Trajectory Planning for a Bicycle

Joshua Ren, Zac Gong, Yifan Wang

INTRODUCTION/MOTIVATION

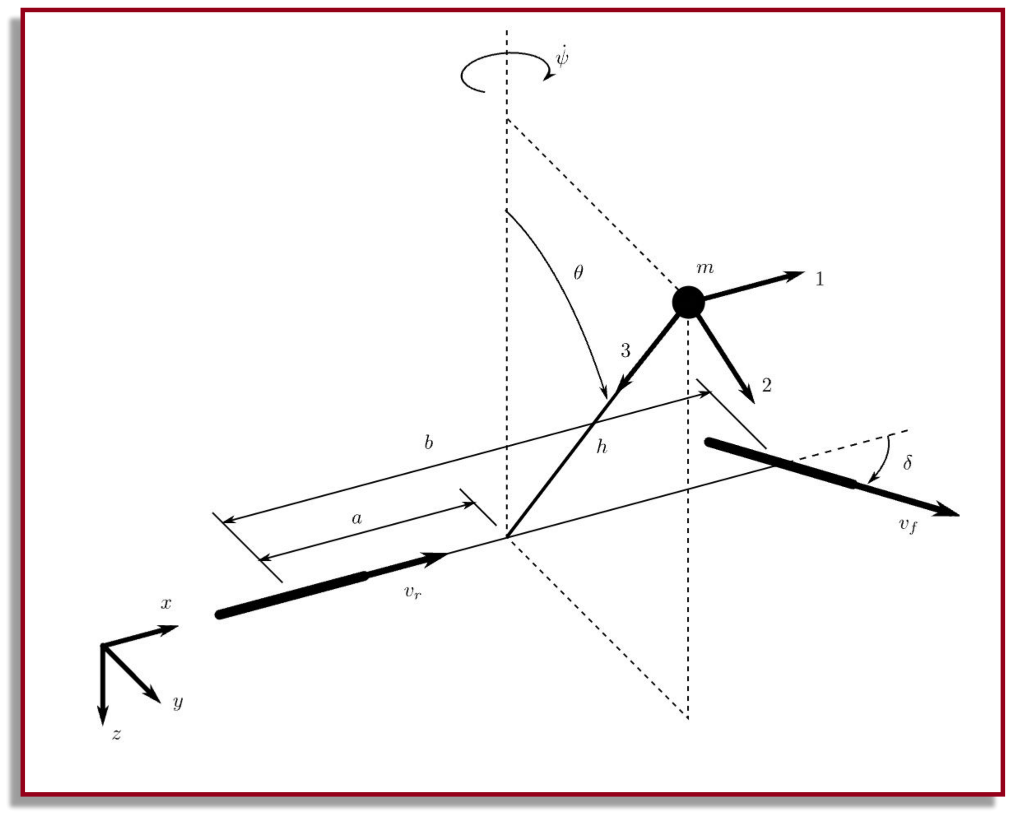
- **We decided to investigate the dynamics of a bicycle by implementing and comparing various controllers (LQR, Proportional)**
- **Bicycles are complex systems, requiring control to continuously maintain balance. Furthermore, the same control (steering) must also be used to direct the bicycle.**
- **The bicycle is also a non-minimum phase system. To turn the bike to the right, you must first steer to the left to get the bike leaning to the right.**

SYSTEM INTRODUCTION

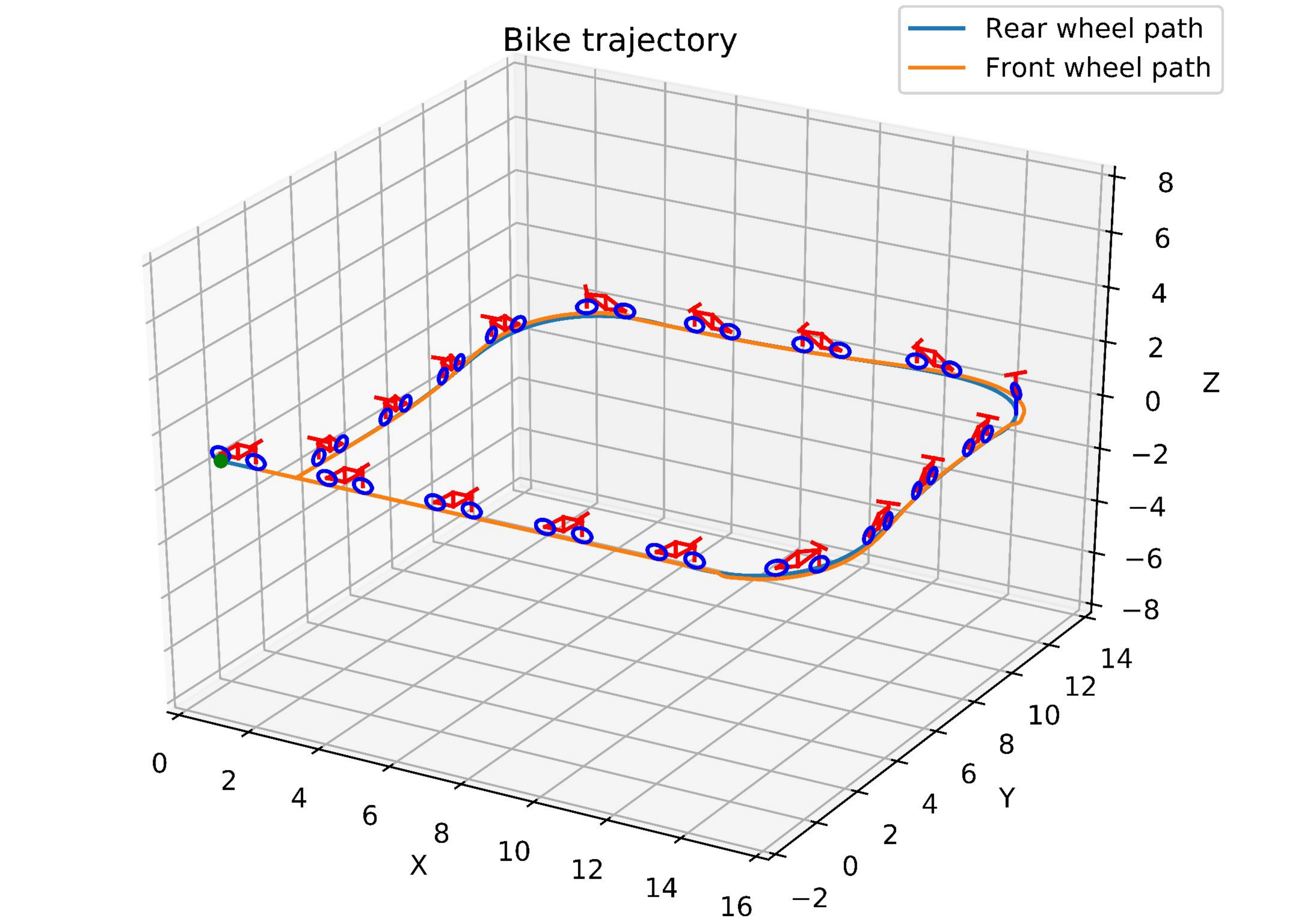
- 4 State Variables: Lean Angle, Lean Angle Rate, Heading Angle, Steering Angle
- 1 Input Variable: Steering Angle Rate
- Assume constant speed, set specific variables
- Calculate cartesian position from heading angle
- Non-Linear Dynamics are complex:

$$(I_x + mh^2)\ddot{\theta} + (I_3 - I_2 - mh^2)\left(\frac{v_r \tan\delta}{b}\right)^2 \sin\theta\cos\theta - mgh\sin\theta = -mh\cos\theta\left(\frac{a v_r}{b\cos^2\delta}\dot{\delta} + \frac{v_r^2}{b}\tan\delta\right)$$

- Can linearize dynamics about upright equilibrium, or linearize about a trajectory



BICYCLE TRAJECTORY PLOT



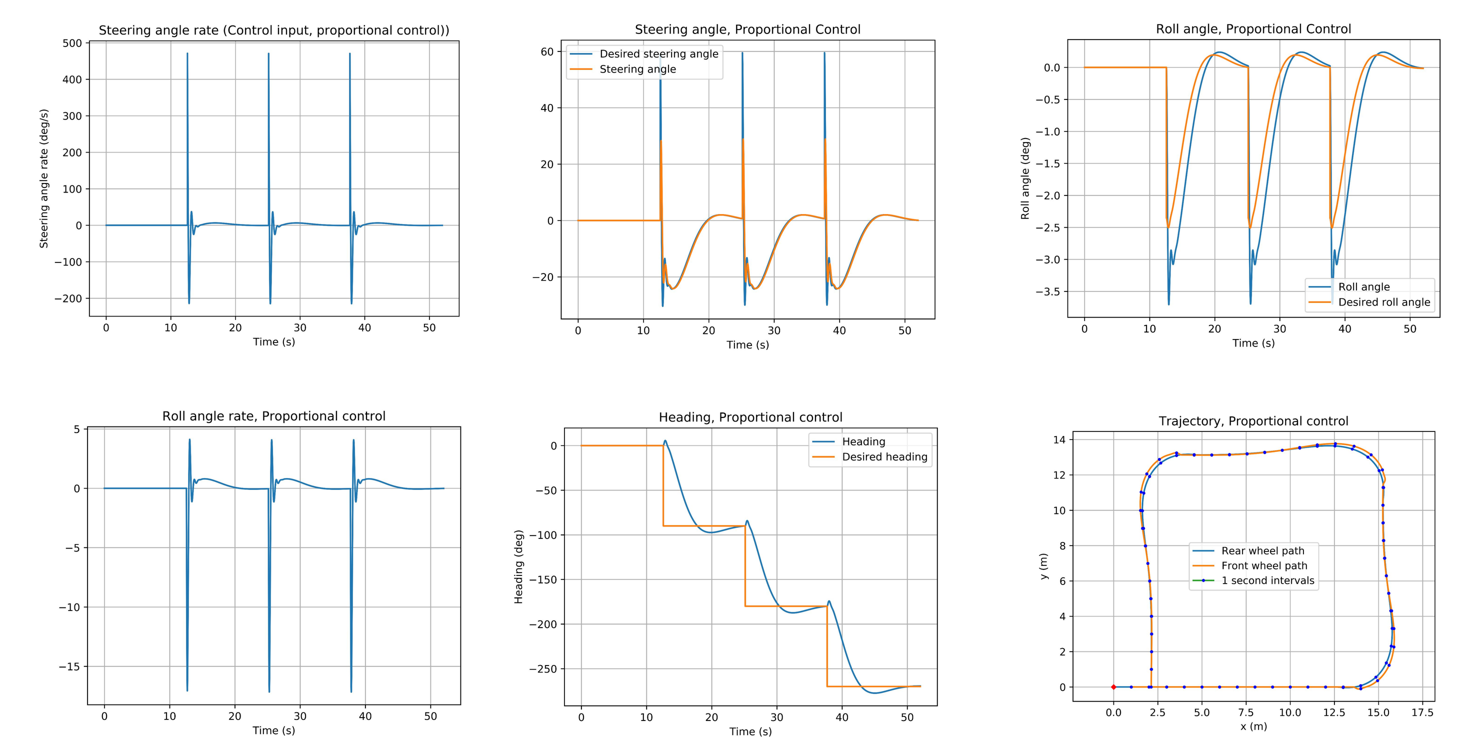
METHODS

- Implemented LQR and proportional controller to achieve goals of stabilizing bike and also steering bike to desired heading and trajectory
- Two versions of LQR depending on linearization
  - Linearization about 0 equilibrium leads to time-constant LQR, sends bike to x = 0 from any initial condition
  - Linearization about a trajectory leads to a time-varying LQR, sends bike along the trajectory
- Proportional controller details
  - Dual loop feedback controller with inner roll stabilization loop and outer heading tracking loop (Reference: Jason K. Moore)
  - Implemented parameter sweep of gains to maximize distance traveled in given time
- Compared by running reference trajectory on each controller in heading hold mode. Drive straight for x seconds, turn and hold 90 degree heading change, drive for x seconds

RESULTS

- The LQR controller required less control effort than the proportional controller to maintain the same approximate trajectory
- We attempted to implement derivative feedback on heading error but couldn't get it to work. We also tried iLQR, but not successfully.
- Potential extensions include:
  - iLQR to utilize the trajectory of the bicycle as part of planning
  - Respecting input limits (steer angle limits, rate limits) with model predictive control
  - Integrating path planning for obstacle avoidance and/or other guidance

Proportional Control



LQR Control