# Towards a Verified Range Analysis for JavaScript JITs

Steve Gustaman

Original paper by Brown et al. PLDI 2020

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  - Crucial for browser to be fast

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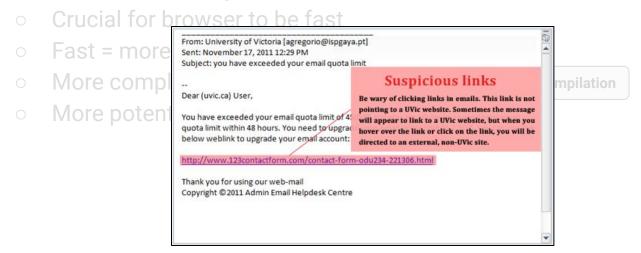
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Browser JITs <u>mis</u>compile and execute web JS code



https://www.uvic.ca/systems/status/features/phishing.php

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  - More potential bugs = more security problems

- Browser JITs <u>can be forced to mis</u>compile and execute <u>malicious</u> web JS code
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  - More potential bugs = more security problems

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- Every JIT compilation is security critical

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JS code

https://www.theregister.com/2019/08/09/coinbase\_pwned/

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Using those two vulnerabilities to achieve arbitrary code execution, the attacker's shellcode issued a curl command to download and run the stage-one implant, a <u>Netwire variant</u>. Used for reconnaissance and credential theft on victims' machines, the malicious code was detected by Coinbase at this point based on unusual behavior, specifically Firefox spawning a shell.

The stage-one payload then transitioned to a stage two implant, identified by Martin as a variant of the Mokes malware family. It's a remote access trojan (RAT) and was operated under direct human control. Martin speculates that the attackers moved to stage two when they believed they had compromised a target of value.

Once aware of the hack, Coinbase's security team collected data artifacts related to the break-in, revoked affected credentials, and contacted Mozilla's security team, which

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How do JIT bugs happen?

Javascript is memory-safe language

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x = x+1

arr = [0,1,2,3,4]

return arr[x]
```

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\begin{array}{c} x=[0,3] \\ \hline x=[1,4] \\ \hline \end{array}
\begin{array}{c} x=\ldots \\ x=x+1 \\ \text{arr}=[0,1,2,3,4] \\ \text{if } (x>=\text{arr.len}) \\ \text{return undefined} \\ \text{return } \underline{\text{arr}[x]} \\ \end{array}
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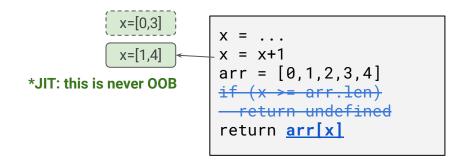
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- What will happen if JIT engine range analysis is wrong?

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\*buggy range analysis

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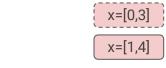
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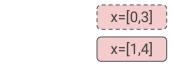
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Browser JITs can be forced to miscompile and execute malicious web JS code

JIT engine range applyois' correctness is critical.

How to verify JIT engine range analysis' correctness?

# VeRA:

a system for verifying the range analysis pass in browser JIT compilers

• Goal: Verify correctness of JIT engine range analysis

• **Goal**: In actual JS semantics, is it possible to get a value outside of JIT engine's range?

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- 3. Ask SMT solver: is it possible to get value from (1) outside of (2)?

```
// exponent computation have to be over-estimations of
// the Int32 this over approximation is rectified.
MOZ INIT OUTSIDE CTOR int32 t lower;
                                                                           utside of JIT
MOZ INIT OUTSIDE CTOR int32 t upper ;
MOZ INIT OUTSIDE CTOR bool hasInt32LowerBound;
MOZ INIT OUTSIDE CTOR bool hasInt32UpperBound;
MOZ INIT OUTSIDE CTOR FractionalPartFlag canHaveFractionalPart : 1;
MOZ INIT OUTSIDE CTOR NegativeZeroFlag canBeNegativeZero : 1;
MOZ INIT OUTSIDE CTOR uint16 t max exponent ;
                                                                           tside of (2)?
// Any symbolic lower or upper bound computed for this term.
const SymbolicBound* symbolicLower ;
```

integer, float, NaN, inf, 0, -0, ...

```
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MOZ INIT OUTSIDE CTOR int32 t lower;
                                                                           outside of JIT
                                           → x=[lower,upper]
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                               int32 t 1 = op -> lower;
                         109
                               int32 t u = op->upper ;
                         1110
                               FractionalPartFlag canHaveFractionalPart = op->canHaveFractionalPart;
                         1112
                               // Abs never produces a negative zero.
                         1113
   // Any symbolic lower 1114
                               NegativeZeroFlag canBeNegativeZero = ExcludesNegativeZero;
                         1115
   const SymbolicBound*
                         1116
                               return new (alloc) Range(
                                   std::max(std::max(int32 t(0), 1), u == INT32 MIN ? INT32 MAX : -u), true,
                         1117
                                   std::max(std::max(int32 t(0), u), 1 == INT32 MIN ? INT32 MAX : -1),
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flow function for: x = abs(y)

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flow function for: x = abs(y)

 Goal: In actual JS semantics, is it possible to get a value outside of JIT engine's range?

#### Key Ideas:

- Encode actual JS semantics in SMT
- 2. Encode JIT engine range analysis routine in SMT
- 3. Ask SMT solver: is it possible to get value from (1) outside of (2)?

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#### Key Ideas:

- Encode actual JS semantics in SMT
- Automatically generate JIT engine range analysis routine in SMT
- 3. Ask SMT solver: is it possible to get value from (1) outside of (2)?

round up value to nearest integer

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- 2. Automatically generate JIT engine range analysis routine in SMT
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#### Encode actual JS semantics in SMT

- Trivial: use SMT ceil operator
- 2. Automatically generate JIT engine range analysis routine in SMT
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disallows several construct e.g. loops

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#### Range\* Range::ceil(TempAllocator& alloc, const Range\* op) Range\* copy = new(alloc) Range(\*op); // We need to refine max exponent because ceil may have incremente // If we have got int32 bounds defined, just deduce it using the de // Else we can just increment its value, // as we are looking to maintain an over estimation. if (copy->hasInt32Bounds()) copy->max exponent = copy->exponentImpliedByInt32Bounds(); else if (copy->max exponent < MaxFiniteExponent) copy->max\_exponent\_++; copy->canHaveFractionalPart = ExcludesFractionalParts; copy->assertInvariants(); return copy;

#### ceil range flow function

```
e analysis routine in SMT ic in VeRA C++
oblem
le from (1) outside of (2)?
```

#### Firefox original implementation (C++)

#### ceil range flow function

```
Range*
Range::ceil(TempAllocator& alloc, const Range* op)
    Range* copy = new(alloc) Range(*op);
    // We need to refine max exponent because ceil may have incre
    // If we have got int32 bounds defined, just deduce it using
    // Else we can just increment its value,
    // as we are looking to maintain an over estimation.
    if (copy->hasInt32Bounds())
        copy->max exponent = copy->exponentImpliedByInt32Bounds()
    else if (copy->max exponent < MaxFiniteExponent)
        copy->max_exponent_++;
    copy->canHaveFractionalPart = ExcludesFractionalParts;
    copy->assertInvariants();
    return copy;
```

```
range copy = op;
// missing fract check
if (hasInt32Bounds(copy)) {
  copy.maxExponent = exponentImpliedByInt32Bounds(copy);
} else if (copy.maxExponent < maxFiniteExponentS) {</pre>
  copy.maxExponent += (uint16 t) 1;
copy.canHaveFractionalPart = excludesFractionalPartsS;
return copy;
```

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#### Rewritten (VeRA C++)

range ceil(range const& op) {

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    - Subset of C++ specific for this problem
  - Vera C++  $\rightarrow$  IR (SSA)  $\rightarrow$  SMT
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    - value = 5
    - range.lower = 1; range.upper = 10;

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      inRange = true

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      inRange = false

## $inRange(R, v) \triangleq$

$$R.\exp < e_{INF} \implies \neg isInf(v)$$

$$\land R. \exp \neq e_{INF_OR_NAN} \implies \neg isNaN(v)$$

$$\wedge \neg R$$
.canBeNegZero  $\Longrightarrow v \neq -0.0$ 

$$\wedge \neg R$$
.canHaveFraction  $\implies$  round $(v) = v$ 

$$\land R.$$
hasInt32LowerBound  $\implies$  (isNaN( $\upsilon$ )  $\lor \upsilon \ge R.$ lower)

$$\land R.$$
hasInt32UpperBound  $\Longrightarrow$  (isNaN $(v) \lor v \le R.$ upper)

$$\land R.\exp \ge \exp Of(v)$$

## $wellFormed(R) \triangleq$

```
R.lower \geq JS INT MIN \wedge R.lower \leq JS INT MAX
\land R.upper \ge JS_INT_MIN \land R.upper \le JS_INT_MAX
\land \neg R.hasInt32LowerBound \implies R.lower = JS_INT_MIN
\land \neg R.hasInt32UpperBound \implies R.upper = JS_INT_MAX
\land R.canBeNegZero \implies contains(0, R)
\land (R.\exp = e_{INF} \lor R.\exp = e_{INF_{OR}} \land AN \lor R.\exp \le 1023)
\land (R.\text{hasInt32LowerBound} \land R.\text{hasInt32UpperBound})
        \implies R.\exp = \exp Of(\max(|R.lower|, |R.upper|))
\land R.hasInt32LowerBound \implies R.exp \ge \exp Of(R.1ower)
\land R.hasInt32UpperBound \implies R.exp \ge \exp Of(R.upper)
```

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- JS semantics in SMT
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| Verification SMT |  |  |
|------------------|--|--|
|                  |  |  |
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|                  |  |  |

- JS semantics in SMT
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#### **Verification SMT**

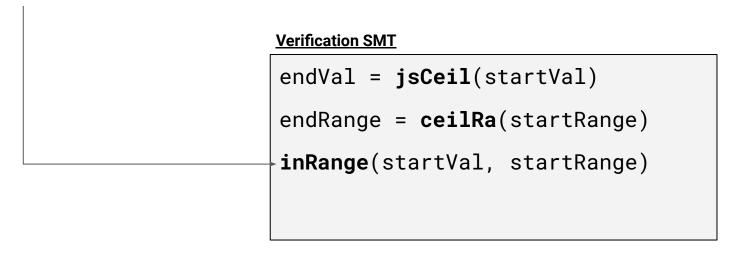
```
endVal = jsCeil(startVal)
```

- JS semantics in SMT
- JIT engine range analysis routine in SMT
- inRange(value, range)

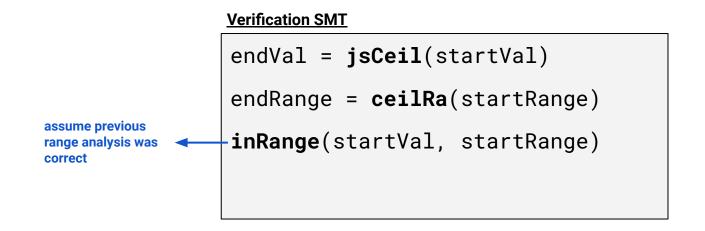
#### **Verification SMT**

```
endVal = jsCeil(startVal)
endRange = ceilRa(startRange)
```

- JS semantics in SMT
- JIT engine range analysis routine in SMT
- inRange(value, range)



- JS semantics in SMT
- JIT engine range analysis routine in SMT
- inRange(value, range)



- JS semantics in SMT
- JIT engine range analysis routine in SMT
- inRange(value, range)

#### **Verification SMT**

```
endVal = jsCeil(startVal)
endRange = ceilRa(startRange)
inRange(startVal, startRange)
NOT inRange(endVal, endRange) ??
```

- JS semantics in SMT
- JIT engine range analysis routine in SMT
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```
verification SMT

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# Implementation

# **Implementation**

- Test VeRA-rewritten flow functions with Firefox's test suites
  - Passed all ~147k tests

# **Implementation**

- Test VeRA-rewritten flow functions with Firefox's test suites
  - Passed all ~147k tests
- Verify 21 top-level Firefox range analysis flow functions using VeRA

| Operation | R1       | R2 | R3 | R4  | R5.i32   | R5.double | R6.i32   | R6.double | R7  | W1 | W2 | W3       | W4       | Undef |
|-----------|----------|----|----|-----|----------|-----------|----------|-----------|-----|----|----|----------|----------|-------|
| add       | 15       | 2  | 5  | 386 | 2        | <u></u>   | 2        | ∞         | 21  | 2  | 1  | 2        | 80       | 1     |
| sub       | 13       | 2  | 11 | 445 | 8        | $\infty$  | 5        | $\infty$  | 14  | 2  | 2  | 2        | 78       | 1     |
| and*      | -        | -  | -  | -   | 2        | -         | 1        | -         | -   | -  | -  | -        | -        | 1     |
| or*       | -        | -  | -  | -   | 2        | -         | 2        | -         | -   | -  | -  | -        | _        | 1     |
| xor*      | -        | -  | -  | -   | 2        | -         | 2        | -         | -   | -  | 1- | 1-       | -        | 1     |
| not*      | -        | -  | -8 | -   | 2        | -         | 1        | -         | -   | -  | -  | 15-      | -        | 1     |
| mul       | 92       | 65 | 22 | 362 | $\infty$ | $\infty$  | $\infty$ | $\infty$  | 94  | 4  | 4  | 4        | $\infty$ | 11    |
| lsh*      | _        | _  |    | _   | 1        | -         | 1        | -         | _   | -  | _  | -        | _        | 1     |
| rsh*      | -        | -  |    | -   | 1        | -         | 1        | -         | -   | -  | 1- | 1        | -        | 1     |
| ursh*     | -        | _  | -  | -   | X        | -         | X        | -         | -   | -  | -  | -        | -        | 1     |
| lsh'*     | _        | _  | _  | -   | 1        | -         | 1        | -         | -   | -  | -  | _        | -        | 1     |
| rsh'*     |          | _  | -  | -   | 1        | -         | 1        | -         | -   | -  | -  | -        | -        | 1     |
| ursh'*    | _        | =  | -  | -   | X        | _         | X        | -         | -   | -  | -  | -        | -        | 1     |
| abs       | 1        | 1  | 1  | 5   | 1        | $\infty$  | 1        | 224       | 1   | 1  | 1  | 4        | $\infty$ | 1     |
| min       | 2        | 20 | 2  | 2   | 5        | 224       | 2        | $\infty$  | 3   | 2  | 1  | 2        | $\infty$ | 1     |
| max       | 3        | 17 | 2  | 2   | 15       | $\infty$  | 2        | $\infty$  | 4   | 3  | 2  | 12       | $\infty$ | 1     |
| floor     | 4        | 2  | 1  | 5   | 1        | 146       | 1        | 9         | 54  | 1  | 1  | 5        | $\infty$ | 1     |
| ceil      | $\infty$ | 1  | X  | 8   | 1        | 5         | 1        | 9         | 266 | 1  | 1  | $\infty$ | $\infty$ | 1     |
| sign      | 1        | 1  | 1  | 1   | 1        | 2         | 1        | 2         | 1   | 1  | 1  | 1        | 2        | 1     |

| Operation | R1       | R2 | R3 | R4  | R5.i32   | R5.double | R6.i32 | R6.double                  | R7        | W1     | W2    | W3       | W4       | Undef   |    |
|-----------|----------|----|----|-----|----------|-----------|--------|----------------------------|-----------|--------|-------|----------|----------|---------|----|
| add       | 15       | 2  | 5  | 386 | 2        | $\infty$  | 2      | 00                         | 21        | 2      | 1     | 2        | 80       | 1       |    |
| sub       | 13       | 2  | 11 | 445 | 8        | $\infty$  | 5      | 000                        | 14        | 2      | 2     | 2        | 78       | 1       |    |
| and*      | -        | -  | -  | -   | 2        | -         | 1      | -                          | -         | -      | -     | -        | -        | 1       |    |
| or*       | _        | -  | _  | -   | 2        | -         | 2      | lunchia laft               | anana     | nd io  | uin+2 | 2 no+    | in+22    | but for | 10 |
| xor*      | -        | -  | -  | -   | 2        | -         | .,     | ursh's left<br>analysis we |           |        |       |          |          |         |    |
| not*      | -        | -  | -8 | -   | 2        | -         | 1      | here that t                |           |        |       |          |          |         | a  |
| mul       | 92       | 65 | 22 | 362 | $\infty$ | $\infty$  | ∞      | nere that t                | iic i aii | ge mas | direc | ady be   | cii daj  | ustcu   |    |
| lsh*      | _        | _  | _  | -   | 1        | -         | 1      | -                          | _         | _      | _     | _        | _        | 1       |    |
| rsh*      | -        | -  |    | -   | 1        | -         | 1      | -                          | -         | -      | 1-    | 1-       | -        | 1       |    |
| ursh*     | _        | _  | -  | -   | X        | -         | X      | -                          | -         | -      | -     | -        | -        | 1       |    |
| lsh'*     | _        | -  | _  | -   | 1        | -         | 1      | -                          | -         | -      | -     | -        | _        | 1       |    |
| rsh'*     | -        | _  | _  | -   | 1        | -         | _1_    | -                          | _         | _      | -     | -        | -        | 1       |    |
| ursh'*    | -        | _  | -  | -   | X        | -         | X      | -                          | -         | -      | -     | -        | -        | 1       |    |
| abs       | 1        | 1  | 1  | 5   | 1        | $\infty$  | 1      | 224                        | 1         | 1      | 1     | 4        | $\infty$ | 1       |    |
| min       | 2        | 20 | 2  | 2   | 5        | 224       | 2      | 00                         | 3         | 2      | 1     | 2        | $\infty$ | 1       |    |
| max       | 3        | 17 | 2  | 2   | 15       | $\infty$  | 2      | 00                         | 4         | 3      | 2     | 12       | $\infty$ | 1       |    |
| floor     | 4        | 2  | 1  | 5   | 1        | 146       | 1      | 9                          | 54        | 1      | 1     | 5        | $\infty$ | 1       |    |
| ceil      | $\infty$ | 1  | X  | 8   | 1        | 5         | 1      | 9                          | 266       | 1      | 1     | $\infty$ | $\infty$ | 1       |    |
| sign      | 1        | 1  | 1  | 1   | 1        | 2         | 1      | 2                          | 1         | 1      | 1     | 1        | 2        | 1       |    |

| Operation | R1       | R2 | R3 | R4    | R5.i32       | R5.double | R6.i32       | R6.double | R7  | W1 | W2 | W3       | W4       | Undef |
|-----------|----------|----|----|-------|--------------|-----------|--------------|-----------|-----|----|----|----------|----------|-------|
| add       | 15       | 2  | 5  | 386   | 2            | 00        | 2            | 00        | 21  | 2  | 1  | 2        | 80       | 1     |
| sub       | 13       | 2  | 11 | 445   | 8            | $\infty$  | 5            | $\infty$  | 14  | 2  | 2  | 2        | 78       | 1     |
| and*      | -        | -  | -  | -     | 2            | -         | 1            | -         | -   | -  | -  | -        | -        | 1     |
| or*       | -        | _  | -  | -     | 2            | -         | 2            | -         | -   | -  | -  | -        | -        | 1     |
| xor*      | _        | -  | -  | -     | 2            | _         | 2            | -         | -   | -  | -  | -        | _        | 1     |
| not*      | -        | -  | -8 | -     | 2            |           | 1            | -         | -   | -  | -  | -        | -        | 1     |
| mul       | 92       | 65 | 22 | 362   | $\infty$     | $\infty$  | $\infty$     | $\infty$  | 94  | 4  | 4  | 4        | $\infty$ | 11    |
| lsh*      | _        | _  | _  | -     | 1            | -         | 1            | -         | _   | -  | _  | _        | _        | 1     |
| rsh*      | -        | =: | -  | -     | 1            | _         | 1            | -         | -   | -  | -  | -        | -        | 1     |
| ursh*     | _        | -  | -  | -     | $\mathbf{X}$ | -         | X            | -         | -   | -  | -  | -        | _        | 1     |
| lsh'*     | _        | -  | _  | -     | 1            | -         | 1            | -         | _   | -  | -  | _        | _        | 1     |
| rsh'*     | _        | -  | _  | _     | 1            | -         | 1            | -         | _   | -  | -  | -        | -        | 1     |
| ursh'*    | -        | -  | -  | -     | X            | -         | $\mathbf{X}$ | -         | -   | -  | -  | -        | -        | 1     |
| abs       | 1        | 1  | 1  | 5     | 1            | $\infty$  | 1            | 224       | 1   | 1  | 1  | 4        | $\infty$ | 1     |
| min       | 2        | 20 | 2  | 2     | 5            | 224       | 2            | $\infty$  | 3   | 2  | 1  | 2        | $\infty$ | 1     |
| max       | 3        | 17 | 2  | 2     | 15           | $\infty$  | 2            | $\infty$  | 4   | 3  | 2  | 12       | $\infty$ | 1     |
| floor     | 4        | 2  | 1  | 5     | 1            | 146       | 1            | 9         | 54  | 1  | 1  | 5        | $\infty$ | 1     |
| ceil      | $\infty$ | 1  | X  | 8     | 1            | 5         | 1            | 9         | 266 | 1  | 1  | $\infty$ | $\infty$ | 1     |
| sign      | 1        | 1  | 1  | acţua | l bug        | 2         | 1            | 2         | 1   | 1  | 1  | 1        | 2        | 1     |

• Bug in Firefox JIT range analysis flow function for **ceil** 

- Bug in Firefox JIT range analysis flow function for ceil
- inRange $(R, v) \triangleq$

$$R.\exp < e_{INF} \implies \neg isInf(v)$$
 (R1)

$$\land R. \exp \neq e_{INF_OR_NAN} \implies \neg isNaN(v)$$
 (R2)

$$\land \neg R$$
.canBeNegZero  $\Longrightarrow \upsilon \neq -0.0$  (R3)

$$\land \neg R. \mathsf{canHaveFraction} \implies \mathsf{round}(v) = v$$
 (R4)

$$\land R.$$
 hasInt32LowerBound  $\implies$  (isNaN( $v$ )  $\lor v \ge R.$  lower) (R5)

$$\land R.$$
 hasInt32UpperBound  $\implies$  (isNaN( $v$ )  $\lor v \le R.$  upper) (R6)

$$\land R. \exp \ge \exp Of(v)$$
 (R7)

- Bug in Firefox JIT range analysis flow function for ceil
- inRange $(R, v) \triangleq$

```
R. \exp < e_INF \implies \neg isInf(v) (R1) \land R. \exp \neq e_INF_OR_NAN \implies \neg isNaN(v) (R2) \land \neg R. \text{canBeNegZero} \implies v \neq -0.0 (R3) \land \neg R. \text{canHaveFraction} \implies \text{round}(v) = v (R4) \land R. \text{hasInt32LowerBound} \implies (isNaN(v) \lor v \geq R. \text{lower}) (R5) \land R. \text{hasInt32UpperBound} \implies (isNaN(v) \lor v \leq R. \text{upper}) (R6) \land R. \exp \geq \expOf(v) (R7)
```

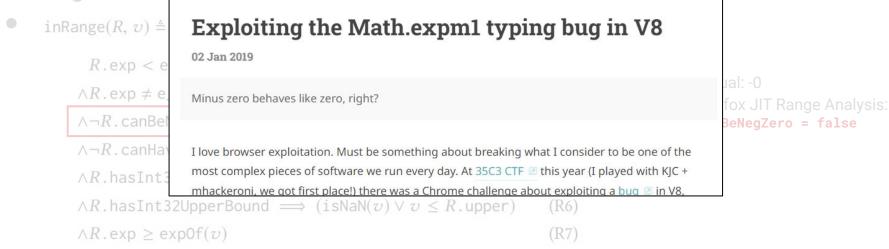
- Bug in Firefox JIT range analysis flow function for ceil
- inRange $(R, v) \triangleq$

```
R.\exp < e_{INF} \implies \neg isInf(v)
                                                                         (R1)
                                                                                         ceil(-0.5)
                                                                                                 actual: -0
\land R. \exp \neq e_{INF_{OR}} = \neg isNaN(v)
                                                                         (R2)
\land \neg R.canBeNegZero \implies v \neq -0.0
                                                                         (R3)
\land \neg R.canHaveFraction \implies round(v) = v
                                                                         (R4)
\land R. hasInt32LowerBound \implies (isNaN(v) \lor v \ge R. lower)
                                                                         (R5)
\land R. hasInt32UpperBound \implies (isNaN(v) \lor v \le R. upper)
                                                                         (R6)
\land R.\exp \ge \exp Of(v)
                                                                         (R7)
```

- Bug in Firefox JIT range analysis flow function for ceil
- inRange(R, v)  $\triangleq$

```
R.\exp < e_{INF} \implies \neg isInf(v)
                                                                        (R1)
                                                                                       ceil(-0.5)
                                                                                               actual: -0
\land R. \exp \neq e_{INF_{OR}} = \neg isNaN(v)
                                                                        (R2)
                                                                                               Firefox JIT Range Analysis:
\land \neg R.canBeNegZero \implies v \neq -0.0
                                                                        (R3)
                                                                                               canBeNegZero = false
\land \neg R.canHaveFraction \implies round(v) = v
                                                                        (R4)
\land R. hasInt32LowerBound \implies (isNaN(v) \lor v \ge R. lower)
                                                                        (R5)
\land R. hasInt32UpperBound \implies (isNaN(v) \lor v \le R. upper)
                                                                        (R6)
\land R.\exp \ge \exp Of(v)
                                                                        (R7)
```

Bug in Firefox JIT range analysis flow function for ceil



#### Conclusion

- VeRA: a system for verifying the range analysis pass in browser JIT compilers
  - Rewrite range analysis flow function in VeRA C++
  - Verify by SMT solver
- Verified 21 top-level Firefox range analysis flow functions
  - Detected Firefox bug (existed for 6 years)

## My Review

#### Strengths

- Targets critical part of browser
- Presents end-to-end system: C++ to SMT
- Easy to adapt to logic change

#### Weaknesses

- SMT solver could not verify everything (timeouts)
- Current implementation is tightly coupled to Firefox
- Hard to extend to other RA engine (e.g. Chrome)

# Thank you

Steve Gustaman

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## $inRange(R, v) \triangleq$

$$R. \exp < e_INF \implies \neg isInf(v)$$
 (R1)  
 $\land R. \exp \neq e_INF_OR_NAN \implies \neg isNaN(v)$  (R2)  
 $\land \neg R. \text{canBeNegZero} \implies v \neq -0.0$  (R3)  
 $\land \neg R. \text{canHaveFraction} \implies \text{round}(v) = v$  (R4)  
 $\land R. \text{hasInt32LowerBound} \implies (isNaN(v) \lor v \geq R. \text{lower})$  (R5)  
 $\land R. \text{hasInt32UpperBound} \implies (isNaN(v) \lor v \leq R. \text{upper})$  (R6)  
 $\land R. \exp \geq \expOf(v)$  (R7)

## $wellFormed(R) \triangleq$

$$R. \text{lower} \ge \text{JS\_INT\_MIN} \land R. \text{lower} \le \text{JS\_INT\_MAX}$$
 (W1)  
 $\land R. \text{upper} \ge \text{JS\_INT\_MIN} \land R. \text{upper} \le \text{JS\_INT\_MAX}$  (W1)  
 $\land \neg R. \text{hasInt32LowerBound} \implies R. \text{lower} = \text{JS\_INT\_MIN}$  (W2)  
 $\land \neg R. \text{hasInt32UpperBound} \implies R. \text{upper} = \text{JS\_INT\_MAX}$  (W2)  
 $\land R. \text{canBeNegZero} \implies \text{contains}(0, R)$   
 $\land (R. \text{exp} = \text{e\_INF} \lor R. \text{exp} = \text{e\_INF\_OR\_NAN} \lor R. \text{exp} \le \text{1023})$  (W3)  
 $\land (R. \text{hasInt32LowerBound} \land R. \text{hasInt32UpperBound})$   
 $\implies R. \text{exp} = \text{expOf}(\text{max}(|R. \text{lower}|, |R. \text{upper}|))$   
 $\land R. \text{hasInt32LowerBound} \implies R. \text{exp} \ge \text{expOf}(R. \text{lower})$  (W4)  
 $\land R. \text{hasInt32UpperBound} \implies R. \text{exp} \ge \text{expOf}(R. \text{upper})$  (W4)

| Operation | R1       | R2 | R3 | R4  | R5.i32   | R5.double | R6.i32   | R6.double | R7  | W1 | W2 | W3       | W4           | Under |
|-----------|----------|----|----|-----|----------|-----------|----------|-----------|-----|----|----|----------|--------------|-------|
| add       | 15       | 2  | 5  | 386 | 2        | 00        | 2        | ∞         | 21  | 2  | 1  | 2        | 80           | 1     |
| sub       | 13       | 2  | 11 | 445 | 8        | $\infty$  | 5        | $\infty$  | 14  | 2  | 2  | 2        | 78           | 1     |
| and*      | -        | -  | -  | -   | 2        | -         | 1        | -         | -   | -  | _  | -        | -            | 1     |
| or*       | -        | -  | -  | -   | 2        | -         | 2        | -         | -   | -  | -  | -        | -            | 1     |
| xor*      | _        | -  | -  | -   | 2        | -         | 2        | -         | -   | -  | -  | -        | -            | 1     |
| not*      | -        | -  | -8 | -   | 2        | -         | 1        | -         | -   | -  | -  | 10-      | =2           | 1     |
| mul       | 92       | 65 | 22 | 362 | $\infty$ | $\infty$  | $\infty$ | $\infty$  | 94  | 4  | 4  | 4        | $\infty$     | 11    |
| lsh*      | _        | _  | _  | _   | 1        | -         | 1        | -         | _   | _  | _  | -        | _            | 1     |
| rsh*      | -        | =0 |    | -   | 1        | -         | 1        | -         | -   | -  |    | -        | -            | 1     |
| ursh*     | -        | _  | -  | -   | X        | -         | X        | -         | -   | -  | -  | -        | <del>-</del> | 1     |
| lsh'*     | _        | -  | _  | -   | 1        | -         | 1        | -         | -   | -  | _  | _        | -            | 1     |
| rsh'*     | -        | -  | -  | -   | 1        | -         | 1        | -         | -   | -  | -  | -        |              | 1     |
| ursh'*    | -        | -  | -0 | -   | X        | -         | X        | -         | -   | -  | -  |          | -            | 1     |
| abs       | 1        | 1  | 1  | 5   | 1        | $\infty$  | 1        | 224       | 1   | 1  | 1  | 4        | $\infty$     | 1     |
| min       | 2        | 20 | 2  | 2   | 5        | 224       | 2        | $\infty$  | 3   | 2  | 1  | 2        | $\infty$     | 1     |
| max       | 3        | 17 | 2  | 2   | 15       | $\infty$  | 2        | $\infty$  | 4   | 3  | 2  | 12       | $\infty$     | 1     |
| floor     | 4        | 2  | 1  | 5   | 1        | 146       | 1        | 9         | 54  | 1  | 1  | 5        | $\infty$     | 1     |
| ceil      | $\infty$ | 1  | X  | 8   | 1        | 5         | 1        | 9         | 266 | 1  | 1  | $\infty$ | $\infty$     | 1     |
| sign      | 1        | 1  | 1  | 1   | 1        | 2         | 1        | 2         | 1   | 1  | 1  | 1        | 2            | 1     |

# Evaluation - Time to Verify

We successfully prove or refute 137 conditions out of a possible 159, for a success rate of  $\approx 86\%$ ; the shortest proofs complete in under a second, while the longest takes ≈ten minutes. The results suggest that **R5.double**, **R6.double**, and W4 are particularly challenging to verify. R5.double and R6.double are more challenging than their integer counterparts because they involve reasoning about floating-point values, which is generally more expensive. W4 is challenging because it involves proving a relationship between two properties of the range, both of which may be modified by the range analysis. Finally, **R1** and **W3** of Math.ceil may timeout because they involve bounding the size of an exponent, since Math.ceil involves extracting the exponent from the absolute value of the range bounds.