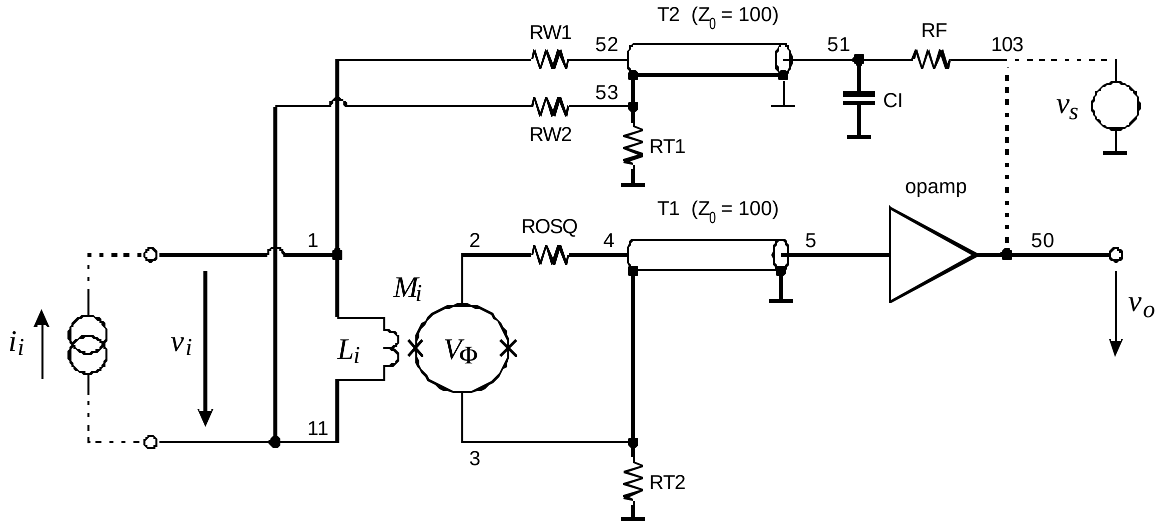
SPICE Simulations of a shunt feedback SQUID series array

Circuit diagram for simulations:



The diagram shows components for two types of simulations, depending on the connections shown by dashed lines.

a) Loop gain

The feedback loop is split at the output of the op-amp (node 50) and a signal v_s is applied to the feedback resistor RF (node 103). The loop gain is measured at the op-amp output. In principle, the feedback loop can be split at any point to determine the loop gain. This configuration drives the feedback path RF from a low impedance, as presented by the operational amplifier (opamp) in closed loop operation.

b) Input impedance

The loop is closed and a unit current is applied to the input (nodes 1 and 11). The resulting voltage drop v_i at the input is numerically equal to the input impedance.

The SQUID model is chosen to match the parameters of the Seiko 100-SQUID series array. The operational amplifier is modeled with a single-pole response with a cutoff frequency of 400 kHz and a gain-bandwidth product of 4 GHz to match the Burr-Brown OPA 687. The wires from the cold stage to the room-temperature electronics are shown as transmission lines. In reality they are not coaxial, but twisted pair. However, SPICE does not distinguish between the two types of lines. The impedance is set to 100 Ω, corresponding to a typical twisted-pair. The propagation delay is 50 ps/cm, so a 10 cm long line has a delay of 500 ps. The resistance of these connecting wires is shown as RW1 and RW2 in the feedback path. At the SQUID output it is included in ROSQ, which can be chosen to include the dynamic output resistance of the SQUID. A small-signal model is used for the SQUID, characterized by its transresistance $M_i V_\Phi$ and the input inductance L_i. For the Seiko SQUID array $M_i V_\Phi$ = 140 V/A and L_i = 20 nH.

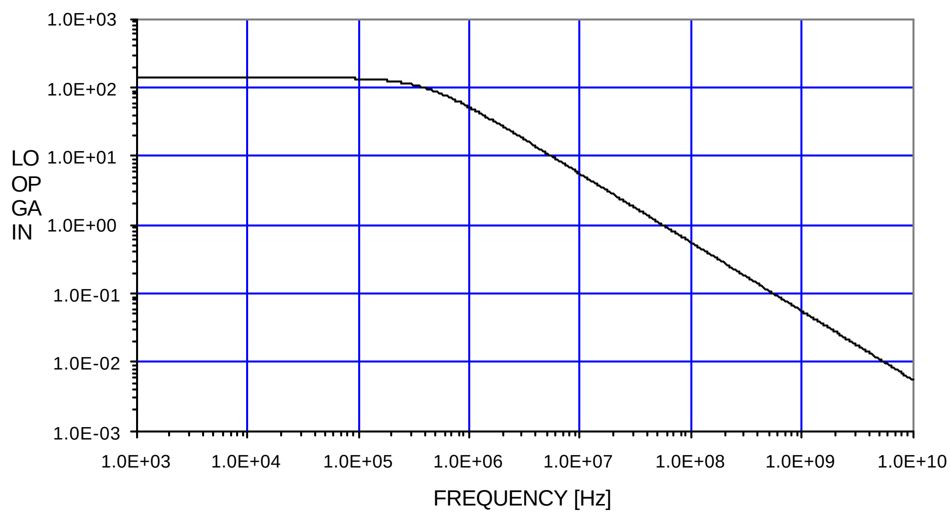
A shunt capacitance CI is shown at node 51 between the feedback resistor and the transmission line TD2. This represents the capacitance to ground of a MOSFET array used to switch feedback resistors. RT1 and RT2 are required by SPICE to provide DC referencing; the values are set to be so large to have no effect on the AC response.

1. Loop gain

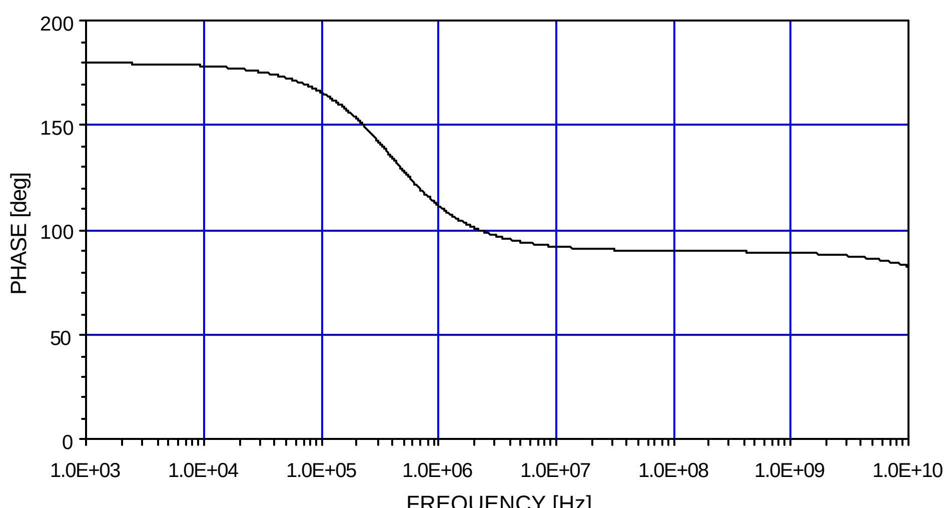
1.1 Loop gain without wiring propagation delay

In this simulation the delay times of both transmission lines T1 and T2 are set to 10^{-18} s (for comparison, 10 cm of twisted pair have a propagation delay of 500 ps). The magnitude and phase of the loop gain are shown below.

LOOP GAIN w. no INPUT LOAD - MAGNITUDE
(GBW= 4 GHz, $A_{V0}=10^4$, R_F=10K, R_{wf}=16 ohms, R_w=16 ohms)

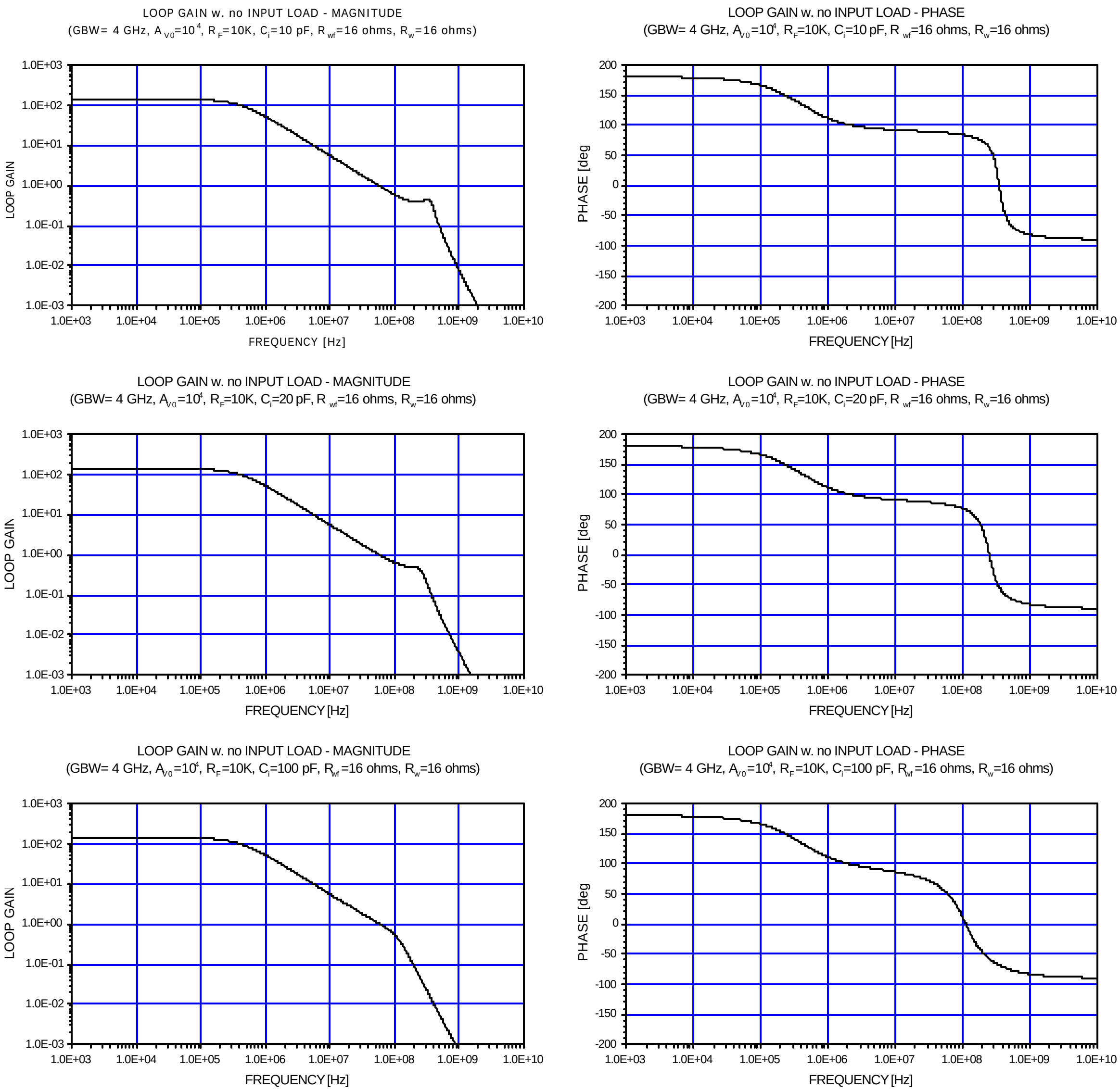


LOOP GAIN w. no INPUT LOAD - PHASE
(GBW= 4 GHz, $A_{V0}=10^4$, R_F=10K, R_{wf}=16 ohms, R_w=16 ohms)



The slight downturn in phase above 1 GHz is due to the time constant L_i / R_F of the feedback network, which corresponds to *f* = 80 GHz..

In the final circuit a MOSFET switch will be used to select feedback resistors and set the loop gain. The substrate capacitance is in the range 10 – 20 pF in suitable switches and can be as high as 300 pF in switches with a very low on-resistance. The next set of figures shows the loop gain and phase for shunt capacitances CI of 10, 20 and 100 pF.



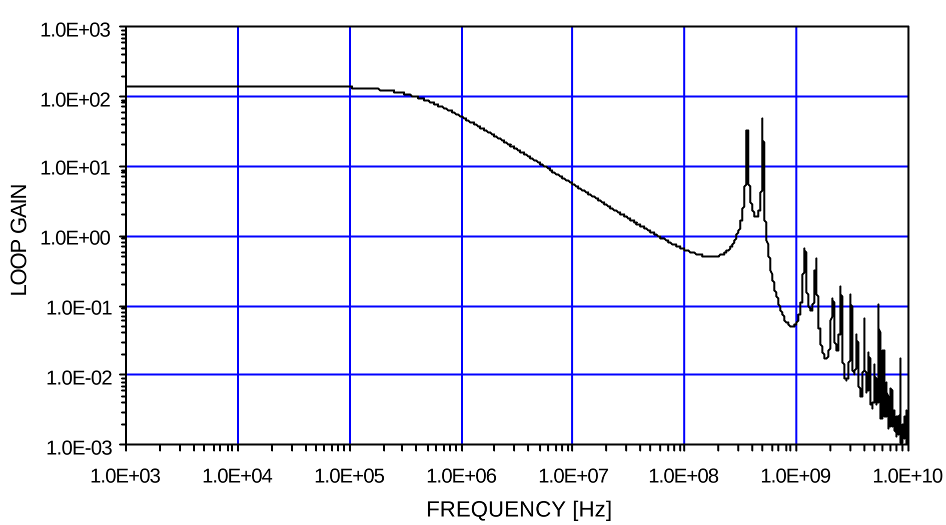
With 10 pF capacitance the resonance formed by the parallel combination of CI and the SQUID's input inductance of 20 nH is clearly visible ($f = 1/2\pi\sqrt{L_i C_i}$ = 356 MHz). Increasing CI to 20 pF moves the resonance down, as does CI= 100. Since the wiring resistance of 16 Ω is comparable to the capacitive reactance $X_C = 1/2\pi f C_i$, which for 100 pF at 100 MHz is 16 Ω, the circuit *Q* is low and the shift in resonant frequency is less than predicted by the simple resonance formula.

Despite the resonance the feedback is unconditionally stable even with a capacitance of 100 pF.

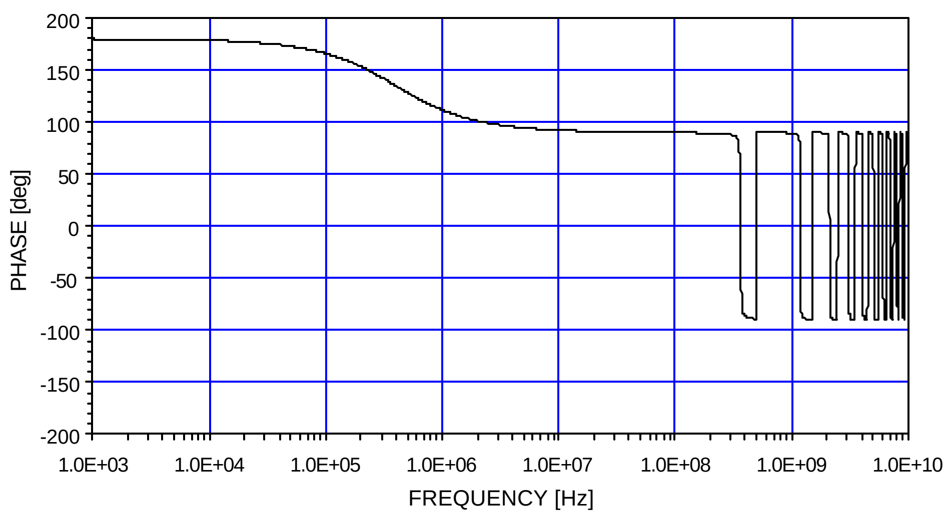
1.2 Loop gain with wiring propagation delay

This picture is complicated when the propagation delay due the wiring is introduced. For 10 cm long connections the propagation delay is 500 ps, i.e. 1 ns total "round trip". The next set of figures shows the effect of the propagation delay, first without wiring resistance and no shunt capacitance CI.

LOOP GAIN w. no INPUT LOAD - MAGNITUDE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, R_{wire}= 0 ohms)



LOOP GAIN w. no INPUT LOAD - PHASE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, R_{wire}= 0 ohms)

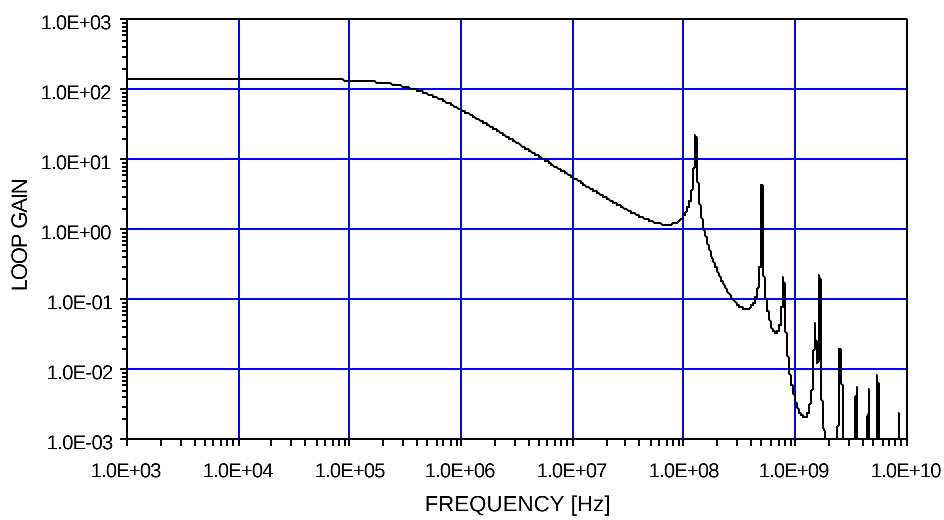


The transmission lines introduce peaks at 360 and 500 MHz, and then many additional peaks at higher frequencies. The lowest peaks are much lower than expected based on the propagation delay alone. Furthermore, the phase alternates between +90 and –90 degrees, which indicates resonance behavior.

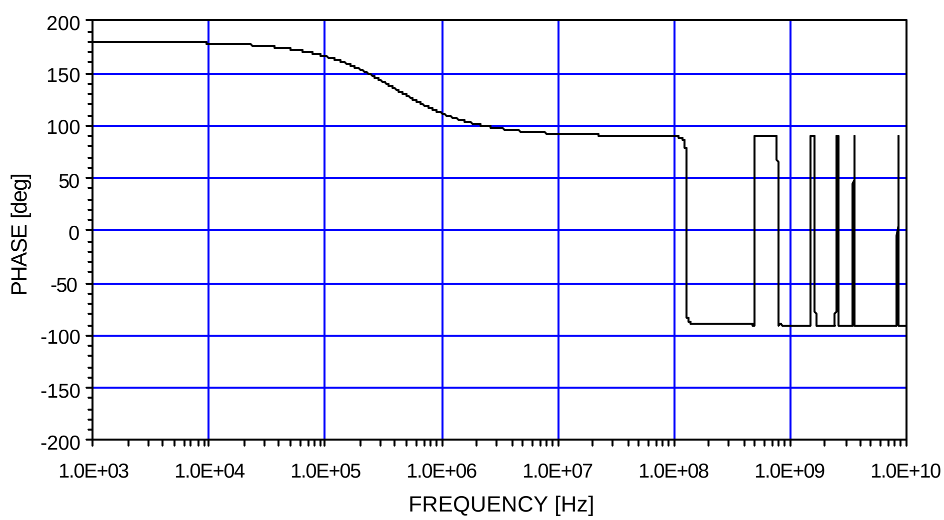
The peak at 500 MHz is explained simply. The operational amplifier presents an infinite load to the transmission line T1. At the frequency where the electrical line length is a quarter wavelength, it presents a short circuit to the SQUID, so the current is maximum. For a propagation delay of 500 ps, the line is an electrical wavelength at 2 GHz, so a quarter wavelength occurs at 500 MHz.

This explains the 500 MHz peak, but not the peak at 360 MHz. Transmission line T2 has the same length as T1, but it is inductively loaded by the SQUID input coil, so maximum current transfer occurs at a lower frequency. Introducing a shunt capacitance CI= 20 pF shifts the 360 MHz peak down to 130 MHz, but leaves the 500 MHz peak intact, as shown in the next set of figures.

LOOP GAIN w. no INPUT LOAD - MAGNITUDE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, C_i=20pF)

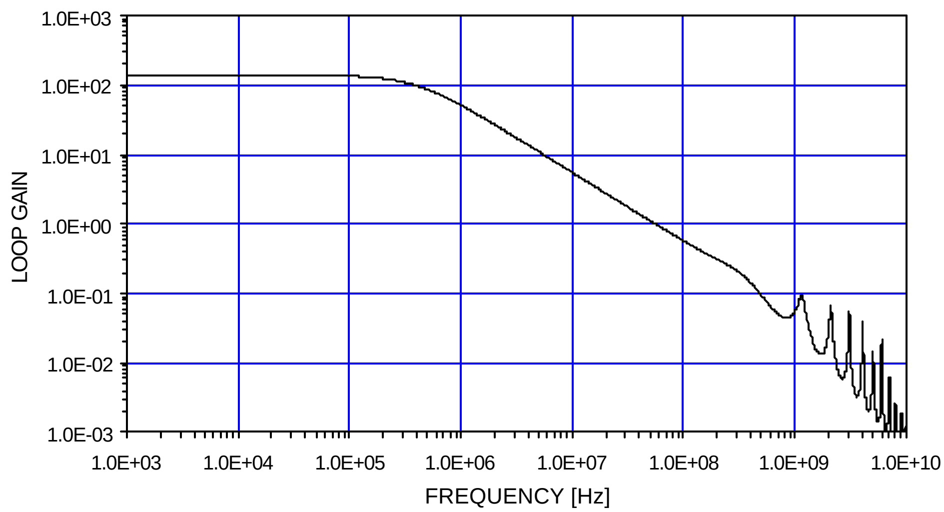


LOOP GAIN w. no INPUT LOAD - PHASE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, C_i=20pF)

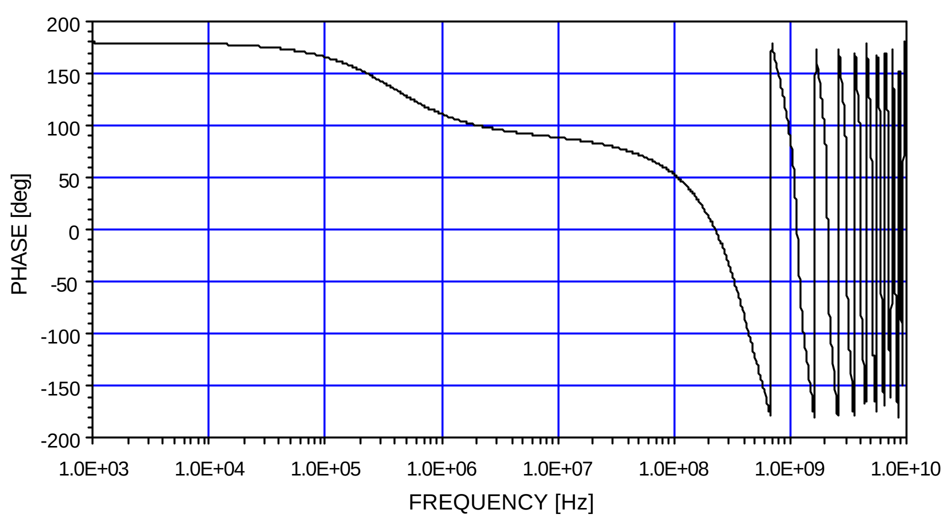


The resonance can be damped by terminating the line in its characteristic impedance. The next set of figures shows the results when the sum RW1+RW2 is made equal to the line impedance, i.e. 100 Ω. Furthermore, a 100 Ω resistor is connected from output of T1 (node 5) to ground. The 20 pF shunt capacitor CI has been removed.

LOOP GAIN w. no INPUT LOAD - MAGNITUDE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, terminated lines)



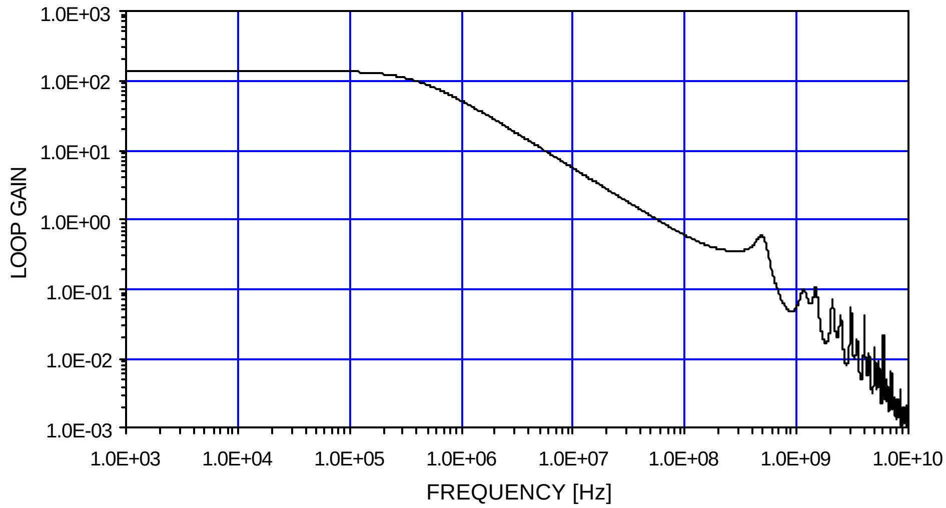
LOOP GAIN w. no INPUT LOAD - PHASE
(GBW= 4 GHz, $A_{V0}=10^4$, L= 2x 10cm, R_F=10K, terminated lines)



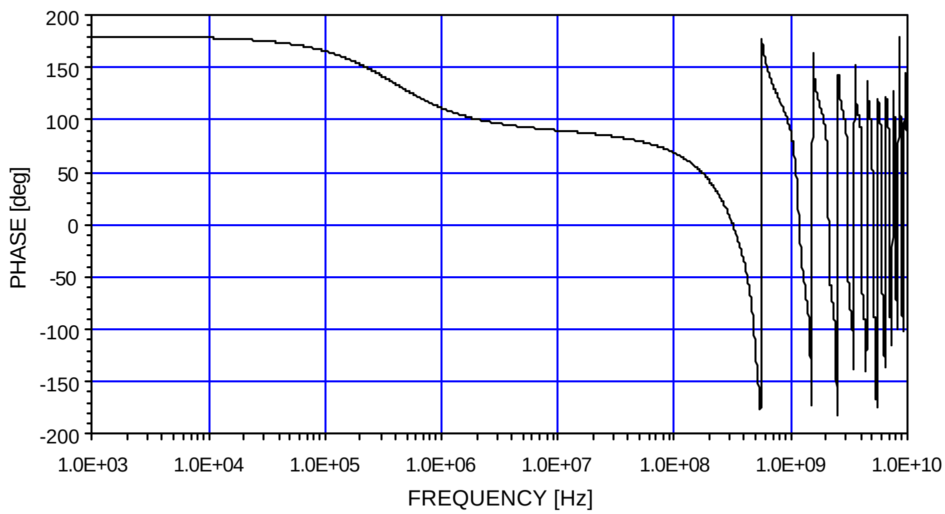
The resonances from T1 are gone. The low frequency resonances from T2 are suppressed (the lowest resonance is still visible as a slight "bump" in the spectrum), but the high frequency resonances remain. This is because the 20 nH input inductance of the SQUID is in series with the termination. At several hundred MHz the inductive reactance is sufficiently small, but at high frequencies it is significant, so the line is severely mismatched. Nevertheless, the effect of the resonances on the overall phase response is small, so now the phase alternates between +180 and –180 degrees, with a period given by the propagation delay.

Introducing a 100 Ω termination in the feedback path is no problem. RW1 simply adds to the feedback resistance. RW2 has no effect, as will be demonstrated below in the section on input impedance. Terminating the SQUID output line is not practical in general. However, the wiring resistance alone is sufficient to damp this resonance so that it is not a problem, as shown below.

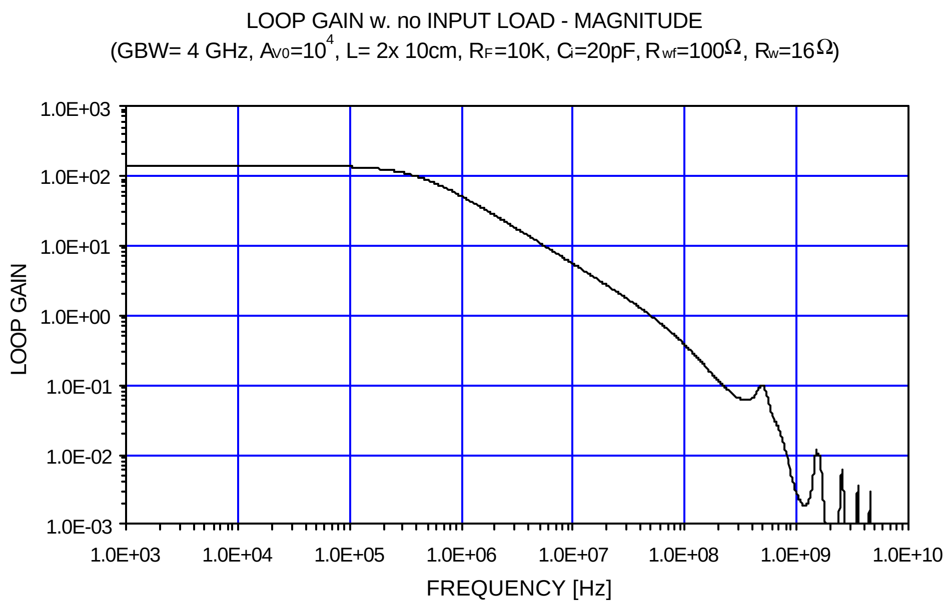
LOOP GAIN w. no INPUT LOAD - MAGNITUDE
(GBW= 4 GHz, A_{V0}=10^4, L= 2x 10cm, R_F=10K, R_{wf}=100 Ω , R_w= 16 Ω)

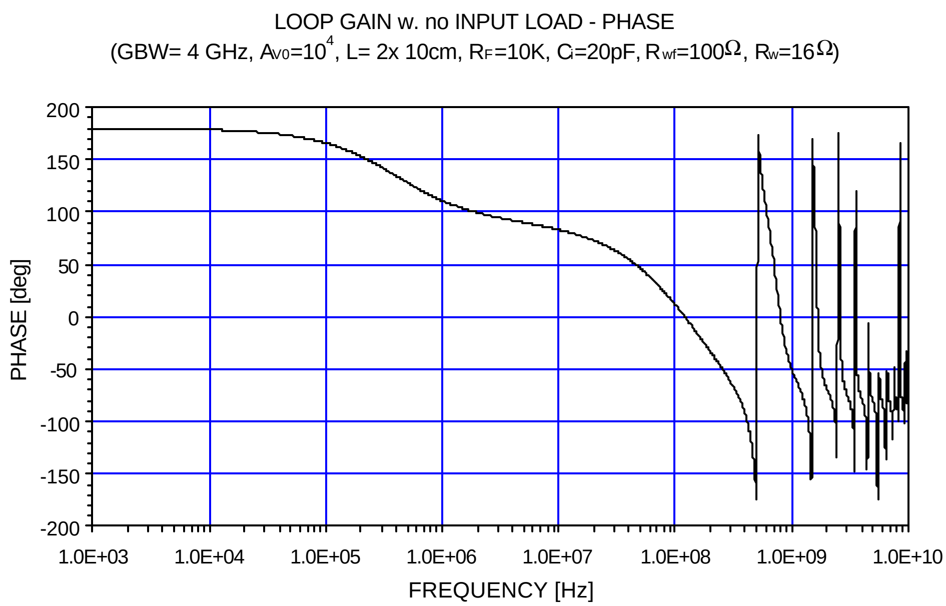


LOOP GAIN w. no INPUT LOAD - PHASE
(GBW= 4 GHz, A_{V0}=10^4, L= 2x 10cm, R_F=10K, R_{wf}=100 Ω, Rw= 16 Ω)

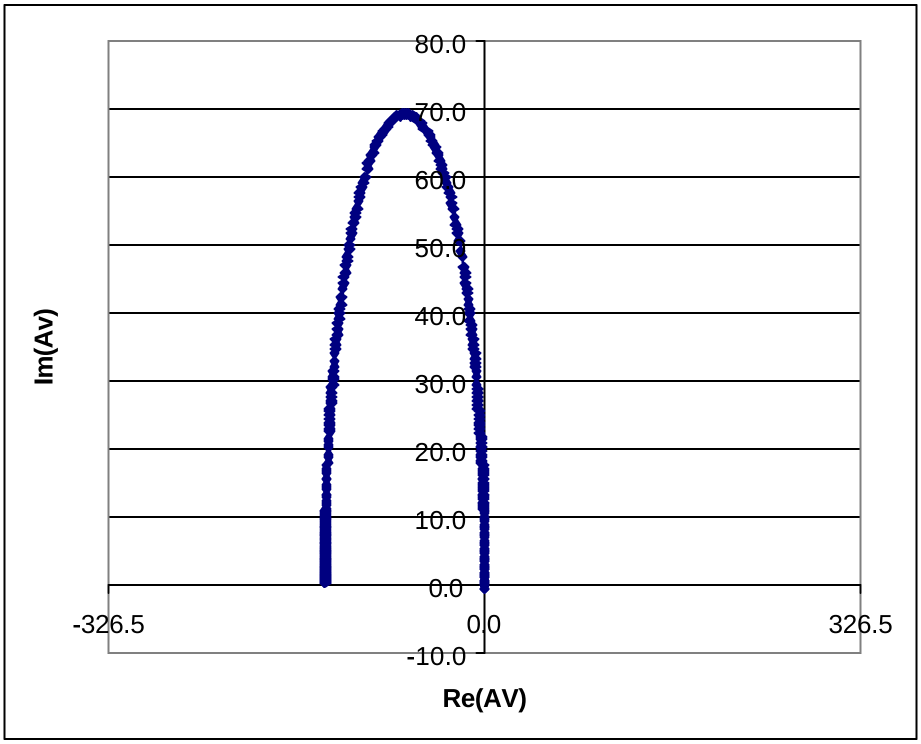


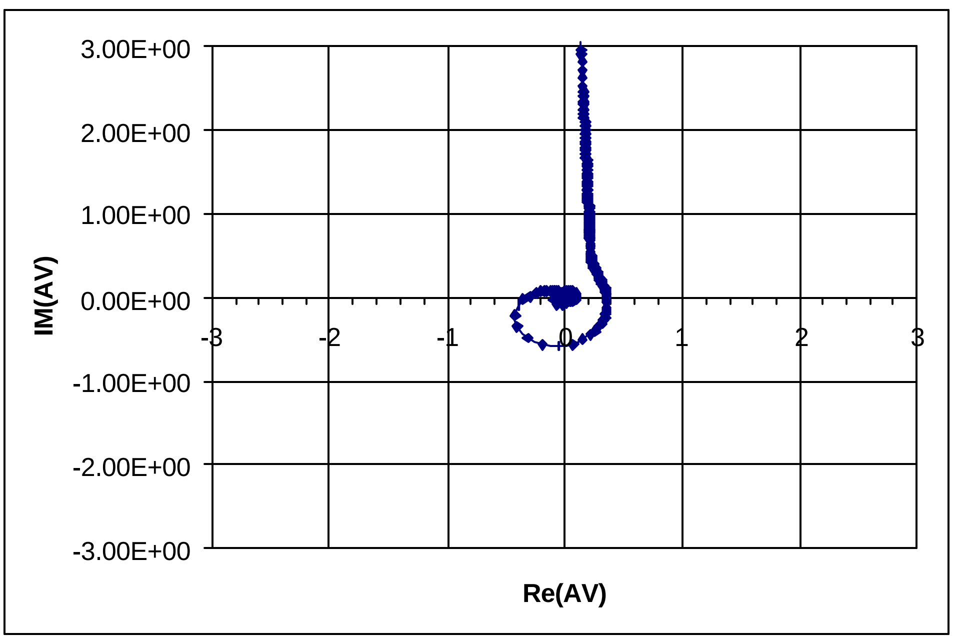
Including the full circuit, i.e. 10 cm line lengths for T1 and T2 together with a 100 Ω termination and a 20 pF shunt capacitance in the feedback path, and including the 16 Ω wire resistance in the output circuit of the SQUID yields the following response.



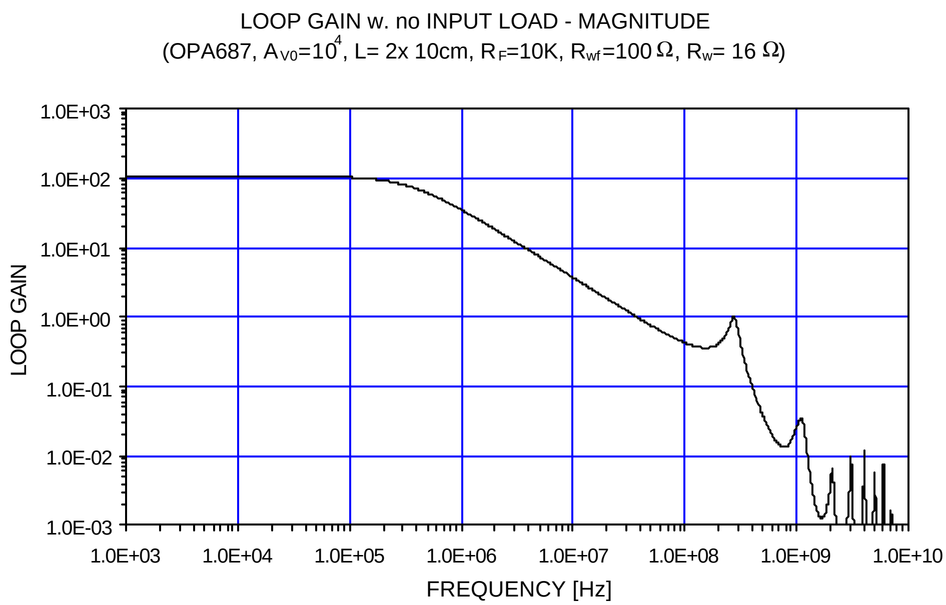


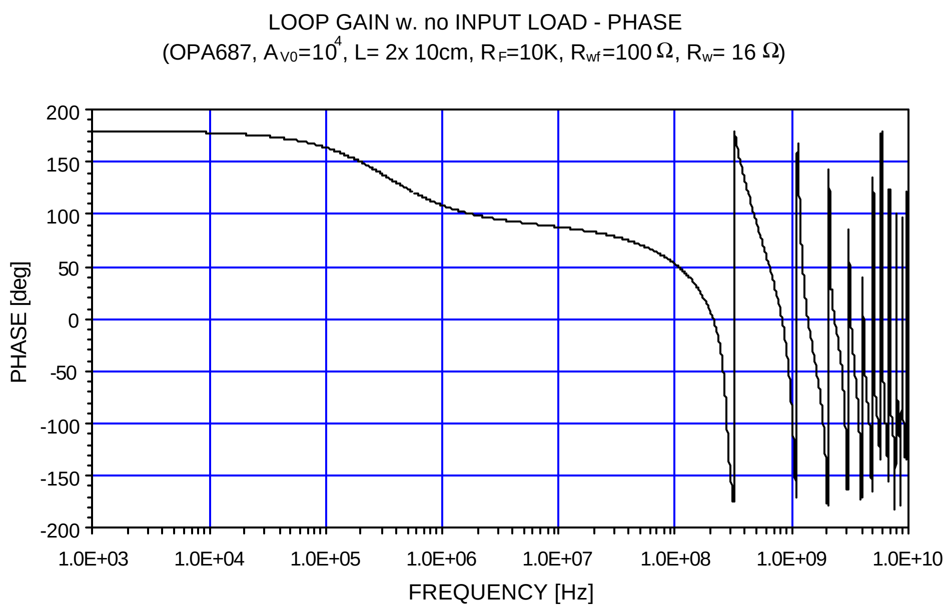
At the feedback loop's unity gain frequency of 50 MHz the phase margin is 45°, so the loop is unconditionally stable. This is also borne out by the Nyquist plots shown below. The imaginary part of the loop gain is plotted versus the real part. If the resulting contour does not enclose the point (1,0), i.e. 1 on the real axis, the feedback loop is stable. The upper plot shows the full frequency range, with individual frequency points progressing from the left leg of the contour towards the origin. The lower plot shows an expanded view of the contour near the origin.





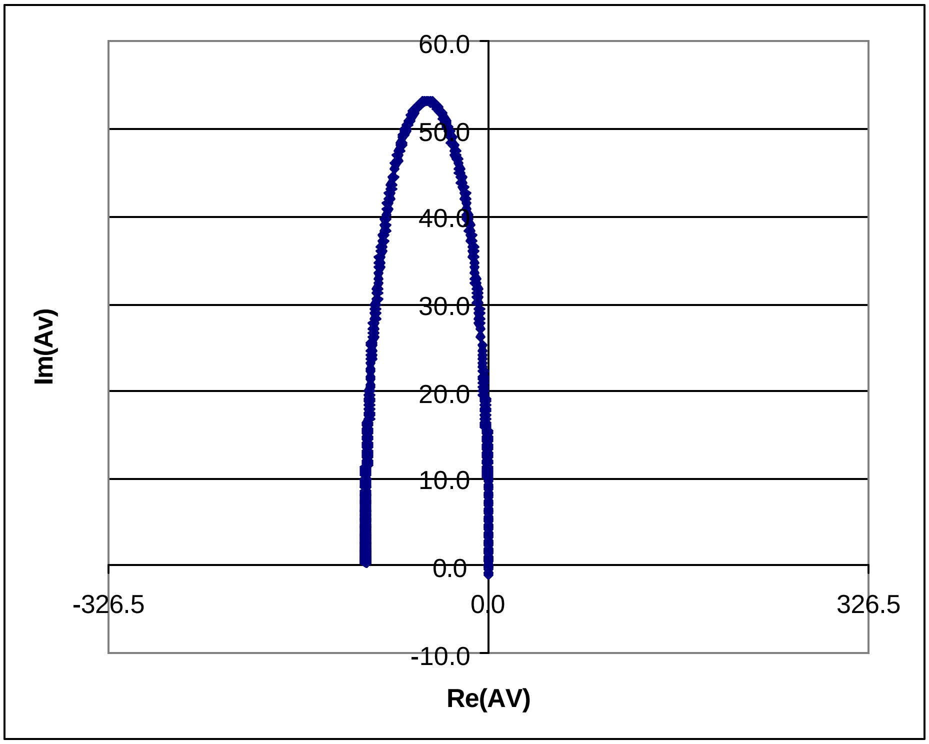
This simulation was repeated, except that the model of the ideal single-pole operational amplifier was replaced by the macromodel for the Burr-Brown OPA687. The results are shown below.

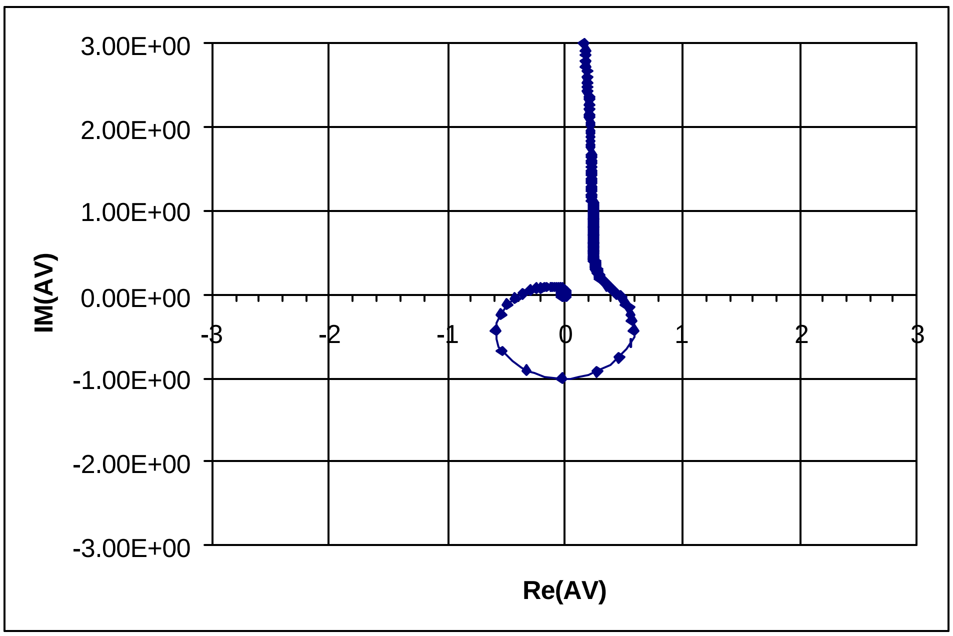




The complex input impedance of the OPA687 moves the first two resonance peaks lower in frequency, so that the first peak attains unity gain. The corresponding phase shift is negative, so potentially this configuration is unstable.

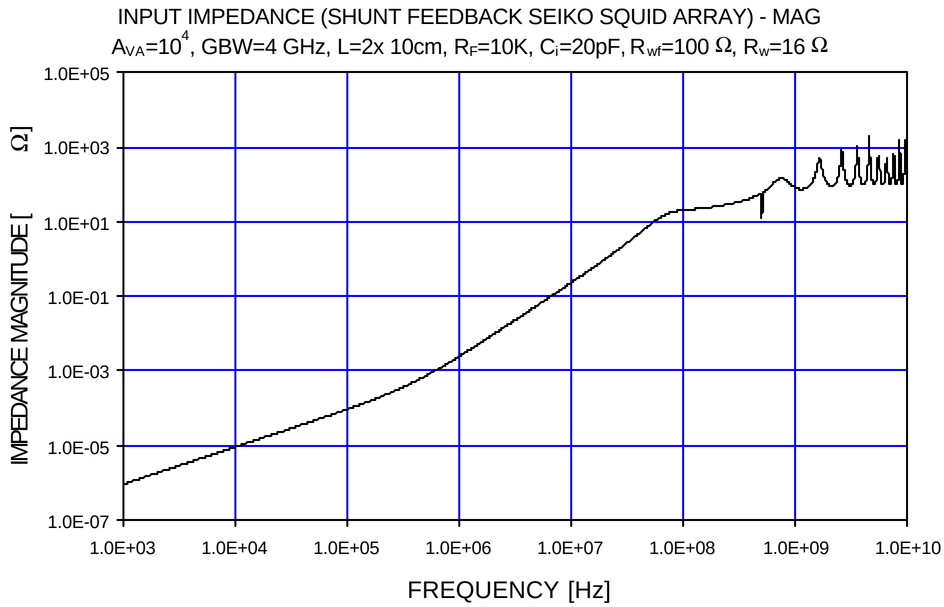
However, the Nyquist plots below show that the loop is stable. Nevertheless, increasing the wiring resistance between the SQUID and the amplifier can damp the resonance peak further.

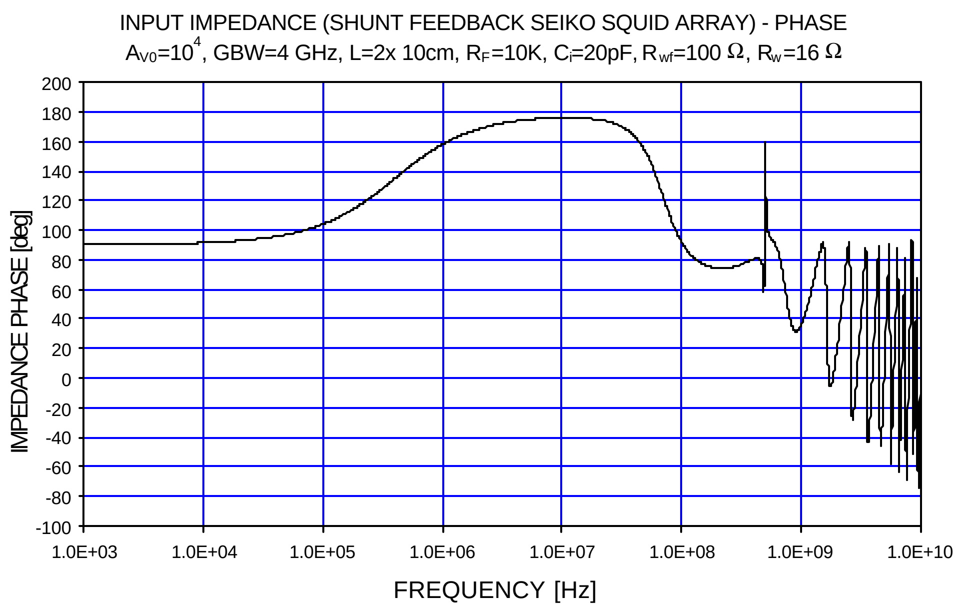




2. Input Impedance

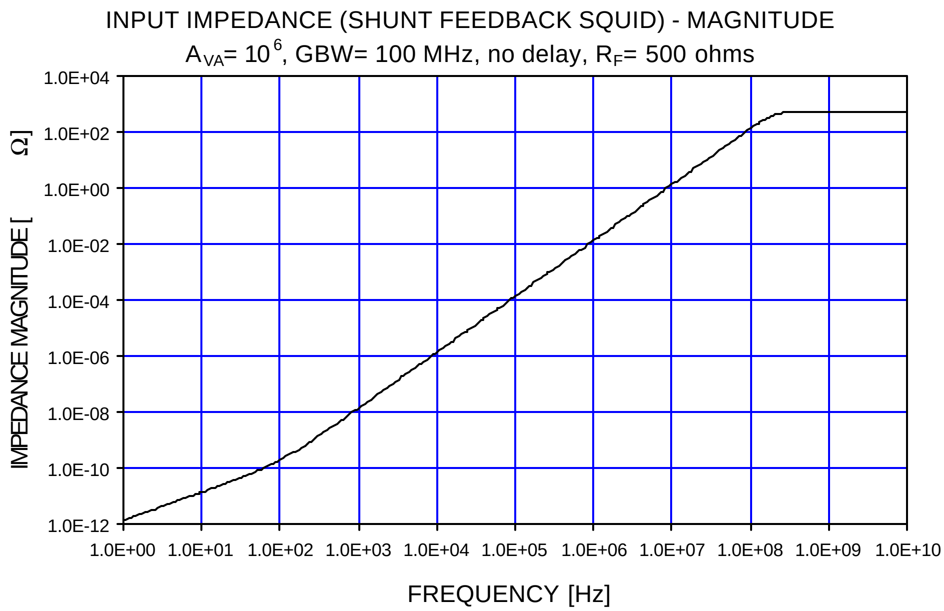
The input impedance was simulated with the same parameter set used for the loop gain in the preceding plots. T1 and T2 are both 10 cm long (each line has 500 ps propagation delay). T2 is terminated with 100 Ω. The shunt resistance CI is 20 pF. The output circuit of the SQUID includes 16 Ω wire resistance. The magnitude and phase of the input impedance vs. frequency are shown below.

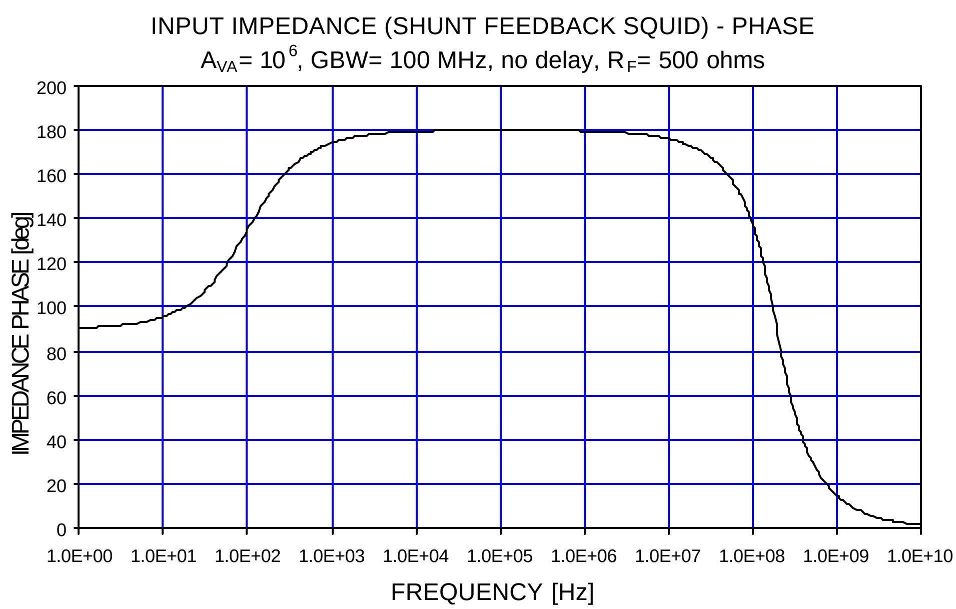




As predicted analytically, the input impedance is inductive at low frequencies. Above the amplifier's upper cutoff frequency the phase approaches 180°. This could lead to an oscillatory response to an applied input signal, but inserting a series resistor gives a net positive input resistance. In the frequency range of primary interest, this series resistance is provided by the bolometer. At higher frequencies a series RC combination shunting the input could be included.

For comparison, the plots below show the simulated input impedance for an idealized feedback loop without transmission line effects and an amplifier with a lower cutoff frequency.





With the OPA687 macromodel the SPICE simulation shows the following input impedance.