lpw

```
(* Mathematica notebook for
 spectra. Following [Esarey]'s paper in sections A. and B.
  date : 04/03/2019
  author: Óscar L. Amaro
*)
```

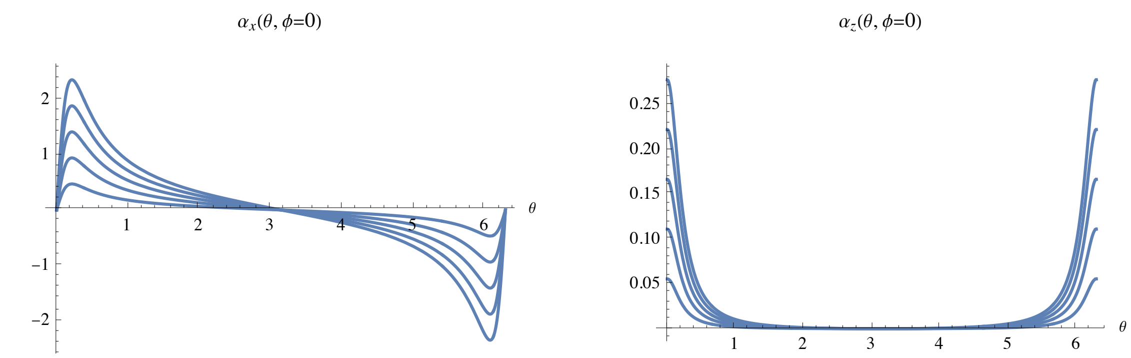
A. Linear polarization p. 3007

```
(*Parameters*)
γ0 = 5; (*p. 48*)
a0 = 0.5;
N0 = 7;

β0 = √(1 - 1 / γ0^2); (*normalizations*)
c = 1;
ω0 = 1;
k0 = ω0 / c;

h0 = γ0 (1 + β0); (* p.3006 (8c)*)

k = ω / c;(*p.3007 (26)*)
L0 = (2 π)/(ω0) * β0; (*much larger than 1/k0???*)
η0 = L0 / 2;

r1 = a0 / (h0 k0); (*p. 3006 (16a)*)
z1 = -a0^2 / (8 h0^2 k0);
β1 = (1 - 1 / M0) / 2;
M0 = h0^2 / (1 + a0^2 / 2);
```

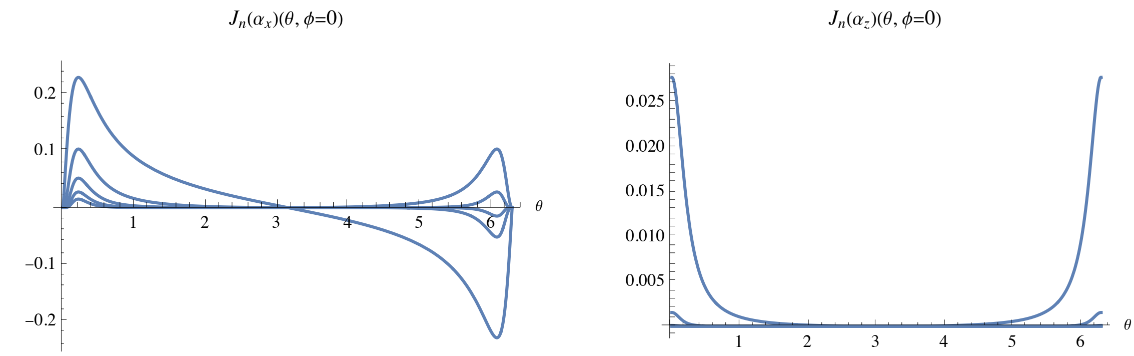
In[∘]:=
```
(*Functions p. 3008*)
kbar = Function[{θ, n, k}, k (1 - β1 (1 + Cos[θ])) - n k0]; (*(30a)*)

(*alpha*)
αz = Function[{θ, ϕ, n}, (n a0^2 (1 + Cos[θ]))/(8 h0^2 (1 - β1 (1 + Cos[θ])))]; (*(38a)*)
αx = Function[{θ, ϕ, n}, (n a0 Sin[θ] Cos[ϕ])/(h0 (1 - β1 (1 + Cos[θ])))]; (*(38b)*)
(*C*)
sumlim = 10; (*parameter that controls the sum*)
Cx = Function[{θ, ϕ, n},
   k0 r1 Sum[(-1)^m BesselJ[m, αz[θ, ϕ, n]]
      (BesselJ[n - 2 m - 1, αx[θ, ϕ, n]] + BesselJ[n - 2 m + 1, αx[θ, ϕ, n]]),
     {m, -sumlim, sumlim}]]; (*(37a)*)
Cz = Function[{θ, ϕ, n},
   2 Sum[(-1)^m BesselJ[m, αz[θ, ϕ, n]] (β1 * BesselJ[n - 2 m, αx[θ, ϕ, n]] + k0 z1 (
         BesselJ[n - 2 m - 2, αx[θ, ϕ, n]] + BesselJ[n - 2 m + 2, αx[θ, ϕ, n]] ) ),
     {m, -sumlim, sumlim}]] ; (*(37b)*)

(*main*)
dIdωdΩ = Function[{ω, θ, ϕ},
   k^2/(4 π^2) Sum[ ((Sin[kbar[θ, n, k] η0 ])/(kbar[θ, n, k] η0))^2 ((Cx[θ, ϕ, n]^2) (1 - Sin[θ]^2 Cos[ϕ]^2) +
       Cz[θ, ϕ, n]^2 Sin[θ]^2 - Cx[θ, ϕ, n] × Cz[θ, ϕ, n] Sin[2 θ] Cos[ϕ])
     , {n, 1, 10}]]; (*(36*)
```

In[∘]:=
```
(*Auxiliary functions*)
fig1 = Show[Table[Plot[αx[θ, 0, n], {θ, 0, 2 π},
     PlotLabel → "αx(θ, ϕ=0)", PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
fig2 = Show[Table[Plot[αz[θ, 0, n], {θ, 0, 2 π}, PlotLabel → "αz(θ, ϕ=0)",
     PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
GraphicsRow[{fig1, fig2}, ImageSize → 600]

fig1 = Show[Table[Plot[BesselJ[n, αx[θ, 0, n]], {θ, 0, 2 π},
     PlotLabel → "Jn(αx)(θ, ϕ=0)", PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
fig2 = Show[Table[Plot[BesselJ[n, αz[θ, 0, n]], {θ, 0, 2 π}, PlotLabel → "Jn(αz)(θ, ϕ=0)",
     PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
GraphicsRow[{fig1, fig2}, ImageSize → 600]

fig1 = Show[Table[Plot[Cx[θ, 0, n], {θ, 0, 2 π},
     PlotLabel → "Cx(θ, ϕ=0)", PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
fig2 = Show[Table[Plot[Cz[θ, 0, n], {θ, 0, 2 π}, PlotLabel → "Cz(θ, ϕ=0)",
     PlotRange → All, AxesLabel → {"θ", ""}], {n, 1, 5}]];
GraphicsRow[{fig1, fig2}, ImageSize → 600]
```

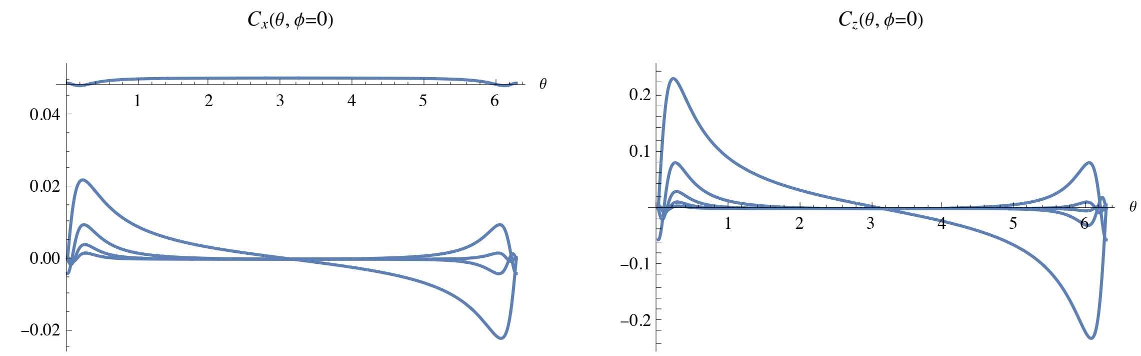
Out[∘]=



Out[∘]=



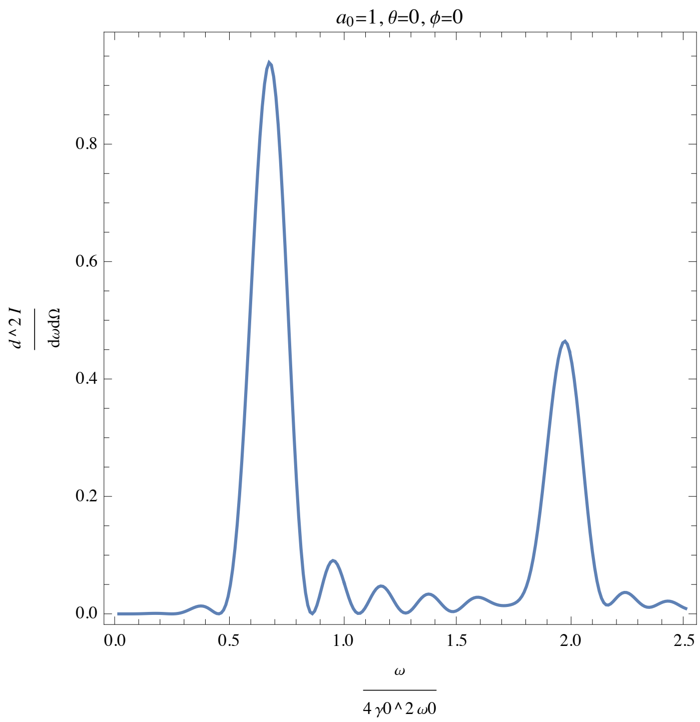
Out[∘]=



In[•]:=
```
(*1d*)
lst = Table[{y, dIdωdΩ[4 γ0^2 ω0 y, 0, 0] /. ω → (4 γ0^2 ω0 y)}, {y, 0.01, 2.5, 0.01}];
```

In[•]:=
```
ListPlot[lst, ImageSize → Medium, AspectRatio → 1,
 Joined → True, Frame → True, FrameLabel → {"ω/(4 γ0^2 ω0)", "d^2 I/(dωdΩ)"},
 PlotLabel → "a₀=" <> ToString[a0] <> ", θ=0, ϕ=0", PlotRange → All]
```

Out[•]=



In[•]:=
```
(*2d*)
tab = Table[{y, gθ, dIdωdΩ[4 γ0^2 ω0 y, gθ / γ0, 0] /. ω → (4 γ0^2 ω0 y)},
   {y, 0.01, 3, 0.05}, {gθ, 0, 1, 0.05}];
Dimensions[
 tab]
```

Out[•]= {60, 21, 3}

In[•]:=
```
ListPlot3D[Flatten[tab, 1], PlotLabel → "a₀=" <> ToString[a0] ,
  AxesLabel → {"ω/(4 γ0^2 ω0)", "γ₀ θ", "d^2 I/(dω dΩ)"}, PlotRange → All]
```

Out[•]=

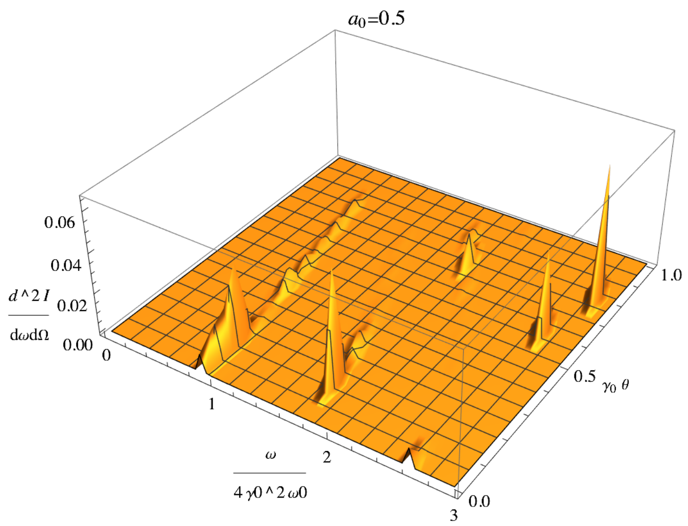


Fig 2: Normalized intensity ($a_0 = 5$, $N_0 = 7$, $\phi = 0$)

```
(*Solve for variable in horizontal axis. Keep the largest root.*)
Clear[y]
Solve[y == (a0^2 / 4) (1 + a0^2 / 2), a0][[4, 1, 2]] (*the article is somewhat ambiguous*)
```

Out[•]= $\sqrt{-1+\sqrt{1+8\,y}}$

In[•]:=
```
(*Define functions*)
Clear[n]
αn = Function[{n, a0}, (n a0^2 / 4) (1 + a0^2 / 2)];
Fn =
  Function[{n, a0}, n αn[n, a0] (BesselJ[(n - 1)/2, αn[n, a0]] - BesselJ[(n + 1)/2, αn[n, a0]])^2];
```

In[ ]:=
```
(*Plot*)
Show[Table[Plot[Fn[n, Sqrt[-1 + Sqrt[1 + 8 y]]],
  {y, 0, 0.5}, AspectRatio → 1, Frame → True, PlotRange → All,
  FrameLabel → {"(a0^2 / 4) (1 + a0^2 / 2)", "Fn(a0)"}], {n, 1, 19, 2}]]
```

Out[ ]=

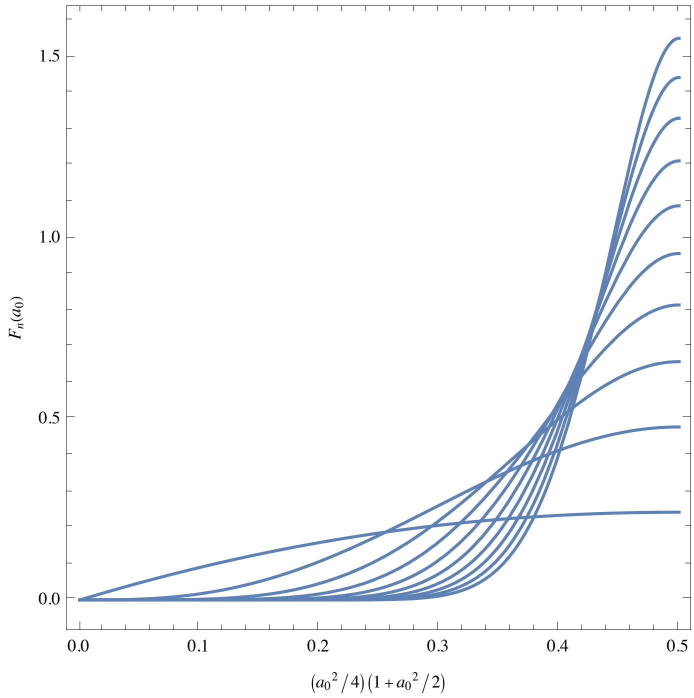


Fig 4: $F_n(a_0)$ as a function of $a_0^2/4\left(1+a_0^2/2\right)$

B. Circular polarization p. 3010

In[ ]:=
```
Y = Function[ξ, ξ^2 BesselK[2 / 3, ξ] ^2];
```

```
In[•]:= LogLogPlot[Y[ξ], {ξ, 10^-1, 10^1}, AspectRatio → 1,
  Frame → True, FrameLabel → {"ξ=ω/ω_c", "Y(ξ)"}]
```

Out[•]=

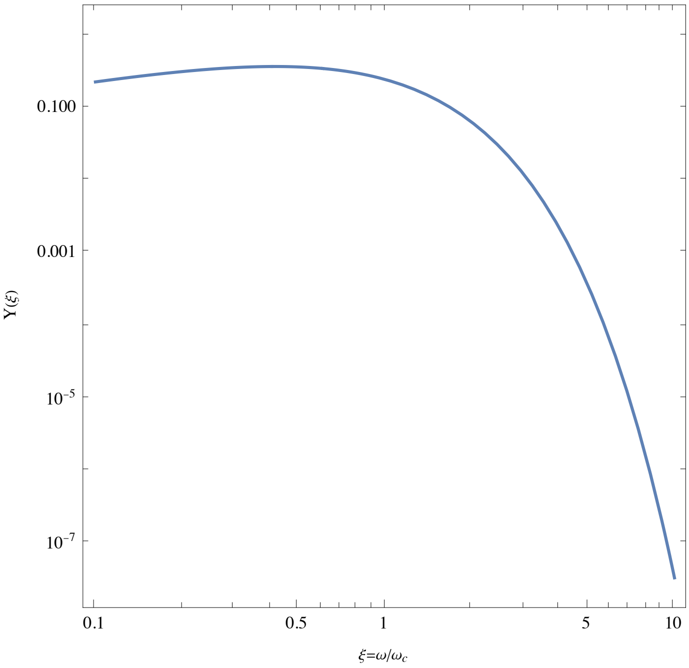


Fig 7: Y= $\xi^2\, K_{2/3}(\xi)^2$

---

2. Linear polarization p. 3014

In[∘]:=
```
(*FIG 8*)
Clear[a0]

h0 = 1;(*arbitrário?*)
ω0 = 1;
γ0 = 5;
M0 = 1;(*arbitrário?*)
nc = 3 a0^3 / (2 × √2); (*approx*)
γ = a0 (M0 + 1) / (2 × (2 M0)^0.5);
ωc = nc (M0 + 1) ω0 / 2;
ξ = ω/ωc (1 + γ^2 θ^2)^1.5;
ωNOR = ω/ω0;
d2IdodO = Function[{ωNOR, a1},
    γ^2 ξ^2/(1 + γ^2 θ^2) (γ^2 θ^2/(1 + γ^2 θ^2) BesselK[1 / 3, ξ]^2 + BesselK[2 / 3, ξ]^2) /. a0 → a1];
θ = Pi / 2;
Plot[{d2IdodO[ωNOR, 6], d2IdodO[ωNOR, 4]}, {ω, 0, 250},
 PlotRange → All, PlotLegends → Placed[{"a₀=4", "a₀=6"}, Above]]
```

Out[∘]=

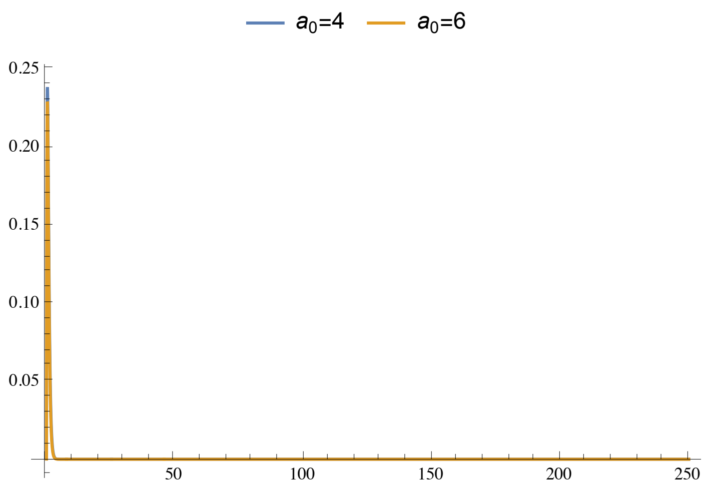
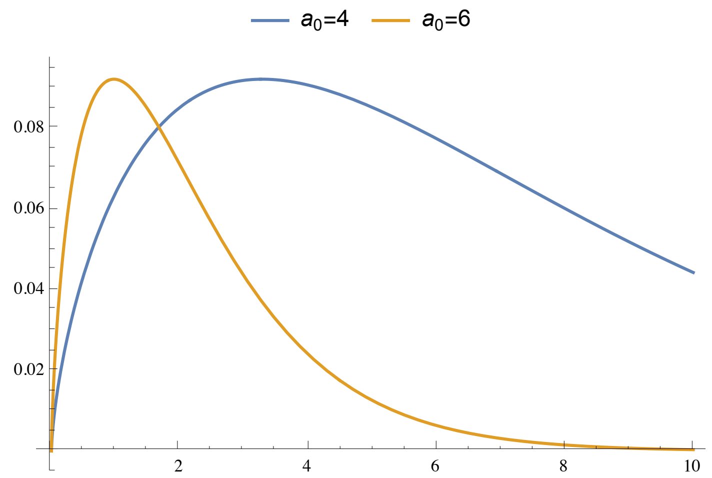


Fig 8:

```
Clear[a0]

h0 = 1;(*arbitrário?*)
ω0 = 1;
γ0 = 5;
M0 = 0.1; (*arbitrário?*)
nc = 3 a0^3 / 4; (*approx*)
γh = h0 / 2;
ωc = nc M0 ω0;
ξ = ω/ωc (1 + γh^2 θ^2)^1.5;
ωNOR = ω/(4 γ0^2 ω0^2);
d2Idod0 = Function[{ωNOR, a1},
   γh^2 ξ^2/(1 + γh^2 θ^2) (γh^2 θ^2/(1 + γh^2 θ^2) BesselK[1 / 3, ξ]^2 + BesselK[2 / 3, ξ]^2) /. a0 → a1];
θ = 0;
Plot[{d2Idod0[ωNOR, 6], d2Idod0[ωNOR, 4]}, {ω, 0, 10},
 PlotRange → All, PlotLegends → Placed[{"a₀=4", "a₀=6"}, Above]]
```

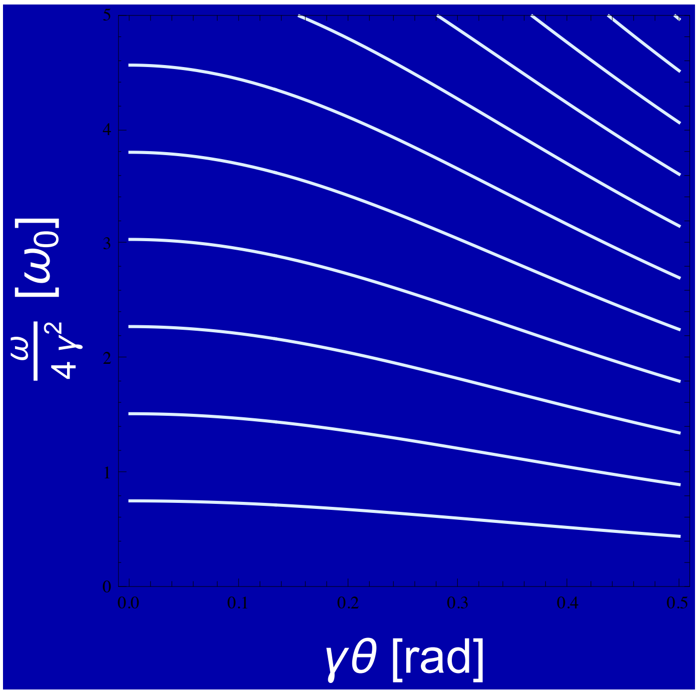
Out[ ]=




```
(*where to look for harmonics*)
```

```
In[•]:= γ0 = 2;
a0 = 0.1;

ω0 = 1;
β0 = √(1 - 1 / γ0^2);
M0 = γ0^2 (1 + β0^2)^2 / (1 + a0^2 / 2);
β1 = (1 - 1 / M0) / 2;
ωn[θ_, n_] := n ω0 / (1 - β1 (1 + Cos[θ]))

Show[ Table[Plot[ωn[θ, n] / (4 γ0^2), {θ, 0, 1 / γ0}, PlotRange → {0, 5},
   PlotStyle → LightBlue, AspectRatio → 1, Axes → False, Frame → True, FrameLabel →
    {Text[Style["γθ [rad]", White, 25]], Text[Style["ω/(4 γ^2) [ω0]", White, 25]]},
   Background → Darker[Blue]], {n, 1, 30, 1}] ]
```

Out[•]=



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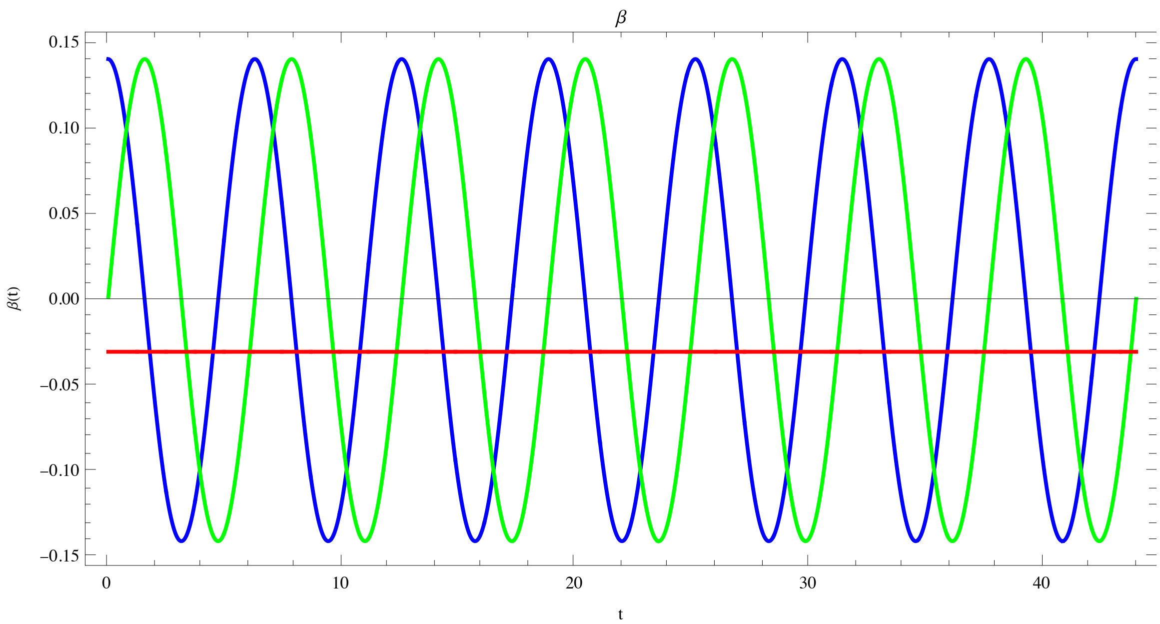
lpw

```
(* Mathematica notebook for trajetory in linear polarized
   plane wave. Following [Esarey]'s paper in section II. Electron
   Motion in Intense Laser Fields (p. 3005) 1) Linear Polarization
  date : 17/01/2019
  author: Óscar L. Amaro
*)
```

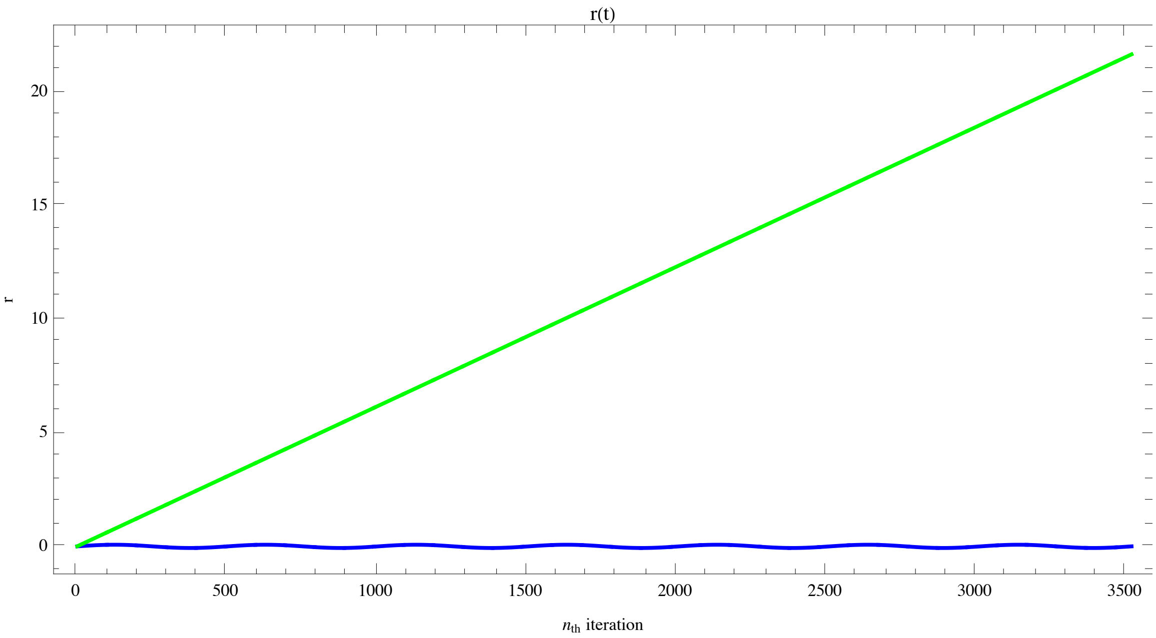
```
(*Parameters*)
N0 = 7;
dt = 0.1 / 8;
tmax = Round[N0 * 2 π / dt]; (* normalized time*)
η = Table[t * dt, {t, 1, tmax, 1}]; (*eta*)

a0 = 1; (*normalized amplitude*)
δp = 0; (*1-linear, 0-circular*)
γ0 = 5; (*initial gamma *)
γ0L = γ0 / (1 + a0^2/2); (*average gamma *)
β0 = Sqrt[1 - 1 / (γ0 * γ0)]; (*initial β *)
h0 = γ0 (1 + β0);
k0 = 1;
M0 = h0^2 / (1 + a0^2 / 2);

(*p.3006*)
r1 = a0 / (h0 k0);
z1 = -a0^2 / (8 h0^2 k0);
β1 = (1 - 1 / M0) / 2;
β1avg = (M0 - 1) / (M0 + 1);

(*Arrays*)
a = Table[a0/√2 {(1 + δp)^0.5 Cos[ η[[t]] ], (1 - δp)^0.5 Sin[ η[[t]] ], 0}, {t, 1, tmax, 1}];
βz = Table[(h0^2 - (1 + Norm[a[[t]]]^2)) / (h0^2 + (1 + Norm[a[[t]]]^2)), {t, 1, tmax, 1}];
γ = Table[(h0^2 + 1 + Norm[a[[t]]]^2) / (2 h0), {t, 1, tmax, 1}];
βprp = Table[{a[[t, 1]] / γ[[t]], a[[t, 2]] / γ[[t]]}, {t, 1, tmax, 1}];
(*linear*)
x = Table[r1 Sin[k0 η[[t]] ], {t, 1, tmax, 1}];
y = Table[0, {t, 1, tmax, 1}];
z = Table[β1 η[[t]] + z1 Sin[2 k0 η[[t]] ], {t, 1, tmax, 1}];

(*circular*)
x = Table[r1/√2 Sin[k0 η[[t]] ], {t, 1, tmax, 1}];
y = Table[-r1/√2 Cos[k0 η[[t]] ], {t, 1, tmax, 1}];
z = Table[β1 η[[t]] , {t, 1, tmax, 1}];
```

In[•]:=
```
(*Plot*)
ListPlot[{Transpose[{η, βprp〚All, 1〛}],
   Transpose[{η, βprp〚All, 2〛}],
   Transpose[{η, βz - 1}]},
  PlotStyle → {Directive[Thick, Blue], Directive[Thick, Green], Directive[Thick, Red]},
  Joined → True, AspectRatio → 0.5, ImageSize → 600, Frame → True,
  FrameLabel → {"t", "β(t)"}, PlotRange → All, PlotLabel → "β"]

ListPlot[{x, z},
  PlotStyle → {Directive[Thick, Blue], Directive[Thick, Green], Directive[Thick, Red]},
  Joined → True, AspectRatio → 0.5, ImageSize → 600, Frame → True,
  FrameLabel → {"nth iteration", "r"}, PlotRange → All, PlotLabel → "r(t)"]

ListPlot[Transpose[{z, x}],
  PlotStyle → {Directive[Thick, Blue], Directive[Thick, Green], Directive[Thick, Red]},
  Joined → True, AspectRatio → 0.5, ImageSize → 600, Frame → True,
  FrameLabel → {"z", "x"}, PlotRange → All, PlotLabel → "x(z)"]
```

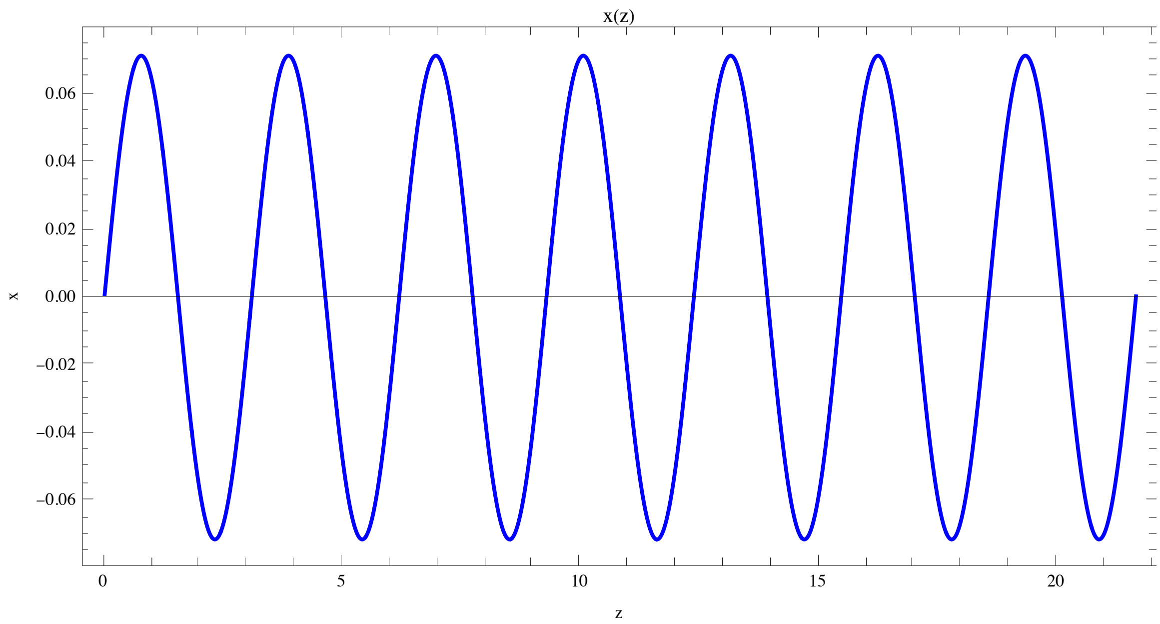
Out[•]=



Out[ ]=


r(t)
r
20
15
10
5
0
0
500
1000
1500
2000
2500
3000
3500
n_{th} iteration

Out[ ]=


x(z)
x
0.06
0.04
0.02
0.00
–0.02
–0.04
–0.06
0
5
10
15
20
z

```
In[•]:= (*Export*)
lst = η;
Dimensions[lst]
Export["dataT.txt", lst, "Table"]

lst = Transpose[{βprp[[All, 1]], βprp[[All, 2]], βz}];
Dimensions[lst]
Export["dataBETA.txt", lst, "Table"]

lst = Transpose[{x, y, z}];
Dimensions[lst]
Export["dataR.txt", lst, "Table"]

Out[•]= {3519}

Out[•]= dataT.txt

Out[•]= {3519, 3}

Out[•]= dataBETA.txt

Out[•]= {3519, 3}

Out[•]= dataR.txt
```