

Paper Review: On-The-Fly Static Analysis via Dynamic Bidirected Dyck Reachability

Krishna et al., POPL '24.

Jung Hyun Kim

*SoftSec Lab., KAIST
IS661 Spring, 2024*

Dyck Reachability

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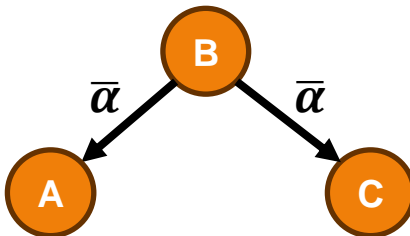
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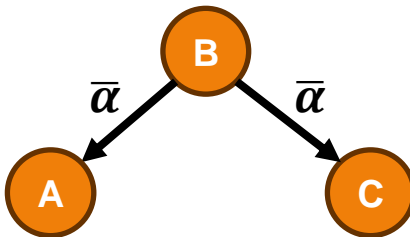
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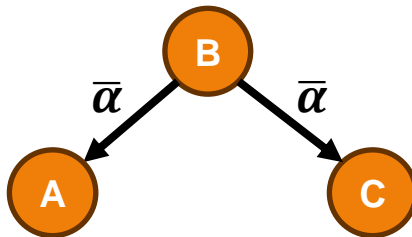


Bidirected graph

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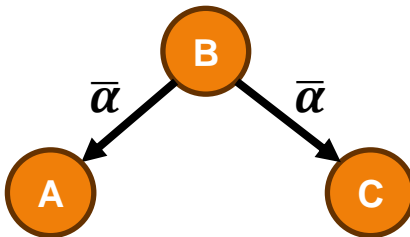
Bidirected graph

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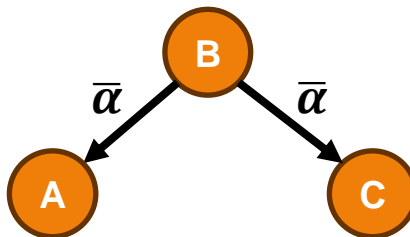
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A, C: *Dyck reachable*

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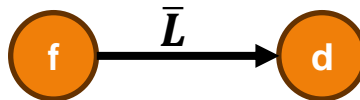
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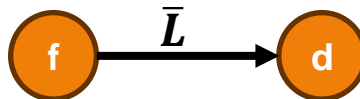
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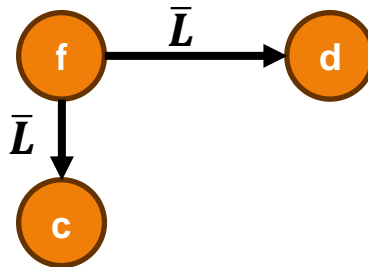
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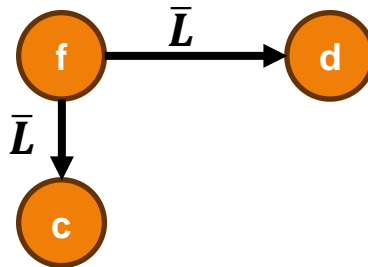
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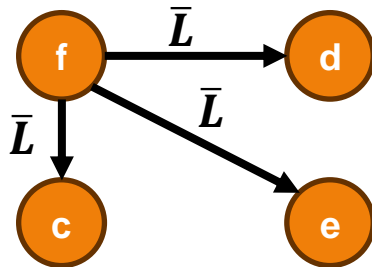
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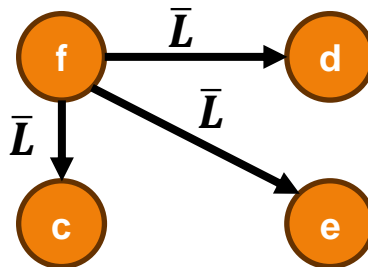
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c, d, e: Dyck reachable



Dyck Reachability with Continuous Analysis

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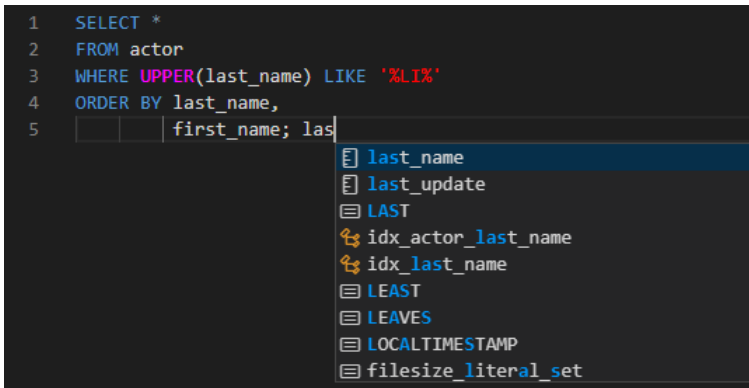
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```
1 SELECT *
2 FROM actor
3 WHERE UPPER(last_name) LIKE '%L%'
4 ORDER BY last_name,
5      first_name; las
```

A screenshot of a code editor with a dark background. It shows a SQL query with five lines. The first line is 'SELECT *', the second is 'FROM actor', the third is 'WHERE UPPER(last_name) LIKE '%L%'' (with 'UPPER' in pink and 'LIKE' in red), the fourth is 'ORDER BY last_name,', and the fifth is 'first_name; las'. A dropdown menu is open from the end of the fifth line, showing a list of column names: 'last_name' (highlighted), 'last_update', 'LAST', 'idx_actor_last_name', 'idx_last_name', 'LEAST', 'LEAVES', 'LOCALTIMESTAMP', and 'filesize_literal_set'.

Type hints from an IDE

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Is *offline* algorithm efficient for constant changes?

Type hints from an IDE

Time Complexity of Dyck Reachability

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	Chatterjee ^[a]	Krishna
Offline time	$O(m + n \cdot \alpha(n))$	
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→ n times faster than before!

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 - Try not to recalculate DSCCs from scratch.

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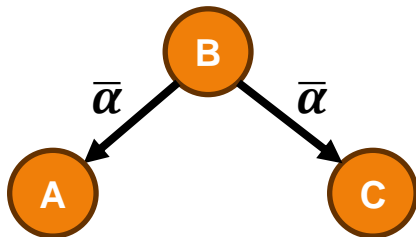
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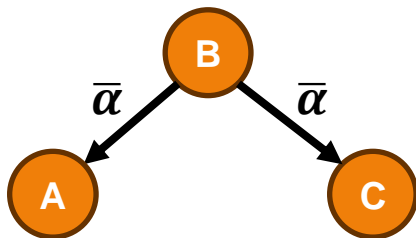
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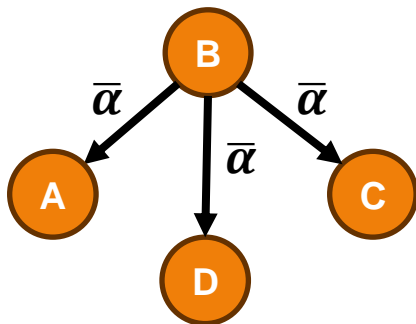
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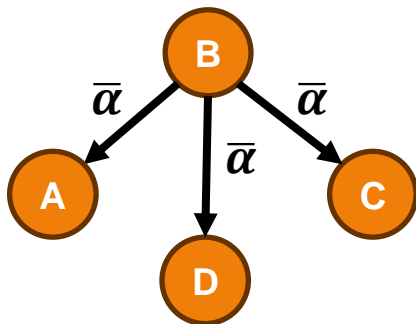
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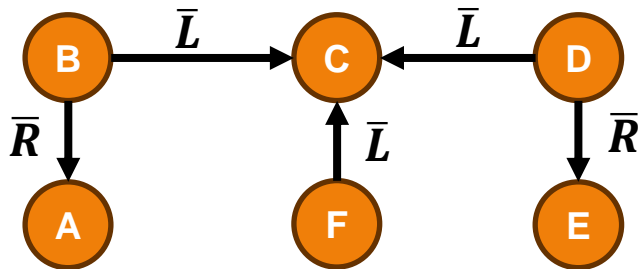
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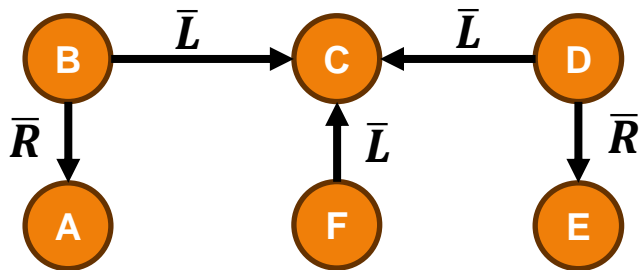
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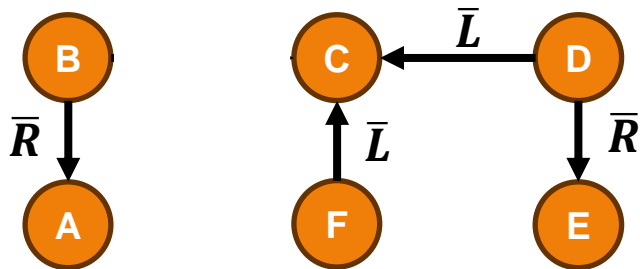


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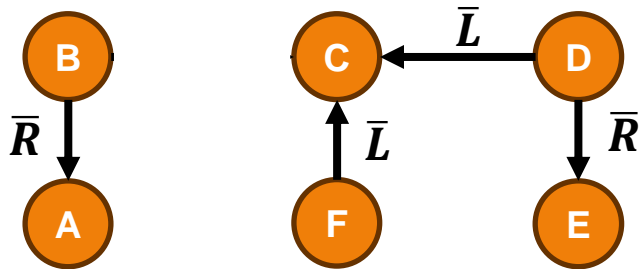


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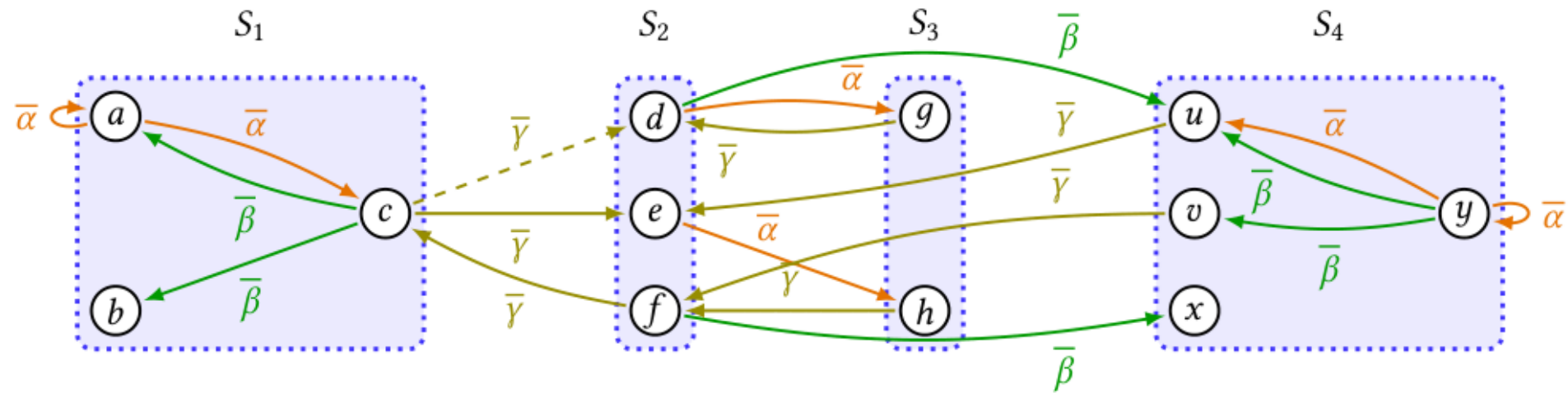
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- After splitting DSCCs into PDSCCs, recalculate the fixpoint.

Example on Deletion

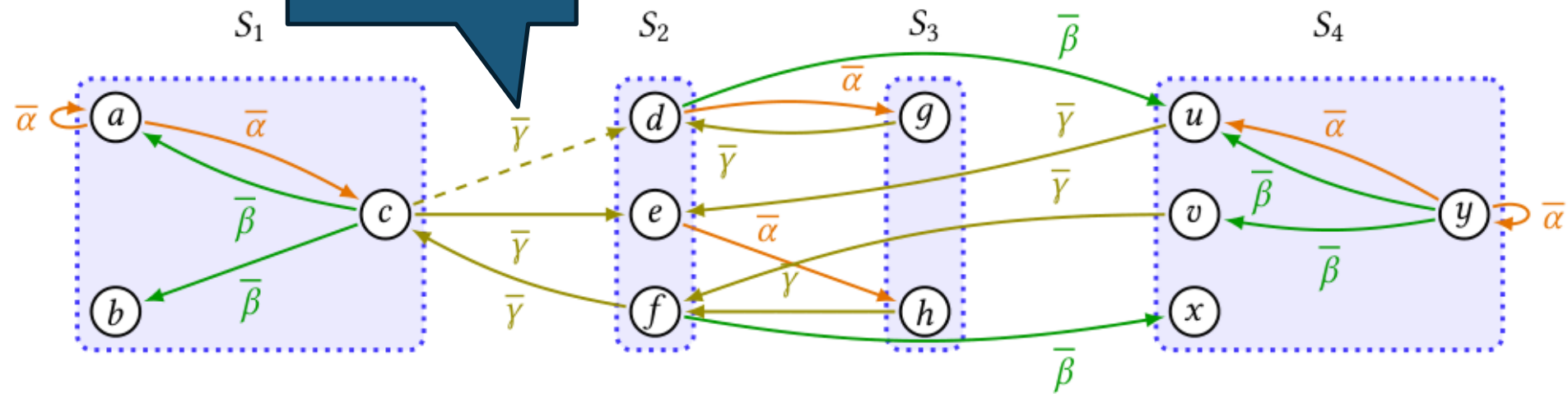
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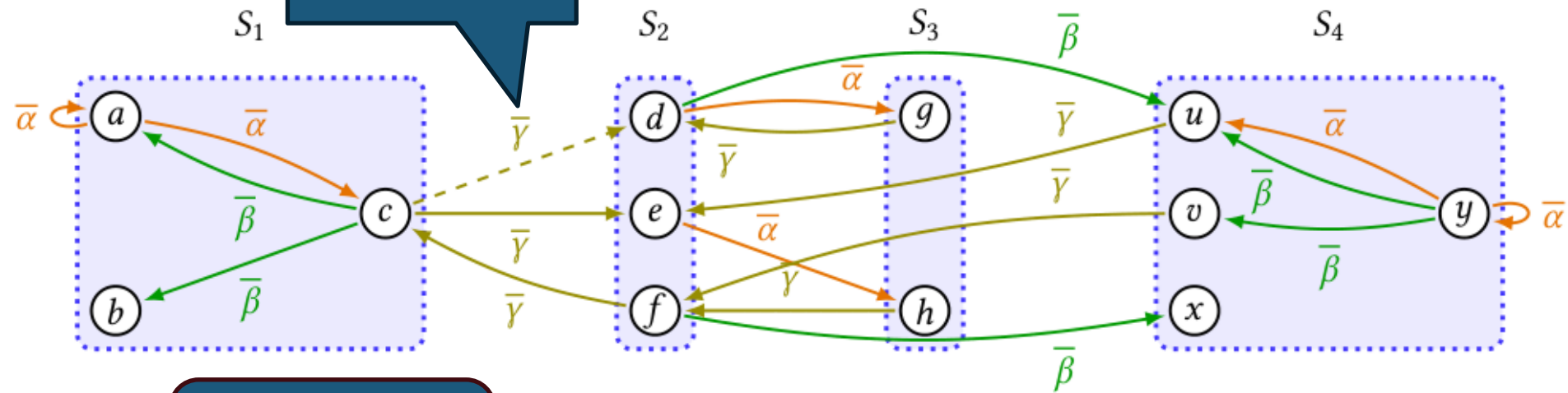
Deletion of $c \rightarrow d$



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[Candidates]

S_2 : $\text{DSCC}(d) = S_2$

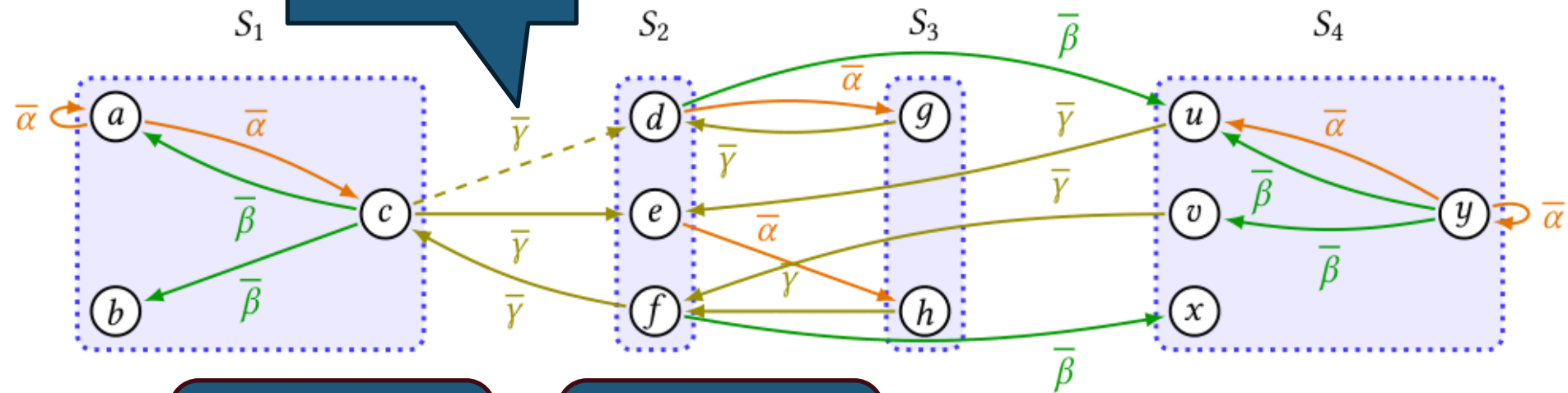
S_3 : $d \rightarrow g$ and $e \rightarrow h$

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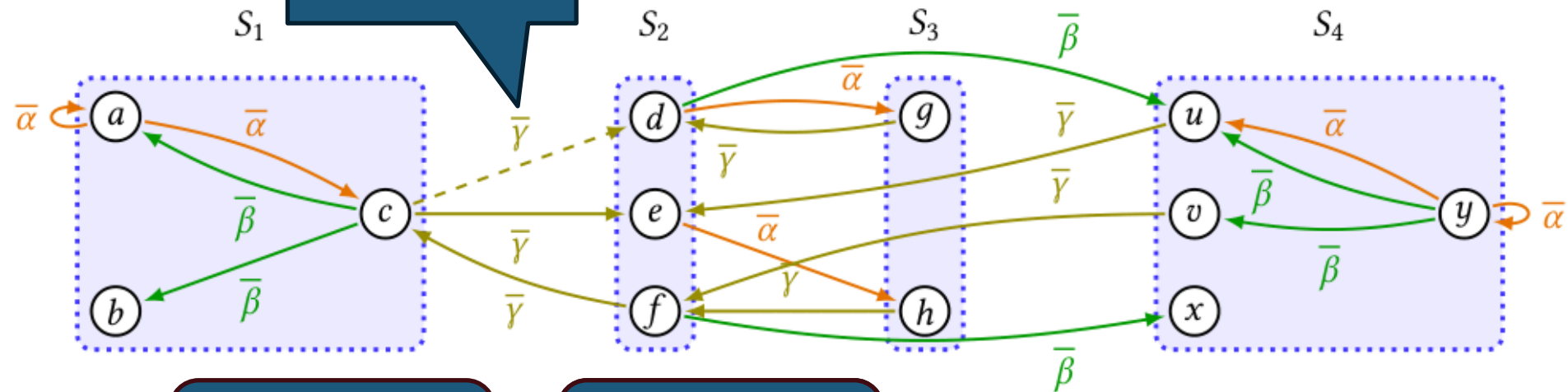
[PDSCC]

P_1 : { d, e }
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 : DSCC

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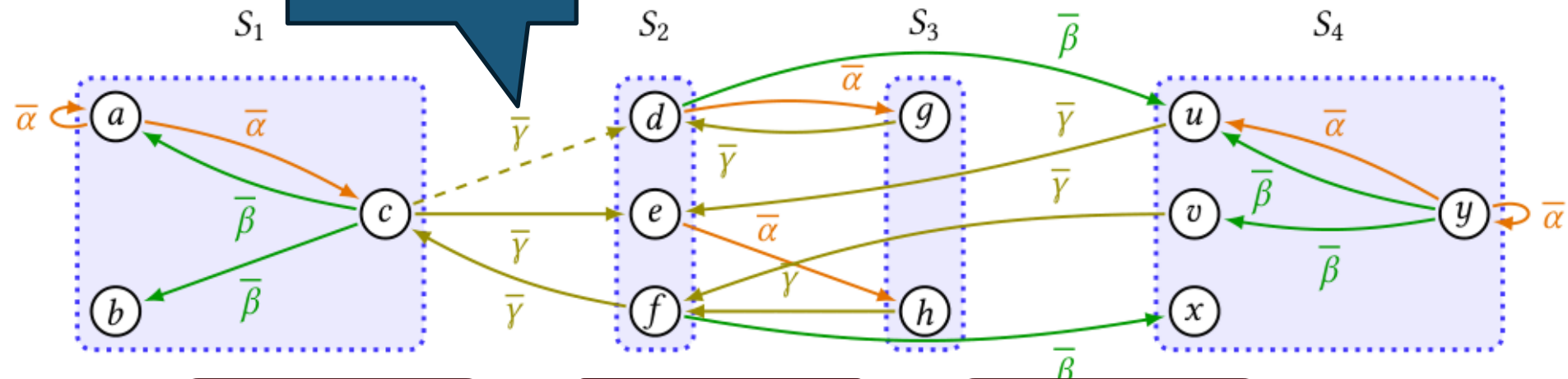
[PDSCC]

P_4 : $\{d, e\}$
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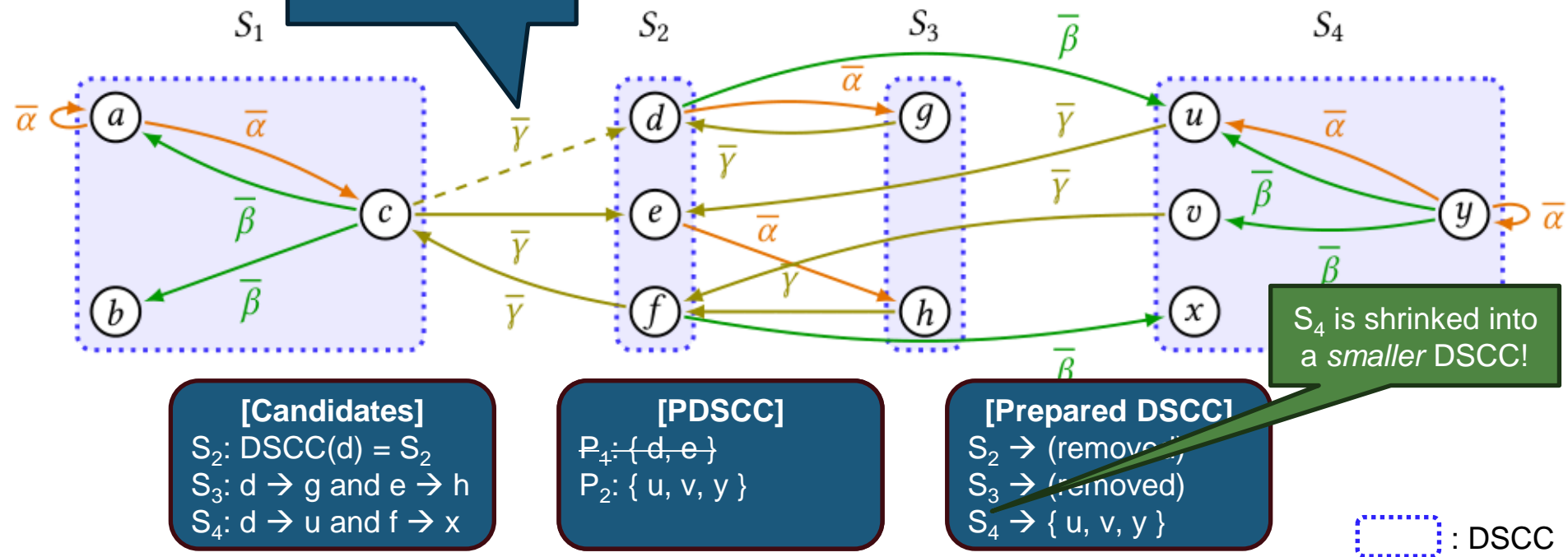
[Prepared DSCC]

$S_2 \rightarrow$ (removed)
 $S_3 \rightarrow$ (removed)
 $S_4 \rightarrow \{u, v, y\}$

 : DSCC

Example on Deletion

Deletion of $c \rightarrow d$



Evaluation Setup

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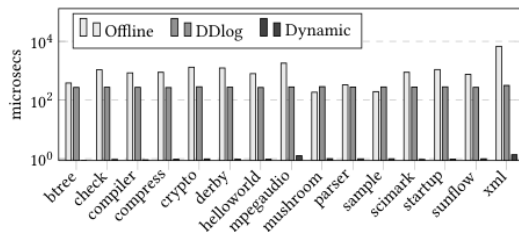
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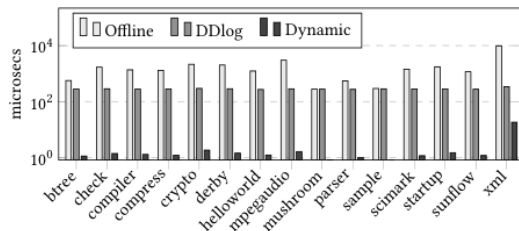
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 - DDlog: the state-of-the-art datalog engine.
 - Dynamic: the proposed method.

* The authors have not specified the details on the figures.

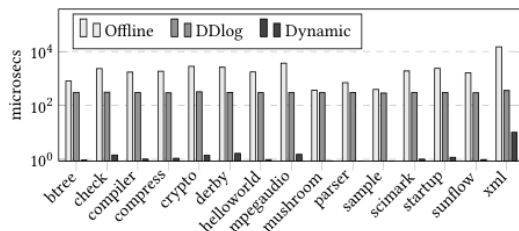
Data Dependence Analysis



(a) Incremental updates.

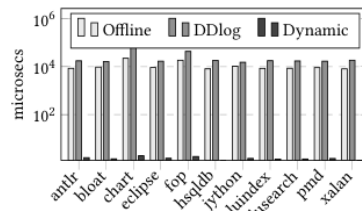


(c) Decremental updates.

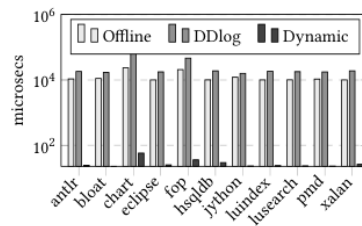


(e) Mixed updates.

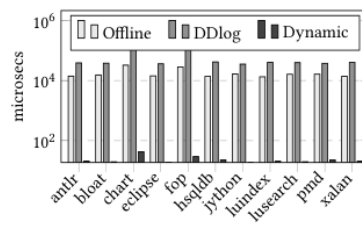
Alias Analysis



(b) Incremental updates.



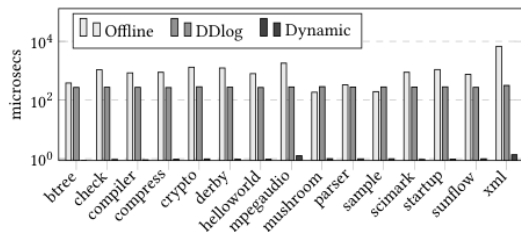
(d) Decremental updates.



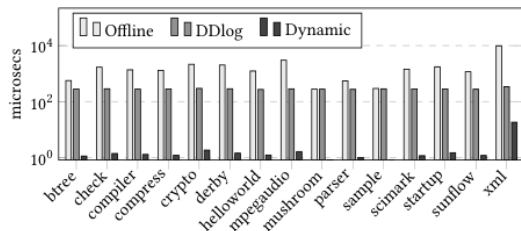
(f) Mixed updates.

* The authors have not specified the details on the figures.

Data Dependence Analysis



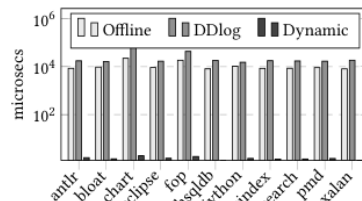
(a) Incremental updates.



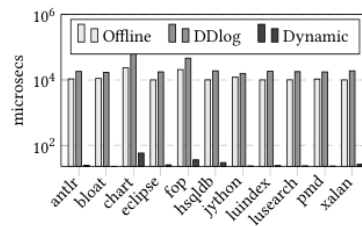
(c) Decremental updates.



Alias Analysis



(b) Incremental updates.



(d) Decremental updates.



The online algorithm is much faster than the offline algorithm!

The authors have not specified the details on the figures.

Questions

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- To build an optimal online algorithm, we should observe some properties and introduce a *proper data structure* in order to optimize the *necessary recalculation* until reaching its fixpoint.
- Going forward from the previous step is sometimes better than going backward incrementally (c.f. datalog engines).

Question?