Optimal Program Synthesis via Abstract Interpretation

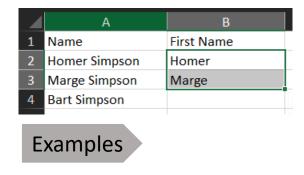
Geon Park

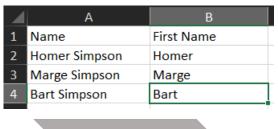
KAIST Programming Systems Laboratory



Motivation

FlashFill

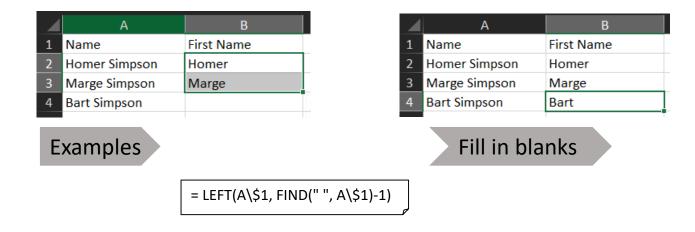




Fill in blanks

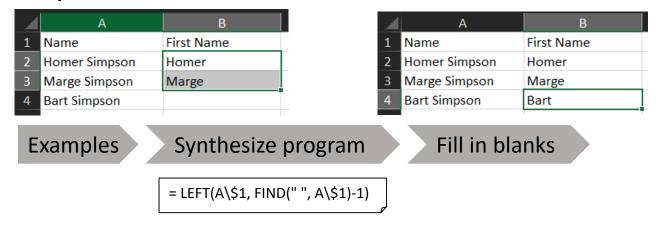
Motivation

FlashFill



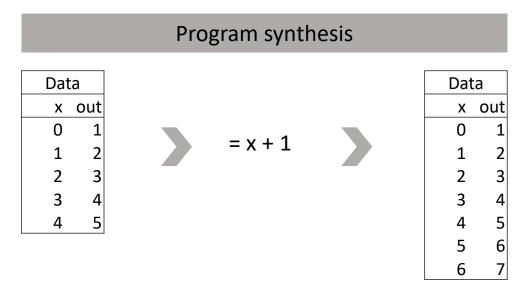
Motivation

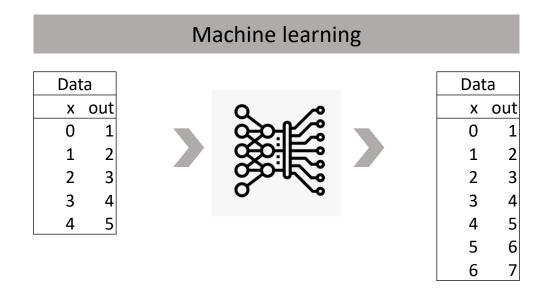
• FlashFill is program synthesis and it's useful



Program Synthesis

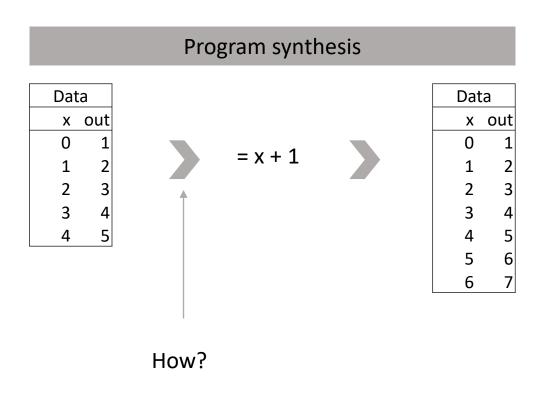
• Program synthesis is robust and interpretable (v.s. machine learning)

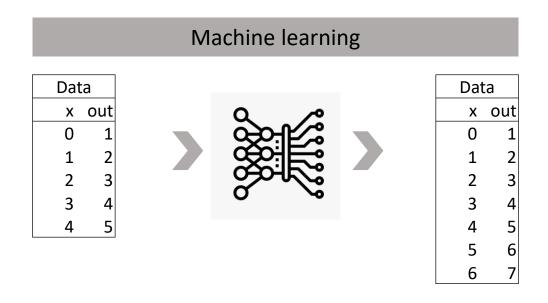




Program Synthesis

• Program synthesis is robust and interpretable (v.s. machine learning)

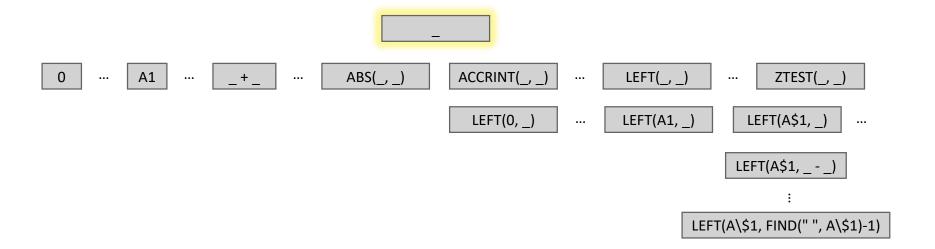




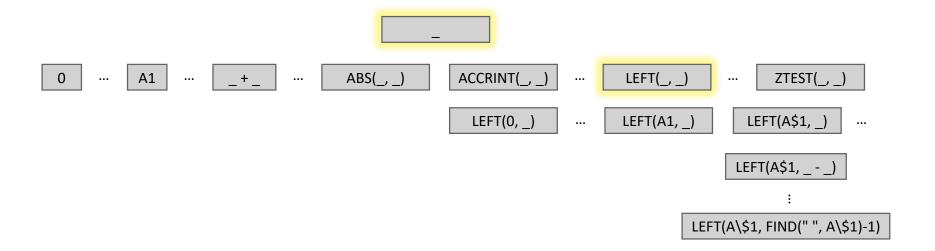
- Many program synthesis, like FlashFill, uses top-down enumeration
 - Filling in the blanks, starting from initial blank



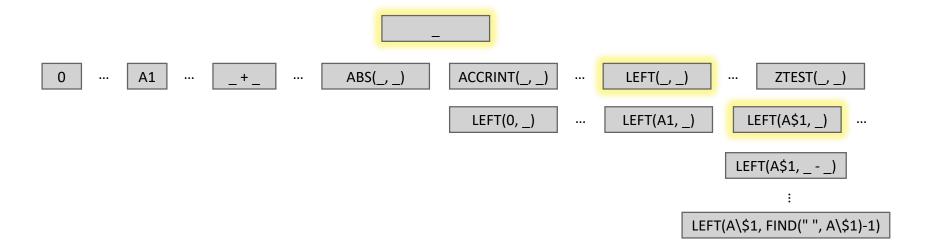
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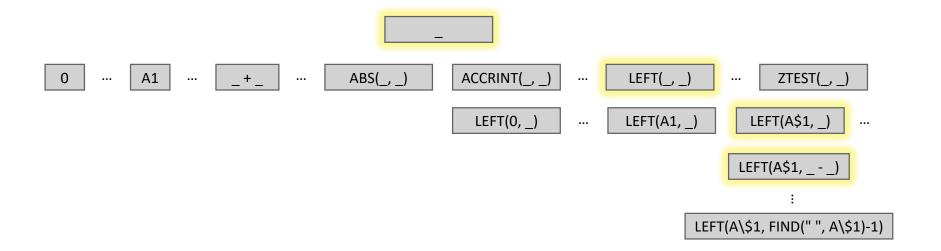
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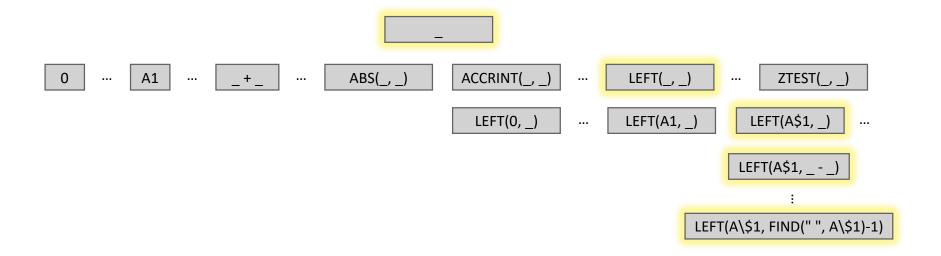
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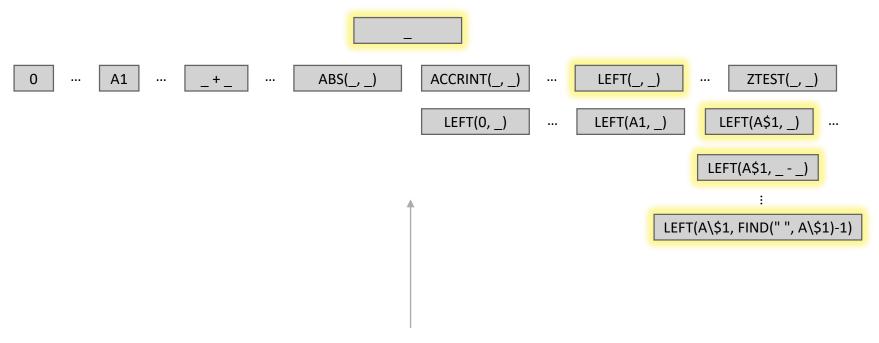
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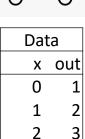


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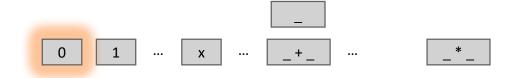


How to traverse this whole (infinite) set?





- Traverse the tree in own standard
- If the node does not fit data, prune out





Data		
Х	out	
0	1	
1	2	
2	3	
3	4	
4	5	

- Traverse the tree in own standard
- If the node does not fit data, prune out





Data		
Х	out	
0	1	
1	2	
2	3	
3	4	
4	5	

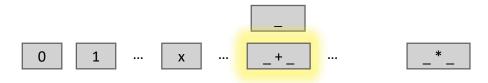
- Traverse the tree in own standard
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Data		
Х	out	
0	1	
1	2	
2	2 3	
3	4	
4	5	

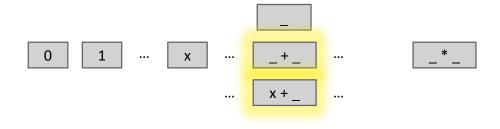
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Data		
Х	out	
0	1	
1	2	
2	2 3	
3	4	
4	5	

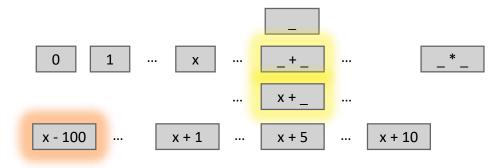
- Traverse the tree in own standard
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0	ij
Da	ta
Х	out
0	1
1	2

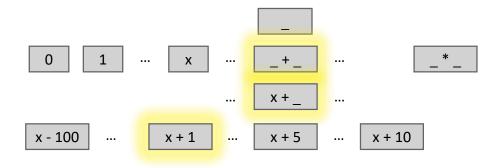
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0	U
Dat	ta
Х	out
0	1
1	2

- Traverse the tree in own standard
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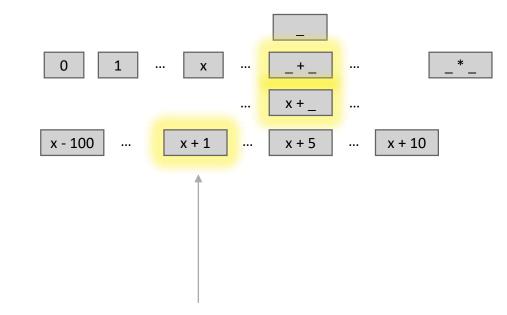


Pruning is usually used



\mathcal{O}
:a
out
1
2

- Traverse the tree in own standard
- If the node does not fit data, prune out

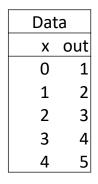


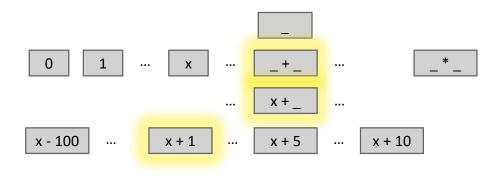
How do we "efficiently" traverse highly likely path first?

Pruning is usually used

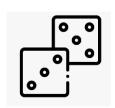


- Traverse the tree in own standard
- If the node does not fit data, prune out





• For traversing order, probabilistic model is well used



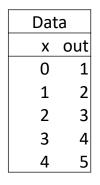
- Provide probability for each production rule
- Search first the path with highest likelihood

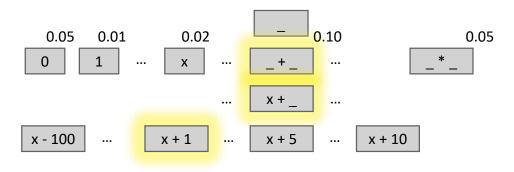
Pruning is usually used



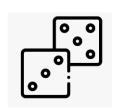
	_			•			
•	Traverse	the	tree	ın	own	stand	dard

• If the node does not fit data, prune out





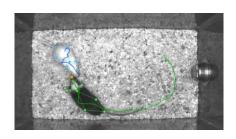
For traversing order, probabilistic model is well used



- Provide probability for each production rule
- Search first the path with highest likelihood

Problem: Synthesis for fuzzy, new data





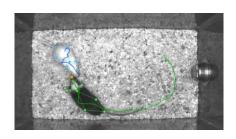
Scene 1			S	cene	2
Time	Dist.	Label	Time	Dist.	Label
0	100	N	0	20	Υ
1	80	N	1	13	Υ
2	2	Υ	2	60	N
3	33	Υ	3	94	N
4	60	N	4	100	N

- 1. Biologist want to know mice' behavior
- 2. Videos mice, extract features (like distance)
- 3. Labels some of them for training set

^{*}Jennifer J Sun et al., The Multi-Agent Behavior Dataset: Mouse Dyadic Social Interactions, arXiv preprint, 2021.

Problem: Synthesis for fuzzy, new data





S	cene	1	S	cene	2
Time	Dist.	Label	Time	Dist.	Label
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- 1. Biologist want to know mice' behavior
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- Goal to make formula that best (not perfectly) describes the behavior
- Still can prune out formula that is worse than temporal best

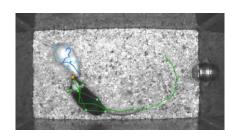


New data cannot give any probability for production rule

^{*}Jennifer J Sun et al., The Multi-Agent Behavior Dataset: Mouse Dyadic Social Interactions, arXiv preprint, 2021.

Problem: Synthesis for fuzzy, new data





S	cene	1	S	cene	2
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New data cannot give any probability for production rule

Need a novel approach

^{*}Jennifer J Sun et al., The Multi-Agent Behavior Dataset: Mouse Dyadic Social Interactions, arXiv preprint, 2021.

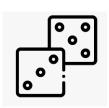
Idea

Pruning by heuristic



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out

For traversing order,



- Traverse the tree in order given by A* search
- Search first the node that is likely to have optimal node as child
- Makes faster to find optimal node



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out

Dat	Data		
Х	ou		
1	10		
2	20		
3	-10		



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out

Data		a						
	x	out			_			
	1	10						
	2	20	10 * x				c*x (c	: < 0)
	3	-10		_		'		



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out

Data		
x out	_	
1 10		
2 20	10 * x	c * x (c < 0)
3 -10		
	accuracy = 0.67	accuracy at most 0.33



- For each node, calculate highest accuracy it and its child can get
- If another node has accuracy higher than that, prune this out

Data		
x out		
1 10		
2 20	10 * x	c * x
3 -10		
	accuracy = 0.67	accommost 0.33

Appealing result: A new best attempt

Novel approach finds optimal program significantly faster than Metasketches.

Time to identify the optimal program and prove its optimality



Metasketches (SMT solver)

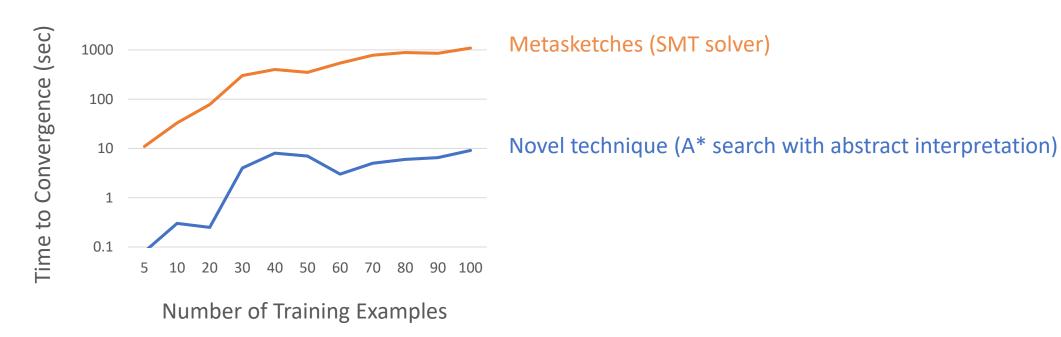
- Adds SMT constraint that programs' score > temporal best
- Change temporal best score if "SAT" made

^{*}Stephen Mell et al., Optimal Program Synthesis via Abstract Interpretation, POPL, 2024.

Appealing result: A new best attempt

Novel approach finds optimal program significantly faster than Metasketches.

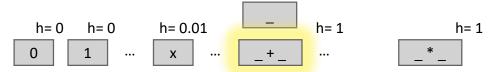
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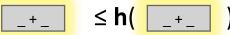
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A* search

• Search first the path with highest heuristic

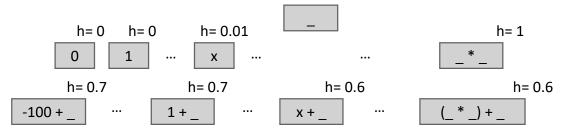


- Heuristic is a function that **overapproximates** the accuracy
 - Maximum accuracy possible for any program filling



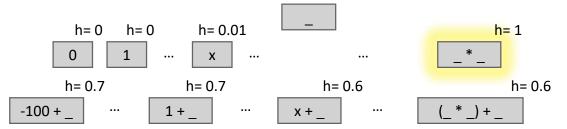
A* search

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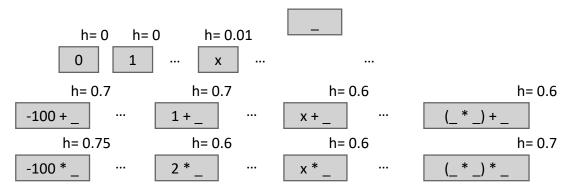
- Heuristic is a function that **overapproximates** the accuracy
 - Maximum accuracy possible for any program filling __+_ ≤ h(__+_)

• Search first the path with highest heuristic



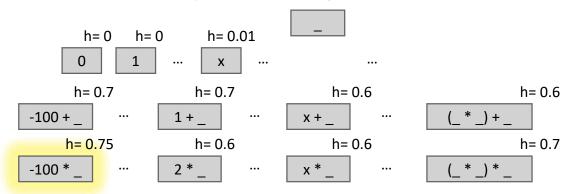
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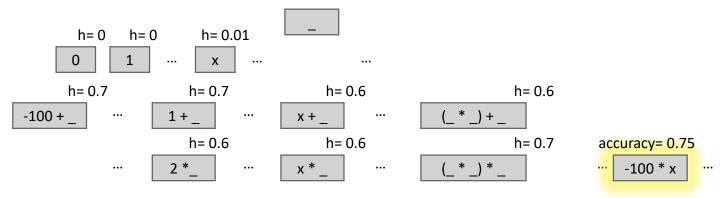
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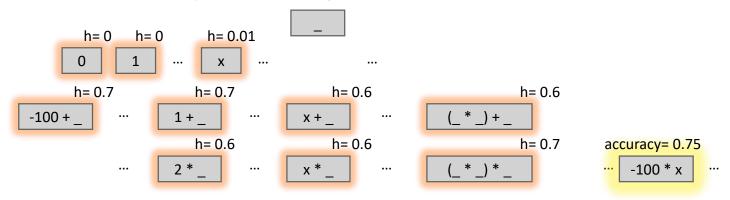
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Search first the path with highest heuristic



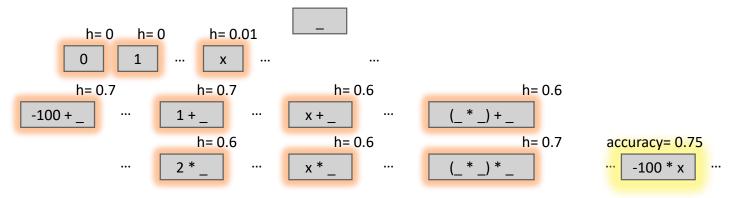
- Heuristic is a function that **overapproximates** the accuracy
 - Maximum accuracy possible for any program filling __+_ ≤ h(__+_)
- When formula is complete, can prune other nodes with low heuristic

Search first the path with highest heuristic



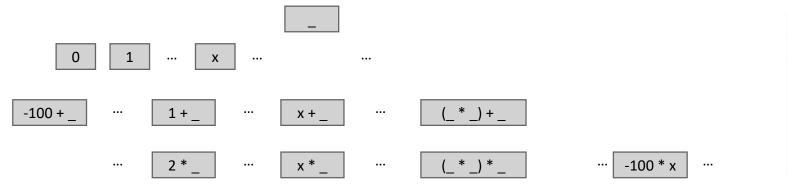
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 - Maximum accuracy possible for any program filling __+_ ≤ h(__+_)
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Search first the path with highest heuristic



- Heuristic is a function that **overapproximates** the accuracy
 - Maximum accuracy possible for any program filling __+_ ≤ h(__+_)
- When formula is complete, can prune other nodes with low heuristic
- If completed formula's accuracy is higher than any other node's heuristic, it's optimal

Define heuristic to be the highest accuracy possible for any completion



Data	
Х	out
0	1
1	2
2	3
3	4
4	5

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x

Data		
	Х	out
	0	1
	1	2
	2	3
	3	4
	4	5

-100 * x

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x

Data	
Х	out
0	1
1	2
2	3
3	4
4	5

Evaluation	
х	out
0	0
1	-100
2	-200
3	-300
4	-400

-100 * x

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
 - h(-100 * x) = 0

-100 * x	

Data	
Х	out
0	1
1	2
2	3
3	4
4	5

Evaluation		
х	out	
0	0	
1	-100	
2	-200	
3	-300	
4	-400	

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
 - h(-100 * x) = 0
- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100

-100 * x

Data	
Х	out
0	1
1	2
2	3
3	4
4	5

Evaluation		
Х	out	
0	0	
1	-100	
2	-200	
3	-300	
4	-400	

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 - Example 1: completed node -100 * x
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- Problem : need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation



Data	
Х	out
0	1
1	2
2	2 3
3	
4	4 5

Evaluation		
х	out	
0	0	
1	-100	
2	-200	
3	-300	
4	-400	

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 - Example 1: completed node -100 * x
 - h(-100 * x) = 0
- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)



Data		
	Х	out
	0	1
	1	2
	2	3
	3	4
	4	5

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
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 - c * x (c is constant)
 - Split the interval for c



Data	
Х	out
0	1
1	2
2	3
3	4
4	5

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
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- Problem: need to evaluate for a lot number of constants
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 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node $c_{[0, 100]} * x$

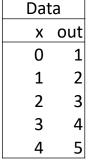


Data	
Х	out
0	1
1	2
2	3
3	4
4	5

Evalu	uation
Х	out
0	0
1	[0, 100]
2	[0, 200]
3	[0, 300]
4	[0, 400]

c_[0, 100] * x

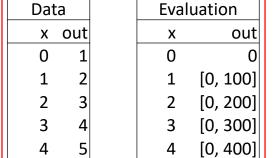
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 - Example 1: completed node -100 * x
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- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node c_[0, 100] * x
 - $h(c_{[0,100]} * x) = max([0,0.8]) = 0.8$



Evaluation		
Х	out	
0	0	
1	[0, 100]	
2	[0, 200]	
3	[0, 300]	
4	[0, 400]	

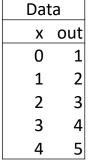
c_[0, 100] * x

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
 - h(-100 * x) = 0
- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node c_[0, 100] * x
 - $h(c_{[0,100]} * x) = max([0,0.8]) = 0.8$



C_[-100, 0] * x

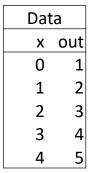
- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
 - h(-100 * x) = 0
- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node c_[0, 100] * x
 - $h(c_{[0,100]} * x) = max([0,0.8]) = 0.8$
 - Example 3: incomplete note c_[-100, 0] * x



Evaluation		
out	Х	
0	0	
[-100, 0]	1	
[-200, 0]	2	
[-300, 0]	3	
[-400, 0]	4	

C_[-100, 0] * x

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
 - h(-100 * x) = 0
- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node c_[0, 100] * x
 - $h(c_{[0,100]} * x) = max([0,0.8]) = 0.8$
 - Example 3: incomplete note $c_{[-100,\,0]}$ * x
 - $h(c_{[-100, 0]} * x) = max([0, 0]) = 0$



Evaluation		
Х	out	
0	0	
1	[-100, 0]	
2	[-200, 0]	
3	[-300, 0]	
4	[-400, 0]	

c_[-100, 0] * x

- Define heuristic to be the highest accuracy possible for any completion
 - Example 1: completed node -100 * x
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- Problem: need to evaluate for a lot number of constants
 - -100, -99, -98, ..., 100
- Solution : use abstract interpretation
 - c * x (c is constant)
 - Split the interval for c
 - Example 2: incomplete node c_[0, 100] * x
 - $h(c_{[0,100]} * x) = max([0,0.8]) = 0.8$
 - Example 3: incomplete note $c_{[-100,\,0]}$ * x
 - $h(c_{[-100, 0]} * x) = max([0, 0]) = 0$
 - When heuristic 0.8 is the remaining highest, split [0, 100] to [0, 50], [50, 100] and repeat

Data	
Х	out
0	1
1	2
2	2 3
3	4
4	5

Eva	luation
Х	out
0	0
1	[-100, 0]
2	[-200, 0]
3	[-300, 0]
4	[-400, 0]

- Define heuristic to be the highest accuracy possible for any completion
 - Example 4: incomplete node



Data	
out	
1	
2	
3	
4	
5	

- Define heuristic to be the highest accuracy possible for any completion
 - Example 4: incomplete node
 - Annotate blank with $(-\infty, \infty)$



Data		
	X	out
	0	1
	1	2
	2	3
	3	4
	4	5

- Define heuristic to be the highest accuracy possible for any completion
 - Example 4: incomplete node
 - Annotate blank with $(-\infty, \infty)$
 - $h(x * _) = max([0, 0.8]) = 0.8$



Data	
Х	out
0	1
1	2
2	3
3	4
4	5

Evaluation		
Х	out	
0	0	
1	(-∞, ∞)	
2	(-∞, ∞)	
3	(-∞, ∞)	
4	(-∞, ∞)	

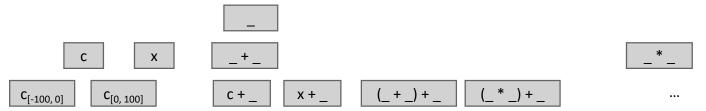
• In conclusion, the space for synthesis looks like below



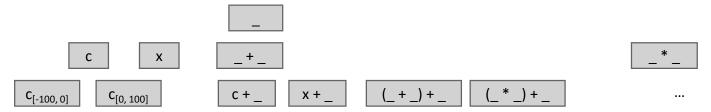
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• Each node is evaluated, and the one with highest heuristic will get expanded

- Test on domain specific languages (which means, language with grammar)
- Each domain specific language is for describing specific situation

NEAR DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

(returns true if the value is nonnegative)

Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

(classify whole data to one true / false)

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NEAR DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

(returns true if the value is nonnegative)

map(z_i - 50) z out (0, 100) (f, t)

Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

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out

(f, t)

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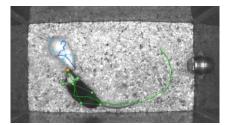
(returns true if the value is nonnegative)

map(
$$z_i$$
 - 50) z (0, 100)

Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

(classify whole data to one true / false)

(useful for labeling animal behavior per time)



Scene 1			Scene 2		
Time	Dist.	Label	Time	Dist.	Label
0	100	N	0	20	Υ
1	80	N	1	13	Υ
2	2	Υ	2	60	N
3	33	Υ	3	94	N
4	60	N	4	100	N

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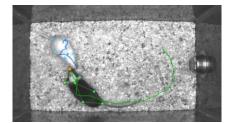
Z	out
(0, 100)	(f, t)

Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

(classify whole data to one true / false)

(fun x
$$\rightarrow$$
 x₂ > 0) \land (fun x \rightarrow x₁ + x₂ \ge 100) $\frac{z}{(0, 100)}$ out

(useful for labeling animal behavior per time)



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Time	Dist.	Label	Time	Dist.	Label
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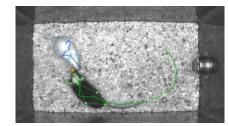
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Z	out
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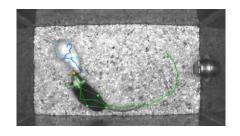
(useful for clarifying if object satisfied certain condition)



- Neurosymbolic program synthesis benchmarks
 - features (numbers) are extracted from video by neural networks

NEAR DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)





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NEAR DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

CRIM13 dataset

Goal: clarify when mice sniff each other

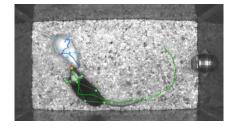
Quivr DSL (for trajectory-per-time $\mathbf{x} \in (\mathbb{R}^n)^*$)

MABe22 dataset

Goal: clarify if three mice interact

YTStreams dataset

Goal: clarify if car caught in traffic camera makes a right turn





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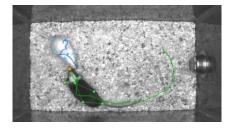
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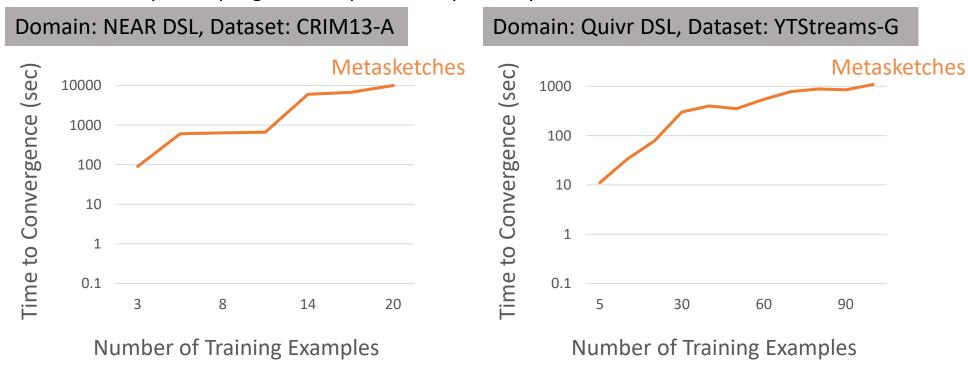


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- Q2. In constrained time, how well does approach's answer evaluate, compared to BFS?

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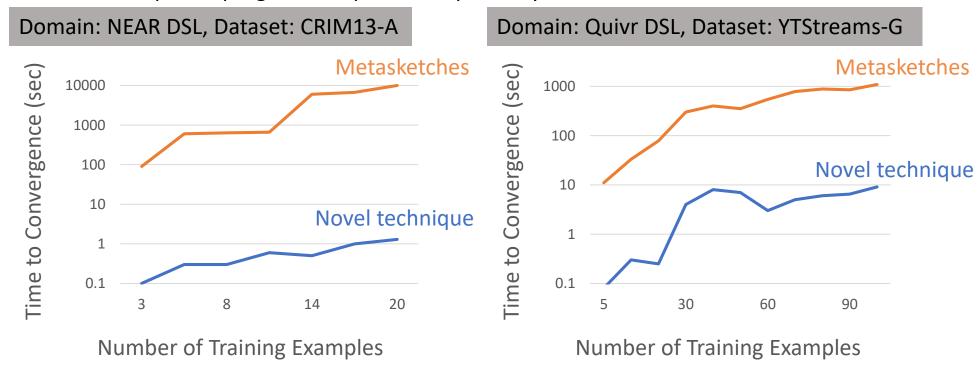
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Time to find optimal program and prove its optimality



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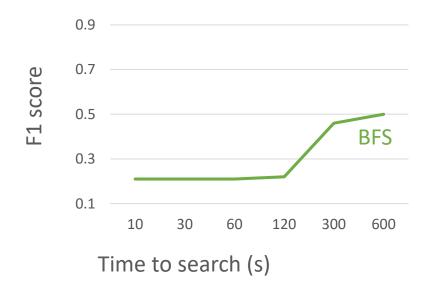


- Q2. In constrained time, how well does approach's answer evaluate, compared to BFS?
- A2. On most tasks, novel approach achieves higher F1 scores more quickly than BFS.

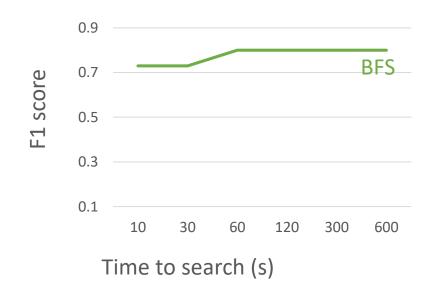
- Q2. In constrained time, how well does approach's answer evaluate, compared to BFS?
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Best F1 score found

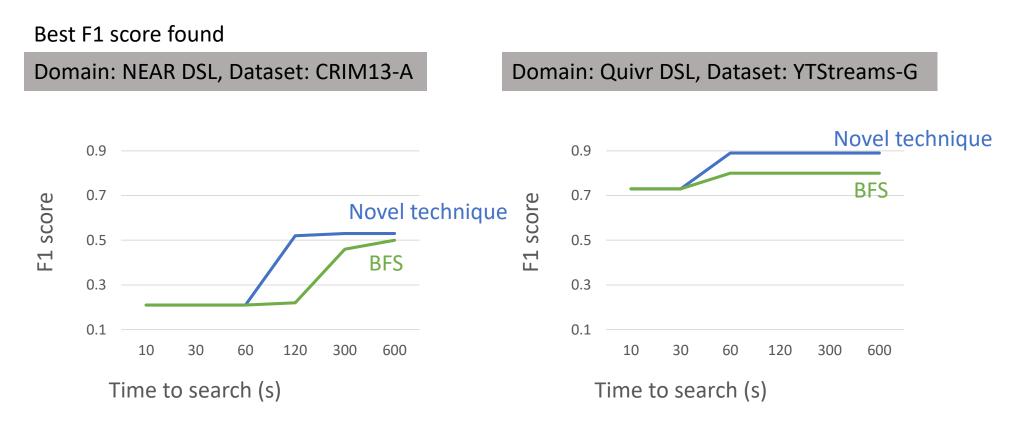
Domain: NEAR DSL, Dataset: CRIM13-A



Domain: Quivr DSL, Dataset: YTStreams-G



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Conclusion

- Synthesizing program is a powerful, interpretable tool for classifying behaviors
- But for fuzzy and new data, traditional techniques are not available
- The novel approach utilizes A* search with abstract interpretation for scalability
- Experimental evaluation demonstrates superior results from other approaches