

Abstract Interpretation of Fixpoint Iterators with Applications to Neural Networks

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Outline

- **Background of Deep Learning**
- **DEQs**
- **Mono DEQs**
- **Combine Abstraction interpolation with DEQs**
- **Result**

Outline

Background of Deep Learning

DEQs

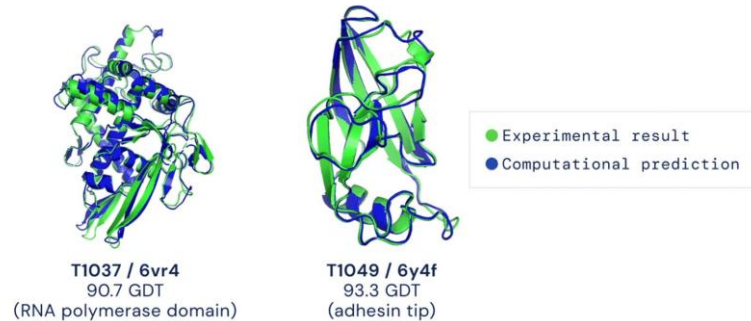
Mono DEQs

Combine Abstraction interpolation with DEQs

Result

The deep learning revolution (recent examples)

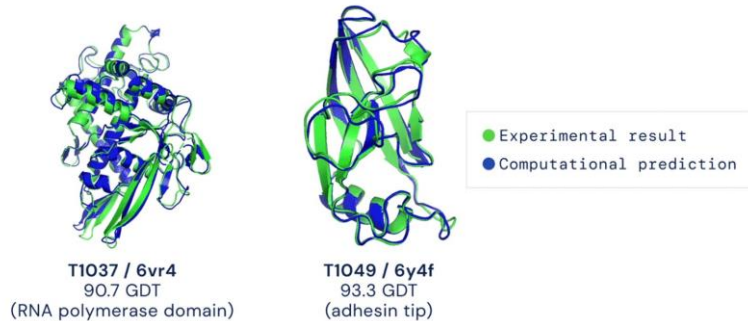
Median Free-Modelling Accuracy



AlphaFold: Jumper et al., 2021

The deep learning revolution (recent examples)

Median Free-Modelling Accuracy



Explaining a joke

Prompt

Explain this joke:

Joke: Did you see that Google just hired an eloquent whale for their TPU team? It showed them how to communicate between two different pods!

Model Response

Prediction: TPUs are a type of computer chip that Google uses for deep learning. A "pod" is a group of TPUs. A "pod" is also a group of whales. The joke is that the whale is able to communicate between two groups of whales, but the speaker is pretending that the whale is able to communicate between two groups of TPUs.

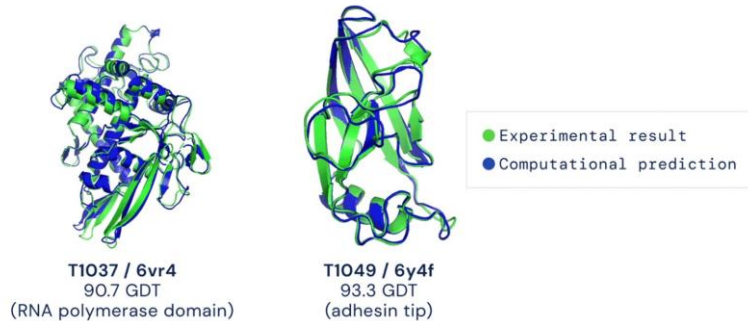
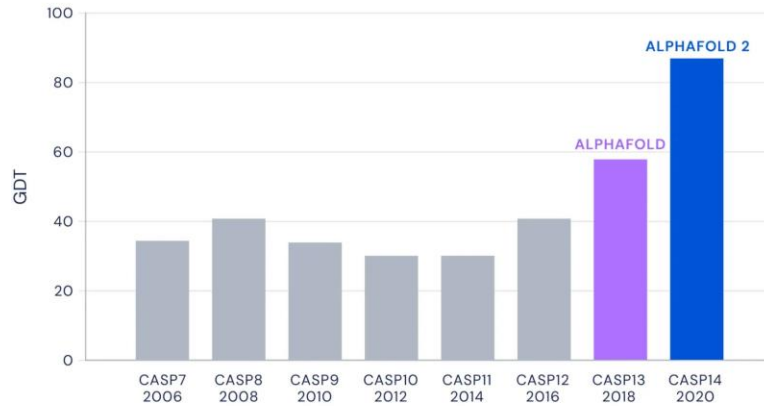
PaLM explains an original joke with two-shot prompts.

PaLM: Chowdhery et al., 2022

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vibrant portrait painting of Salvador Dalí with a robotic half face

a shiba inu wearing a beret and black turtleneck



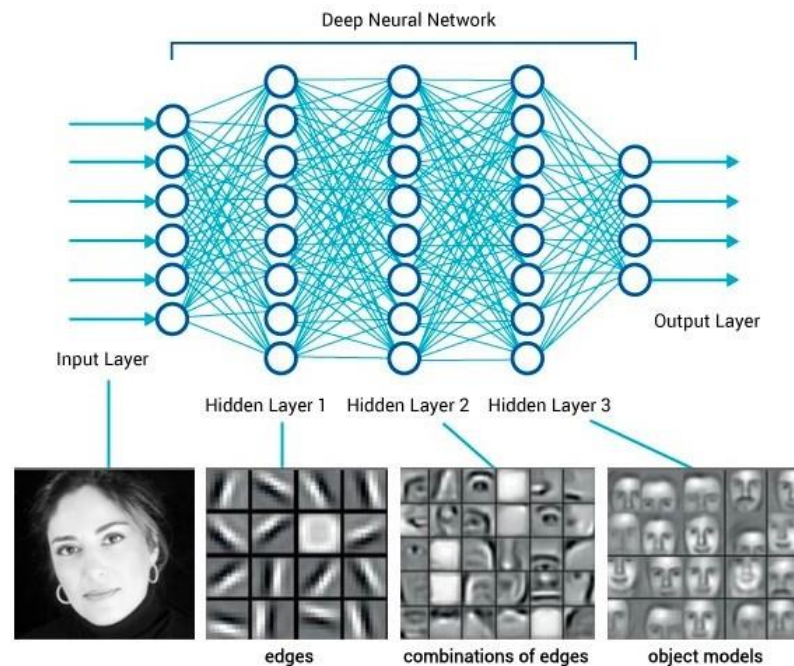
an espresso machine that makes coffee from human souls, artstation

panda mad scientist mixing sparkling chemicals, artstation

DALL-E 2: Ramesh et al., 2022

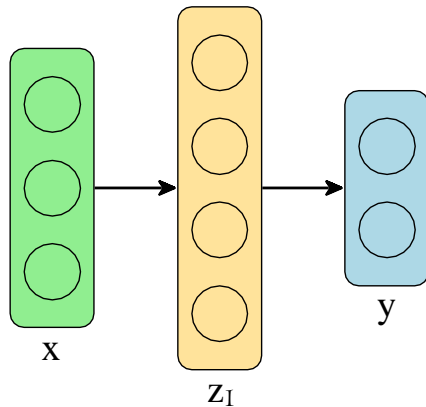
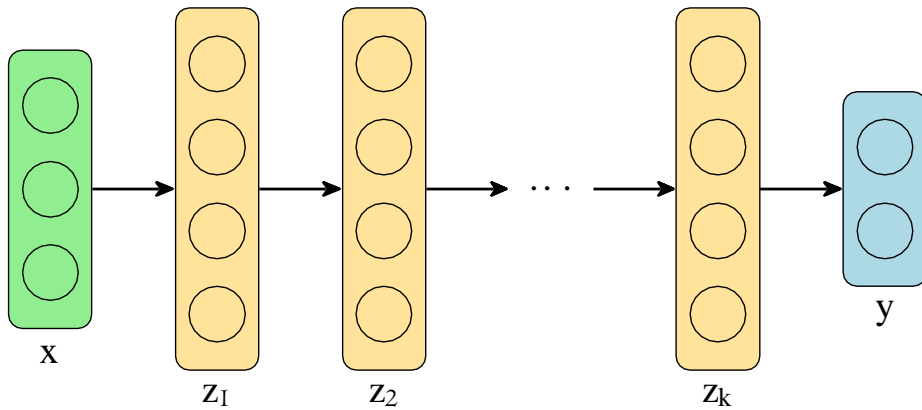
Deep Learning

The story we all tell: deep learning algorithms build hierarchical models of input data, where the earlier layers create “simple” features and hidden layers create high-level abstractions of the data.



Can we lightweight the model?

Deep Learning



DEQs can replace traditional depth in deep networks with a single (implicit) layer.

- Simpler architectural design.
- Vastly reduced memory requirements.
- Matches or exceeds accuracy of comparable fixed-depth networks.

Offers a new perspective on what deep networks are “really” computing.

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Mono DEQs

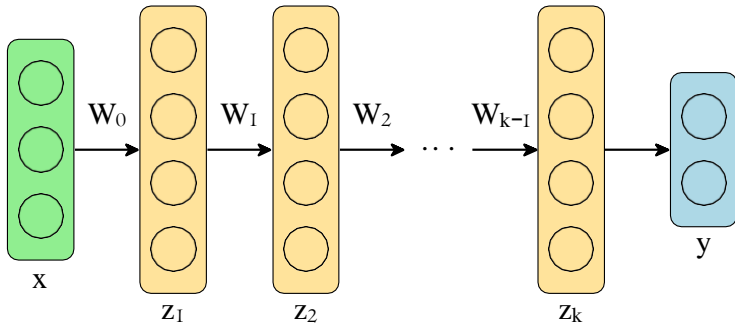
Combine Abstraction interpolation with DEQs

Result

From deep network to DEQs

Traditional network

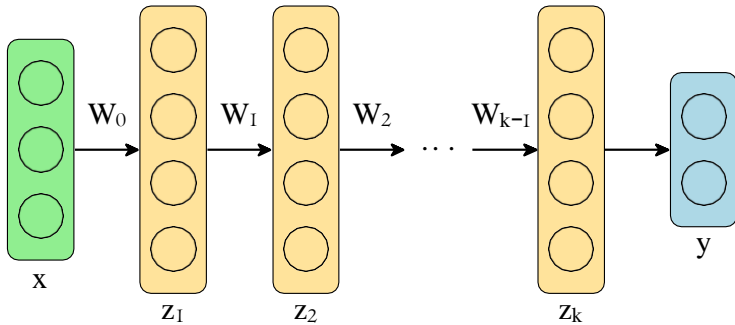
$$x_{i+1} = \sigma(W_i x_i + b_i)$$



From deep network to DEQs

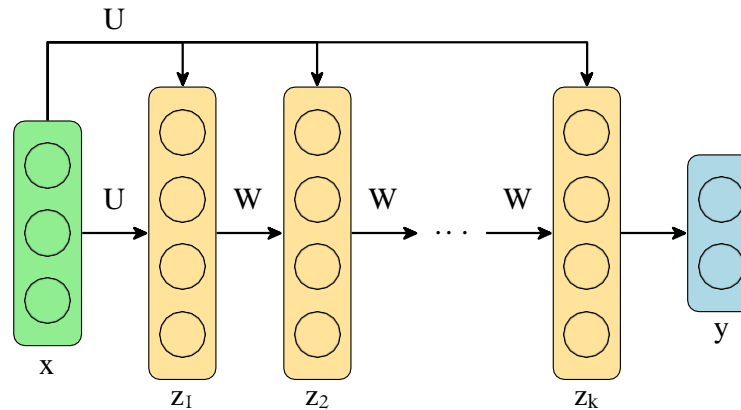
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Weight-tied, input-injected network

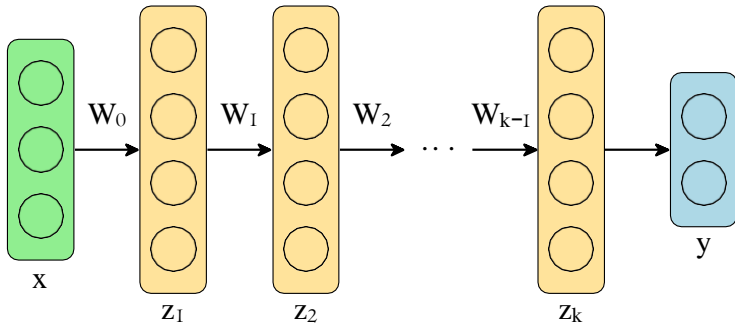
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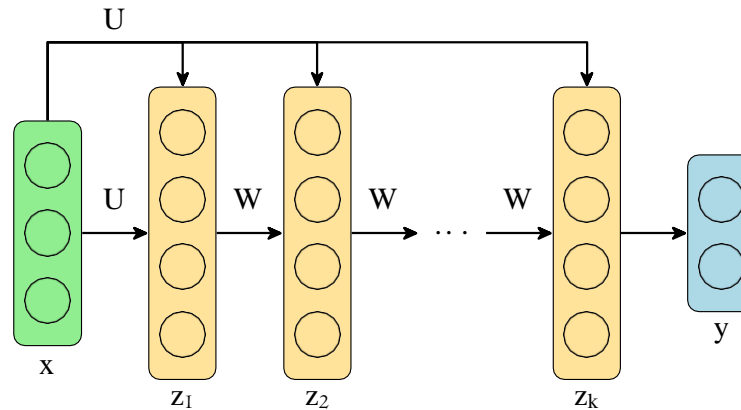
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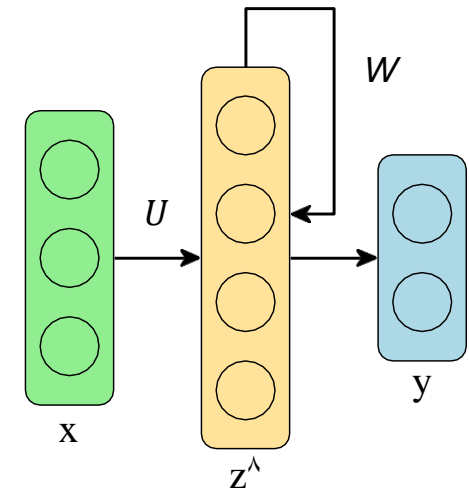
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Deep Equilibrium (DEQ) model

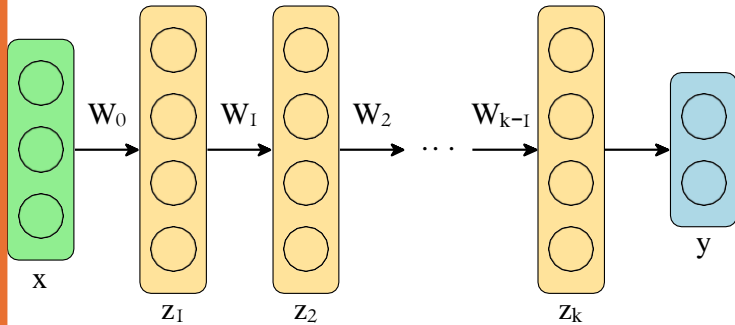
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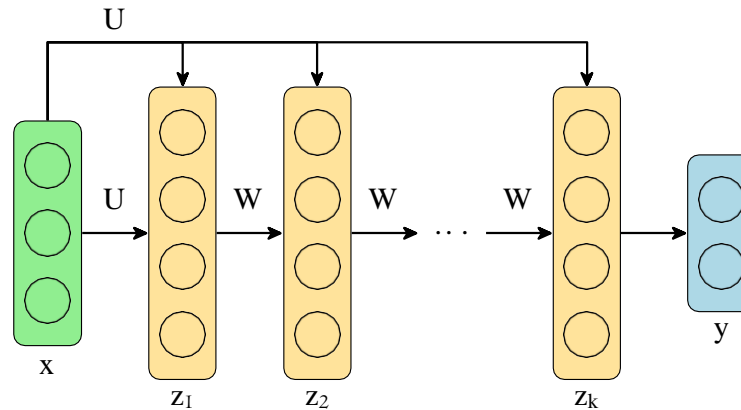
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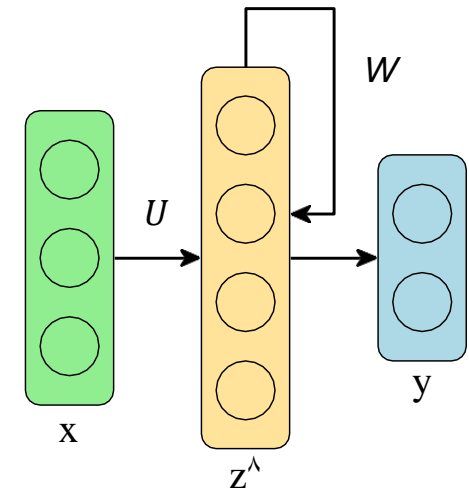
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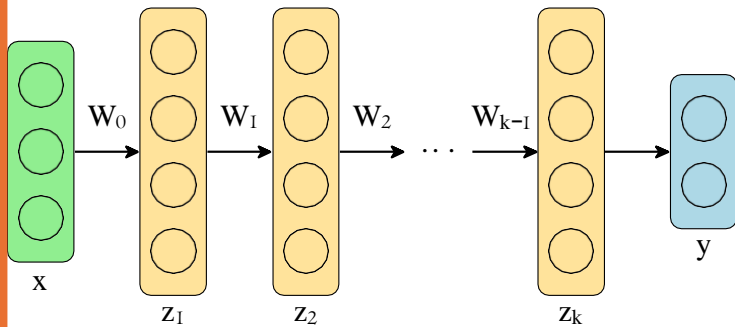


Threshold based

From deep network to DEQs

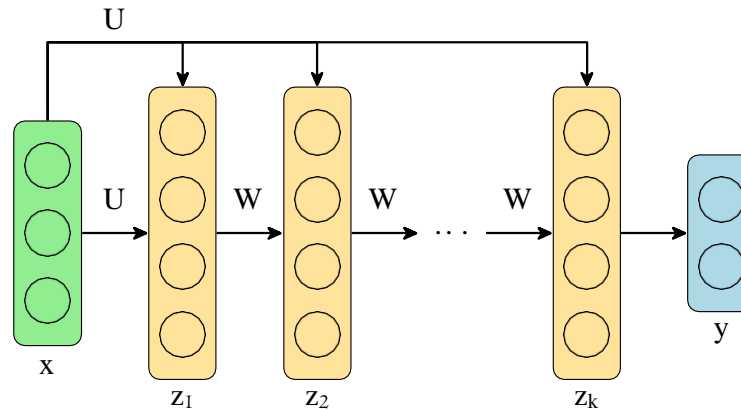
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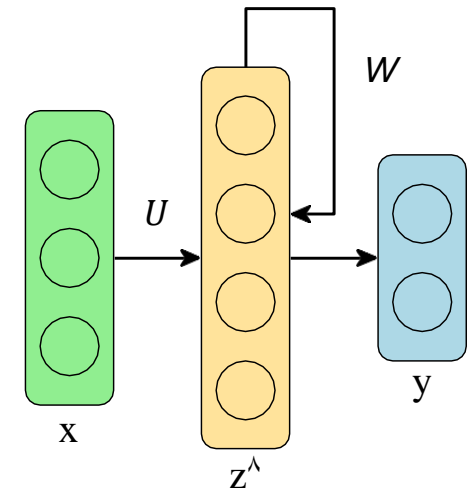
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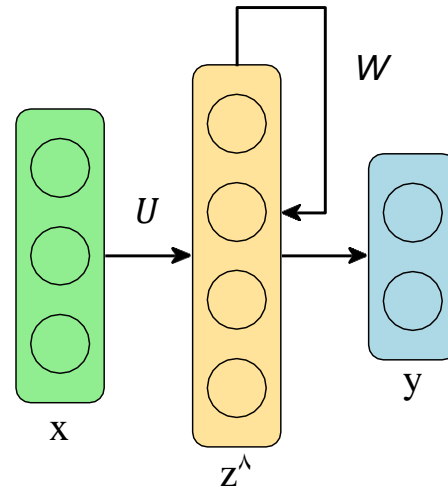
Fixed point based

Fixed point in DEQ

- Fixed point in AI
The End of learning
Consistence input and output

Learning and adjust the weight
until
 $F(x) = y$ always consistence.

Deep Equilibrium (DEQ) model



Outline

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DEQs

Mono DEQs

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Result

Mono DEQs

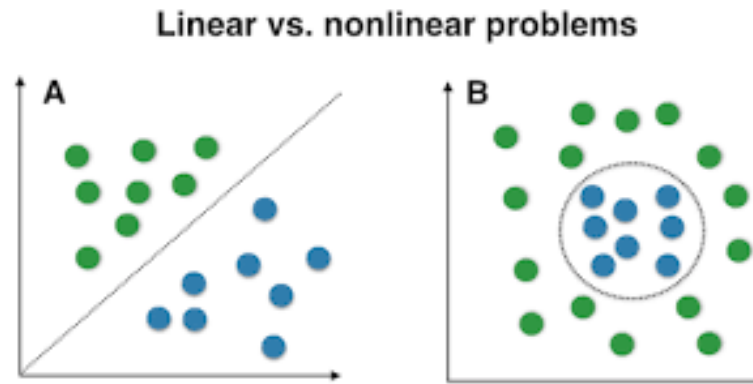
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Mono DEQs

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Mono DEQs

- What can we say about the fixed point?
- Finding linear model's fixed point is easy.
- In general, a non-linear dynamical system can be difficult to establish existence, uniqueness and stability of the fixed point iteration.

Finding fixed point in non-linear is hard.



Mono DEQs

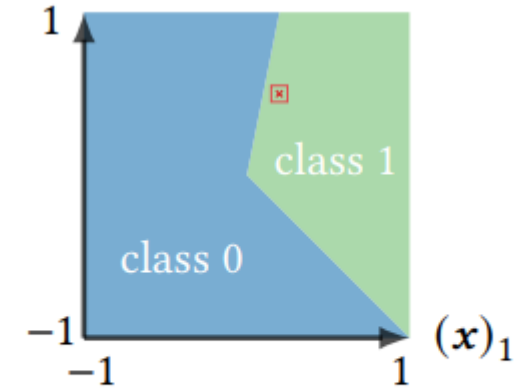
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Mono DEQs

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- Prior Research (Kleene) introduced abstract interpretation to get fixed point

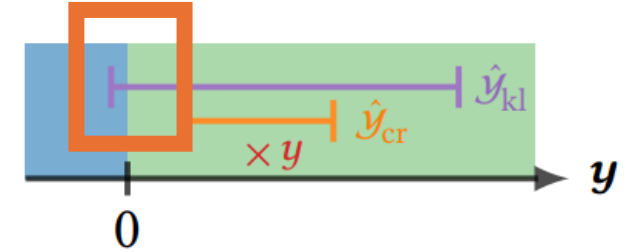
Mono DEQs

- Finding mono DEQs's fixed point is hard
- Prior Research (Kleene) introduced abstract interpretation to get fixed point
 - slow & inaccurate



(a) Classification of concrete inputs in $[-1, 1]^2$ by \hat{f} .

Inaccurate



(c) Abstractions of the output y and resulting classification.

Mono DEQs

- Finding mono DEQs's fixed point is hard
- Prior Research (Kleene) introduced abstract interpretation to get fixed point
 - slow & inaccurate
- Domain are not suitable for DEQs

	Iteration	Inclusion	Precision
Box	✓	✓	✗
(Hybrid) Zonotope	✗	✗	✓
Polyhedra	✗	✗	?
CH-Zonotope	✓	✓	✓

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Combine abstraction interpretation with DEQs

Result

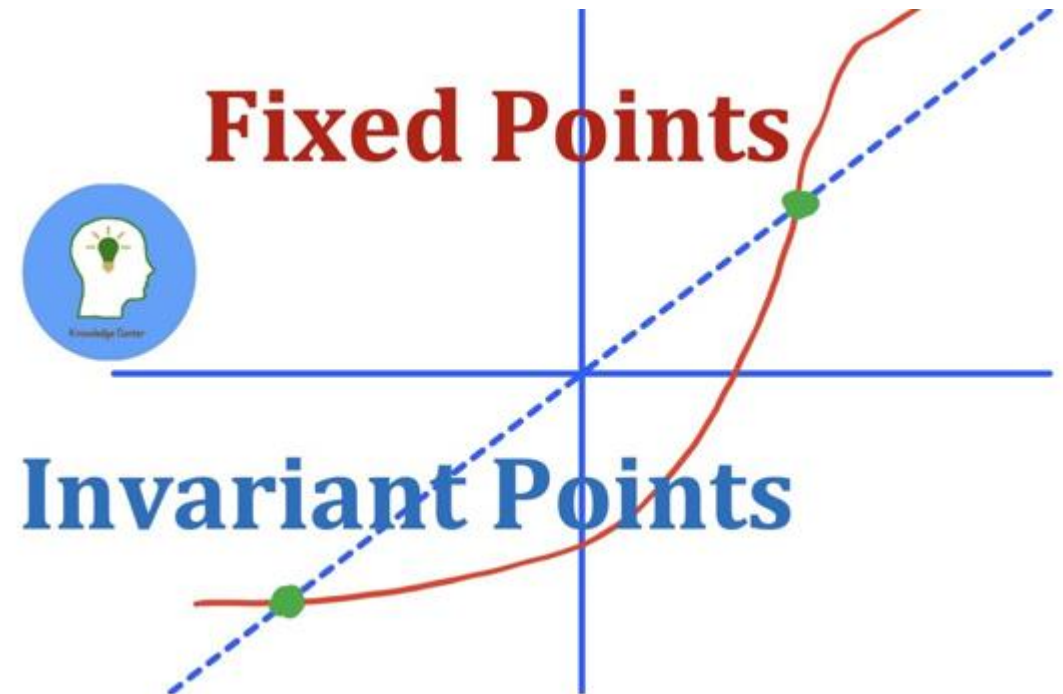
Abstract interpretation in Mono DEQs

They want to evaluate and know the fixed point

Apply Abstract interpretation

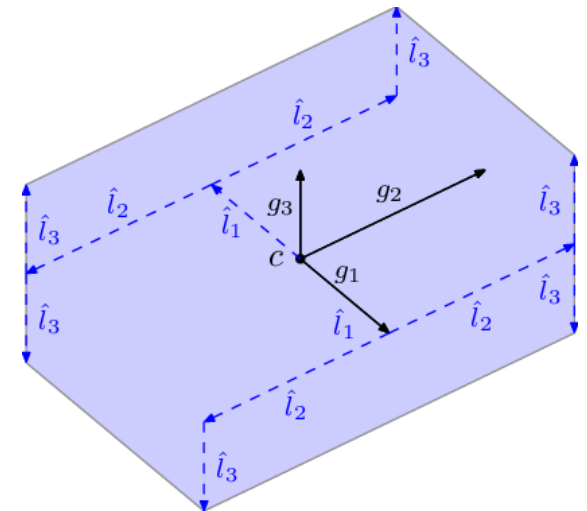
→ Static analysis technique

→ The fixed point was predicted using abstract interpretation.



Abstract interpretation in Mono DEQs

- They apply the zonotope for fixed point.
 - But the neural network is complex.
- Introduce **CH-zonotope**.
 - Apply new domain based on zonotope
 - When dealing with complex problems, it is advantageous.
 - efficient propagation of abstract elements
 - fast (abstract) inclusion checks



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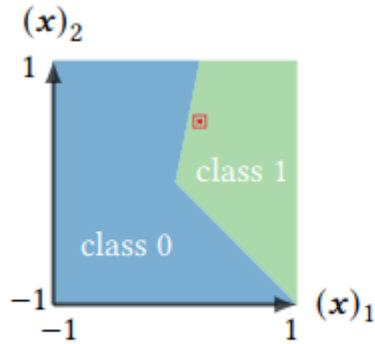
DEQs

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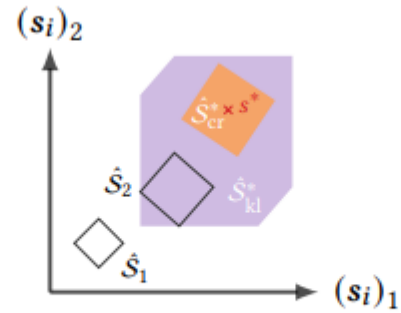
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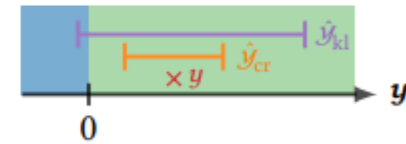
Result



(a) Classification of concrete inputs in $[-1, 1]^2$ by h .



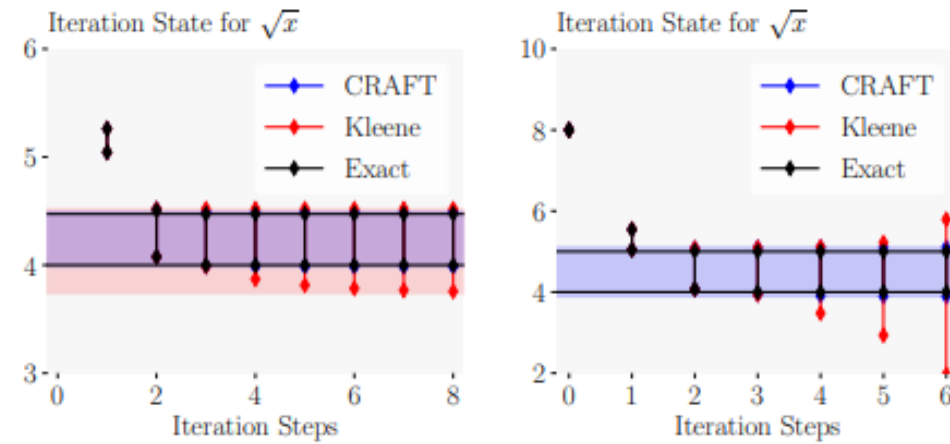
(b) Abstractions of iteration steps s_i .



(c) Abstractions of the output y and resulting classification.

The tool they developed, named Craft,
exhibits better performance than the existing Kleene iteration.

Result



CRAFT provides more accurate and stable convergence results compared to Kleene iteration

Conclusion

- Interpretation the range of correct fixed points with CH-zonotope
 - Convergence Detection
 - Early Stopping Criterion
 - Prevention of Overfitting
 - Resource Efficiency

Thank you