WALNUT CAPITAL

THE BEST IN CITY LIVING

A Robust Optimization Approach to Affordable Housing Through Maximin Decision-Making Process

GUI Manual

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1. Introduction

Walnut Capital (WC), a private residential and commercial real estate developer/manager, prides itself on providing the “Best In City Living” for Pittsburgh residents. In recent years, however, the company has faced significant constraints on any new builds. In April of 2022, the Pittsburgh City Council approved a measure unanimously, broadening existing affordable housing regulations and mandating private developers to incorporate below-market-rate units in any future projects. As per the new ordinance, developers constructing 20 or more units are obligated to allocate a minimum of 10 percent of these new units at affordable rates for lower-income buyers and renters (Britschgi, 2022). This adds considerable burden for developers like WC, as they are required to make up for the portion of rent lost by integrating affordable units.

In response, many firms have refrained from starting new construction to avoid this additional strain. To further add into the complexity, traditional analysis in affordable housing lacks applicability. In essence, the lack of specific and relevant data in the housing market, entails a more sophisticated approach to analyze relationships between decisions and outcomes. This challenge in producing quantitative estimates of our decisions makes the affordable housing initiative ever more complicated.

To mitigate this issue, our student team has championed a Robust Optimization Model aimed to allocate resources in a way that allows WC to achieve and exceed a market rate return on capital, while also abiding by local regulations. Moreover, this same model allows us to produce estimates in the form of sensitivity analysis, making sure that we are now able to measure the impact of our decisions, on a semi-real time basis.

The model interface has been created through Python, utilizing the Tkinter module to create a compatible but valuable interface for ease of use for stakeholders. This allows us to export the program as an executable file, making sure that it is downloadable and easily accessible across multiple devices.

Leveraging a Python Graphical User Interface (GUI) for our optimization model enhances accessibility and user engagement. Rather than grappling with intricate commands or inputting extensive numerical data, users can effortlessly interact with the model. Through visceral button clicks and data input in designated fields, users can readily visualize results in an intuitive manner. A GUI is comparable to employing a user-friendly smartphone app instead of having users type commands into a computer terminal.

The purpose of this manual is to discuss model logistics, inputs, assumptions, benefits, and basic technical guidance for future adjustments.

2. Model Overview

Our chosen model type is referred to as a Maximin Optimization model and functions behind the scenes of the tool's GUI. Maximin is aimed at maximizing the least favorable outcome or payoff among various scenarios or decision options. Essentially, it carefully considers various scenarios and decision options to identify the decision that ensures the most favorable result in the most adverse scenario possible. In the realm of affordable housing, this involves optimizing yearly income in light of the highest maintenance expenses incurred from all tenants within a particular year, along with unforeseen costs.

A Maximin model evaluates multiple scenarios that represent different combinations of resource allocations, market conditions, and regulatory constraints. In our case, behind the scenes of our interface, the model considers resources such as unit layout specifications (square footage by unit type) among others. Our model's "market conditions" can be considered Average Mean Income, target rental rates by unit type, housing demand,etc. Constraints include the 10% of affordable housing units, land size, and market demand by unit type. All model metrics will all be explained in detail later in this manual.

a. Objective Function

The Maximin optimization model requires an objective function to maximize the minimum possible outcome or payoff across all scenarios. In the context of Walnut Capital, this means maximizing the market rate return on capital while ensuring compliance with affordable housing regulations, even in the worst-case scenario. Our model's objective function is as follows:

$$Max\ Annual\ Market\ Rate\ Return = \sum_{i=1}^{5}(R_{M,i} * x_i + R_{A,i} * y_i) - \sum_{i=1}^{5}(Z_i * (x_i + y_i))$$

This is essentially a profit function that subtracts cost from revenue where:

Revenue = Sum of # of units for each unit type multiplied by their respective rent

Cost = Sum of # of units for each unit type multiplied by their respective maximum costs

The function identifies the highest possible market rate return on investment while reducing the overall revenue shortfall from affordable housing units. It achieves this by calculating the optimal quantity of each type of unit to construct, such as 1 Bedroom, 2 Bedroom, Affordable 1 Bedroom, and so on.

Given the nature of the maximin strategy, it would be excessively pessimistic to provide only our top estimate of the annual market rate of return in the least favorable scenario. Therefore, in our user interface, we present two calculations of the annual market rate of return. The first, labeled

"Maximum Expected Return," represents the most probable "profit" Walnut will achieve with the existing configuration. This figure is crucial as it more accurately predicts the likely outcome for that year. The "Minimum Expected Return" represents the lower bound of our projections. This figure does not aim to mirror real-world outcomes but serves as a hypothetical measure designed to enhance our layout's resilience in challenging situations. Thus, in such scenarios, the decrease in profit would be minimized with our current design, considering this factor during the planning phase. Furthermore, adopting this method does not affect the practicality or validity of our model, as the optimal configuration remains relatively unchanged whether or not the maximin approach is applied.

The variables considered are defined below.

b. Decision Variables

$$Let\ k = \{1, \ldots, n\}\ to\ denote\ 'Unit\ Types'$$
$$x_i = Number\ of\ Market - Rate\ Units\ for\ each\ Unit\ Type\ i, where\ i \epsilon k$$
$$y_i = Number\ of Affordable\ Units\ (< 50\%\ AMI)\ for\ each\ Unit\ Type\ i, where\ i \epsilon k$$

A decision variable is a variable whose value can be adjusted or determined by the optimization process to achieve the desired outcome. Decision variables represent the choices or decisions that the model seeks to optimize. This model identifies decision variables that represent the allocation of resources, such as the number and types of units to be developed. These variables are subject to constraints imposed by the new regulations, budget limitations, and other operational considerations, which will be further discussed in latter sections of this manual.

Our decision variables fundamentally concern the number of units to construct for both market-rate and affordable-rate categories.

c. *Parameters*

A_i : $Area\ (sq.ft.)\ per\ unit\ of\ type\ i$
N : $Gross\ Construction\ Area\ (sq.ft.)$
L : $Net\ Residential\ Area\ (sq.ft.)$
$R_{M,i}$: $Market - Rate\ Rental\ Income\ for\ each\ Unit\ Type\ i$
$R_{A,i}$: $Affordable -\ Rate\ Rental\ Income\ for\ each\ Unit\ Type\ i$
$where\ R_{A,i} = (30\% * AMI) - Au$
$D_{max,i}$: $Target\ Demand\ Ratio\ of\ Units$ for each Unit Type i
$D_{min,i}$: $Target\ Demand\ Ratio\ of\ Units$ for each Unit Type i
S_i : $Maintenance\ costs\ per\ unit\ of\ type\ i\ according\ to\ the\ 'Square\ Foot\ Rule'$
Q_i : $Maintenance\ costs\ per\ unit\ of\ type\ i\ according\ to\ the\ '50\%\ Rule'$
$Z_i = Max\{S_i, Q_i\}$: $Represents\ the\ maximum\ maintenance\ cost\ for\ unit\ type\ i$
$c = Affordable\ Housing\ Set - aside\ Requirement\ (\%)$
p = AMI (%) Limit for Affordable Housing
$AMI = 100\%\ Area\ Median\ Income\ (AMI)\ Table$
$Au = Average\ Monthly\ Utility\ Expenses$

A parameter is a constant value that influences the behavior of the optimization model. Parameters are fixed values that are known before the optimization process begins and remain unchanged throughout the optimization. Some of these parameters are immediately accessible, while others need straightforward transformations or mathematical operations, all of which we've incorporated into the backend logic of our model. These, of course, can be changed to reflect the conditions of different properties, but remain stagnant for a singular project's optimization process. They will be entered into the interface each time so that the model may optimize according to inputs for a specific project.

Though many of our model's parameters are self-explanatory, it is important to further define the more nuanced considerations. In particular, the "Square Foot Rule" (variable S_i) is a tool that can be used to estimate a property's annual cost of maintenance. This rule simply assumes $1 per square foot for yearly maintenance costs. For example, in the case of Bakery Living, the property consumes around 300,000 square feet. This rule would allocate an annual maintenance cost of $300,000 for that property. Similarly, the "50% rule" (variable Q_i) recommends allocating half of a property's rental income for repairs, maintenance, taxes, and insurance. Subtracting tax and insurance costs allows one to quickly determine what remaining amount to attribute to maintenance costs. Variable Z_i simply selects the maximum cost among the two calculated estimates for maintenance cost, thus allowing the model to optimize while considering the "worst case scenario." Finally, variable *c* allows users to change the required percentage of affordable housing requirements according to the most up-to-date laws.

d. Constraints

$\sum_{i=1}^{n} A_i * (x_i + y_i) \leq L$ *(Land Size Constraint)*
$y_i \geq c(y_i + x_i), \forall i \epsilon k$ *(Affordable Housing Constraint)*
$x_i + y_i \geq D_{min,i}$ *(Minimum Demand Ratio Constraint)*
$x_i + y_i \leq D_{max,i}$ *(Maximum Demand Ratio Constraint)*
$\sum_{i=1}^{n} A_i * (x_i + y_i)/L \geq 0.9$ *(Minimum Utilization Constraint)*
x_i, y_i *integer for all* $i \in k$ *(Integer Constraint)*
$x_i, y_i \geq 0$ *(Non-negativity Constraint)*

Constraints ensure that the resource allocation decisions comply with regulatory requirements, budget limits, and other operational boundaries. For example, our constraints certify that model outputs do not exceed the total amount of land available and that *c*% of units abide by affordable housing rules. Additionally, the latter constraints guard against the model's intuition to suggest building 0 affordable units (to save money) or building only the most profitable unit type. This ensures units are assigned proportionately to demand by unit type, and prevent negative outputs. This is accomplished by establishing minimum and maximum limits for demand, based on the historical demand for Walnut Capital's properties. Additionally, we impose a minimum utilization requirement to guarantee that our blueprint does not result in a significantly low rate of land use.

e. Model Benefits

Using a Maximin optimization model is the preferred approach for WC in this scenario because it allows the company to ensure the best outcome even in the face of poor or unexpected conditions. The model can help identify the best allocation of resources that maximizes returns under not only the constraints imposed by the housing mandates, but also considering other restrictions such as land costs, demand by unit type, and more. By focusing on maximizing the least favorable outcome, WC can ensure that its investments are robust against potential losses or impacts resulting from the integration of affordable units. In essence, the Maximin optimization model enables WC to navigate the challenges posed by the new regulations while still striving to provide the "Best In City Living" for Pittsburgh residents and maintaining profitability in its real estate development projects.

Furthermore, as mentioned earlier, our model enables us to predict how adjustments to our decision variables influence the maximum profit. It also provides insights into the effects of relaxing our constraints.

3. Model Inputs

Beyond simple variable definitions, it is important to consider why our model inputs are relevant to WC analysis and the problem at hand.

a. *Unit Types by Demand ($D_{max,i}$, $D_{min,i}$):*

Understanding the mix of unit types (e.g studios, one-bedrooms) and their respective demand is pivotal for modeling because each type warrants different levels of demand. This directly influences the project's ability to attract a diverse tenant base and maximize occupancy rates. We care about this because it affects both the revenue potential and the community profile of the development.

b. *Square Footage (A_i, N, L):*

Distinct characteristics of each unit type like square footage determine the value proposition and, hence, ensure that pricing strategies are finely tuned to what a unit or building offers. For instance, a unit with greater square footage can command higher rent, impacting the overall profitability of the development.

c. *Affordable Housing Criteria (c, AMI, Au):*

This input ensures compliance with legal obligations and reflects the company's social responsibility efforts. Given the changing regulations, it's essential to dynamically adjust the proportion of affordable units. Additionally, as the median income levels in an area change, AMI figures must be updated to ensure the affordability criteria used in the model are current. Utility expenses are encompassed within the mandates governing rent prices for affordable housing units, thereby exerting influence on pricing determinations and unit allocations. These dynamic entries ensure that the affordable housing units are priced correctly relative to the area's economic conditions.This is a matter of legal compliance, ethical practice, and social reputation, which can affect Walnut Capital’s brand and their longevity as they look to continue expanding.

d. *Maintenance Costs (S_i, Q_i, Z_i):*

These inputs are the computational outcomes based on the primary model inputs. They are essential for a realistic simulation of the development's performance under different conditions. Consideration of rental income and maintenance cost by unit allows our model to more accurately estimate financial returns and profitability of a new building.

e. Rents ($R_{M,i}$, $R_{A,i}$):

Similar to affordable housing metrics, market-based rents must be current within the model. As economic factors shift, so too does the ability to command certain rental prices. Updated rent metrics allow the model to adapt to market conditions, ensuring financial forecasts are accurate and strategic pricing decisions are informed.

4. Decision Points

The decision points, or decisions made by the model, namely the number of market-rate units by type (for *i* types) and number of affordable units by type (for *i* types), are more obviously relevant to WC. These outputs will allow the firm to optimally allocate resources, abide by local regulations, and respect budget limitations. They also can inform decisions regarding construction timelines, investments in amenities, and other operational considerations such as sustainability efforts.

Overall, these decision points matter because they directly impact the feasibility, compliance, and success of development projects. Strategic decisions made at these points can significantly influence Walnut Capital's ability to achieve its financial objectives while fulfilling its requirement to provide affordable housing.

5. Model Outputs

In addition to the optimal number of units to produce for each unit type, our model also provides optimized values that are calculated according to the recommended number of the units.

a. Minimum and Maximum Expected Returns:

The "Minimum Expected Returns" represent a highly conservative and somewhat unrealistic "profit" prediction, based on the assumption that every unit incurs the highest possible maintenance costs within a year. This scenario is highly improbable and represents a cautious method of projecting our investment returns. Nevertheless, this approach is critical for creating a resilient layout that protects our income in the unlikely event that such a scenario occurs.

On the other hand, the "Maximum Expected Return" provides a pragmatic and probable estimate of our profit. In this calculation, we adopt a conventional profit estimation method common in the industry, reflecting what we anticipate earning from our design. This figure is the result of optimizing across potentially millions of layout possibilities arising from our initial problem setup. Therefore, we can be assured that, within the bounds of affordable housing and demand constraints, we're not just reducing our income deficit but also reinvesting efficiency gains to enhance our layout and revenue wherever feasible. This level of optimization is unattainable without such advanced techniques, ensuring a swift and superior solution far surpassing the outcomes of traditional, non-data-driven decision-making processes.

b. *Land Utilization Rate:*

This output is crucial as it provides insights into our spatial efficiency. Given that the net residential area encompasses the total area available for construction within a specific building, understanding the extent of space utilization versus space wastage is essential. A utilization percentage nearing 100% indicates optimal use of every buildable square foot, thereby maximizing profit per square foot.

c. *Total Number of Units:*

The total number of units serves as a straightforward measure for swiftly contrasting our optimized outcomes with initial projections or past choices. An increase in the "Total Number of Units" suggests an enhancement in our decision-making processes. Yet, relying solely on this metric could be deceptive, as achieving a significantly higher count of units might be possible through the construction of studio apartments rather than 1 Bedrooms and 2 Bedrooms, which could breach many of our established constraints.

The model also provides a sensitivity analysis, comprising shadow prices and reduced costs, and a precision value for model variables. A sensitivity analysis assesses the influence of changes in input factors on a model's outcomes.

More specifically, the shadow price reveals the extent to which the objective function's value would rise or fall in response to a slight adjustment in one of the constraints, with all other constraints held constant. In the case of our model, the shadow price associated with a particular constraint indicates the increase or decrease in annual market rate return for each additional unit of that constraint.

Reduced cost, on the other hand, represents the amount by which the coefficient of a decision variable in the objective function would need to change in order for that decision variable to enter the optimal solution. Essentially, they reflect the potential improvement in the objective function's value by including or excluding decision variables.

Precision in this context refers to the level of confidence we have in the validity of our sensitivity analysis results. A precision rate approaching 100% indicates that our sensitivity calculations are increasingly accurate, which is crucial given that these figures are estimations and inherently possess a small margin of error. Therefore, a precision rate nearing 100% suggests that our margin of error is almost negligible.

d. LP Metrics

A section of our model includes a range of supplementary metrics derived from the idealized, yet unachievable outcomes discussed in the preceding section. This is essentially our "hidden output," that shapes the underlying logic behind our sensitivity analysis. It's crucial to recognize that this doesn't represent the actual output of our model but serves merely as a benchmark for calculating our precision. The figures produced in this section will invariably surpass those of our model's real solution, attributable once again to the allowance for fractional units in the calculation.

e. More on Sensitivity Analysis, Shadow Prices, and Reduced Costs:

By understanding how the market rate return is influenced by adjustments in constraints or decision variables, WC can make informed decisions about resource allocation. For instance, a higher shadow price associated with a particular constraint suggests that additional investment in that resource could potentially lead to a greater increase in market rate return. Similarly, reduced costs provide insights into which resources or activities offer the most significant potential for improving market rate return. For instance, decision variables with negative reduced costs in a maximization problem indicate that a one unit increase will reduce objective function (“profit” or “expected returns”). Conversely, for a minimization problem, negative reduced costs suggest that increasing the variable's value from zero would increase the objective function value. Here, we are dealing with a maximization problem.

When dealing with reduced cost, a reduced cost of zero means that all the decision variables (the number of units for each unit type) have non-zero values. Moreover, a zero reduced cost for every variable aligns with our constraints enforcing that every decision variable must actively contribute to the solution. This means every decision variable's current level is exactly right for maximizing our objective function under the given constraints. Meaning, they are all part of the solution. This makes sense since we have constraints making sure that we will at least produce several market rate units for each unit type and also allocate at least 10% of those market rate units to the affordable housing rate. Hence, for most cases, we will see a zero in our reduced cost output.

This number gets interesting when we see non zero values. In the case where reduced costs contain values that are not zero, that means we will be able to change the objective function ("profit" or "expected return") by the value of the reduced cost for every unit of increase or decrease in that variable. For example if the reduced costs for 1 Bedroom is 11,000, hence, an additional unit of 1Bedroom would increase our expected return by $11000. If the value of the reduced cost is negative, an increase of one unit will lower our expected return by the specified amount in the reduced costs. For example, if the reduced costs for 1 Bedroom is -11,000, hence, an additional unit of 1Bedroom would decrease our expected return by $11,000.

In the realm of linear programming (LP), shadow prices, or dual values, provide critical insight into how the value of the objective function—whether it's "profit" or expected returns—alters with a slight increase in the right-hand side (RHS) values of constraints. These figures are pivotal in sensitivity or post-optimality analysis, offering a gauge on the potential change in the optimal solution's value (the objective function's value) for every unit increase in a resource's availability tied to a constraint. Taking the 'Land Size Constraint' as an example, a positive shadow price indicates that adding one square foot to our Net Residential Area would roughly boost profit by the shadow price's value. For instance, if the shadow price for our Land Size Constraint is $13, it implies that each additional square foot of Net Residential Area could elevate annual profit by $13. Similarly, for the 'Maximum Units Constraint' concerning 1-bedroom apartments, a positive shadow price of 2265.9 suggests that increasing the maximum units of 1 Bedroom apartment by one could potentially raise yearly profit by about $2265.9. Conversely, a negative shadow price is anticipated for all our affordable rate units, indicating that increasing the minimum required number of affordable units would decrease our expected returns by the specified shadow price amount. This outcome is logical, given that affordable units represent our least profitable unit category. A zero shadow price reflects a situation where the constraint in question does not limit the optimal solution of the linear programming problem. This insight can guide decision-making, especially in resource allocation and understanding the constraints' roles in achieving the optimal outcome.

f. Precision:

This value simply aids decision-makers in gauging a model's reliability. Precision percentage values assess the overall quality and effectiveness of the optimization model. It gives us an indication of the error rate in our sensitivity analysis. It is important to hold our model accountable and ensure it is performing adequately.

So, does the presence of an error rate detract from the value of our analysis? The straightforward answer is no. This is because the sensitivity analysis is based on the premise that it's possible to achieve an optimal layout that includes fractional units, such as suggesting that we could construct 182.3 units of a 1 Bedroom apartment. Such precision is, of course, impractical in real-life scenarios. Therefore, what we're essentially doing is comparing our tangible, applicable outcomes against an idealized, though unattainable, benchmark. This approach might seem odd, but it's a necessary aspect of employing linear programming for sensitivity analysis and is, in fact, a common industry practice. Furthermore, we can trust that our practical solution (which excludes fractional units) is indeed the best possible outcome, as guaranteed by our algorithm's design.

Moreover, a low precision, will help us decide whether or not we want to go about with the estimates found in the sensitivity analysis. This is important as sensitivity analysis is one of the main outputs of our model.

6. Optimization Insights

Optimization insights in the context of the given model pertain to the evaluation of the model's effectiveness and efficiency in allocating resources under the given constraints. The before and after analysis specifically looks at how the optimization model changes the decision-making landscape and improves outcomes.

a. Before Optimization

Before the application of the Maximin optimization model, decisions would be based on less structured methods, such as intuition, conventional wisdom, or simple financial calculations that do not fully account for the complex interplay of market conditions, regulatory requirements, and financial goals. In this stage, the potential for suboptimal decision-making is higher due to:

<u>Incomplete Market and Regulatory Analysis:</u> Decisions are made without a thorough examination of market dynamics and regulatory influences, potentially overlooking crucial factors.

Neglect of Worst-Case Scenarios: Failure to consider worst-case scenarios results in riskier investment approaches, leaving projects vulnerable to unforeseen challenges.

Limited Exploration of Investment Strategies: The full spectrum of investment and development strategies remains unexplored, limiting opportunities for innovation.

b. *After Optimization*

After implementing the optimization model, decision-making is enhanced by a systematic, data-driven approach. The model uses mathematical and computational techniques to explore a wide array of scenarios, identifying the strategy that offers the best minimum outcome. Key improvements include:

Comprehensive Scenario Analysis: The model conducts detailed scenario analyses, providing insights into the impact of different decisions across a range of conditions.

Strategic Resource Allocation: By strategically allocating resources, the model ensures adherence to affordable housing mandates while maximizing profitability.

Robust Risk Mitigation: The model identifies strategies that mitigate risk effectively, even in challenging market conditions, safeguarding against adverse outcomes.

c. *Insights Gained*

The transition from before to after optimization provides several critical insights:

Improved Resource Allocation: The model provides clarity on how to allocate resources effectively, balancing between different types of housing units, amenities, and other project components to meet both financial and regulatory objectives.

Risk Management: By considering worst-case scenarios, the model helps Walnut Capital prepare for and mitigate potential adverse market conditions, reducing the likelihood of financial underperformance.

Regulatory Compliance: The insights ensure that all development projects comply with affordable housing regulations, integrating these requirements seamlessly into the broader financial and operational strategy.

Market Adaptability: The model's outputs can be updated with new market data and regulatory information, allowing for dynamic adjustment of strategies in response to changing conditions

7. Accessing Application

a. For Windows:

i. *Locating the Application:*

1. Extract the contents of **WalnutCapitalOptiModel.zip.**
2. Open the extracted folder and find the **dist** folder.
3. Look for **Walnut_Finalexe.exe** inside the **dist** folder.
4. Double-click on **Walnut_Finalexe.exe** to run it.

ii. *Creating a Desktop Shortcut:*

To make it easier for you to access the application, you can create a shortcut to the application in your Desktop. To do so:

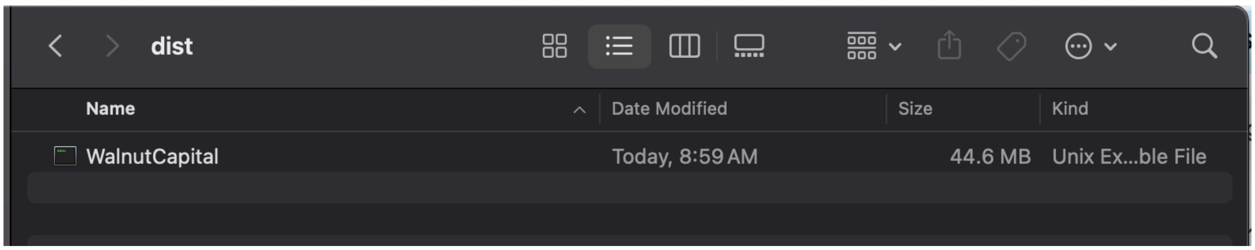
1. Find the **Walnut_Finalexe.exe** file as described earlier.
2. Right-click on **Walnut_Finalexe.exe.**
3. Hover over **Send To** in the menu that appears.
4. Click on **Desktop (create shortcut)** in the submenu.

And that's it! You'll have a shortcut to the .exe file on your desktop.

b. <u>For Mac: (Newer Versions: Non-Intel processors):</u>

i. *Locating the Application:*

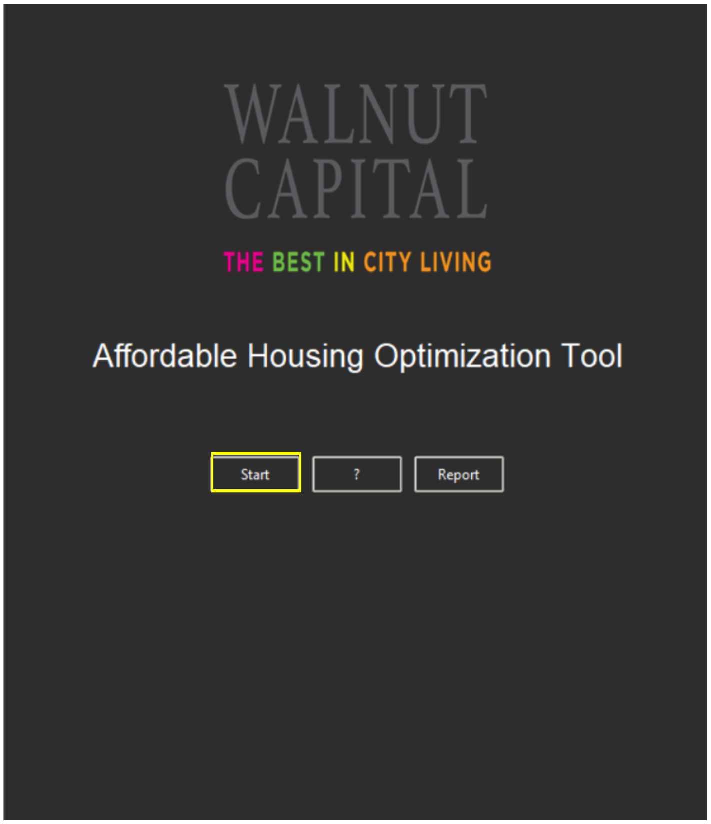
1. Extract the contents of **WalnutCapitalOptiModel_Mac.zip.**
2. Open the extracted folder and find the **dist** folder.
3. Look for **WalnutCapital.exe** inside the **dist** folder.



4. Double-click to run it. *Note: This may take a few seconds*.

8. Interface Tour

a. Home Page:

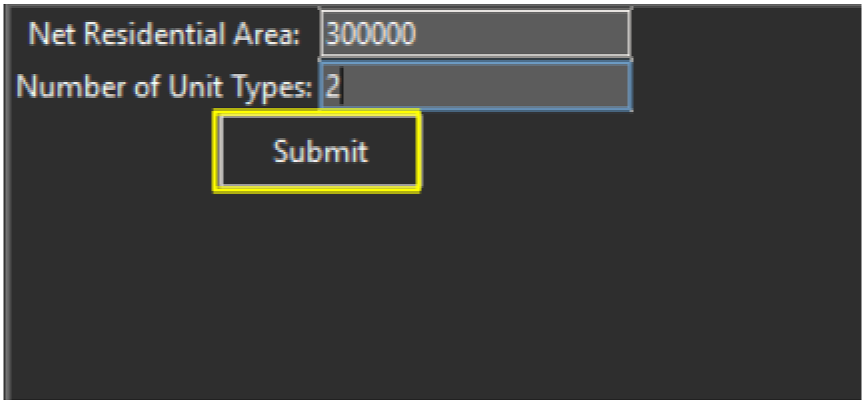


Report Button: Contains useful contact information for reporting major required changes in the model logic or interface. This report button should only be used when an answer cannot be found in other sections of the manual.

"?" Button: Clicking this button will bring you to this exact manual. This manual contains all the specifications needed to run the model successfully and interpret the results accurately.

Start Button: Clicking this button will take you to our main program.

b. Main Program:

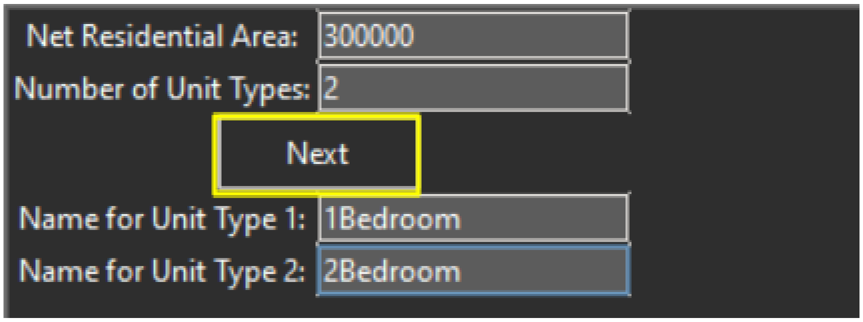


The very first set of inputs are the "Net Residential Area" and "Number of Unit Types." This is the main decision point as it determines the overall constraints and capacity of the model. It is important to only include the buildable area (excluding hallways, parking garages, etc.). This area represents the dedicated square footage for all the different unit types. No commas are needed. In other words, only insert values such as 300000 instead of 300,000.

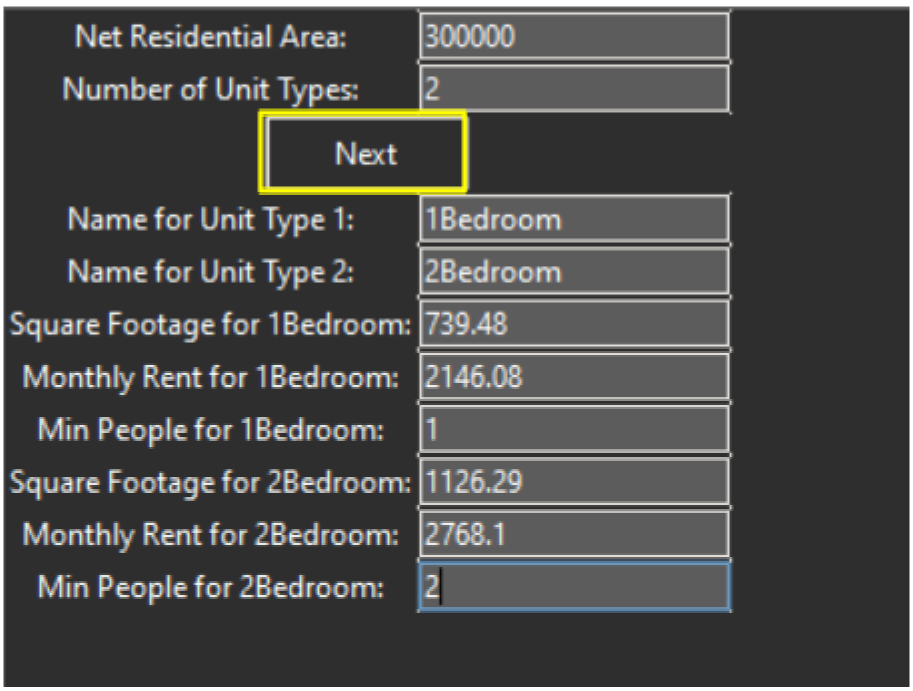
Acceptable Inputs	**Unacceptable Inputs**
‘300000’	‘300,000’
‘300000.45’	‘300000 sq. ft.’

The 'Number of Unit Types' **must be an integer**. This determines the number of unit types we are interested in offering.

Clicking the "Next" button will prompt us to name the unit types. Since in our example we entered "2" for the "Number of Unit Types," we will be prompted to name two unit types. It is important to note that there should not be any spaces between the names. For example, if we want to denote 1-bedroom apartments, it is acceptable to write '1Bedroom', ‘1bedroom’ or '1B'. However, any other variations such as "1 Bedroom" or "1-Bedroom" will crash the program.



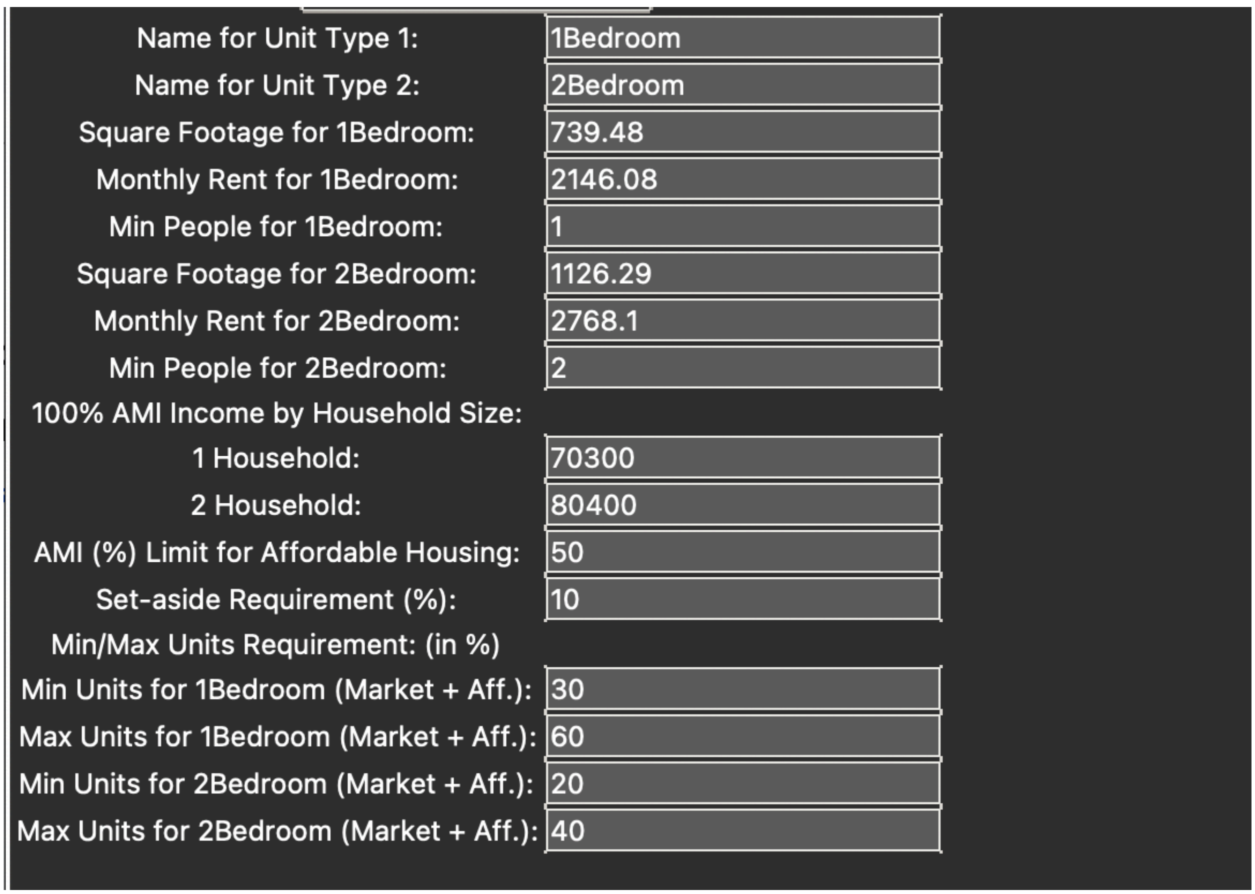
Clicking the "Next" button again will prompt us for further inputs. Sticking with our example of two unit types, we are prompted to specify the desired square footage for each unit, as well as the monthly rent we want to charge. For each unit type, we will be asked for the minimum number of people. In our example, the "Min People for 1Bedroom" is one person. This is because we are assigning the 1-household AMI values to the 1-bedroom apartment units. The same logic is used for other unit types, such as the 2-bedroom. This is one of our main model assumptions.



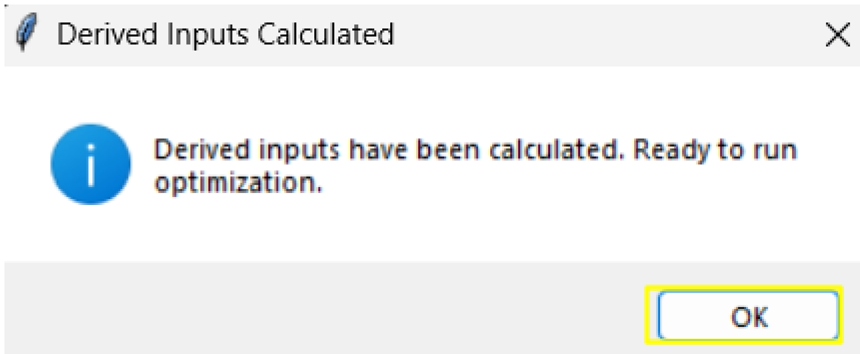
Clicking the "Next" button again will prompt yet another set of inputs. This is where we are supposed to enter the 100% AMI values for the respective household type. This information can be easily found on the URA website (https://www.ura.org/pages/housing-resources).

The "AMI Percentage" placeholder is where we actually define our own definition of affordability. For Pittsburgh in 2024, affordability is defined using 50% AMI for individuals as a benchmark. Hence, we can simply input '50' in this section. Again, it is best not to include any additional characters such as "%" after any numerical input. The "Set-aside Requirement (%)" is related to the affordable housing mandate specification. In Pittsburgh, as of 2024, this requirement is 10%. Hence, we should enter "10".

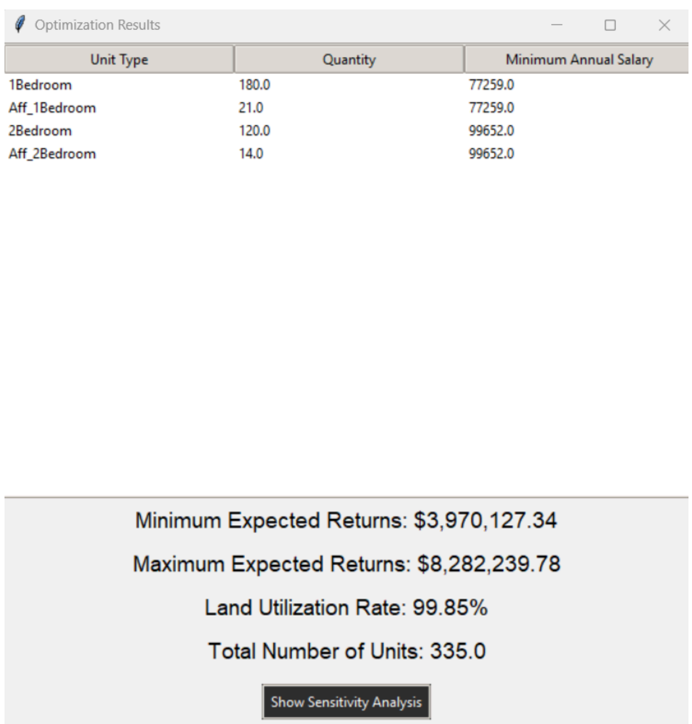
Below is another set of inputs, coming in the form of percentages. These are our minimum and maximum demand percentages that correspond to the historical demand for each unit type. Although these inputs are percentages, it is best to only type the number. Hence, ‘50%’ should be written as ‘50’.



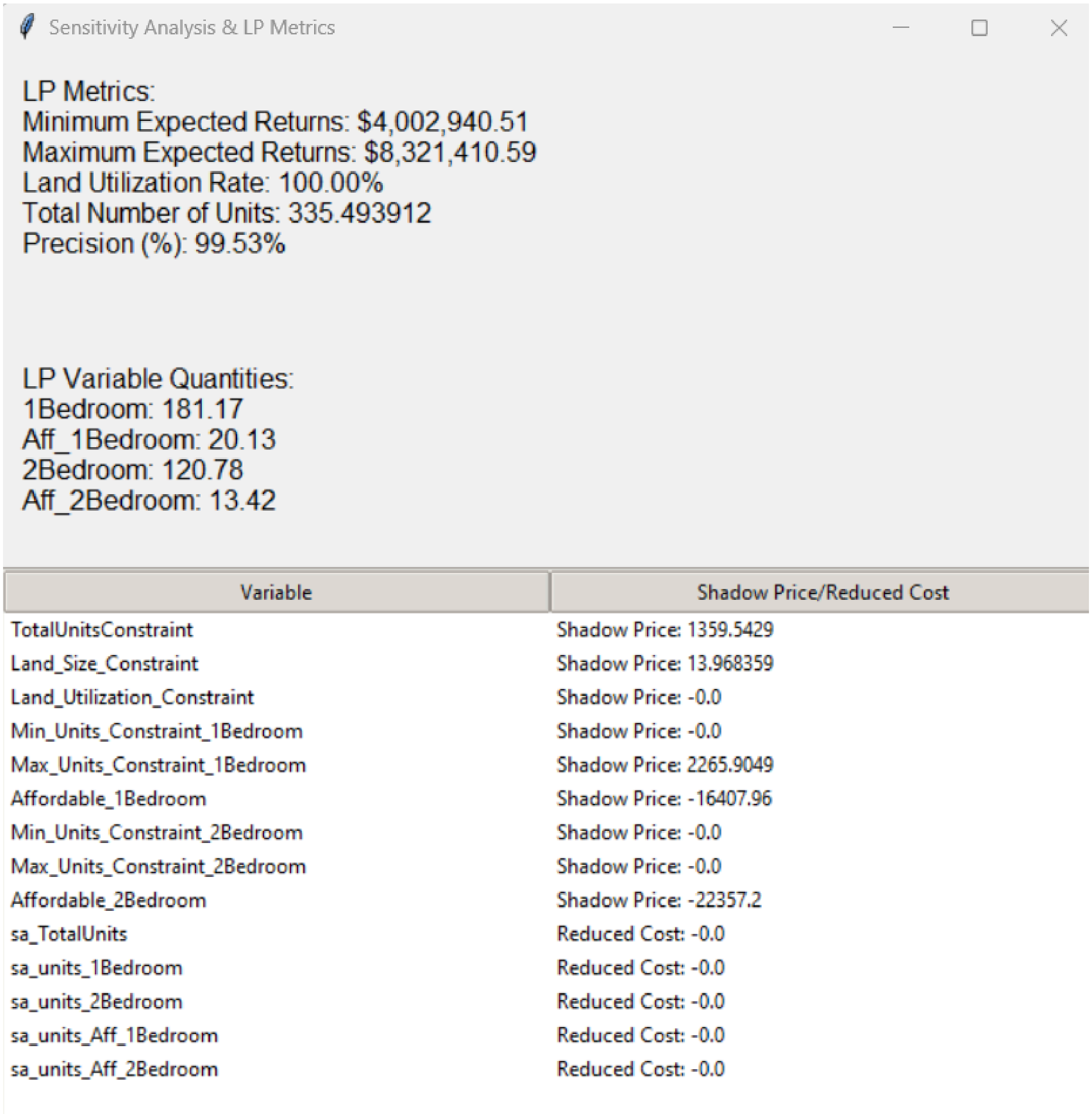
Once done, we can click the “Calculate Derive Inputs” Button (not pictured) which should show us a pop-up window similar to the one below.



Upon seeing this pop-up, we should proceed to click the “Run Optimization” button (not pictured). We will then be able to view our optimized results, as shown below.



Clicking the "Show Sensitivity Analysis" button will bring us to a page consisting of additional metrics and our very own sensitivity analysis.


Sensitivity Analysis & LP Metrics
LP Metrics:
Minimum Expected Returns: $4,002,940.51
Maximum Expected Returns: $8,321,410.59
Land Utilization Rate: 100.00%
Total Number of Units: 335.493912
Precision (%): 99.53%
LP Variable Quantities:
1Bedroom: 181.17
Aff_1Bedroom: 20.13
2Bedroom: 120.78
Aff_2Bedroom: 13.42
Variable
Shadow Price/Reduced Cost
TotalUnitsConstraint
Shadow Price: 1359.5429
Land_Size_Constraint
Shadow Price: 13.968359
Land_Utilization_Constraint
Shadow Price: -0.0
Min_Units_Constraint_1Bedroom
Shadow Price: -0.0
Max_Units_Constraint_1Bedroom
Shadow Price: 2265.9049
Affordable_1Bedroom
Shadow Price: -16407.96
Min_Units_Constraint_2Bedroom
Shadow Price: -0.0
Max_Units_Constraint_2Bedroom
Shadow Price: -0.0
Affordable_2Bedroom
Shadow Price: -22357.2
sa_TotalUnits
Reduced Cost: -0.0
sa_units_1Bedroom
Reduced Cost: -0.0
sa_units_2Bedroom
Reduced Cost: -0.0
sa_units_Aff_1Bedroom
Reduced Cost: -0.0
sa_units_Aff_2Bedroom
Reduced Cost: -0.0

8. Conclusion

In conclusion, this Maximin optimization model shows a significant advancement in strategic planning for urban development projects constrained by new affordable housing regulations. By integrating robust mathematical frameworks with practical decision-making processes, this model not only ensures compliance with legal mandates but also maximizes returns in a market that is increasingly challenging for developers.

The model's strength lies in its ability to navigate through the worst-case scenarios, ensuring that Walnut Capital can sustain its commitment to providing "Best In City Living" even under stringent conditions. This approach mitigates risks and leverages opportunities, aligning the company's financial objectives with social responsibilities.

Furthermore, several iterations of the model demonstrated that increasing its complexity generally resulted in higher expected returns and maintained a stable utilization rate. This indicates that we have successfully reallocated efficiency gains to optimize expected returns. This represents an added benefit over traditional heuristic-based decision-making methods.

Moreover, the user-friendly GUI interface of the model democratizes the usage of complex optimization processes, allowing stakeholders to interact with the model efficiently and effectively. This interface ensures that adjustments to the model can be made seamlessly, reflecting changes in market conditions or regulatory requirements, thereby maintaining the model's relevance and utility over time.

As urban landscapes evolve and the demand for affordable housing increases, Walnut Capital's adoption of such innovative tools will not only enhance its competitive edge but also contribute positively to community development. The insights gained from the model will guide strategic decisions, optimize resource allocation, and ultimately, forge a path towards sustainable and profitable urban development.

9. References

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