

Time Series Project

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Import Dependencies

```
library(astsa)
library(tidyverse)
library(tswge)
library(tseries)
library(forecast)
library(knitr)
library(Metrics)
```

Import Data

```
# nfl = read.csv("nfl_attendance.csv")
# save(nfl, file = "nfl_attendance.RData")
load("nfl_attendance.RData")

head(nfl)
```

| | team | team_name | year | total | home | away | week | weekly_attendance |
|---|---------|-----------|------|--------|--------|--------|------|-------------------|
| 1 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 1 | 77434 |
| 2 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 2 | 66009 |
| 3 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 3 | NA |
| 4 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 4 | 71801 |
| 5 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 5 | 66985 |
| 6 | Arizona | Cardinals | 2000 | 893926 | 387475 | 506451 | 6 | 44296 |

Format Data

```
nfl.cumulative = nfl %>%  
  group_by(year, week) %>%  
  summarize(attendance = sum(weekly_attendance, na.rm = TRUE)) %>%  
  ungroup()
```

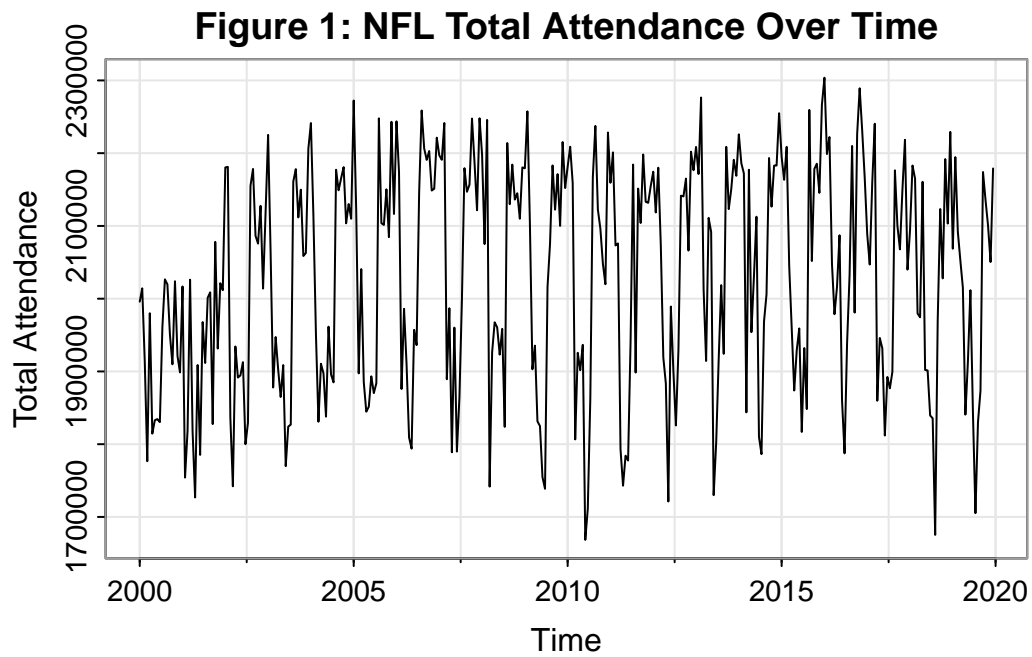
`summarise()` has grouped output by 'year'. You can override using the
`.groups` argument.

```
head(nfl.cumulative)
```

```
# A tibble: 6 x 3  
  year week attendance  
  <int> <int>      <int>  
1  2000     1    1995898  
2  2000     2    2014246  
3  2000     3    1913296  
4  2000     4    1776618  
5  2000     5    1980014  
6  2000     6    1814476
```

Plot Time Series

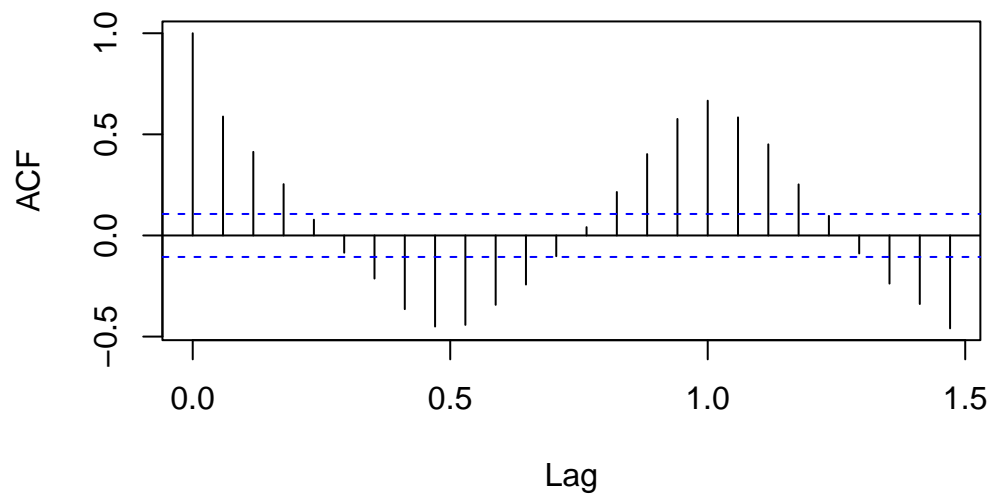
```
nfl.cumulative.ts = ts(nfl.cumulative$attendance, start = c(2000, 1), frequency = 17)  
  
tsplot(  
  nfl.cumulative.ts,  
  main = "Figure 1: NFL Total Attendance Over Time",  
  ylab = "Total Attendance"  
)
```



ACF, PACF and Spectral Density

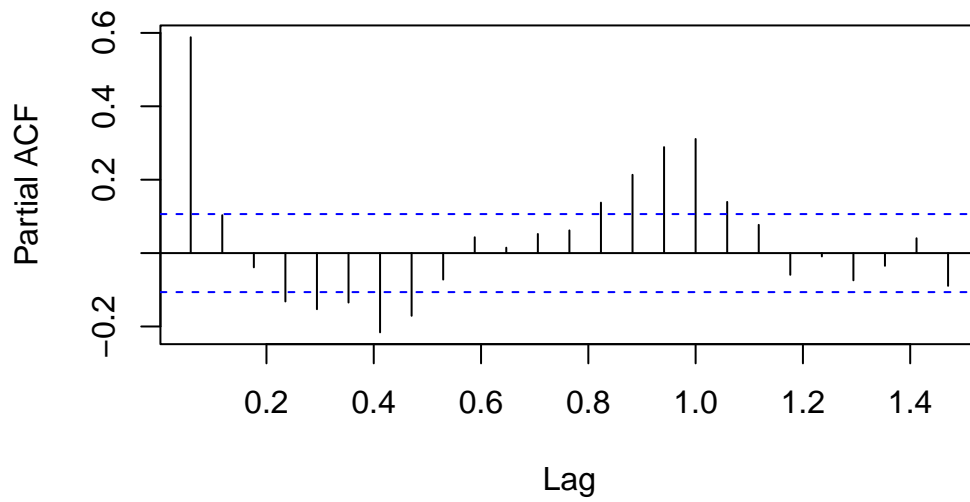
```
acf(  
  nfl.cumulative.ts,  
  main = "Figure 2: NFL Total Attendance ACF"  
)
```

Figure 2: NFL Total Attendance ACF



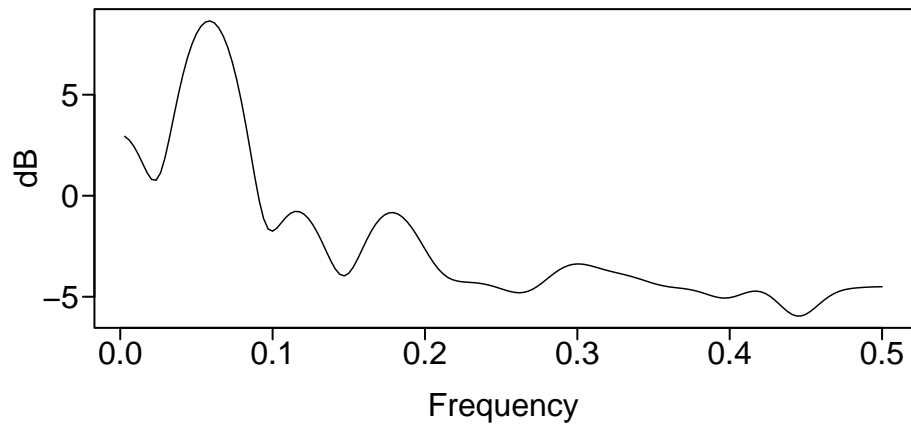
```
pacf(  
  nfl.cumulative.ts,  
  main = "Figure 3: NFL Total Attendance PACF"  
)
```

Figure 3: NFL Total Attendance PACF



```
specDensity = parzen.wge(nfl.cumulative.ts)
```

Parzen Window Truncation point: $M = 36$



Check for stationarity

```
adf.test(nfl.cumulative.ts)
```

Warning in adf.test(nfl.cumulative.ts): p-value smaller than printed p-value

Augmented Dickey-Fuller Test

data: nfl.cumulative.ts

Dickey-Fuller = -10.112, Lag order = 6, p-value = 0.01

alternative hypothesis: stationary

Training/Testing Split

```
nfl.training = nfl.cumulative %>%  
  dplyr::filter(year < 2019)  
  
nfl.training.ts = ts(nfl.training$attendance, start = c(2000, 1), frequency = 17)  
  
nfl.testing = nfl.cumulative %>%  
  dplyr::filter(year == 2019)  
  
nfl.testing.ts = ts(nfl.testing$attendance, start = c(2019, 1), frequency = 17)
```

Holt Winters Model

Fit Model

```
hwModel = HoltWinters(nfl.training.ts)  
hwForecast = forecast(  
  hwModel,  
  h = 17  
)  
  
tsplot(  
  nfl.cumulative.ts,
```

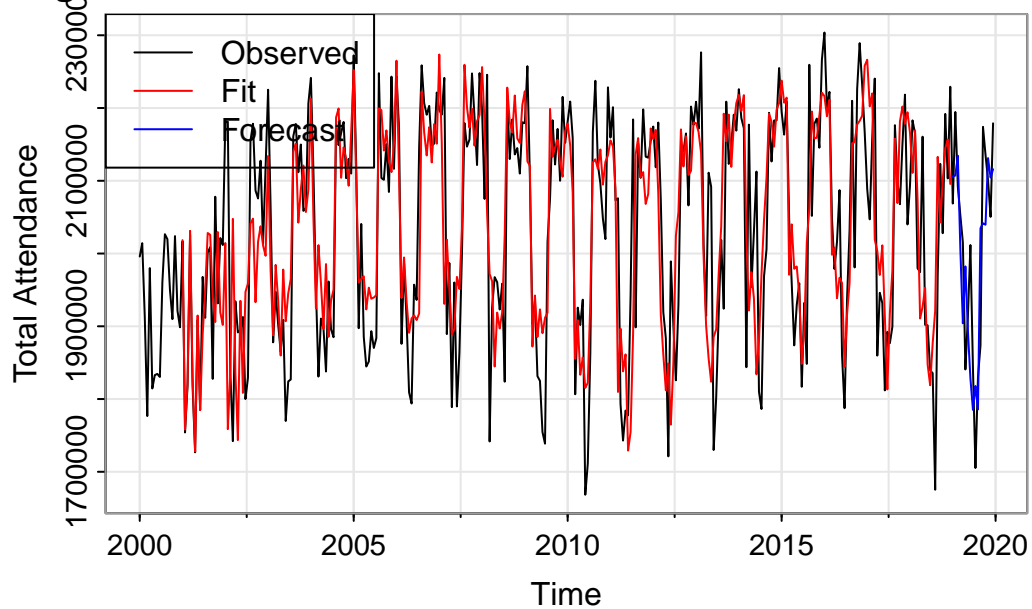
```

    main = "Figure 4: NFL Total Attendance Over Time Holt Winters Model",
    ylab = "Total Attendance"
)
lines(hwForecast$fitted, col = "red")
lines(hwForecast$mean, col = "blue")

legend(
  "topleft",
  legend = c("Observed", "Fit", "Forecast"),
  lty = 1,
  col = c("black", "red", "blue")
)

```

Figure 4: NFL Total Attendance Over Time Holt Winters Mo



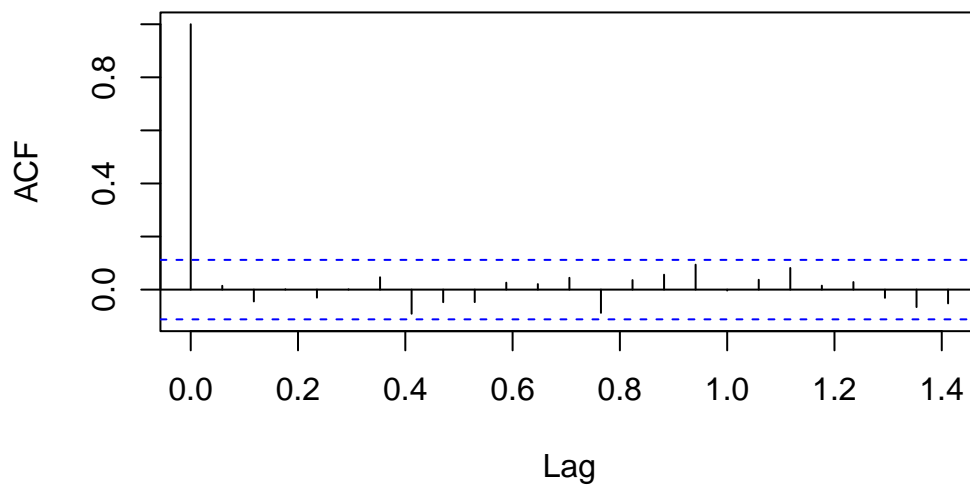
Model Evaluation

```

hwResiduals = window(nfl.training.ts, start = 2001) - hwForecast$fitted
acf(hwResiduals)

```

Series hwResiduals



```
Box.test(hwResiduals)
```

Box-Pierce test

```
data: hwResiduals  
X-squared = 0.063451, df = 1, p-value = 0.8011
```

```
rmse.hw = rmse(nfl.testing$attendance, hwForecast$mean)  
rmse.hw
```

```
[1] 96414.18
```


SARIMA

Determine Model Orders

```
auto.arima(nfl.training.ts, approximation = FALSE)
```

Series: nfl.training.ts
ARIMA(1,0,1)(0,1,1)[17] with drift

Coefficients:

| | ar1 | ma1 | sma1 | drift |
|------|--------|---------|---------|----------|
| | 0.9857 | -0.9412 | -0.6985 | 326.3695 |
| s.e. | 0.0170 | 0.0260 | 0.0466 | 396.4345 |

$\sigma^2 = 9.884e+09$: log likelihood = -3958.77
AIC=7927.55 AICc=7927.75 BIC=7946.17

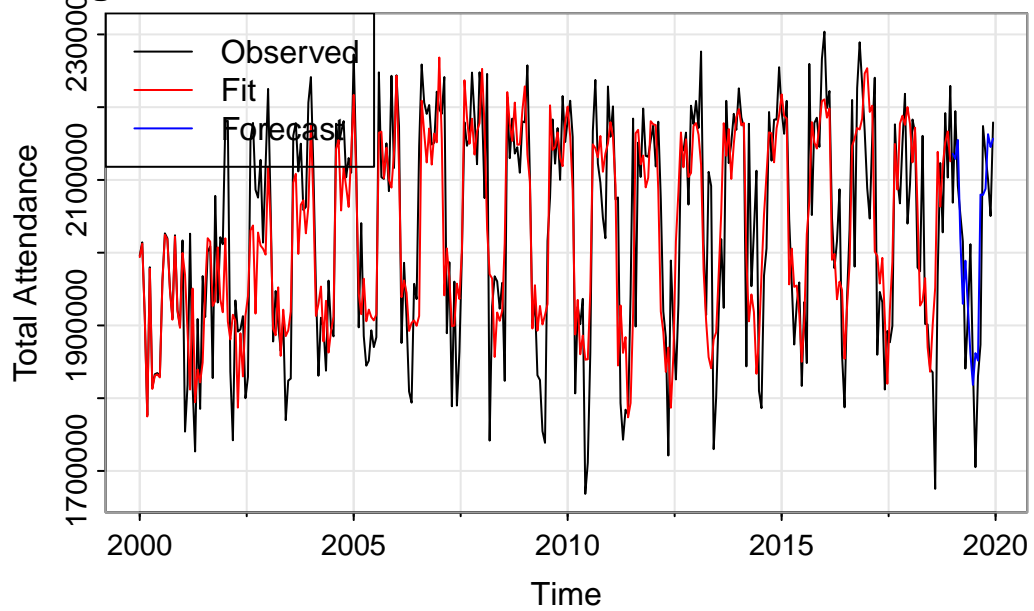
Fit Model

```
sarimaModel = arima(nfl.training.ts, order = c(1, 0, 1), seasonal = list(order = c(0, 1, 1)))  
sarimaForecast = predict(sarimaModel, n.ahead = 17)
```

```
sarimaFit = nfl.training.ts - sarimaModel$residuals
```

```
tsplot(  
  nfl.cumulative.ts,  
  main = "Figure 5: NFL Total Attendance Over Time SARIMA Model",  
  ylab = "Total Attendance"  
)  
lines(sarimaFit, col = "red")  
lines(sarimaForecast$pred, col = "blue")  
  
legend(  
  "topleft",  
  legend = c("Observed", "Fit", "Forecast"),  
  lty = 1,  
  col = c("black", "red", "blue")  
)
```

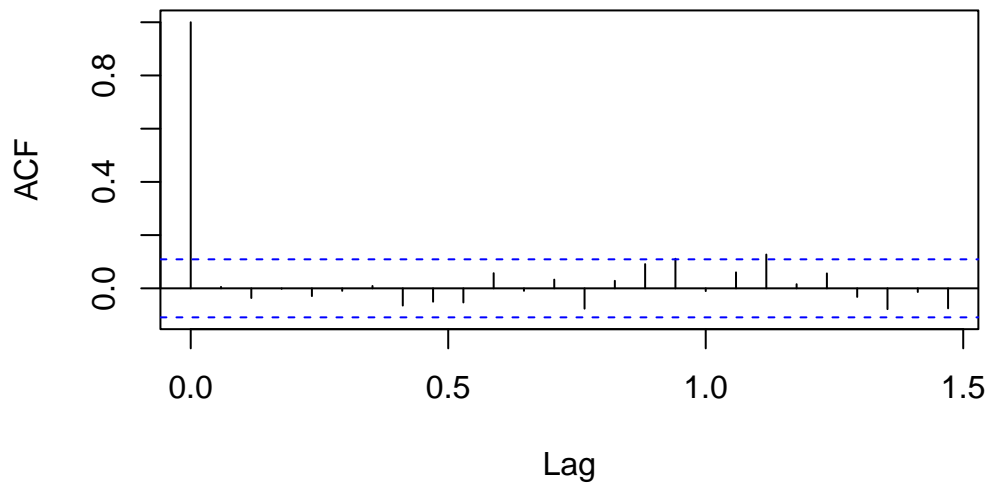
Figure 5: NFL Total Attendance Over Time SARIMA Model



Model Evaluation

```
acf(sarimaModel$residuals)
```

Series sarimaModel\$residuals



```
Box.test(sarimaModel$residuals)
```

Box-Pierce test

```
data: sarimaModel$residuals  
X-squared = 0.0096764, df = 1, p-value = 0.9216
```

```
rmse.sarima = rmse(nfl.testing$attendance, sarimaForecast$pred)  
rmse.sarima
```

```
[1] 96660.92
```

Linear Regression

Fit Model

```
nfl.cumulative2 = nfl.cumulative[18:nrow(nfl.cumulative),] %>%
  mutate(
    t = 18:nrow(nfl.cumulative),
    sin17 = sin((2 * pi * t) / 17),
    cos17 = cos((2 * pi * t) / 17),
    attendance.lag = nfl.cumulative$attendance[t - 17]
  )

head(nfl.cumulative2)
```

A tibble: 6 x 7

| | year | week | attendance | t | sin17 | cos17 | attendance.lag |
|---|-------|-------|------------|-------|-------|--------|----------------|
| | <int> | <int> | <int> | <int> | <dbl> | <dbl> | <int> |
| 1 | 2001 | 1 | 2016864 | 18 | 0.361 | 0.932 | 1995898 |
| 2 | 2001 | 2 | 1754008 | 19 | 0.674 | 0.739 | 2014246 |
| 3 | 2001 | 3 | 1818632 | 20 | 0.895 | 0.446 | 1913296 |
| 4 | 2001 | 4 | 2026086 | 21 | 0.996 | 0.0923 | 1776618 |
| 5 | 2001 | 5 | 1817838 | 22 | 0.962 | -0.274 | 1980014 |
| 6 | 2001 | 6 | 1726716 | 23 | 0.798 | -0.603 | 1814476 |

```
nfl.cumulative2.training = nfl.cumulative2 %>%
  filter(year < 2019)

nfl.cumulative2.testing = nfl.cumulative2 %>%
  filter(year == 2019)

lmModel = lm(attendance ~ sin17 + cos17 + attendance.lag, data = nfl.cumulative2.training)

summary(lmModel)
```

Call:

```
lm(formula = attendance ~ sin17 + cos17 + attendance.lag, data = nfl.cumulative2.training)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|-----|----|--------|----|-----|
|-----|----|--------|----|-----|

-319227 -63226 1791 56355 287738

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|----------------|------------|------------|---------|--------------|
| (Intercept) | 1.257e+06 | 1.061e+05 | 11.845 | < 2e-16 *** |
| sin17 | -3.430e+04 | 8.699e+03 | -3.943 | 9.99e-05 *** |
| cos17 | 9.063e+04 | 1.071e+04 | 8.459 | 1.18e-15 *** |
| attendance.lag | 3.835e-01 | 5.220e-02 | 7.347 | 1.90e-12 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 99220 on 302 degrees of freedom

Multiple R-squared: 0.5847, Adjusted R-squared: 0.5805

F-statistic: 141.7 on 3 and 302 DF, p-value: < 2.2e-16

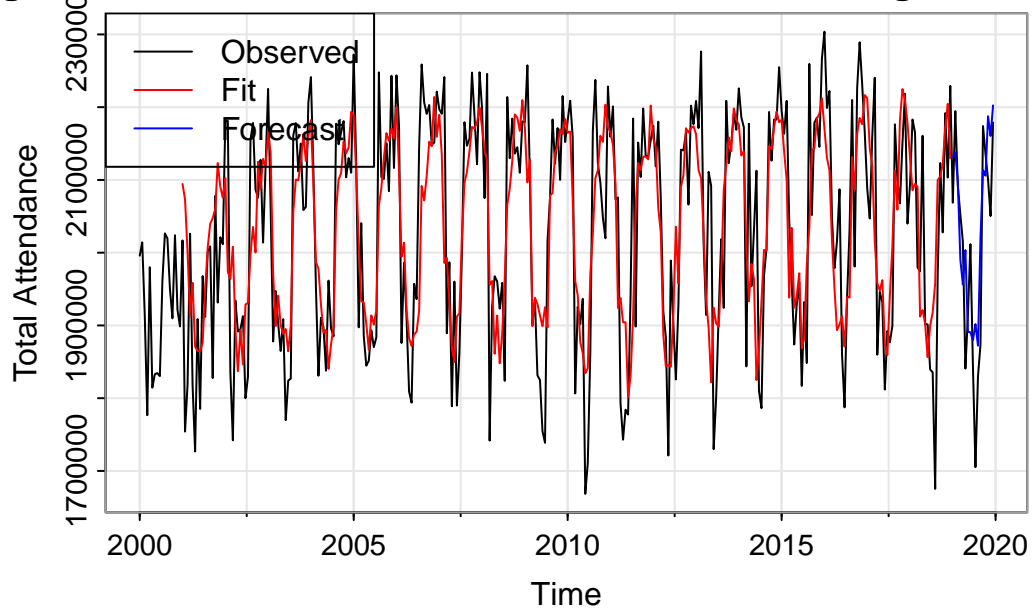
```
lmModel.predict = predict(lmModel, newdata = nfl.cumulative2.testing)

ts.lmModel = ts(lmModel$fitted.values, start = c(2001, 1), frequency = 17)
ts.lmPredict = ts(lmModel.predict, start = c(2019, 1), frequency = 17)

tsplot(
  nfl.cumulative.ts,
  main = "Figure 6: NFL Total Attendance Over Time Linear Regression Model",
  ylab = "Total Attendance"
)
lines(ts.lmModel, col = "red")
lines(ts.lmPredict, col = "blue")

legend(
  "topleft",
  legend = c("Observed", "Fit", "Forecast"),
  lty = 1,
  col = c("black", "red", "blue")
)
```

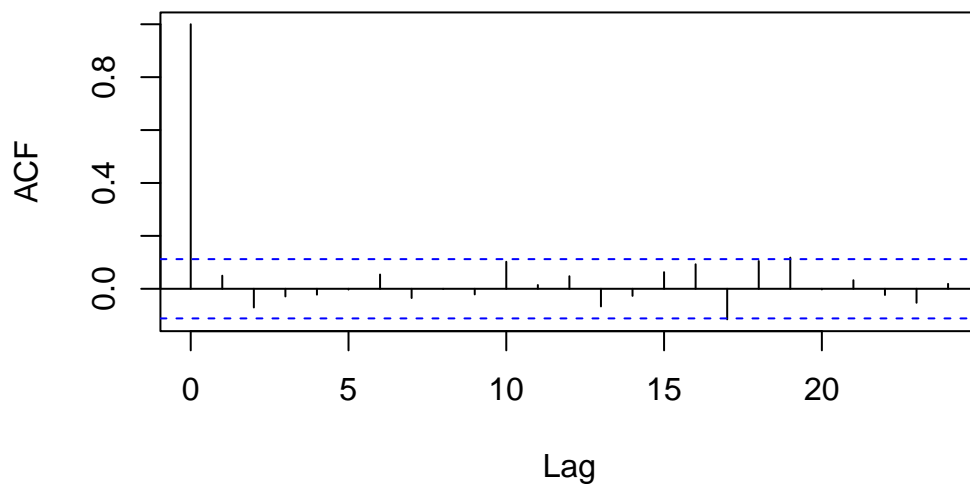
Figure 6: NFL Total Attendance Over Time Linear Regression



Model Evalutation

```
acf(lmModel$residuals)
```

Series lmModel\$residuals



```
Box.test(lmModel$residuals)
```

Box-Pierce test

```
data: lmModel$residuals  
X-squared = 0.75023, df = 1, p-value = 0.3864
```

```
rmse.lm = rmse(nfl.cumulative2.testing$attendance, lmModel.predict)  
rmse.lm
```

```
[1] 92131.05
```

Final Results

```
plot(  
  nfl.testing.ts,  
  main = "Figure 7: 2019 NFL Total Attendance vs Model Forecasts",
```

```

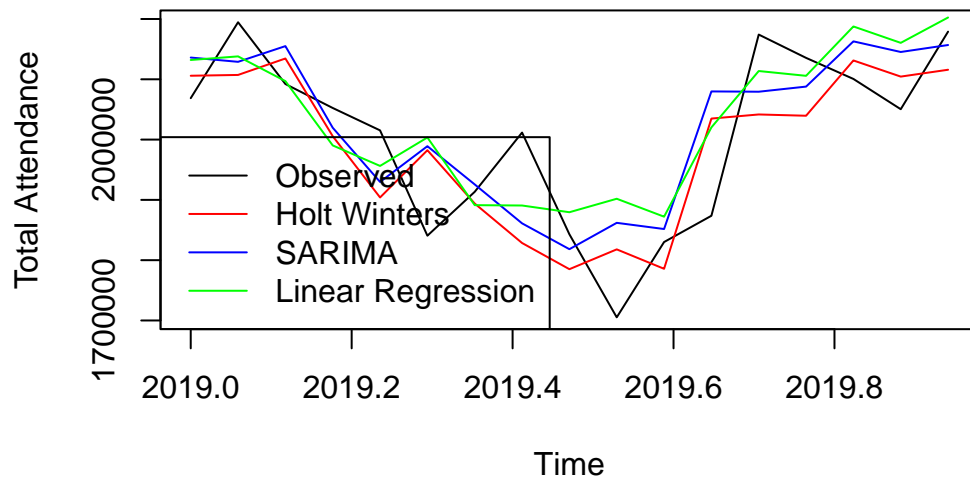
    ylab = "Total Attendance"
)

lines(hwForecast$mean, col = "red")
lines(sarimaForecast$pred, col = "blue")
lines(ts.lmPredict, col = "green")

legend(
  "bottomleft",
  legend = c(
    "Observed",
    "Holt Winters",
    "SARIMA",
    "Linear Regression"
  ),
  lty = 1,
  col = c(
    "black",
    "red",
    "blue",
    "green"
  )
)

```


Figure 7: 2019 NFL Total Attendance vs Model Forecasts



```
data.frame(  
  model = c(  
    "Holt Winters",  
    "SARIMA",  
    "Linear Regression"  
  ),  
  rmse = c(  
    rmse.hw,  
    rmse.sarima,  
    rmse.lm  
  )  
) %>%  
  kable(  
    col.names = c(  
      "Model Type",  
      "RMSE"  
    ),  
    caption = "RMSE of NFL Attendance Forecasts"  
  )
```

Table 1: RMSE of NFL Attendance Forecasts

| Model Type | RMSE |
|-------------------|----------|
| Holt Winters | 96414.18 |
| SARIMA | 96660.92 |
| Linear Regression | 92131.05 |