured well. By looking at the blue line at the top right hand corner of the plot, it can be said that twice the length of the notch doesn't cover the entire peak. Hence, it doesn't get reduced to white noise here. It is very close to being consistent with white noise. It is not well equipped to be a time series model.

```
install.packages ("hwwntest")
library("hwwntest")
```

#Bartlett B test:
bartlettB.test((ts(resid(model2))))

```
        Bartlett B Test for white noise

data:
= 1.0405, p-value = 0.2291
```

OBSERVATION: We notice that the p-value in this case is 0.2291. Also, as the value is greater than 0.05, we cannot reject the null hypothesis of reduction to white noise. Thus, white noise has been captured well according to the Bartlett B test. We can also see that the blue-line test is a visual one and the Bartlett B test is the one that predicts white noise accurately.

C).

90 percent calculation part:

This is our model equation:

S_t = 3.193x10^4 + (4.755x$10^{-1}S_{t-1}$) + (3.376x$10^{-1}S_{t-2}$) + 2.603x$10^{-1}A_t$ + (8.162x$10^{-2}A_{t-1}$) S_t - 4.755x$10^{-1}BS_t$ - 3.376x$10^{-1}B^2S_t$ = 3.193x10^4 + 2.603x$10^{-1}A_t$ + 8.162x$10^{-2}BA_t$
S_t = (1-4.755x10^{-1}B - 3.376x10^{-1}B 2) -1.3.193x10^4 + (1-4.755x10^{-1}B - 3.376x$10^{-1}B^2$) -1 .At..(2.603x10^{-1} + 8.162x10^{-2}B)

As the value of (r<=p),

2.603x10^{-1} + 8.162x10^{-2}B = (1-4.755x10^{-1}B - 3.376x10^{-1}B 2)($\boldsymbol{\delta}$ + $\boldsymbol{\delta}_1$B + $\boldsymbol{\delta}_2B_2$ +...)

```
> deltapartial<-delta<-c(rep(0, times=500))
> delta[1]<-0.2603
> delta[2]<--0.2053
> deltapartial[1]<-delta[1]
> deltapartial[2]<-deltapartial[1] + delta[2]
> for (j in 3:500){
+ k1<-j-1
+k2<-j-2
+ delta[j]<-0.4755*delta[k1]+0.3376*delta[k2]
+ deltapartial[j]<-deltapartial[k1]+delta[j]
+ }
> deltapartial[500]*0.9
[1] -0.3311685
> deltapartial[1:20]
[1] 0.26030000 0.05500000 0.04525713 -0.02868488 -0.06713351 -0.11037865 [7] -0.14392197 -0.17447138 -0.20032185 -0.22292723 -0.24240320 -0.25929560 [13] -0.27390303 -0.28655174 -0.29749766 -0.30697266 -0.31517336 -0.32227155 [19] -0.32841530 -0.33373300
```

90% interval calculation part is as follows:

19 + (-0.3311 + 0.3284)/ (-0.3337 + 0.3284) = 19.5 months approximately.

Q2.

```
rgoods<-read.csv("/Users/renukachintalapati/Downloads/RPCEGoods5219.txt")
attach(rgoods)
head(rgoods)
```

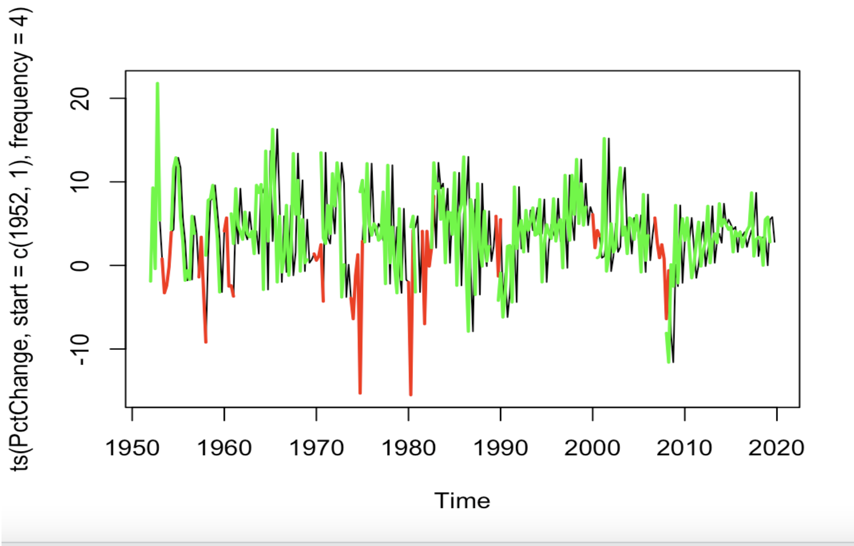
```
> head(rgoods)
     Quarter PctChange
1 1952-01-01      -1.9
2 1952-04-01       9.3
3 1952-07-01      -0.4
4 1952-10-01      21.8
5 1953-01-01       5.4
6 1953-04-01       0.8
```

A). Listing the economic downturns below:

Peak	Trough	Contraction Peak to Trough	Expansion Trough to Peak
Jul 53	May 54	10	45
Aug 57	Apr 58	8	39
Apr 60	Feb 61	10	24
Dec 69	Nov 70	11	106
Nov 73	Mar 75	16	36
Jan 80	Jul 80	6	58
Jul 81	Nov 82	16	12
Jul 90	Mar 91	8	92
Mar 01	Nov 01	8	120
Dec 07	Jun 09	18	73

```
#Economic downturn:

eco_downturn<-c(rep(NA, 5),PctChange[6:10], rep(NA, 11), PctChange[22:25], rep(NA, 7), PctChange[33:37], rep(NA, 34), PctChange[72:76], rep(NA, 11), PctChange[88:93], rep(NA, 19), PctChange[113:115], rep(NA, 3), PctChange[119:124], rep(NA, 26), PctChange[151:153], rep(NA, 39), PctChange[193:196], rep(NA, 23), PctChange[220:226], rep(NA, 46))
eco_upturn<-c(PctChange[1:5], rep(NA, 4), PctChange[11:21], rep(NA, 4), PctChange[26:32], rep(NA, 4), PctChange[38:71], rep(NA, 5), PctChange[77:87], rep(NA, 6), PctChange[94:112], rep(NA, 3), PctChange[116:118], rep(NA, 6), PctChange[125:150], rep(NA, 3), PctChange[154:192], rep(NA, 4), PctChange[197:219], rep(NA, 7), PctChange[227:272])
plot(ts(PctChange, start = c(1952, 1), frequency = 4))
```

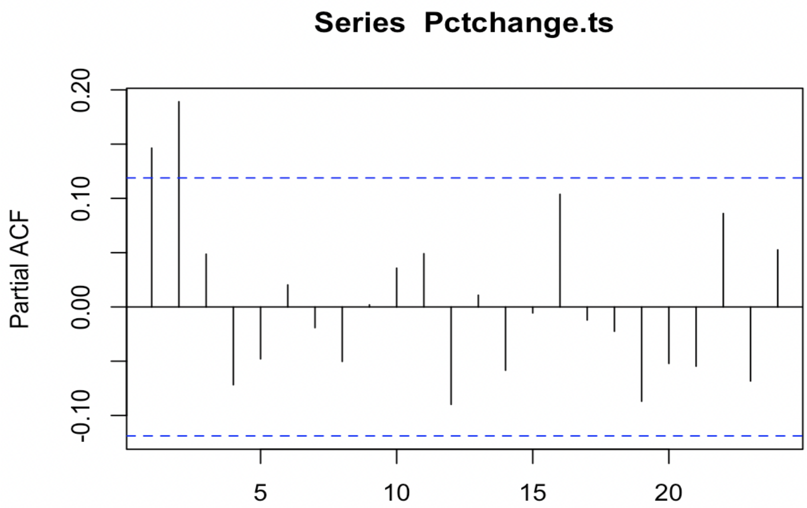
```
lines(ts(eco_downturn, start = c(1952,1), frequency = 4), col='red', lwd=2)
lines(ts(eco_upturn, start = c(1952, 1), frequency = 4), col='green', lwd=2)
```



OBSERVATION: Outliers are present in the ranges of August 1957 - April 1958; November 1973- March 1975, and January 1980 - July 1980. These periods mentioned have drastically impacted the sales of the goods in the country. We observe that the trend is modest and the trend part has not been captured well. Volatility has reduced considerably after 2010. The model has not captured seasonality that well.

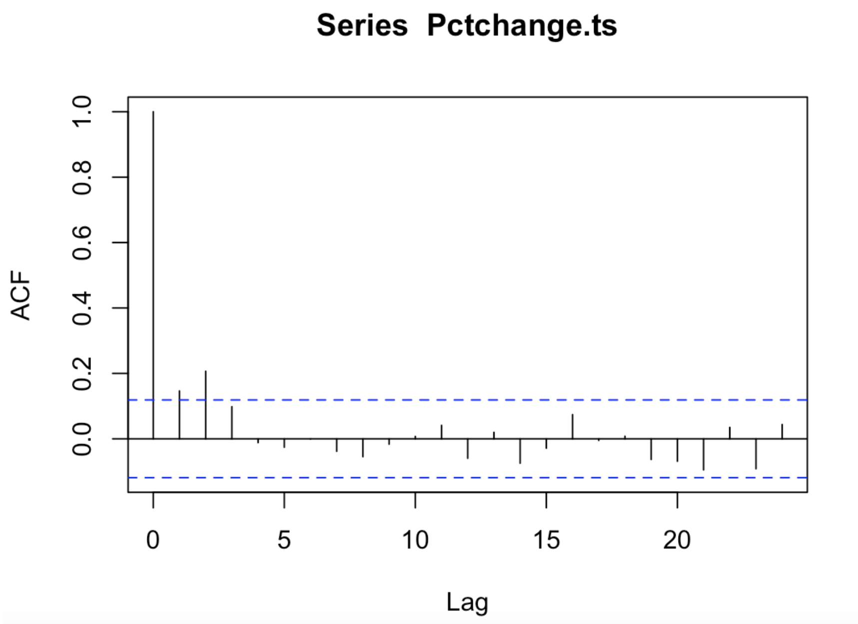
NOTE: We have gotten 4 outliers. They are observations 25, 92, 114, and 228.
So,

```
obs25<-c(rep(0,24),1,rep(0,247))
obs92<-c(rep(0,91),1,rep(0,180))
obs114<-c(rep(0,113),1,rep(0,158))
obs228<-c(rep(0,227),1,rep(0,44))

#Adding the outliers to the dataframe rgoods:
rgoods<-data.frame(obs25, obs92, obs114, obs228)
```



Pctchange.ts<-((ts(PctChange,start=c(1952,1),freq=4))[1:272])
PACF:
pacf(Pctchange.ts)

ACF:
acf(Pctchange.ts)



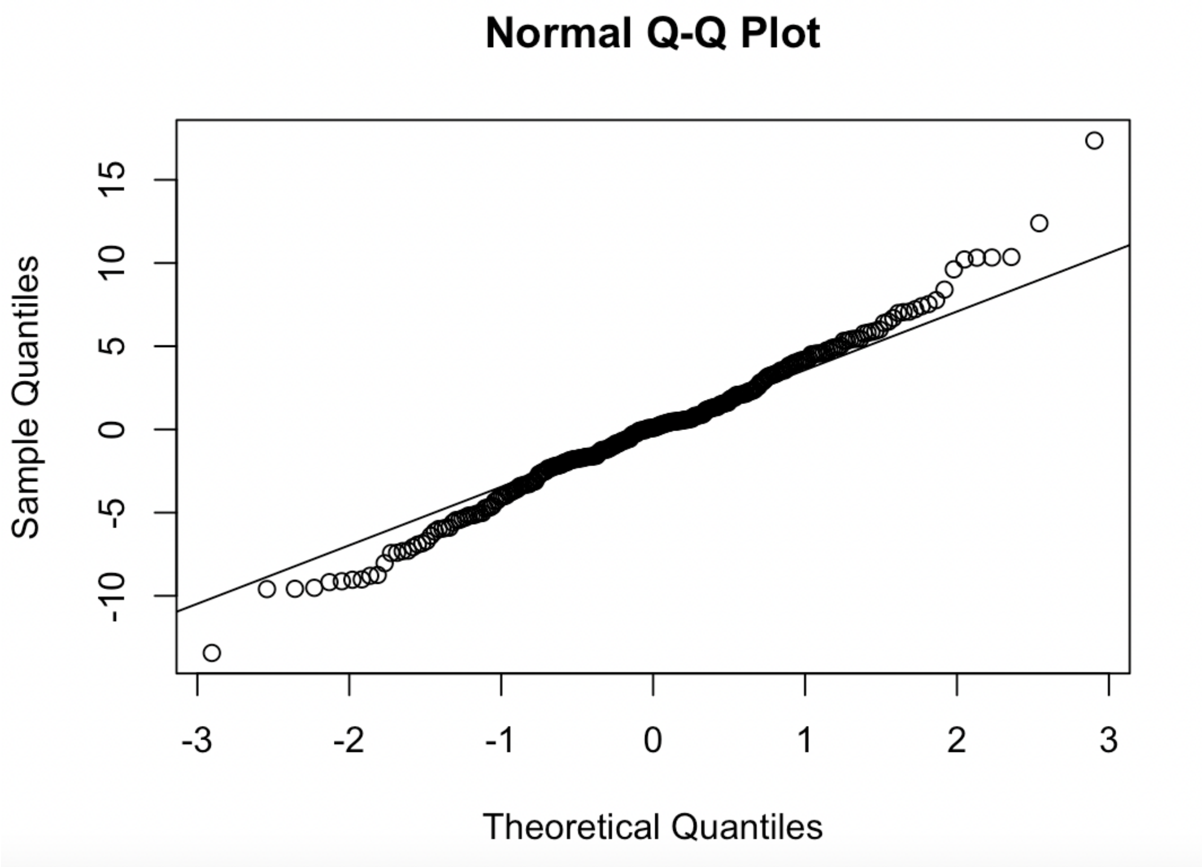
OBSERVATION: We see that almost all the spikes are insignificant here. The plot has been reduced to white noise almost. The value of p is 2 here but we will consider the value to be 4 as it is necessary to estimate two pseudo cycles.
C). arxmodel.ar2<-arima(Pctchange.ts,order=c(4,0,0),xreg=rgoods)
arxmodel.ar2

```
Call:
arima(x = Pctchange.ts, order = c(4, 0, 0), xreg = rgoods)

Coefficients:
         ar1     ar2     ar3      ar4  intercept     obs25     obs92    obs114
      0.0707  0.2393  0.0915  -0.1783     3.7086  -12.4014  -19.9490  -20.6027
s.e.  0.0607  0.0602  0.0602   0.0630     0.3385    4.1666    4.1456    4.1985
        obs228
      -12.7922
s.e.    4.1884

sigma^2 estimated as 18.66:  log likelihood = -784.07,  aic = 1588.14
```

D). Residual analysis of ARX model:

#Normal quantile plot:

qqnorm(resid(arxmodel.ar2))
qqline(resid(arxmodel.ar2))



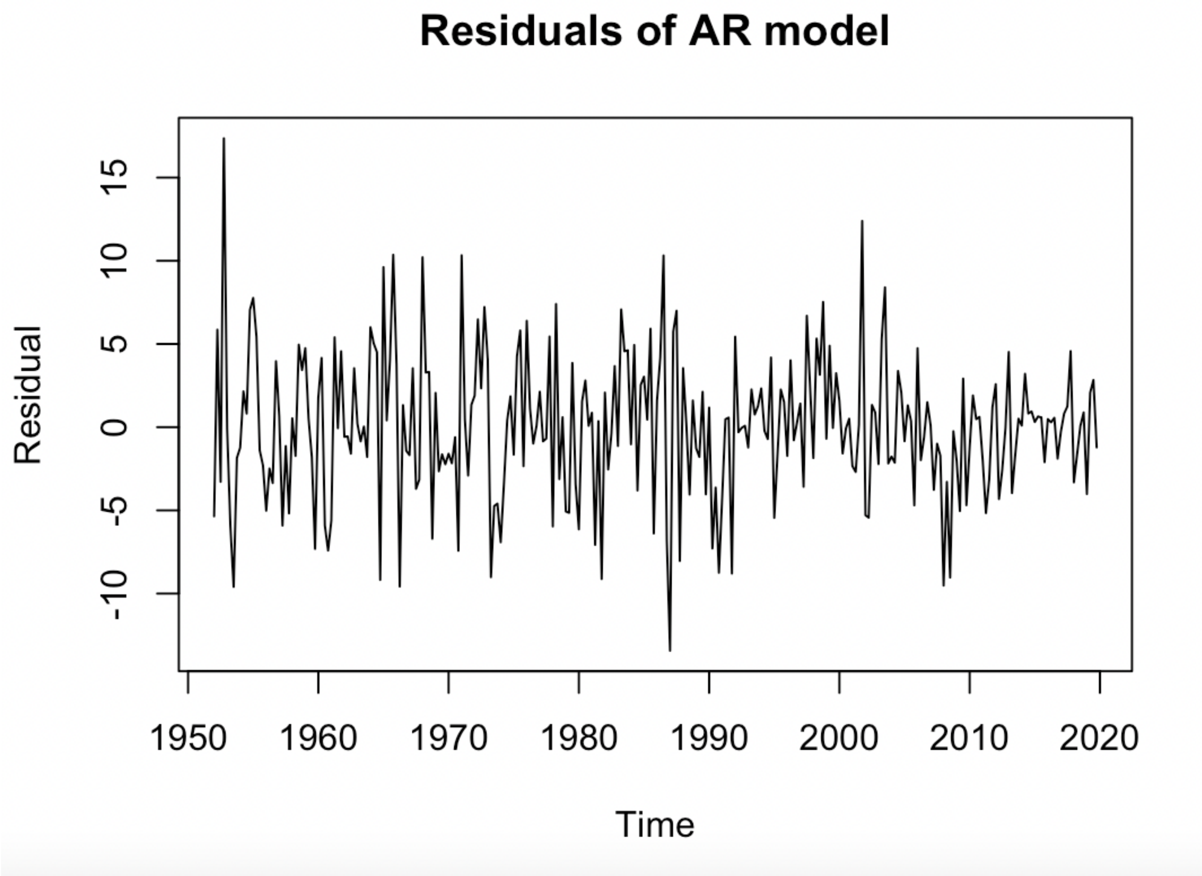
#Test for normality:
shapiro.test(resid(arxmodel.ar2))

OBSERVATION:

```
        Shapiro-Wilk normality test

data:  resid(arxmodel.ar2)
W = 0.98987, p-value = 0.05538
```

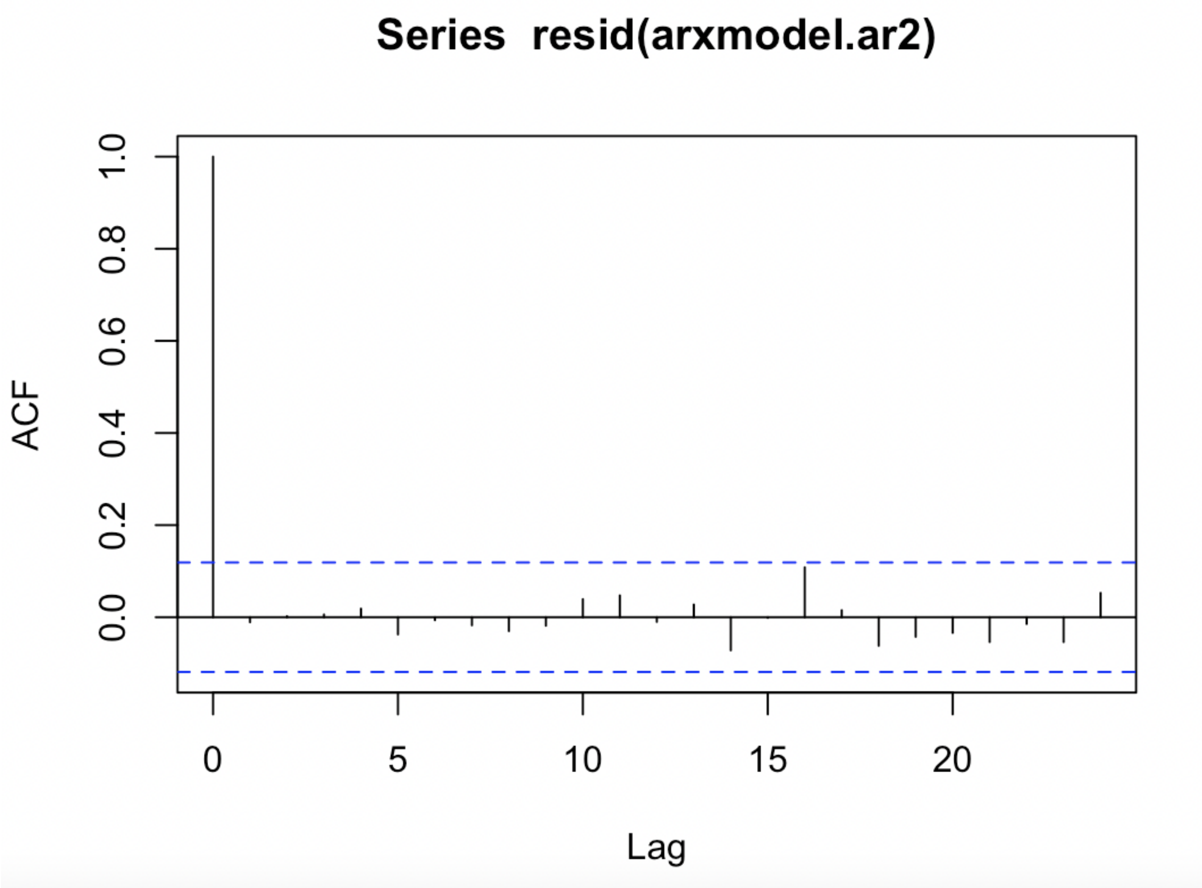
We notice that the p-value is 0.05538. It is non-significant for rejection of normality.

plot(ts(resid(arxmodel.ar2),start=c(1952,1),freq=4),xlab="Time",ylab="Residual",main="Residuals of AR model")



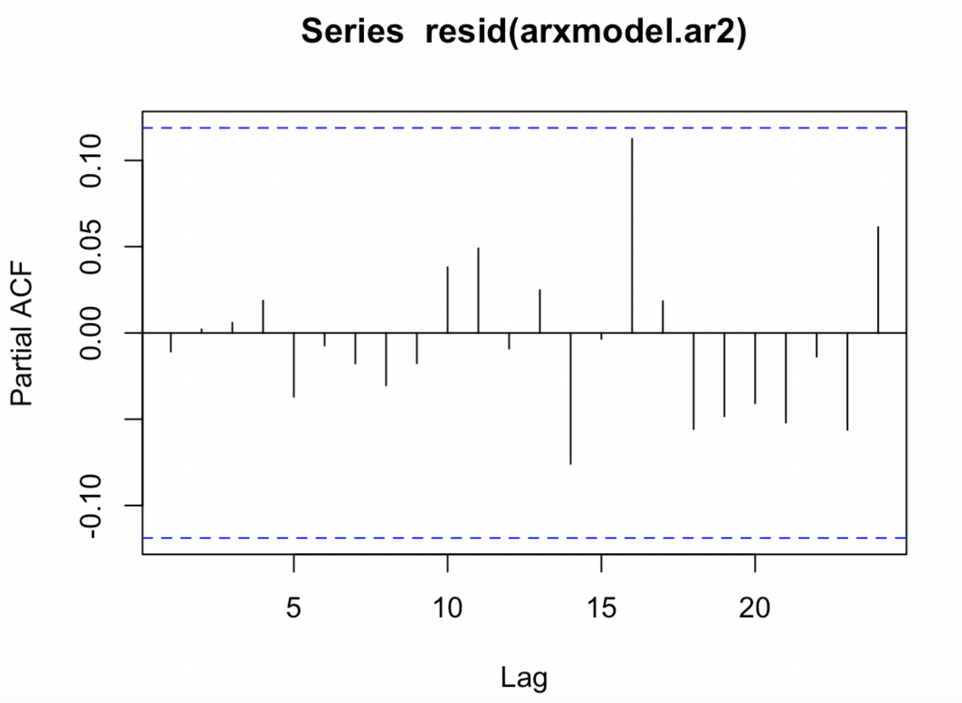
OBSERVATION: The trend part has not been captured well by our model here. We can see that the residuals are modest here.

```
#Autocorrelation of residual plot:
acf(resid(arxmodel.ar2))
```



OBSERVATION: We notice that there are no significant lags in the above plot. Our model has reduced the above residuals to white noise.

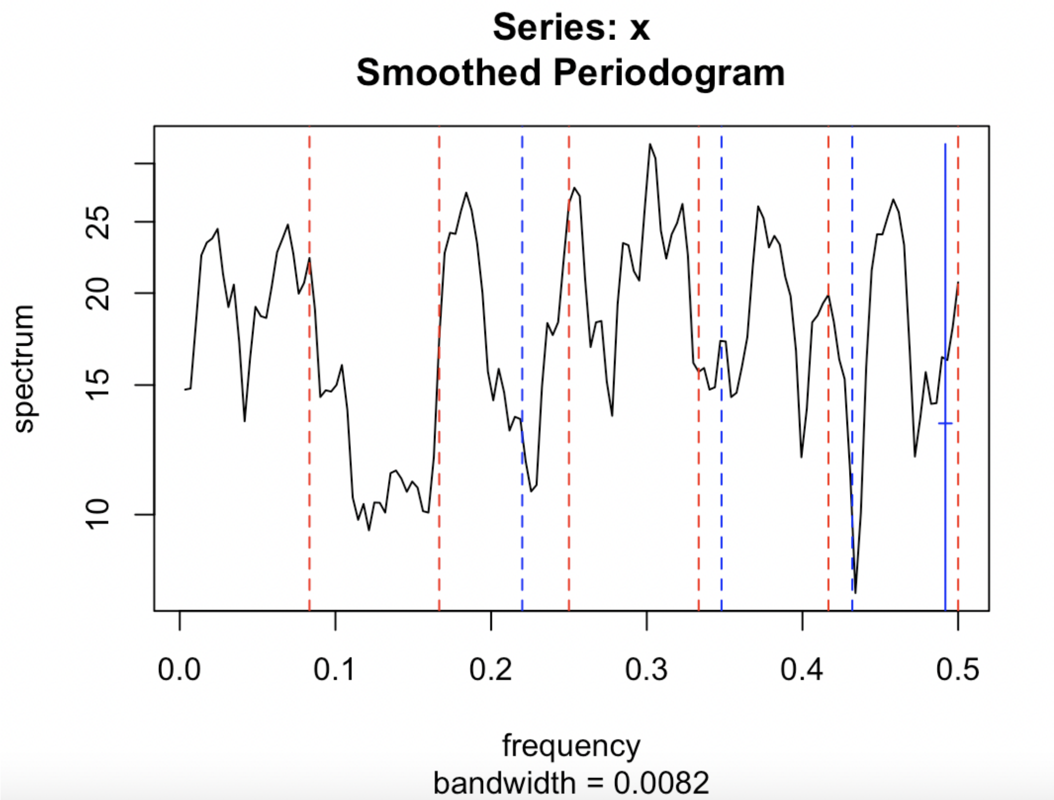
PACF:



OBSERVATION:

#Residual spectral density:

```
spectrum(resid(arxmodel.ar2),span=8)
abline(v=c(1/12,2/12,3/12,4/12,5/12,6/12),col="red",lty=2)
abline(v=c(0.220,0.348,0.432),col="blue",lty=2)
```



OBSERVATION: We observe that the spectral plot is not flat and hence white noise structure has not been conformed by the residuals here. There is activity at the frequencies of 3/12, 5/12, and 6/12 clearly and mostly even at the frequencies of 1/12 and 2/12. We can say that the residuals exhibit unmodelled seasonal structure here. Also, we notice that the frequency 0.348 is prominent in the above plot. This implies that the calendar structure has not been captured well by the residuals.
As we double the blue line above the notch, it is kind of longer than the complete vertical range of the spectral plot. Thus the residuals have been reduced almost to white noise structure above.

#Bartlett B test:

```
bartlettB.test((ts(resid(arxmodel.ar2))))
```

OBSERVATION:

```
        Bartlett B Test for white noise

data:
= 0.34284, p-value = 0.9998
```

We see that the p-value is equivalent to 0.9998 here. As the value of p is greater than 0.05, we can say that white noise structure has been assumed by our model here.

e).
coef(arxmodel.ar2)

```
> coef(arxmodel.ar2)
         ar1          ar2          ar3          ar4    intercept        obs25
  0.07073884   0.23930301   0.09151129  -0.17833248   3.70860915 -12.40141180
       obs92       obs114       obs228
-19.94899990 -20.60273872 -12.79218964
```

zeros<-1/polyroot(c(1,-coef(arxmodel.ar2)[1:4]))
zeros

```
> zeros
[1]  0.5390301-0.3213239i -0.5036607-0.4462886i -0.5036607+0.4462886i
[4]  0.5390301+0.3213239i
```

OBSERVATION: We notice that the 0.5390±0.32i and -0.5036±0.4462i are conjugate pairs. Now let us compute the amplitude and the period of these pairs.

Amplitude calculation:

Mod(zeros[3])

OBSERVATION:

```
> Mod(zeros[3])
[1] 0.6729395
```

Mod(zeros[4])

OBSERVATION:

```
> Mod(zeros[4])
[1] 0.6275368
```

Period calculation:

2*pi/Arg(zeros)[3]

OBSERVATION:

```
> 2*pi/Arg(zeros)[3]
[1] 2.600101
```

2*pi/Arg(zeros)[4]

```
> 2*pi/Arg(zeros)[4]
[1] 11.68839
```

f). We have seen that one of the pairs has a stochastic cyclic component of 11.68 quarters and the second one suggests a weaker stochastic cyclic component with a period of 2.6001. Also, we have done the AR(4) fit here and thus, we can see the shorter (and weaker) cyclical component. Thus, the features of time series are revealed by the AR(4) model. The results obtained are thus comparable to the GNP data as mentioned in our notes. The results are almost similar to those.

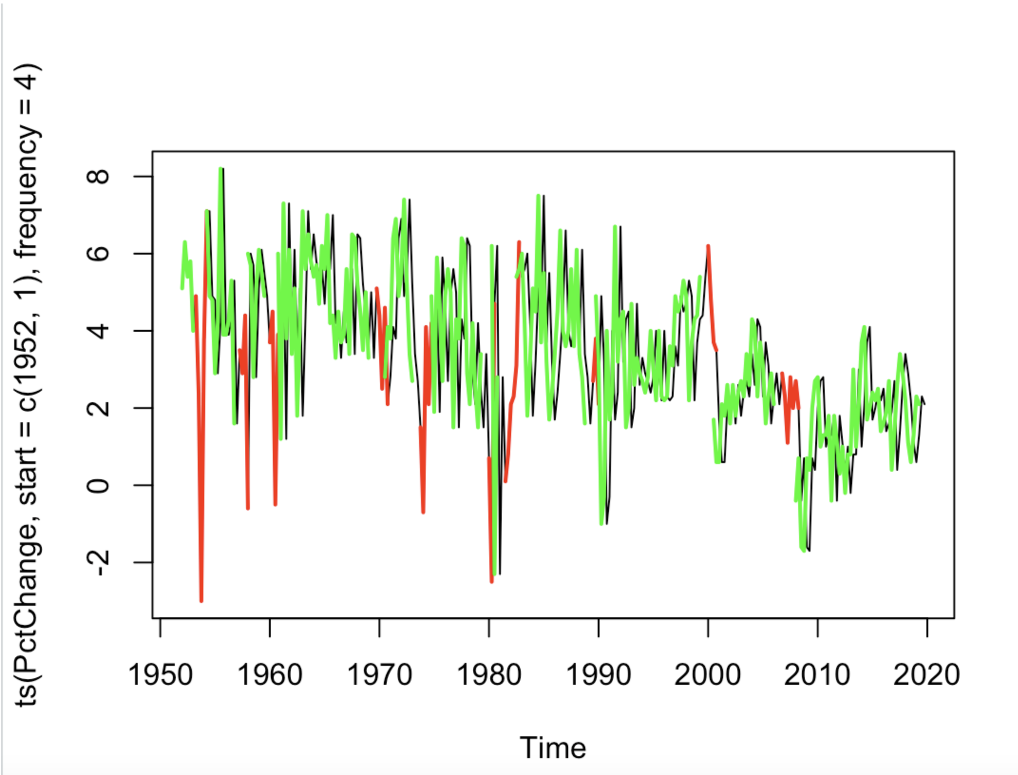
Q3.
rservices<-read.csv("/Users/renukachintalapati/Downloads/RPCEServices5219.txt")
attach(rservices)

Peak	Trough	Contraction Peak to Trough	Expansion Trough to Peak
Jul 53	May 54	10	45
Aug 57	Apr 58	8	39
Apr 60	Feb 61	10	24
Dec 69	Nov 70	11	106
Nov 73	Mar 75	16	36
Jan 80	Jul 80	6	58
Jul 81	Nov 82	16	12
Jul 90	Mar 91	8	92
Mar 01	Nov 01	8	120
Dec 07	Jun 09	18	73

B).

```
economic_downturn1<-c(rep(NA, 5),PctChange[6:10], rep(NA, 11), PctChange[22:25], rep(NA, 7), PctChange[33:37], rep(NA, 34), PctChange[72:76], rep(NA, 11), PctChange[88:93], rep(NA, 19), PctChange[113:115], rep(NA, 3), PctChange[119:124], rep(NA, 26), PctChange[151:153], rep(NA, 39), PctChange[193:196], rep(NA, 23), PctChange[220:226], rep(NA, 46))
economic_upturn1<-c(PctChange[1:5], rep(NA, 4), PctChange[11:21], rep(NA, 4), PctChange[26:32], rep(NA, 4), PctChange[38:71], rep(NA, 5), PctChange[77:87], rep(NA, 6), PctChange[94:112], rep(NA, 3), PctChange[116:118], rep(NA, 6), PctChange[125:150], rep(NA, 3), PctChange[154:192], rep(NA, 4), PctChange[197:219], rep(NA, 7), PctChange[227:272])
plot(ts(PctChange, start = c(1952, 1), frequency = 4))
lines(ts(economic_downturn1, start = c(1952,1), frequency = 4), col='red', lwd=2)
lines(ts(economic_upturn1, start = c(1952, 1), frequency = 4), col='green', lwd=2)
```

OBSERVATION:

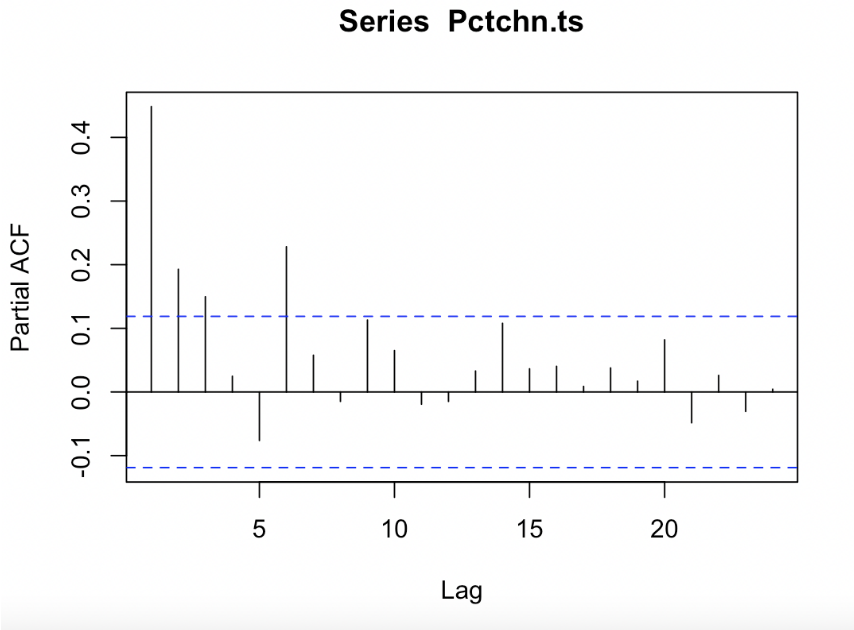


OBSERVATION: We see that the trend decreases gradually as the number of years increases. Also, we can say that the volatility reduces in a similar pattern. It is high during the initial years. Also, the economic downturns can be seen in the above plot. The volatility reduces after the year 2000.

C).
Pctchn.ts<-((ts(PctChange,start=c(1952,1),freq=4))[1:272])

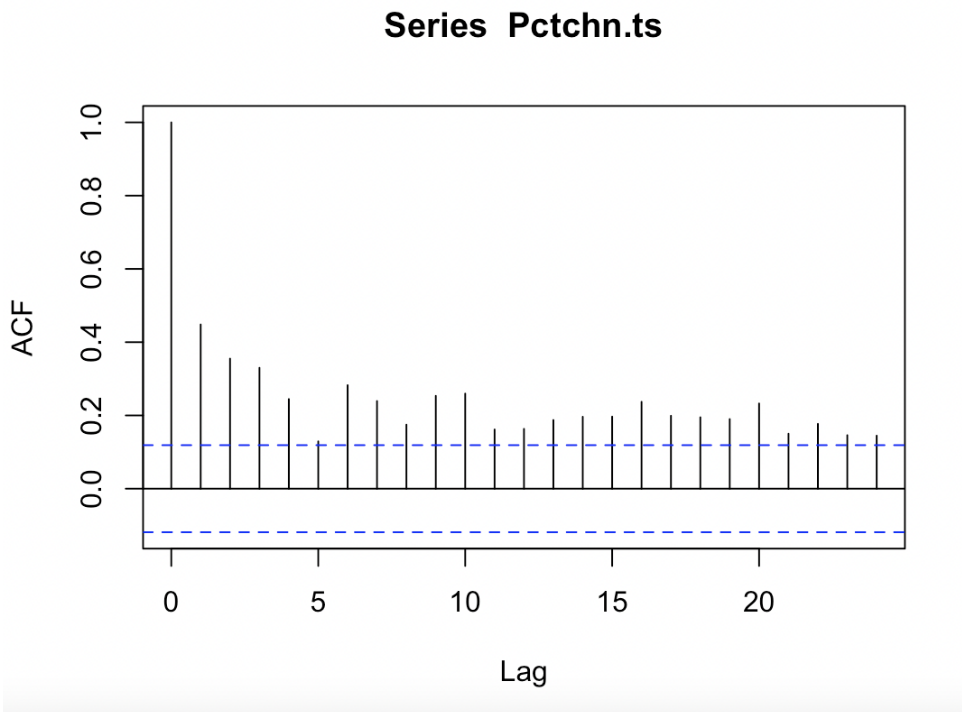
pacf(Pctchn.ts)

OBSERVATION:



ACF:
acf(Pctchn.ts)

OBSERVATION:



By noticing the PACF plot, we see that the p-value is equivalent to 6. The lags are insignificant after this value of 6. Thus, we can say that order is 6 here.

Let us now compute the outliers and create a dataframe using the same. The outliers are as follows:
8, 25, 35, 89, 114, and 117.

```
obs8<-c(rep(0,7),1,rep(0,264))
obs25<-c(rep(0,24),1,rep(0,247))
obs35<-c(rep(0,34),1,rep(0,237))
obs89<-c(rep(0,88),1,rep(0,183))
obs114<-c(rep(0,113),1,rep(0,158))
obs117<-c(rep(0,116),1,rep(0,155))
```

```
rservices<-data.frame(obs8, obs25, obs35, obs89, obs114, obs117)
```

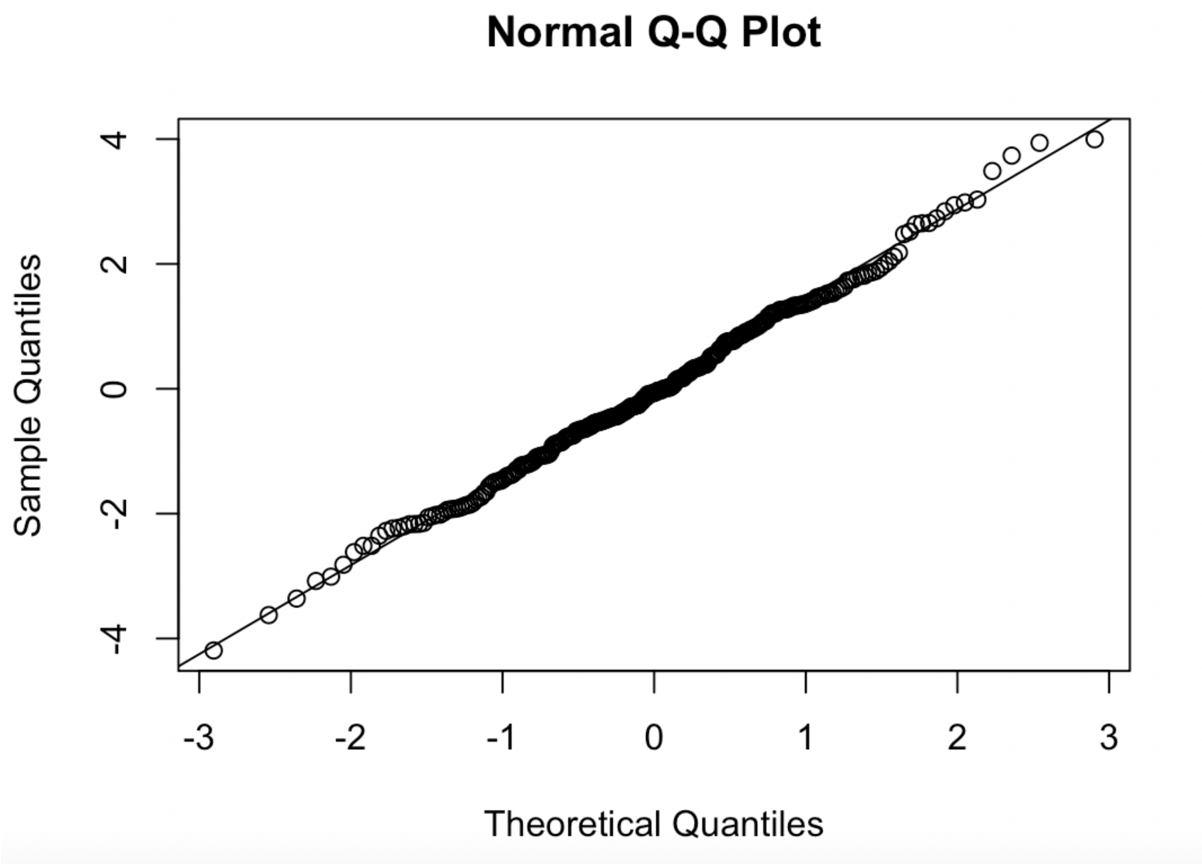
Fitting the model, we get:

```
arxmodel.ar<-arima(Pctchn.ts,order=c(6,0,0),xreg=rservices)
arxmodel.ar
```

```
> arxmodel.ar

Call:
arima(x = Pctchn.ts, order = c(6, 0, 0), xreg = rservices)

Coefficients:
         ar1     ar2     ar3      ar4      ar5     ar6  intercept     obs8
      0.3957  0.2133  0.0510  -0.0442  -0.0931  0.2480     3.4700  -7.1685
s.e.  0.0597  0.0661  0.0649   0.0657   0.0669  0.0597     0.3639   1.2855
        obs25    obs35   obs89   obs114   obs117
      -5.0861  -4.1268  -2.695  -5.6022  -5.6695
s.e.   1.2759   1.3032   1.289   1.2862   1.2706

sigma^2 estimated as 2.003:  log likelihood = -480.89,  aic = 989.78
```

Analysis of residuals:

```
#Normal quantile plot:
qqnorm(resid(arxmodel.ar))
qqline(resid(arxmodel.ar))
```



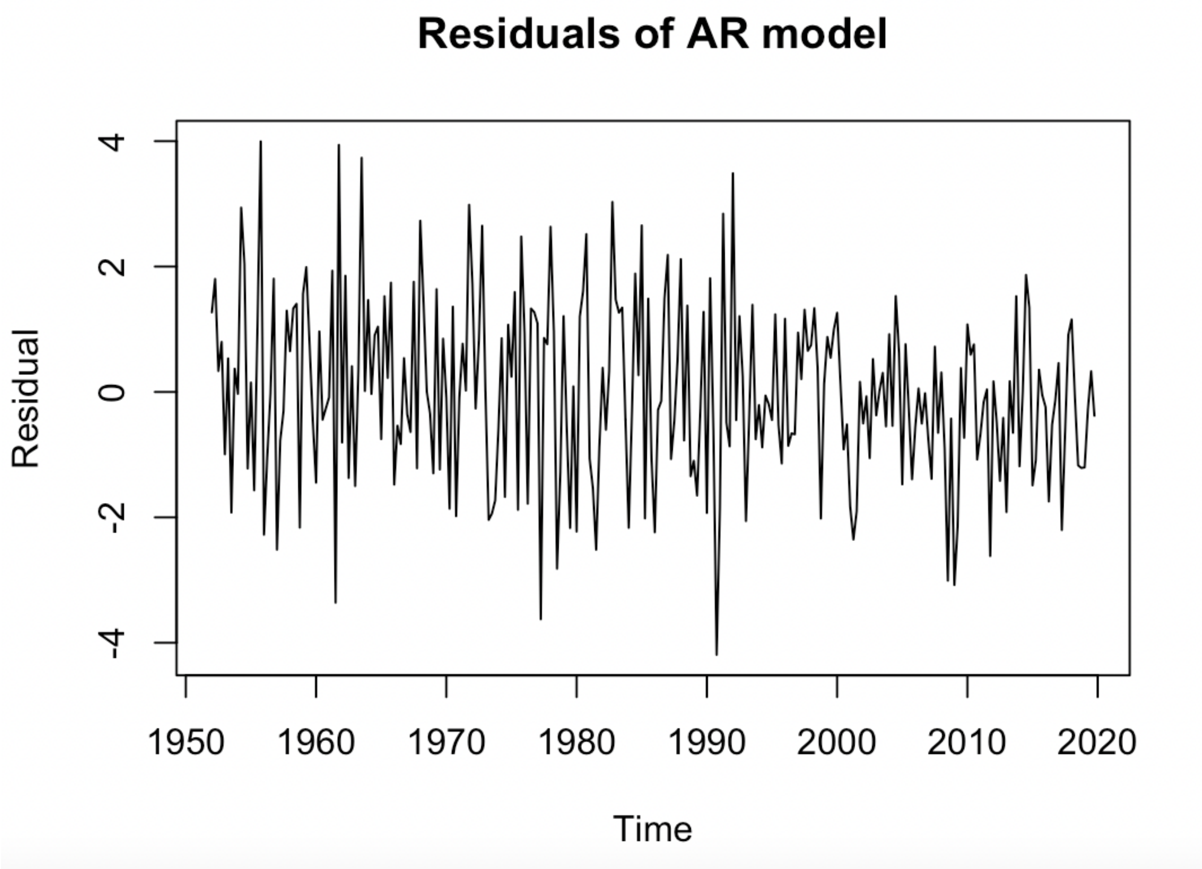
OBSERVATION: There are not many outliers above.

#Test for normality:
shapiro.test(resid(arxmodel.ar))

```
        Shapiro-Wilk normality test

data:  resid(arxmodel.ar)
W = 0.99612, p-value = 0.7421
```

OBSERVATION:
We notice that the p-value is greater than 0.05 here. So, it is non-significant for rejection of normality.

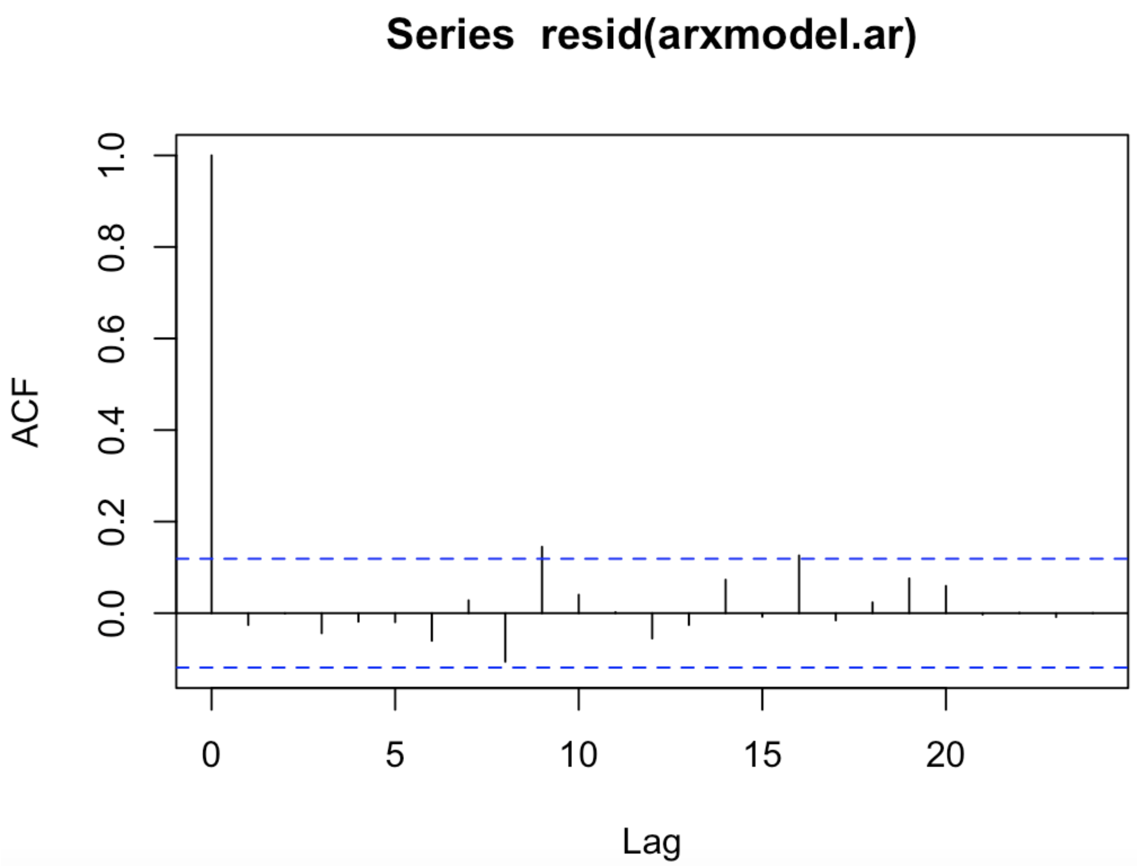
plot(ts(resid(arxmodel.ar),start=c(1952,1),freq=4),xlab="Time",ylab="Residual",main="Residuals of AR model")



OBSERVATION: We can notice that the trend component has not been captured by our model properly. It is sort of modest here.

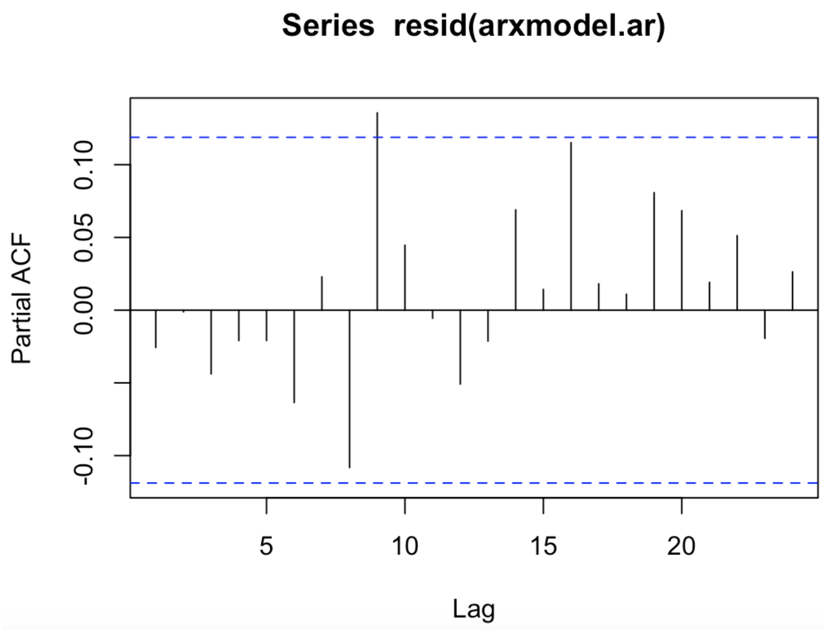
#Autocorrelation of residual plot:
acf(resid(arxmodel.ar))

OBSERVATION:



There are not many significant lags observed in the above plot. The residuals have almost been reduced to white noise here.

PACF:

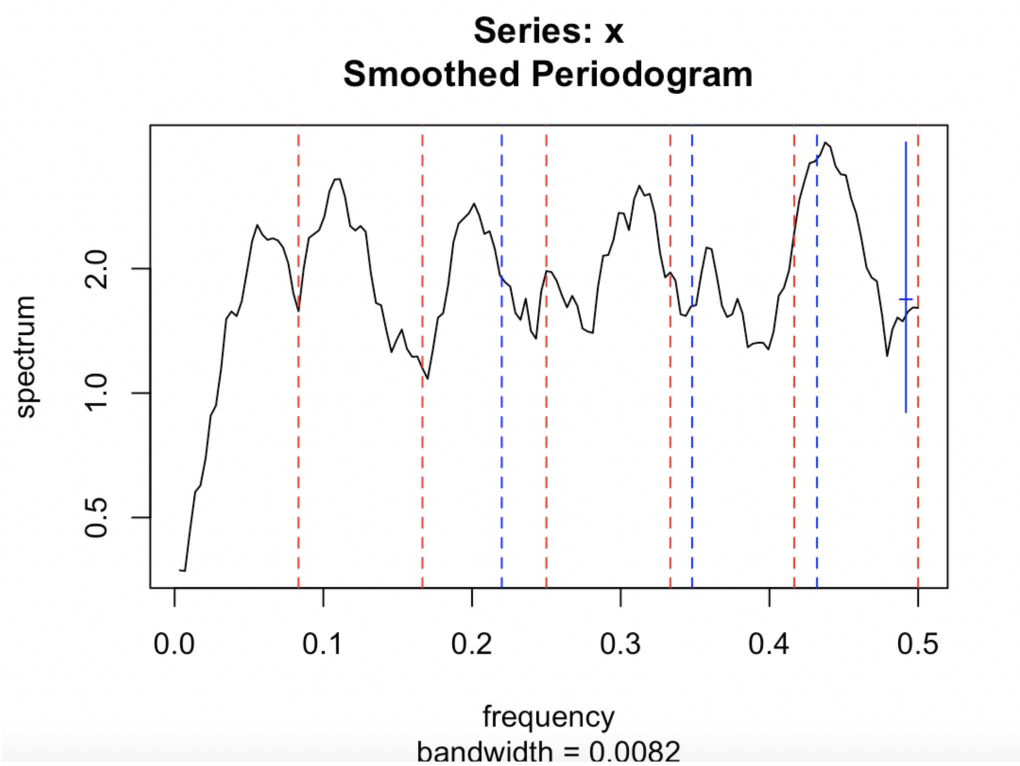


Code used for obtaining the above plot is as follows:

pacf(resid(arxmodel.ar))

#Residual spectral density:

spectrum(resid(arxmodel.ar),span=8)
abline(v=c(1/12,2/12,3/12,4/12,5/12,6/12),col="red",lty=2)
abline(v=c(0.220,0.348,0.432),col="blue",lty=2)



OBSERVATION: There is a spectral peak at low frequency indicating the slow movement of time series. There is some activity at frequencies 3/12 and probably 5/12. Seasonal structure has not been well captured by our model. Double the length of the line above the notch is greater than the entire vertical range of the plot above. Thus, it is not white noise structure. Blue line test is just a visual test for us.
The model can be reduced to white noise structure.

```
        Bartlett B Test for white noise

data:
= 0.53488, p-value = 0.9372
```

OBSERVATION: We see that the p-value is 0.9372 here. It is greater than 0.05. We would thus accept the null hypothesis here and it suggests that it has white noise structure.

e). Zeros calculation:

coef(arxmodel.ar)

```
> coef(arxmodel.ar)
        ar1         ar2         ar3         ar4         ar5         ar6 
 0.39567482  0.21330255  0.05104185 -0.04418748 -0.09314537  0.24797432 
  intercept        obs8       obs25       obs35       obs89      obs114 
 3.46997558 -7.16850755 -5.08606061 -4.12684207 -2.69502273 -5.60215013 
     obs117 
-5.66950030 
```

zeros1<-1/polyroot(c(1,-coef(arxmodel.ar)[1:6]))
zeros1

```
> zeros1
[1]  0.4986627-0.5715416i -0.3610006-0.6908813i -0.3610006+0.6908813i
[4]  0.9045399+0.0000000i -0.7841894-0.0000000i  0.4986627+0.5715416i
```

AMPLITUDE calculation:

Mod(zeros1[3])

```
> Mod(zeros1[3])
[1] 0.7795117
```

Mod(zeros1[4])

```
> Mod(zeros1[4])
[1] 0.9045399
```

Mod(zeros1[6])

```
> Mod(zeros1[6])
[1] 0.7585014
```

Period calculation:

2*pi/Arg(zeros1)[3]

```
> 2*pi/Arg(zeros1)[3]
[1] 3.061536
```

2*pi/Arg(zeros1)[4]

```
> 2*pi/Arg(zeros1)[4]
[1] 3.852132e+14
```

2*pi/Arg(zeros1)[6]

```
> 2*pi/Arg(zeros1)[6]
[1] 7.362608
```

f). We can say that one pair suggests a stochastic cyclical component and the period is 7.3628 quarters. The second pair points to a weaker cyclic component with 3.061 quarters. This is an AR(6) model fit, and we see that there are shorter(and weaker) cyclic components. It reveals the time series' features.