lpw

```
(* Mathematica notebook for spectra
  (to compare with simulation). Following [Esarey]'s paper in sections A. and
  B. analysis of harmonics spectra requires spectra "boris.txt" from a test-
 particle or particle-in-cell simulation.

  date : 05/04/2019
  author: Óscar L. Amaro
*)
```

Intro

In[∘]:=
```
(*Parameters*)
γ0 = 5; (*p. 48*)
a0 = 1;

β0 = √(1 - 1 / γ0^2); (*normalizations*)
ω0 = 2;
k0 = ω0;

h0 = γ0 (1 + β0); (* p.3006 (8c) se se excluir Φ...*)
M0 = h0^2 / (1 + a0^2 / 2);
η0 = 100;(*controls fineness of spectrum*)

β1 = (1 - 1 / M0) / 2;
r1 = a0 / (h0 k0); (*p. 3006 (16a)*)
z1 = -a0^2 / (8 h0^2 k0);
β1 = (1 - 1 / M0) / 2; (*p.3006 16c*)
(*Functions p. 3008*)
kbar = Function[{ω, θ, n}, ω (1 - β1 (1 + Cos[θ])) - n k0];(*(30a)*)
```

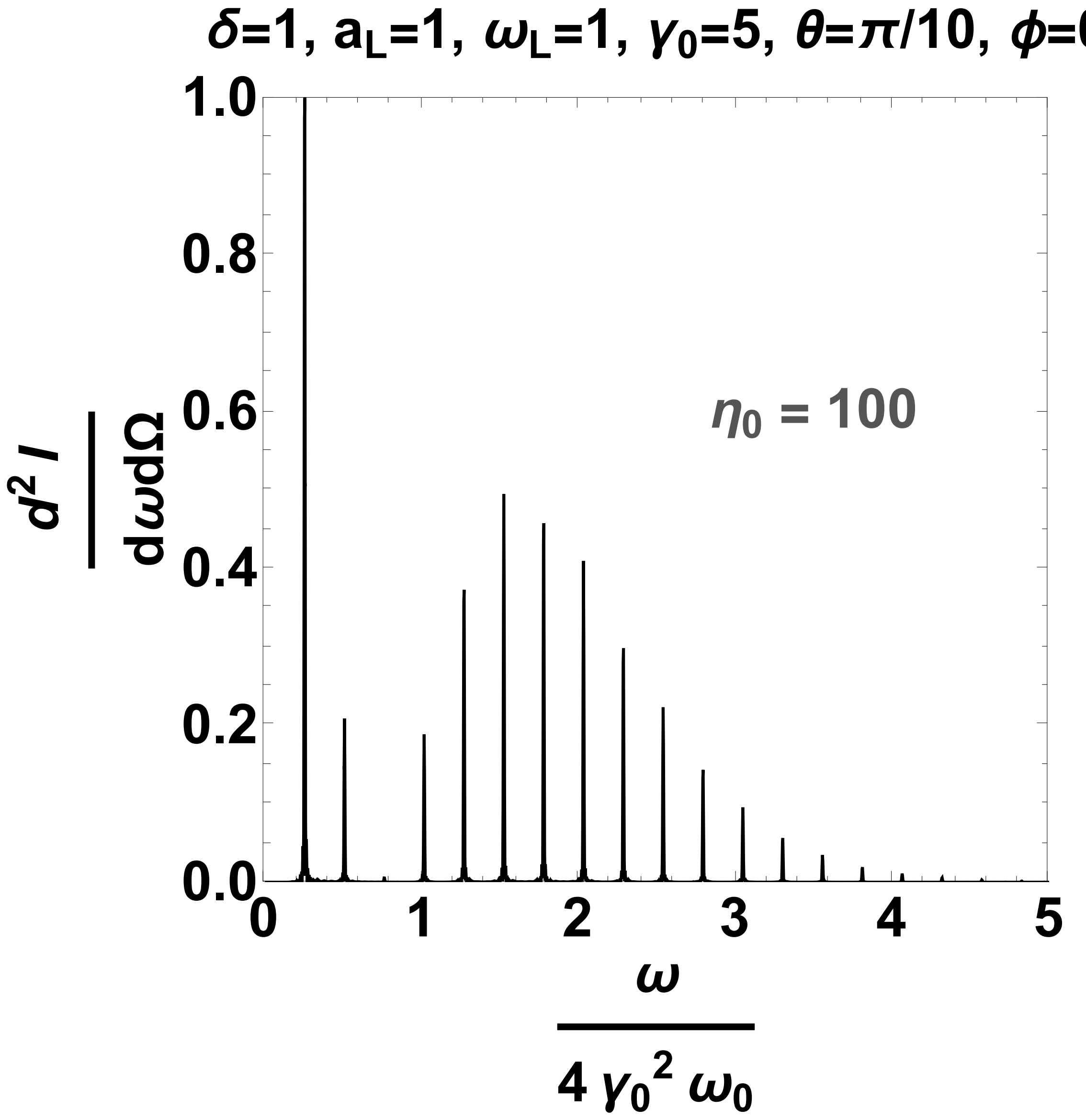
A. Linear polarization p. 3007

```
(*alpha*)
αz = Function[{θ, ϕ, n}, (n a0^2 (1 + Cos[θ]))/(8 h0^2 (1 - β1 (1 + Cos[θ])))]; (*(38a)*)
αx = Function[{θ, ϕ, n}, (n a0 Sin[θ] Cos[0])/(h0 (1 - β1 (1 + Cos[θ])))]; (*(38b)*)
(*C*)
sumlim = 20; (*parameter that controls the sum*)
Cx = Function[{θ, ϕ, n},
   k0 r1 Sum[N[  (-1)^m BesselJ[m, αz[θ, 0, n]]
       (BesselJ[n - 2 m - 1, αx[θ, 0, n]] + BesselJ[n - 2 m + 1, αx[θ, 0, n]])  ],
     {m, -sumlim, sumlim}]]; (*(37a)*)

Cz = Function[{θ, ϕ, n},
   2 Sum[
     N[  (-1)^m BesselJ[m, αz[θ, 0, n]] (β1 * BesselJ[n - 2 m, αx[θ, 0, n]] + k0 z1 (
          BesselJ[n - 2 m - 2, αx[θ, 0, n]] + BesselJ[n - 2 m + 2, αx[θ, 0, n]] )) ],
     {m, -sumlim, sumlim}]] ;(*(37b)*)

(*main*)
dIdωdΩ = Function[{ω, θ, ϕ},
   ω^2/(4 π^2) Sum[ (Sin[kbar[ω, θ, n] η0 ]/(kbar[ω, θ, n] η0))^2 (Cx[θ, 0, n]^2 (1 - Sin[θ]^2 Cos[ϕ]^2) +
       Cz[θ, 0, n]^2 Sin[θ]^2 - Cx[θ, 0, n] × Cz[θ, 0, n] Sin[2 θ] Cos[ϕ])
     , {n, 1, 40}]]; (*(36*)
```

In[•]:=
```
(*plot against simulation*)
wmax = 5;
lst = ParallelTable[{y, dIdωdΩ[4 γ0^2 ω0 y, N[π / 10], 0] /. ω → ( 4 γ0^2 ω0 y)},
   {y, 0.01, wmax, wmax/2000}];
```

In[∘]:=
```
Max[lst⟦All, 2⟧];
lst⟦All, 2⟧ /= Max[lst⟦All, 2⟧];
grp1 = ListPlot[lst, AspectRatio → 1, Joined → True,
   Frame → True, FrameLabel → {"ω/(4 γ0^2 ω0)", "d^2 I/(dωdΩ)"}, PlotLabel → Text[
     Style["δ=1, aL=" <> ToString[a0] <> ", ωL=1, γ0=5, θ=π/10, ϕ=0", 30, Bold]],
   FrameStyle → Directive[Bold, FontSize → 30], PlotStyle →
    Directive[Black, Opacity[1]], ImageSize → 600, PlotRange → {{0, 5}, {0, 1}}];
Show[{grp1, Graphics[{Text[Style[Style["η0 = 100", Darker[Gray]] ,
      FontSize → 30, Bold], {3.5, 0.6}]}]}]
```

Out[∘]=



In[∘]:=
```
Export["esarey.txt", lst]
```

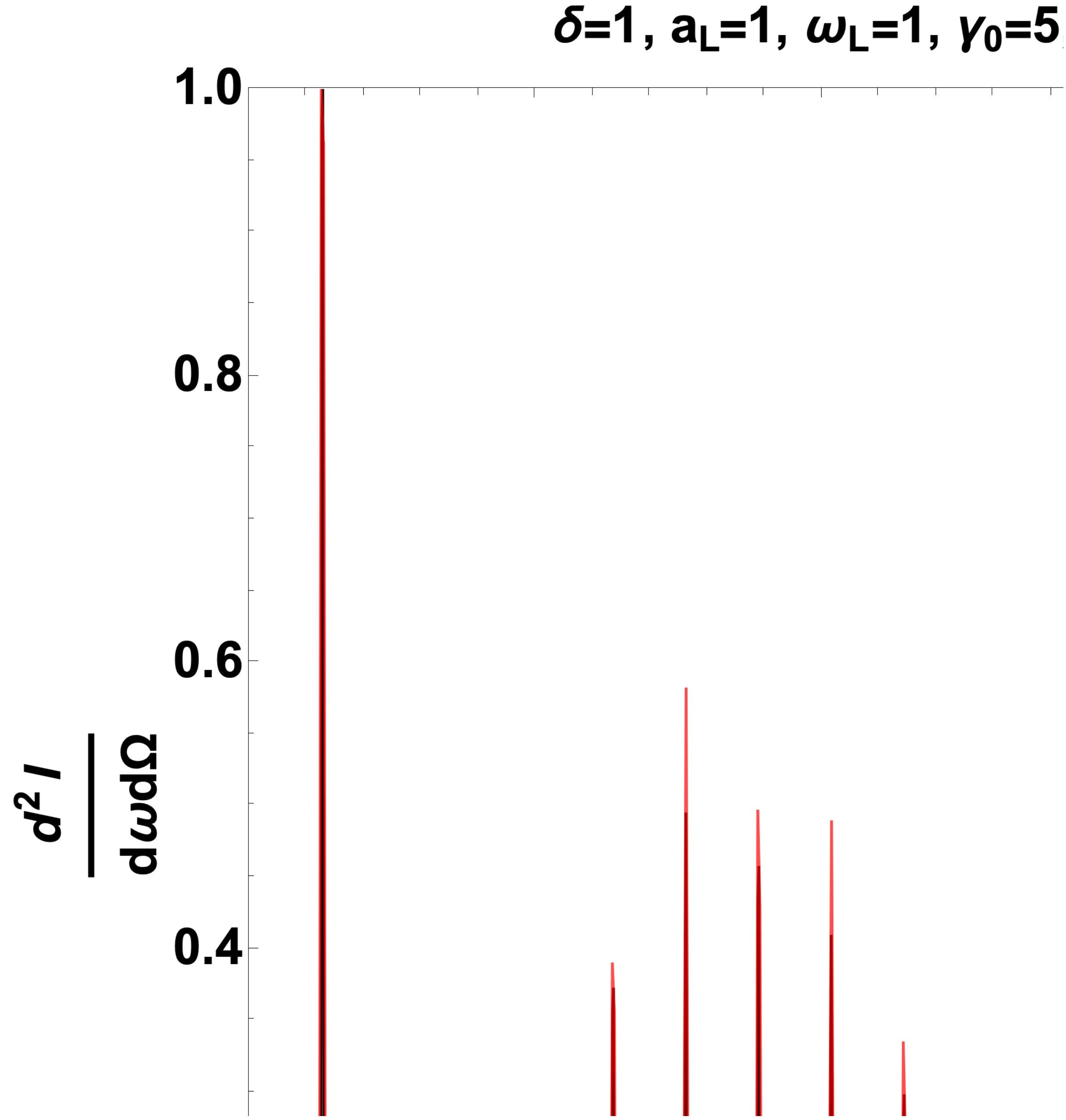
Out[∘]= esarey.txt

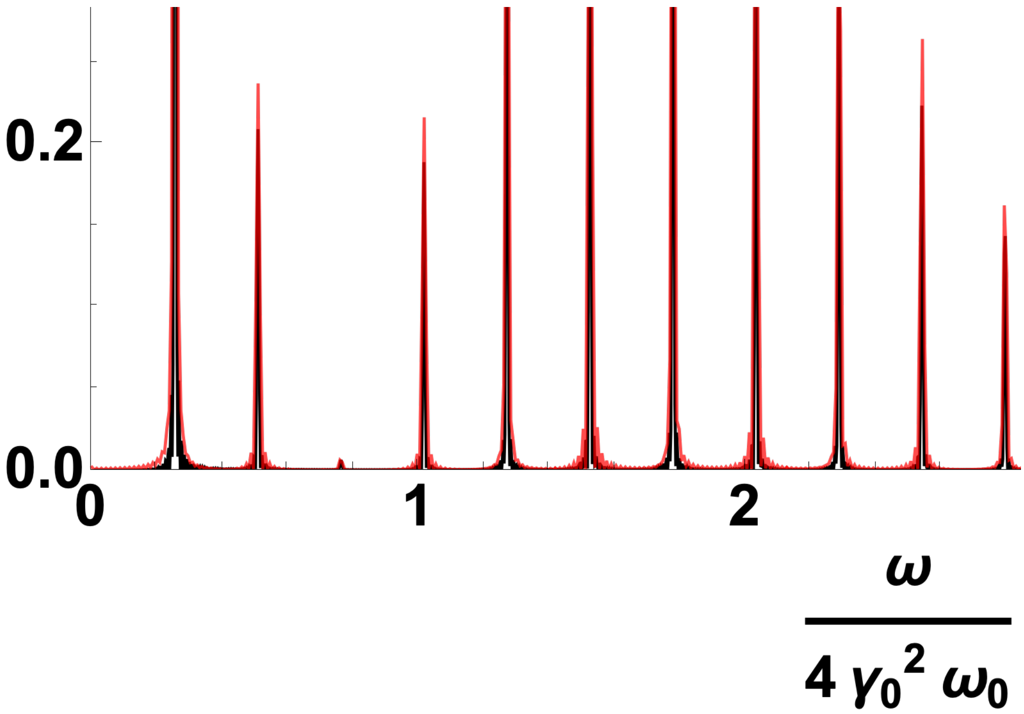
Compare with simulation

In[∘]:=
```
mat = Import["boris.txt", "Table"];
mat = Flatten[mat];
w = Import["dataw.txt", "Table"];
w = Flatten[w];
```

In[∘]:=
```
grp2 = ListPlot[Transpose[{w / (4 * γ0 ^ 2), mat / Max[mat]}],
    ImageSize → Medium, AspectRatio → 1, Joined → True, Frame → True,
    FrameLabel → {" ω / (4 γ0^2 ω0) ", " d^2 I / (dω dΩ) "}, PlotRange → {{0, 2.5}, {0, 1}},
    PlotStyle → {Black, Directive[Red, Opacity[0.7]]}, ImageSize → 600];
Show[{grp1, grp2, Graphics[
    {Text[Style[Style["Red", Red] "- simulation\nBlack - theory",
      FontSize → 30, Bold], {4, 0.6}]}]}, ImageSize → 1000]
```

Out[∘]=



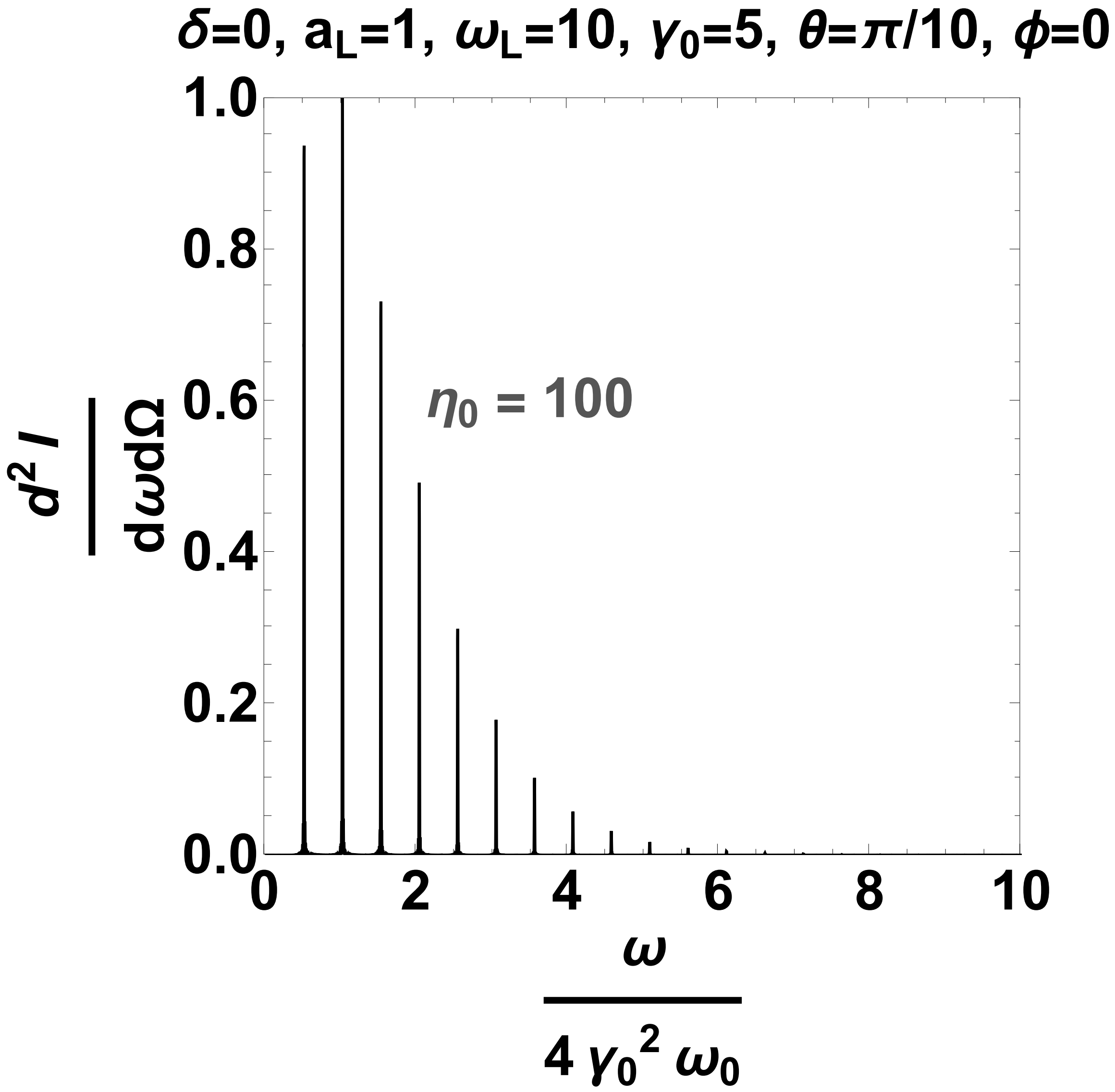


B. Circular polarization p. 3010

```
In[•]:= α = Function[{n, θ}, (n (a0 / √2) Sin[θ]) / (h0 (1 - β1 (1 + Cos[θ])))];
dIdωdΩ = Function[{ω, θ, ϕ},
   ω^2/π^2 Sum[N[(Sin[kbar[ω, θ, n] η0 ] / kbar[ω, θ, n])^2
       ((Cos[θ] - β1 (1 + Cos[θ]))^2 / Sin[θ]^2 BesselJ[n, α[n, θ]]^2 + (k0^2 r1^2)/2
          (((1/2 (BesselJ[-1 + n, x] - BesselJ[1 + n, x])) /. x → α[n, θ])^2)]
     , {n, 1, 80}]];
(*(36*)

In[•]:= lst = ParallelTable[
   {y, dIdωdΩ[4 γ0^2 y, N[π / 10], 0] /. ω → (4 γ0^2 y)}, {y, 0.01, 10, 10/6000}];
```

In[•]:=
```
Max[lst[[All, 2]]];
lst[[All, 2]] /= Max[lst[[All, 2]]];
grp1 = ListPlot[lst, AspectRatio → 1, Joined → True,
   Frame → True, FrameLabel → {"ω/(4 γ0^2 ω0)", "d^2 I/(dωdΩ)"}, PlotLabel → Text[
     Style["δ=0, aL=" <> ToString[a0] <> ", ωL=10, γ0=5, θ=π/10, ϕ=0", 30, Bold]],
   FrameStyle → Directive[Bold, FontSize → 30],
   PlotStyle → Directive[Black, Opacity[1]],
   ImageSize → 600, PlotRange → {{0, 10}, {0, 1}}];
Show[{grp1, Graphics[{Text[Style[Style["η0 = 100", Darker[Gray]] ,
      FontSize → 30, Bold], {3.5, 0.6}]}]}]
```

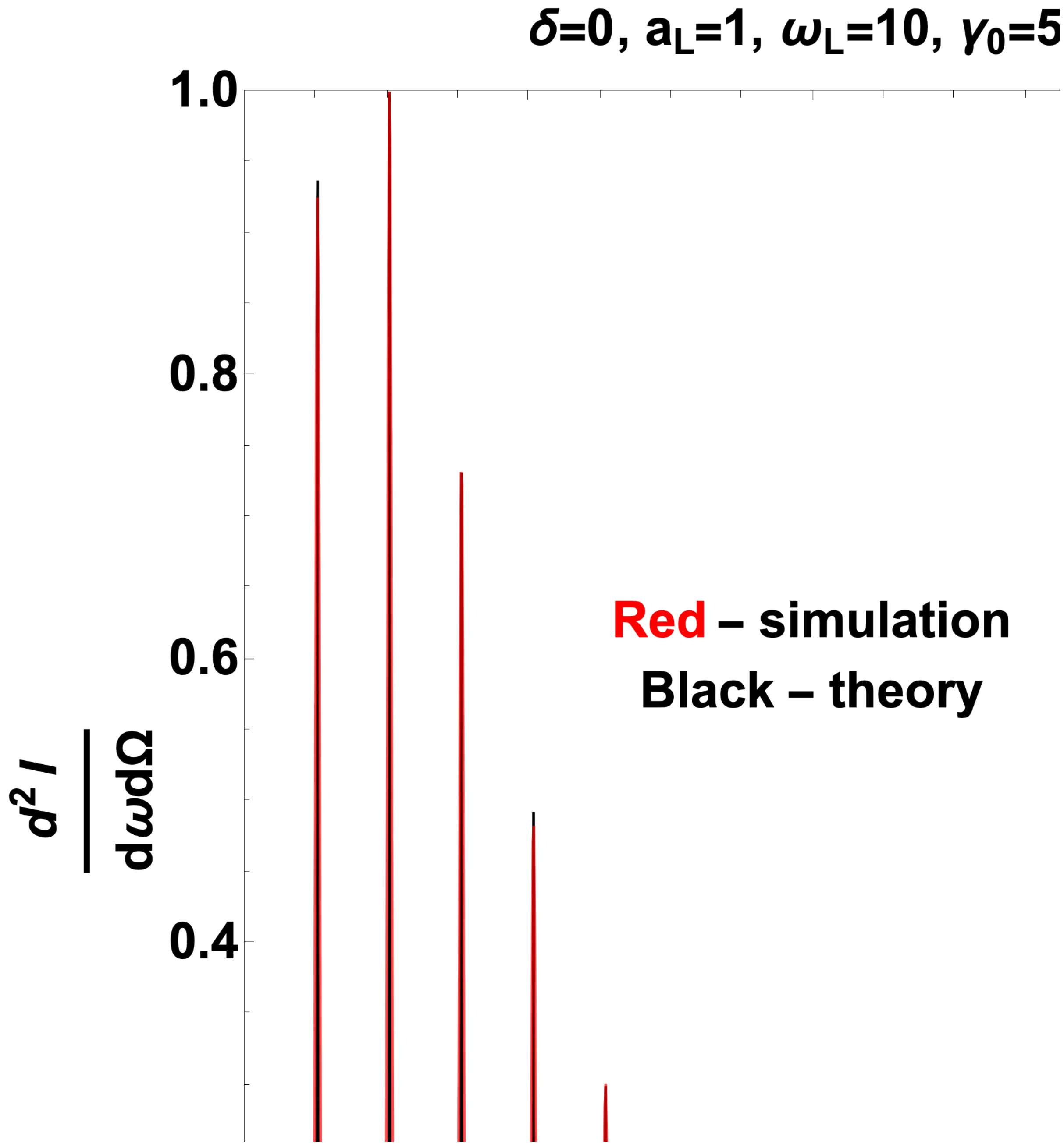
Out[•]=

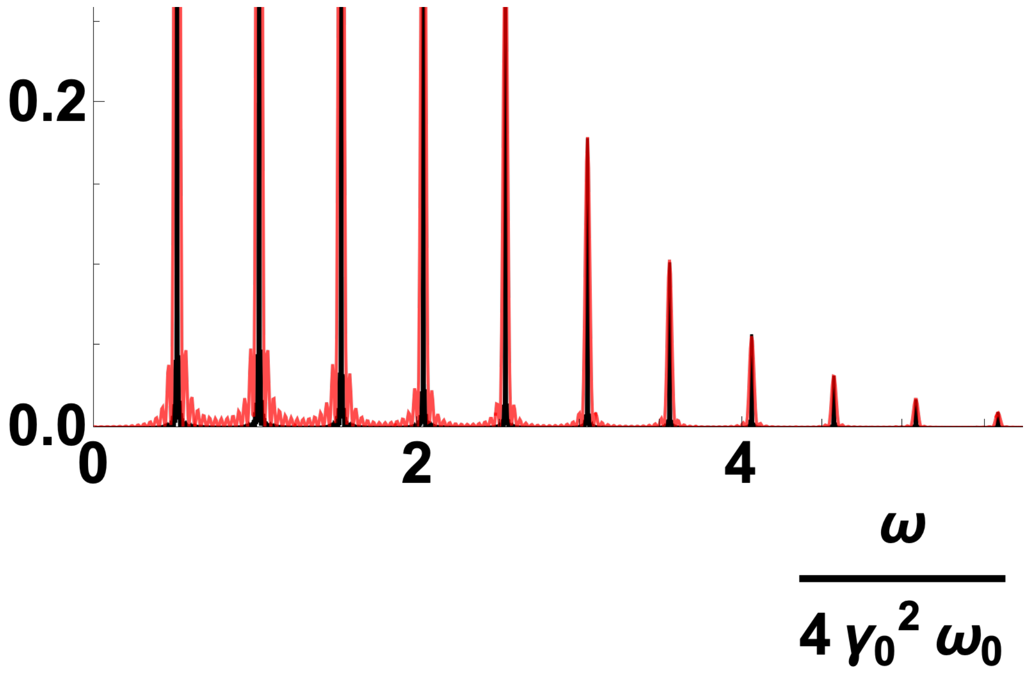


Compare with simulation

```
In[•]:= mat = Import["boris.txt", "Table"];
        mat = Flatten[mat];
        w = Import["dataw.txt", "Table"];
        w = Flatten[w];

In[•]:= grp2 = ListPlot[Transpose[{w / (4 * γ0 ^ 2), mat / Max[mat]}],
           ImageSize → Medium, AspectRatio → 1, Joined → True, Frame → True,
           FrameLabel → {"ω/(4 γ0^2 ω0)", "d^2 I/(dω dΩ)"}, PlotRange → {{0, 2.5}, {0, 1}},
           PlotStyle → {Black, Directive[Red, Opacity[0.7]]}, ImageSize → 600];
        Show[{grp1, grp2, Graphics[
           {Text[Style[Style["Red", Red] "- simulation\nBlack - theory",
              FontSize → 30, Bold], {4, 0.6}]}]}], ImageSize → 1000]
```

Out[•]=




0.2
0.0
0
2
4
$\frac{\omega}{4\,\gamma_0^2\,\omega_0}$