问1.1. $\therefore K = \begin{bmatrix} f_x & 0 & d_x \\ 0 & f_y & d_y \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = K \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

$\therefore x =$

问1.1. $K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore \begin{cases} x = \dfrac{X \cdot f_x}{Z} + c_x \\ y = \dfrac{Y \cdot f_y}{Z} + c_y \end{cases}$$

2. 齐次坐标下 $\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

$$\therefore \begin{cases} x = \dfrac{f_x \cdot X + c_x \cdot Z}{\lambda} \\ y = \dfrac{f_y \cdot Y + c_y \cdot Z}{\lambda} \end{cases} \qquad \lambda = Z$$

问2.1 $H = \begin{bmatrix} 1.207 & 0.586 & 1.0 \\ 2.207 & 3.242 & 2.0 \\ 1.0 & 2.0 & 1.0 \end{bmatrix}$

$|K| = 1$

$$H_S = \begin{bmatrix} s\cdot R & t \\ 0^T & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$H_A = \begin{bmatrix} K & 0 \\ 0^T & 1 \end{bmatrix} \qquad K = \begin{bmatrix} k_{11} & k_{12} \\ 0 & k_{22} \end{bmatrix}, \quad k_{11}\cdot k_{22} = 1$$

$$k_{11} > 0,\ k_{12} > 0,\ k_{22} > 0$$

$$H_P = \begin{bmatrix} I & 0 \\ v^T & v \end{bmatrix}$$

$$H = \begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$

$$H_S\cdot H_A\cdot H_P = \begin{bmatrix} sRK & t \\ 0^T & 1 \end{bmatrix}\cdot\begin{bmatrix} I & 0 \\ v^T & v \end{bmatrix}$$

$$= \begin{bmatrix} sRK + t\cdot v^T & vt \\ v^T & v \end{bmatrix} = \begin{bmatrix} 1.707 & 0.586 & 1.0 \\ 2.707 & 8.242 & 2.0 \\ 1.0 & 2.0 & 1.0 \end{bmatrix}$$

$$\therefore\ sRK + t\cdot v^T = \begin{bmatrix} 1.707 & 0.586 \\ 2.707 & 8.242 \end{bmatrix} \quad vt = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix} \quad v = 1.0,\ v^T = [1.0\ \ 2.0]$$

$$\therefore\ t = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix} \Rightarrow s\cdot R\cdot K = \begin{bmatrix} 0.707 & -1.414 \\ 0.707 & 4.242 \end{bmatrix}$$

$$\therefore\ s^2 \cdot |R|\cdot|K| = 4$$

$$\therefore\ s^2 = 4 \Rightarrow s = 2$$

$$\therefore\ R\cdot K = \frac{1}{2}\cdot\begin{bmatrix} 0.707 & -1.414 \\ 0.707 & 4.242 \end{bmatrix}$$

$$\therefore\ \begin{bmatrix} k_{11}\cdot\cos\theta & k_{12}\cdot\cos\theta - k_{22}\cdot\sin\theta \\ k_{11}\cdot\sin\theta & k_{12}\cdot\sin\theta + k_{22}\cdot\cos\theta \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0.707 & -1.414 \\ 0.707 & 4.242 \end{bmatrix}$$

$\therefore\ k_{11} = \frac{1}{2},\ \theta = \frac{\pi}{4},\ k_{22} = 2,\ k_{12} = 1$

$\therefore$

$$H_S = \begin{bmatrix} \sqrt{2} & -\sqrt{2} & 1 \\ \sqrt{2} & \sqrt{2} & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad H_A = \begin{bmatrix} \frac{1}{2} & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad H_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

问3.1、

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow X' = H_A \cdot X$$

设有 $AB \parallel CD$.

$A:(x_1, y_1)$, $B:(x_2, y_2)$ $C:(x_3, y_3)$ $D:(x_4, y_4)$

$$\therefore\ \frac{|AB|^2}{|CD|^2} = \frac{(x_2-x_1)^2+(y_2-y_1)^2}{(x_3-x_4)^2+(y_3-y_4)^2}$$

$$\therefore\ \frac{|AB|}{|CD|} = \sqrt{\frac{(x_1-x_2)^2+(y_1-y_2)^2}{(x_3-x_4)^2+(y_3-y_4)^2}} = \sqrt{\frac{(X_A-X_B)^T\cdot(X_A-X_B)}{(X_C-X_D)^T\cdot(X_C-X_D)}}$$

$\therefore\ X_A' = H_A \cdot X_A \quad X_B' = H_B \cdot X_B, \quad X_C' = H_A \cdot X_C \cdot X_D' = H_D \cdot X_D$

$$\begin{aligned}
\therefore\ |A'B'|^2 &= (X_A' - X_B')^T \cdot (X_A' - X_B') \\
&= (H_A \cdot (X_A - X_B))^T \cdot (H_A \cdot (X_A - X_B)) \\
&= (X_A - X_B)^T \cdot H_A^T \cdot H_A \cdot (X_A - X_B) \\
&= (X_A - X_B)^T \cdot \begin{pmatrix} A^2 & At \\ tA & 1 \end{pmatrix} \cdot (X_A - X_B)
\end{aligned}$$

同理 $|C'D'|^2 = (X_C - X_D)^T \cdot \begin{pmatrix} A^2 & At \\ tA & 1 \end{pmatrix} \cdot (X_C - X_D)$

设 $(\bar{x}_A - \bar{x}_B)^T = (\bar{x}_{AB}^T, 0)^T$，$(\bar{x}_C - \bar{x}_D)^T = (\bar{x}_{CD}^T, 0)^T$

$$\therefore \frac{|AB|}{|CD|} = \frac{\bar{x}_{AB}^T \cdot \bar{x}_{AB}}{\bar{x}_{CD}^T \cdot \bar{x}_{CD}}$$

$$\left|\frac{A'B'}{C'D'}\right|^2 = \frac{(\bar{x}_{AB}^T \;\; 0)\begin{pmatrix} A^2 & At \\ tA & 1 \end{pmatrix}\begin{pmatrix} \bar{x}_{AB} \\ 0 \end{pmatrix}}{(\bar{x}_{CD}^T \;\; 0)\begin{pmatrix} A^2 & At \\ tA & 1 \end{pmatrix}\begin{pmatrix} \bar{x}_{CD} \\ 0 \end{pmatrix}}$$

$$= \frac{(\bar{x}_{AB}^T A^2 \;\; \bar{x}_{AB}^T A t)\cdot\begin{pmatrix} \bar{x}_{AB} \\ 0 \end{pmatrix}}{(\bar{x}_{CD}^T A^2 \;\; \bar{x}_{CD}^T A t)\cdot\begin{pmatrix} \bar{x}_{CD} \\ 0 \end{pmatrix}}$$

$$= \frac{\bar{x}_{AB}^T \cdot A^2 \cdot \bar{x}_{AB}}{\bar{x}_{CD}^T A^2 \cdot \bar{x}_{CD}}$$

$$= \frac{(x_1 - x_2, \; y_1 - y_2)\cdot A^2 \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}}{(x_3 - x_4 \;\; y_3 - y_4)\, A^2 \begin{pmatrix} x_3 - x_4 \\ y_3 - y_4 \end{pmatrix}}$$

$\because AB // CD$

$$\therefore \frac{|AB|}{|CD|} = k \Rightarrow \bar{x}_{AB} = k \cdot \bar{x}_{CD}$$

$$\therefore \left|\frac{A'B'}{C'D'}\right|^2 = \frac{k \cdot \bar{x}_{CD}^T \cdot A^2 \cdot k \cdot \bar{x}_{CD}}{\bar{x}_{CD}^T \cdot A^2 \cdot \bar{x}_{CD}} = k^2$$

$$\therefore \left|\frac{A'B'}{C'D'}\right| = k \text{ 不变.}$$