CS5321 Numerical Optimization Homework 1

Due Oct 28

1. (30%) For a single variable unimodal function $f \in [0, 1]$, we want to find its minimum. We have introduced the binary search algorithm in the class. But in each iteration, we need two function evaluations, $f(x_k)$ and $f(x_k + \epsilon)$. Here is another type of algorithms, called ternary search. Figure 1 illustrates the idea. The initial triplet of x values is $\{x_1, x_2, x_3\}$.

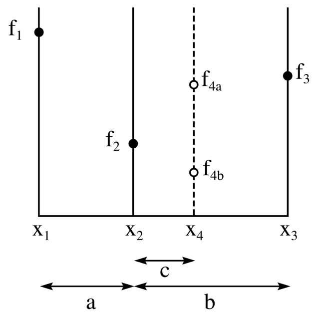

Figure 1: The idea of ternary search.

Answers are put here.
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   (a) (10%) For the search direction, show that to find the minimum point, if $f(x_4) = f_{4a}$, the triplet $\{x_1, x_2, x_4\}$ is chosen for the next iteration. If $f(x_4) = f_{4b}$, the triplet $\{x_2, x_4, x_3\}$ is chosen. (Hint: use the property of unimodal.)

   (b) (10%) For either case, we want these three points keep the same ratio, which means
$$\frac{a}{b} = \frac{c}{a} = \frac{c}{b - c}.$$
Show that under this condition, the ratio of $b/a = (\sqrt{5} + 1)/2$, which is the golden ratio ϕ. (So this algorithm is called the *Golden-section search*).

   (c) (10%) If we let each iteration of the algorithm has two function evaluations, show the convergence rate of the Golden-section search is ϕ^{-2}. (This means it is faster than the binary search algorithm under the same number of function evaluations.)

Answers are put here.

2. (15%) Show that Newton's method for single variables is equivalent to build a quadratic model

$$q(x) = f(x_k) + f'(x_k)(x - x_k) + \frac{f''(x_k)}{2}(x - x_k)^2$$

at the point x_k and use the minimum point of $q(x)$ as the next point. (Hint: to show the next point $x_{k+1} = x_k - f'(x_k)/f''(x_k)$)

Answers are put here.

3. (15%) Matrix A is an $n \times n$ symmetric matrix. Show that all A's eigenvalues are positive if and only if A is positive definite.

Answers are put here.

4. (50%) Consider a function $f(x_1, x_2) = (x_1 - x_2)^3 + 2(x_1 - 1)^2$.

    (a) Suppose $\vec{x}_0 = (1, 2)$. Compute $\vec{x_1}$ using the steepest descent step with the optimal step length.
    Answers are put here.

    (b) What is the Newton's direction of f at $(x_1, x_2) = (1, 2)$? Is it a descent direction?
    Answers are put here.

    (c) Compute the LDL decomposition of the Hessian of f at $(x_1, x_2) = (1, 2)$. (No pivoting)
    Answers are put here.

    (d) Compute the modified Newton step using LDL modification.
    Answers are put here.

    (e) Suppose $\vec{x}_0 = (1, 1)$ and $\vec{x}_1 = (1, 2)$, and the $B_0 = I$. Compute the quasi Newton direction p_1 using BFGS.
    Answers are put here.