# Schrödinger-Poisson–Vlasov-Poisson correspondence

Philip Mocz, Lachlan Lancaster, Anastasia Fialkov, Fernando Becerra, and Pierre-Henri Chavanis, Phys Rev D 97, 083519 (2018)
Notebook: Óscar Amaro, November 2022 @ GoLP-EPP

**Introduction**

In this notebook we reproduce some results from the paper.

## Eq 30 using Husimi representation

```
Clear[m, ℏ, t, ψ, x, p, η, Ψ, n, A, r, ℱ, ρ]
(* ℏm → ℏ/m *)
ψ[x_, t_] :=
 π^-0.25 (I ℏm Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏm - 1/ℏm))/(4 (Cos[t]^2 + ℏm^2 Sin[t]^2))]
(* initial condition eq 29 is correct *)
ψ[x, 0] ⅇ^(x^2/2) - π^-0.25
```

Out[•]= 0.

In[129]:=
```
Clear[m, ℏ, t, ψ, x, p, η, Ψ, n, A, r, ℱ, ρ]
n = 1;
t = π / 2;
A = π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 ;
ψ = A Exp[(-2 x^2 + I Sin[2 t] (ℏ / m - m / ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))] /. {x → r};
(* eq 30 → eq 31 can easily be done by inspection *)
ρ = Refine[Abs[ψ]^2, {m > 0, ℏ > 0, r > 0}] // Simplify
(* 1/Sqrt[π] //N=0.5641895835477563`*)


(* eq 23 Husimi representation *)
int = (1/(2 π ℏ))^(n / 2) (1/(π η^2))^(n / 4) Integrate[ψ Exp[- (x - r)^2/(2 η^2) - I (p (r - x / 2))/ℏ], r]
```

Out[134]= $\dfrac{0.56419\, e^{-\frac{m^2 r^2}{\hbar^2}}\, m}{\hbar}$

Out[135]= $\dfrac{0.282095\, e^{-\frac{(m^2 x + i p \hbar)(-i p \eta^2 + x \hbar)}{2 (m^2 \eta^2 \hbar + \hbar^3)}} \left(\frac{1}{\eta^2}\right)^{1/4} \eta\, \mathrm{Erf}\left[\frac{m^2 r \eta^2 + \hbar\, (i p \eta^2 + (r - x)\, \hbar)}{\sqrt{2}\, \eta\, \hbar\, \sqrt{m^2 \eta^2 + \hbar^2}}\right]}{\sqrt{\frac{1}{\hbar}} \left(\frac{i \hbar}{m}\right)^{0.5} \sqrt{m^2 \eta^2 + \hbar^2}}$

```
(* the Erf component will give +1-(-1)=2 *)
```

In[138]:= $\mathrm{Limit}\left[\mathrm{Erf}\left[\frac{m^2 r \eta^2 + \hbar\, (i p \eta^2 + (r - x)\, \hbar)}{\sqrt{2}\, \eta\, \hbar\, \sqrt{m^2 \eta^2 + \hbar^2}}\right], r \to +\infty\right]$

Out[138]= $1$ if $x \in \mathbb{R}$ && $p == 0$ && $\eta\, \hbar \sqrt{m^2 \eta^2 + \hbar^2} > 0$

In[139]:= $\mathrm{Limit}\left[\mathrm{Erf}\left[\frac{m^2 r \eta^2 + \hbar\, (i p \eta^2 + (r - x)\, \hbar)}{\sqrt{2}\, \eta\, \hbar\, \sqrt{m^2 \eta^2 + \hbar^2}}\right], r \to -\infty\right]$

Out[139]= $-1$ if $x \in \mathbb{R}$ && $p == 0$ && $\eta\, \hbar \sqrt{m^2 \eta^2 + \hbar^2} > 0$

```
(* so Ψ = 2 int *)
Ψ =
```

$$2 \times \left(\left(0.28209479177387814` \, e^{-\frac{\left(m^2 x+i p \hbar\right)\left(-i p \eta^2+x \hbar\right)}{2\left(m^2 \eta^2 \hbar+\hbar^3\right)}} \left(\frac{1}{\eta^2}\right)^{1/4} \eta\right) \Big/ \left(\sqrt{\frac{1}{\hbar}} \left(\frac{i \hbar}{m}\right)^{0.5`} \sqrt{m^2 \eta^2+\hbar^2}\right)\right)$$

```
(* eq 24 ℱ → eq 32 *)
(* there may be a factor of 2 missing when comparing with equation 32 *)
ℱ = Refine[Abs[Ψ]^2 // Simplify // ComplexExpand,
   {m > 0, ℏ > 0, η > 0, r > 0, p > 0}] // Simplify
```

$$\frac{2}{\text{Sqrt}[\pi]} \text{ // N}$$

Out[160]= $$\frac{0.56419 \, e^{-\frac{\left(m^2 x+i p \hbar\right)\left(-i p \eta^2+x \hbar\right)}{2\left(m^2 \eta^2 \hbar+\hbar^3\right)}} \left(\frac{1}{\eta^2}\right)^{1/4} \eta}{\sqrt{\frac{1}{\hbar}} \left(\frac{i \hbar}{m}\right)^{0.5} \sqrt{m^2 \eta^2+\hbar^2}}$$

Out[161]= $$\frac{0.31831 \, e^{-\frac{m^2 x^2+p^2 \eta^2}{m^2 \eta^2+\hbar^2}} m \eta}{m^2 \eta^2+\hbar^2}$$

Out[162]= 1.12838

# Figure 2

In[5]:=
```
Clear[m, ℏ, p, t, ψ]
Show[{LogPlot[{1/2 x^2}, {x, -1.1, 1.1}, PlotRange → {10^-1, 10^1},
   Axes → False, Frame → True, AspectRatio → 1, Filling → Bottom,
   FrameLabel → {"x", "|ψ|^2"}, PlotLabel → "t = π/2, V=1/2 x^2"], LogPlot[
   {Abs[π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏ/m - m/ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))]]^
      2 /. {m → 1, ℏ → 1, p → 0, t → π/2},
    Abs[π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏ/m - m/ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))]]^
      2 /. {m → 1, ℏ → 1/2, p → 0, t → π/2},
    Abs[π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏ/m - m/ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))]]^
      2 /. {m → 1, ℏ → 1/4, p → 0, t → π/2},
    Abs[π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏ/m - m/ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))]]^
      2 /. {m → 1, ℏ → 1/8, p → 0, t → π/2},
    Abs[π^-0.25 (I ℏ/m Sin[t] + Cos[t])^-0.5 Exp[(-2 x^2 + I Sin[2 t] (ℏ/m - m/ℏ))/(4 (Cos[t]^2 + (ℏ/m)^2 Sin[t]^2))]]^
      2 /. {m → 1, ℏ → 1/16, p → 0, t → π/2}},
   {x, -1.2, 1.2}, PlotRange → {10^-1, 10^1}, PlotLegends →
    {"ℏ/m=1", "ℏ/m=1/2", "ℏ/m=1/4", "ℏ/m=1/8", "ℏ/m=1/16"}]}]

(* √2 m η/(π(ℏ^2+2m^2 η^2)) Exp[-(p^2η^2+2x^2m^2)/(ℏ^2+2m^2η^2)] *)
```

Out[6]=