Relativistic plasma aperture for laser intensity enhancement

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Notebook on Github

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Introduction

In this notebook we reproduce some results from the paper.

Style

In[ ]:=
```
fntsz = 20;
imgsz = 400;
```

Contribution of N harmonics

In[•]:=
```
Clear[g, N, glim, n, g10]
g[N_] := Sqrt[Sum[n^(-8/3), {n, 2, N}]]
g10 = g[10];
glim = Limit[g[N], N → ∞] // N;
Plot[{g[N], glim}, {N, 3, 20}, PlotRange → {0.45, 1.01 glim},
 Frame → True, Axes → False, FrameStyle → Directive[Black], AspectRatio → 1,
 FrameLabel → {Style["n", Black, fntsz], Style["∑_{n=2}^{N} n^{-8/3}", Black, fntsz]},
 PlotLabel → Style[glim, Black, fntsz], FrameTicksStyle → fntsz, ImageSize → imgsz]
```

Out[•]=

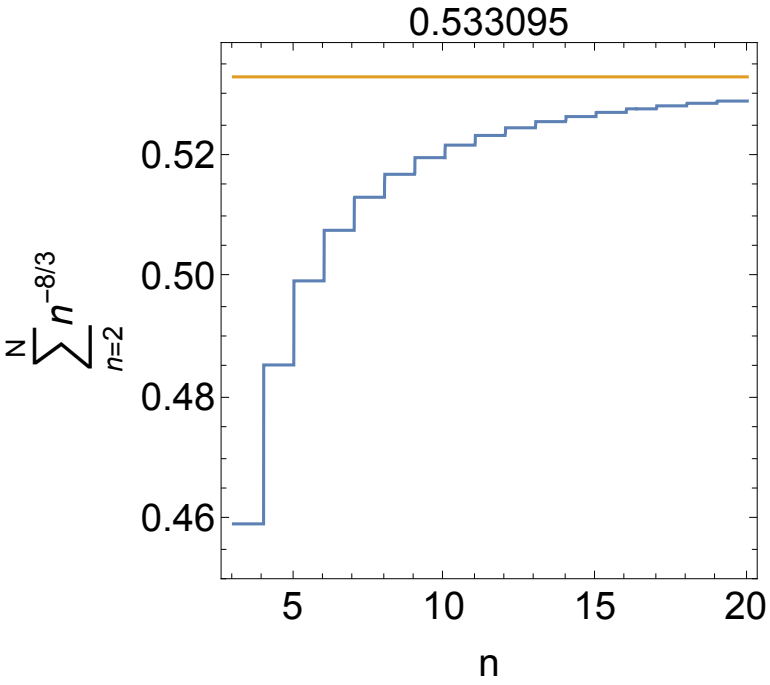

Figure 4 a: find l such that fs=fp

In[•]:=
```
Clear[a0max, a0t, wmin, θ, wavg, fs, ψ, θ2nd, γavg, β, fp]
Clear[τ, W0, λ, c, ω0, a0, k0, np, nc]

(*equation 1*)
a0max = (np π l)/(nc λ);

(*equation 3*)
a0t = (4/3 × √2 k0^2 l^2 np)/((3/4 + 8/3 × √2 k0 l) nc);
```

```
(*equation 6*)
wmin = W0 Sqrt[Abs[Log[a0max / a0]]];

(*equation 7*)
θ = ArcTan[ τ c / (Sqrt[2 Log[2]] W0) wmin / wavg ];

(*equation 8*)
wavg = W0 Sqrt[Abs[Log[a0t / a0]]];

(*equation 9*)
fs = wmin Tan[θ];

(*equation 10*)
ψ = ArcTan[ τ c / (Sqrt[2 Log[2]] W0) ];

(*equation 11*)
θ2nd = ArcCos[ (1 + β Cos[ψ / 2]) / (2 β) ];

(* after equation 11 *)
γavg = Sqrt[1 + (a0t)^2 / 2];
β = Sqrt[1 - 1 / (γavg)^2];

(*equation 12*)
fp = wavg Tan[θ2nd + ψ / 2] // Simplify;


(*parameters*)
τ = 30 × 10^-15;(*s*)
W0 = 1.5 × 10^-6;(*m*)
λ = 805 × 10^-9;(*m*)
c = 3 × 10^8;(*m/s*)
ω0 = 2 π c / λ // N;(*rad/s*)
a0 = 69;(**)
k0 = ω0 / c;(*1/m*)
np = 450 nc;(**)

fs = fs // Simplify
fp = fp // Simplify

leq = FindMinimum[(fs / fp - 1)^2, {l, 10 × 10^-9}][[2, 1, 2]];
Plot[(fs / fp - 1)^2 /. {l → ll 10^-9}, {ll, 10, 30}, Frame → True,
 Axes → False, FrameStyle → Directive[Black], AspectRatio → 1,
 FrameLabel → {Style["l[nm]", Black, fntsz], Style["(fs/fp-1)^2", Black, fntsz]},
```

```
PlotLabel → Style[leq 10^9, Black, fntsz],
ImageSize → imgsz, FrameTicksStyle → fntsz]
```

Out[•]=
$$\frac{9\,\text{Abs}\left[\text{Log}\left[\frac{30\,000\,000\,000\,\text{l}\,\pi}{3703}\right]\right]}{1\,000\,000\,\sqrt{\text{Abs}\left[17.0523+\text{Log}\left[\frac{\text{l}^2}{2.54797\times10^{-8}+1.\,\text{l}}\right]\right]}\,\sqrt{\text{Log}[4]}}$$

Out[•]=
$$1.5\times10^{-6}\,\sqrt{\text{Abs}\left[\text{Log}\left[\frac{2.54517\times10^{7}\,\text{l}^2}{2.54797\times10^{-8}+1.\,\text{l}}\right]\right]}$$

$$\text{Tan}\left[0.688512+\text{ArcCos}\left[\frac{1}{2}\times\left(0.772192+\frac{2.27981\times10^{-18}}{\sqrt{\frac{\text{l}^4}{1.92398\times10^{35}\,\text{l}^4+\left(9+3.53223\times10^{8}\,\text{l}\right)^2}}}\right)\right]\right]$$

Out[•]=

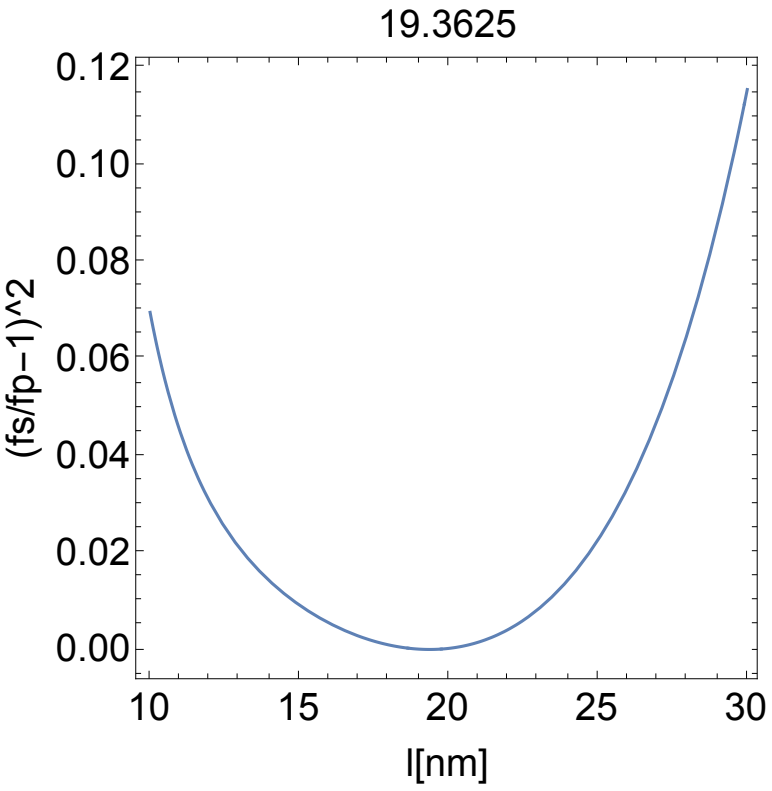

Figure 4 b: harmonics I $\propto n^{-8/3} \rightarrow$ E $\propto n^{-4/3}$

In[•]:=
```
Clear[n]
ListPlot[Table[{n, n^(-4 / 3)}, {n, 1, 10}], PlotRange → {{1, 10}, {0, 1}},
 Frame → True, Axes → False, FrameStyle → Directive[Black], AspectRatio → 1,
 FrameLabel → {Style["n", Black, fntsz], Style["E/E0", Black, fntsz]},
 PlotStyle → PointSize[0.03], FrameTicksStyle → fntsz, ImageSize → imgsz]
```

Out[•]=

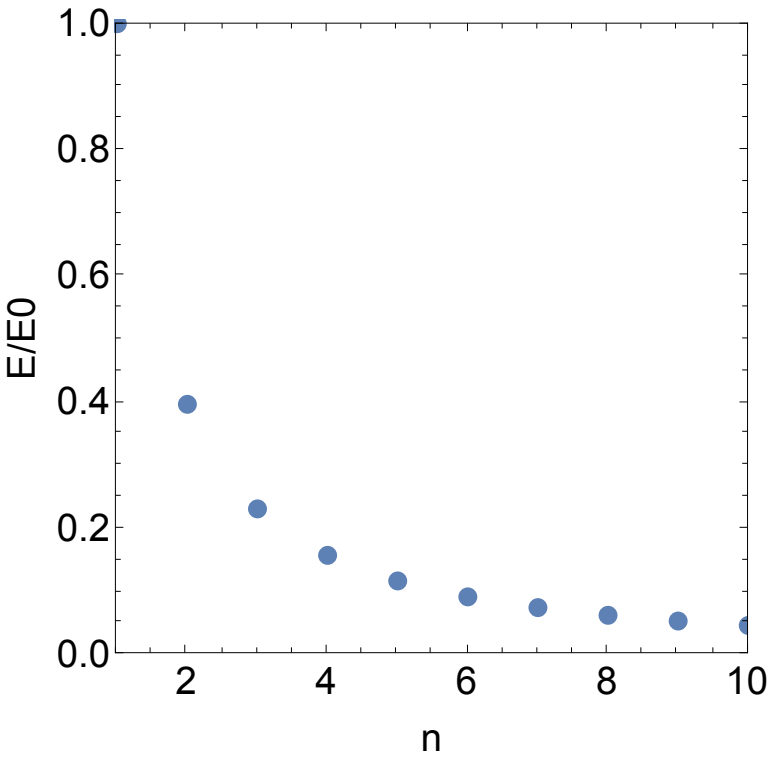

Figure 6: Maximum I

In[•]:=
```
Clear[a0max, a0t, wmin, θ, wavg, fs, ψ, θ2nd, γavg, β, fp]
Clear[τ, W0, λ, c, ω0, a0, k0, np, nc]
Clear[I0, λ, lskin, a0pfun, a0sfun, a0ptop, a0stop]

I0 = 10^22;(*W/cm^2*)
λ = 10^-6;(*m*)

(*def*)
a0 = 0.8855 Sqrt[I0 / 10^18] λ / 10^-6;

(*equation 1*)
a0max = np π l / nc λ;
```

```
(*equation 6*)
wmin = W0 Sqrt[Abs[Log[a0max / a0]]];

(*equation 3*)
a0t = (4 / 3 × √2 k0^2 l^2 np) / ((3 / 4 + 8 / 3 × √2 k0 l) nc);

(*equation 4*)
a0s = 2 a0 Exp[-r^2 / (2 W0)^2] /. {r → wmin};

(*equation 5*)
a0p = 2 g10 a0t + a0;

(*parameters*)
τ = 30 × 10^-15;(*s*)
W0 = 1.5 × 10^-6;(*m*)
λ = 805 × 10^-9;(*m*)
c = 3 × 10^8;(*m/s*)
ω0 = 2 π c / λ // N;(*rad/s*)
k0 = ω0 / c;(*1/m*)
np = 450 nc;(**)

lskin = 42 × 10^-9;
a0ptop = (a0p / a0)^2 /. {l → lskin};
a0pfun =
  ((a0p / a0)^2 HeavisideTheta[lskin - l] + a0ptop HeavisideTheta[l - lskin]);

a0stop = (a0s / a0)^2 /. {l → lskin};
a0sfun =
  ((a0s / a0)^2 HeavisideTheta[lskin - l] + a0stop HeavisideTheta[l - lskin]);

fntsz = 20;
Plot[{a0pfun /. {l → ll 10^-9}, a0sfun /. {l → ll 10^-9}}, {ll, 10, 60 },
 Frame → True, Axes → False, FrameStyle → Directive[Black], AspectRatio → 1 / 2,
 FrameLabel → {Style["l[nm]", Black, fntsz], Style["I/I0", Black, fntsz]},
 PlotRange → {0, 4}, FrameTicksStyle → fntsz, ImageSize → imgsz]
```

Out[•]=