

Efficient high-energy photon production in the supercritical QED regime

Matteo Tamburini and Sebastian Meuren, Phys Rev D **104**, L091903 (2021)

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Introduction

In this notebook we reproduce some results from the paper.

Figure 2

```

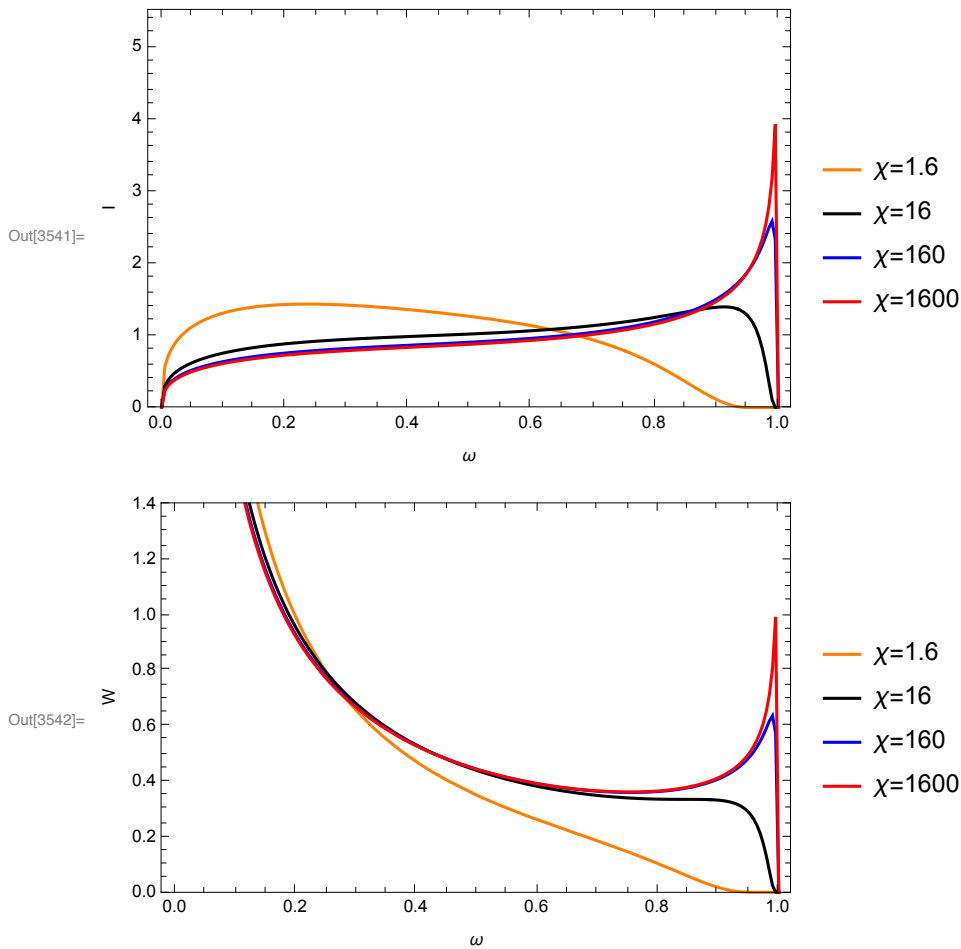
In[3529]:= Clear[τc, ħ, m, c, α, χ, γ, ω, d2Wdtdω, d2Idtdω, dWdt, dIdt]
Clear[dIdt00016, dIdt00160, dIdt01600, dIdt16000]
τc = ħ / (m c^2);
ħ = m = c = 1;
α = 1 / 137;
γ = 1;
d2Wdtdω[ω_?NumericQ, χ_?NumericQ] :=
  d2Wdtdω[ω, χ] = 
$$\frac{\alpha}{\sqrt{3} \pi \tau_c \gamma} \left( \left( 2 + \frac{\omega^2}{1 - \omega} \right) \text{BesselK}\left[2/3, \frac{2 \omega}{3 \chi (1 - \omega)}\right] - \right.$$


$$\left. \text{NIntegrate}\left[\text{BesselK}[1/3, y], \left\{y, \frac{2 \omega}{3 \chi (1 - \omega)}, \infty\right\}\right] \right)$$

d2Idtdω[ω_?NumericQ, χ_?NumericQ] := d2Idtdω[ω, χ] = ω d2Wdtdω[ω, χ]
dWdt[χ_] := NIntegrate[d2Wdtdω[ω, χ], {ω, 0, 1}]
dIdt[χ_] := NIntegrate[d2Idtdω[ω, χ], {ω, 0, 1}]
(* normalization *)
dWdt00016 = dWdt[1.6];
dWdt00160 = dWdt[16];
dWdt01600 = dWdt[160];
dWdt16000 = dWdt[1600];
(* *)
dIdt00016 = dIdt[1.6];
dIdt00160 = dIdt[16];
dIdt01600 = dIdt[160];
dIdt16000 = dIdt[1600];
(* *)
Plot[ $\left\{ \frac{1}{dIdt00016} d2Idtd\omega[\omega, 1.6], \frac{1}{dIdt00160} d2Idtd\omega[\omega, 16], \right.$ 
 $\left. \frac{1}{dIdt01600} d2Idtd\omega[\omega, 160], \frac{1}{dIdt16000} d2Idtd\omega[\omega, 1600] \right\},$ 
{ω, 0, 1}, PlotPoints → 4, Frame → True, FrameLabel → {"ω", "I"},
PlotRange → {0, 5.5}, PlotLegends → {"χ=1.6", "χ=16", "χ=160", "χ=1600"},
PlotStyle → {Orange, Black, Blue, Red}]

Plot[ $\left\{ \frac{1}{dWdt00016} d2Wdtd\omega[\omega, 1.6], \frac{1}{dWdt00160} d2Wdtd\omega[\omega, 16], \right.$ 
 $\left. \frac{1}{dWdt01600} d2Wdtd\omega[\omega, 160], \frac{1}{dWdt16000} d2Wdtd\omega[\omega, 1600] \right\},$ 
{ω, 0, 1}, PlotPoints → 4, Frame → True, FrameLabel → {"ω", "W"},
PlotRange → {0, 1.4}, PlotLegends → {"χ=1.6", "χ=16", "χ=160", "χ=1600"},
PlotStyle → {Orange, Black, Blue, Red}]

```



Prove eq 4

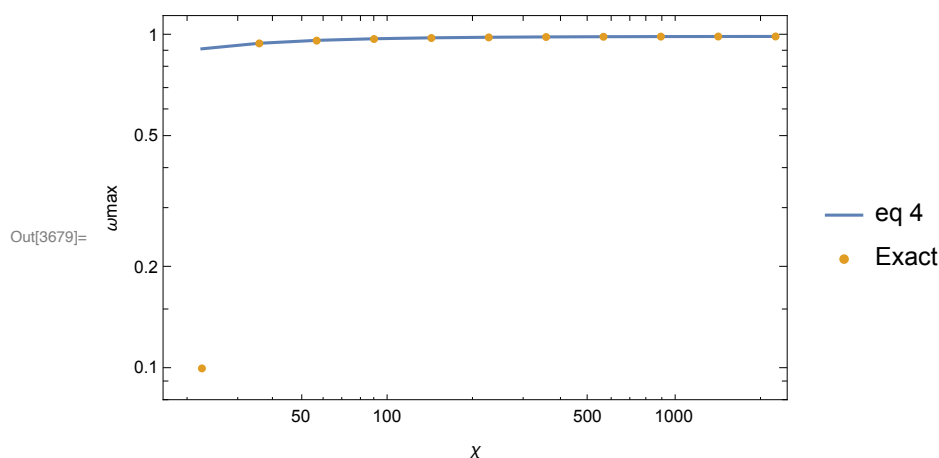
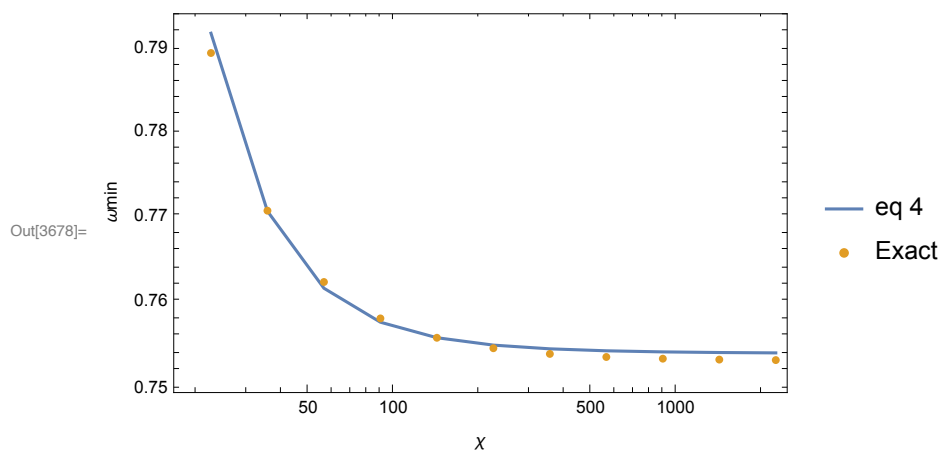
```
In[3669]:= Clear[ωmin, ωmax, χ, ω, getωmin, getωmax, ωminTh, ωminExact, ωmaxTh, ωmaxExact]

ωmin = 0.754 +  $\frac{15.7 + 0.146 \chi}{\chi^2}$ ;

ωmax = 1 -  $\frac{174 + 20 \chi}{15 \chi^2}$ ;

getωmin[χ_] :=
  (FindMinimum[{d2Wdtdω[ω, χ], 0.1 < ω < 1}, {ω, 0.5}] // Quiet)[[2, 1, 2]]
getωmax[χ_] :=
  (FindMaximum[{d2Wdtdω[ω, χ], 0.1 < ω < 1}, {ω, 0.8}] // Quiet)[[2, 1, 2]]
ωminTh = ParallelTable[{10logχ, ωmin /. {χ → 10logχ}}, {logχ, 1.35, 3.5, 0.2}];
ωminExact = ParallelTable[{10logχ, getωmin[10logχ]}, {logχ, 1.35, 3.5, 0.2}];
ωmaxTh = ParallelTable[{10logχ, ωmax /. {χ → 10logχ}}, {logχ, 1.35, 3.5, 0.2}];
ωmaxExact = ParallelTable[{10logχ, getωmax[10logχ]}, {logχ, 1.35, 3.5, 0.2}];
```

```
In[3678]:= ListLogLogPlot[{ $\omega_{\min Th}$ ,  $\omega_{\min Exact}$ }, PlotLegends → {"eq 4", "Exact"},
  Frame → True, FrameLabel → {" $\chi$ ", " $\omega_{\min}$ "}, Joined → {True, False}]
ListLogLogPlot[{ $\omega_{\max Th}$ ,  $\omega_{\max Exact}$ }, PlotLegends → {"eq 4", "Exact"},
  Frame → True, FrameLabel → {" $\chi$ ", " $\omega_{\max}$ "}, Joined → {True, False}]
```



Prove eq 5

```
In[3690]:= Clear[ $\omega_{\min}$ ,  $\omega_{\max}$ ,  $\chi$ ,  $\omega$ , getWmin, getWmax,  $\omega_{\min Th}$ ,  $\omega_{\min Exact}$ ,  $\omega_{\max Th}$ ,  $\omega_{\max Exact}$ ]
getWmin[ $\chi_$ ] := (FindMinimum[{d2Wdtd $\omega$ [ $\omega$ ,  $\chi$ ], 0.1 <  $\omega$  < 1}, { $\omega$ , 0.5}] // Quiet)[[1]]
getWmax[ $\chi_$ ] := (FindMaximum[{d2Wdtd $\omega$ [ $\omega$ ,  $\chi$ ], 0.1 <  $\omega$  < 1}, { $\omega$ , 0.8}] // Quiet)[[1]]
HTh =
  ParallelTable[{10log $\chi$ , (1.315 + 0.315  $\chi$ ) /  $\chi^{2/3}$ }, {log $\chi$ , 1.35, 3.5, 0.2}];
HExact = ParallelTable[{10log $\chi$ , getWmax[10log $\chi$ ] / getWmin[10log $\chi$ ]}, {log $\chi$ , 1.35, 3.5, 0.2}];
```

```
In[3695]:= ListLogLogPlot[{HTh, HExact}, PlotLegends → {"eq 5", "Exact"},
  Frame → True, FrameLabel → {" $\chi$ ", "H"}, Joined → {True, False}]
```

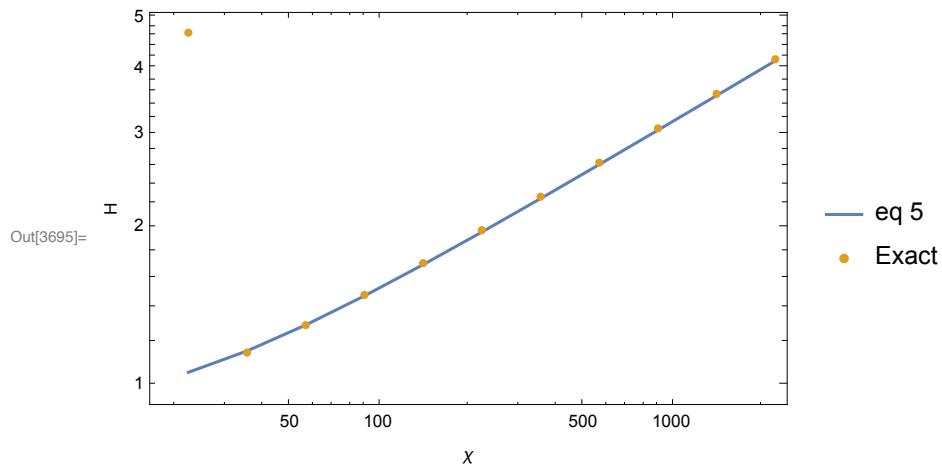


Figure 4

In[3476]:= Clear[W, k, tab1, tab2]

tab1 = ParallelTable[$\left\{k, -1 \frac{W^k}{k!} \text{Exp}[-W]\right\}$, {k, 0, 16}];

tab2 = ParallelTable[$\left\{k, \frac{W^k}{k!} \text{Exp}[-W]\right\}$, {k, 0, 16}];

ListPlot[{tab1, tab2 /. {W → 3.1}}, Filling → Bottom,
PlotMarkers → "OpenMarkers", PlotRange → {{-0.5, 16.5}, All},
Frame → True, FrameLabel → {"k", "Nk"}, PlotLabel → "s=2500nm, W~3.1?"]

ListPlot[{tab1, tab2 /. {W → 0.2}}, Filling → Bottom,
PlotMarkers → "OpenMarkers", PlotRange → {{-0.5, 5.5}, All},
Frame → True, FrameLabel → {"k", "Nk"}, PlotLabel → "s=100nm, W<0.2?"]

