lpw harmonic transformation to lab frame

```
(* Mathematica notebook for analysis of harmonic
 transformation to lab frame. Following [Gibbon]'s lecture 4
  date : 16/02/2019
  author: Óscar L. Amaro
*)
```

In[•]:= $\omega L = \dfrac{m\,\omega 0}{1 + \frac{a0\text{^}2}{2}\,\mathrm{Sin}\left[\frac{\theta L}{2}\right]\text{^}2}$

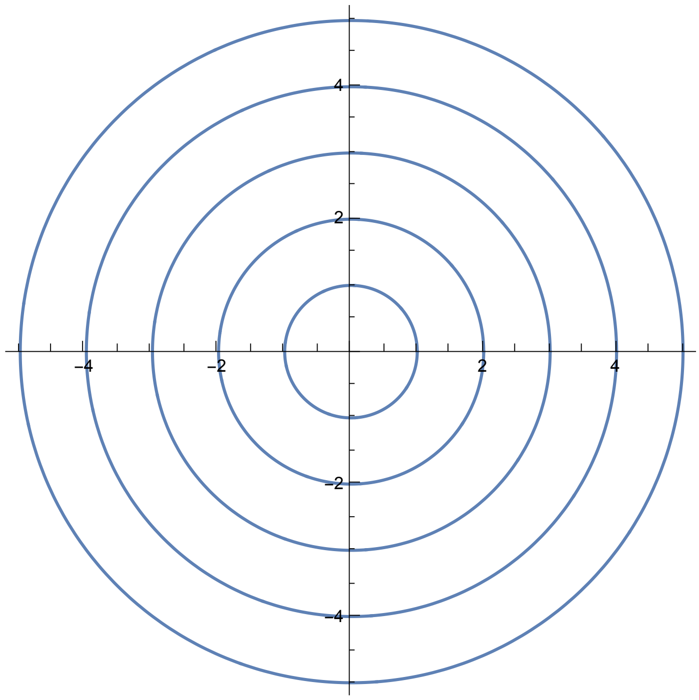
Out[•]= $\dfrac{m\,\omega 0}{1 + \frac{1}{2}\,a0^2\,\mathrm{Sin}\left[\frac{\theta L}{2}\right]^2}$

In[•]:= $Da0 = \left(\dfrac{\gamma 0}{\left(1 + \frac{a0\text{^}2}{2}\,\mathrm{Sin}\left[\frac{\theta L}{2}\right]\text{^}2\right)}\right)\text{^}4$

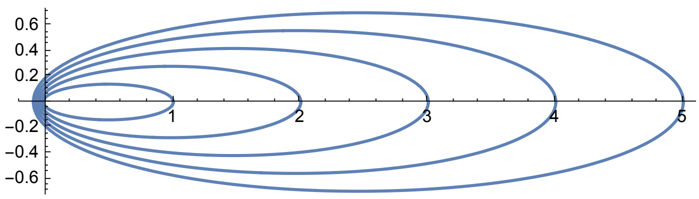
Out[•]= $\dfrac{\gamma 0^4}{\left(1 + \frac{1}{2}\,a0^2\,\mathrm{Sin}\left[\frac{\theta L}{2}\right]^2\right)^4}$

In[∘]:=
```
(*(55)*)

a0 = 0.1; ω0 = 1;
Show[Table[PolarPlot[(m ω0)/(1 + a0^2/2 Sin[θL / 2]^2), {θL, 0, 2 Pi}], {m, 1, 5}]]
Clear[a0, ω0]

a0 = 10; ω0 = 1;
Show[Table[PolarPlot[(m ω0)/(1 + a0^2/2 Sin[θL / 2]^2), {θL, 0, 2 Pi}], {m, 1, 5}]]
Clear[a0, ω0]
```

Out[∘]=



Out[∘]=

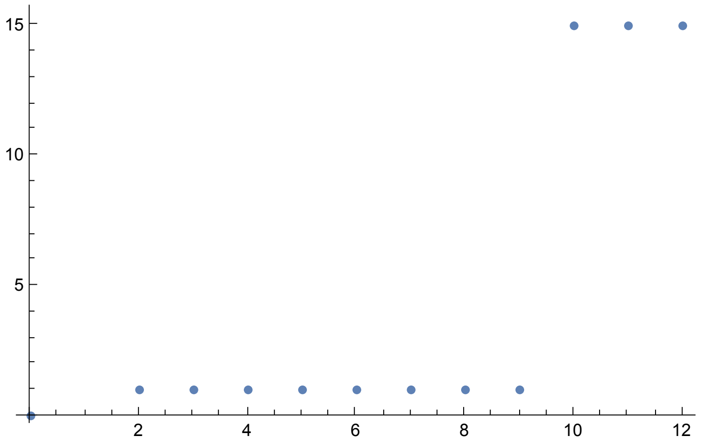


In[∘]:=
```
dPdΩ = Function[{θ, m}, (γ0^2)/(2 a0^2) Cot[θ]^2 BesselJ[m, Sqrt[2] a0/γ0 m Sin[θ]]^2 +
    (a0^2 m^2 (BesselJ[-1 + m, (Sqrt[2] a0 m Sin[θ])/γ0] - BesselJ[1 + m, (Sqrt[2] a0 m Sin[θ])/γ0])^2 Cos[θ]^2)/(2 γ0^2)];
```

```
(*for a0·ω0=cte, we have proportionality*)
γ0 = 10;
dima0 = 12;
dimm = 40;

lst = Table[{0, 0}, {a0, 1, dima0}];

For[a0 = 1, a0 < dima0, a0++;
 lstmax = Table[FindMaximum[dPdΩ[θ, m], {θ, 0.01, π}][[1]], {m, 1, 15}];
 max = Max[lstmax];
 pos = Position[lstmax, max][[1, 1]];
 lst[[a0, 1]] = a0;
 lst[[a0, 2]] = pos;
]
```

In[•]:=
```
lst;
ListPlot[lst]
```

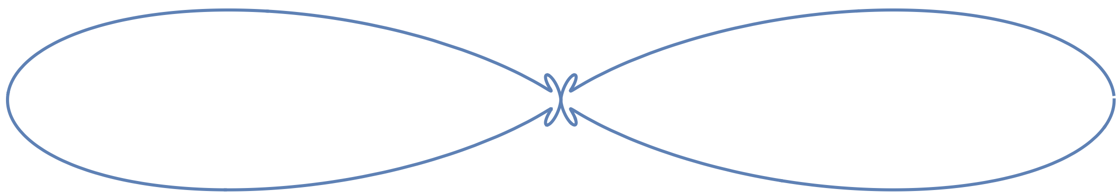
Out[•]=




```
Show[Table[Plot[dPdΩ[θ, m], {θ, 0.01, π}, PlotRange → All], {m, 1, 1}]];

Show[Table[Plot[dPdΩ[θ, m], {θ, 0.01, π}, PlotRange → All], {m, 2, 2}]];

Show[Table[Plot[dPdΩ[θ, m], {θ, 0.01, π}, PlotRange → All], {m, 3, 3}]];

tbl = Table[PolarPlot[dPdΩ[θ, 1, a0],
    {θ, 0.01, 2 π}, ImageSize → 600, Axes → False], {a0, 1, 100}];
```

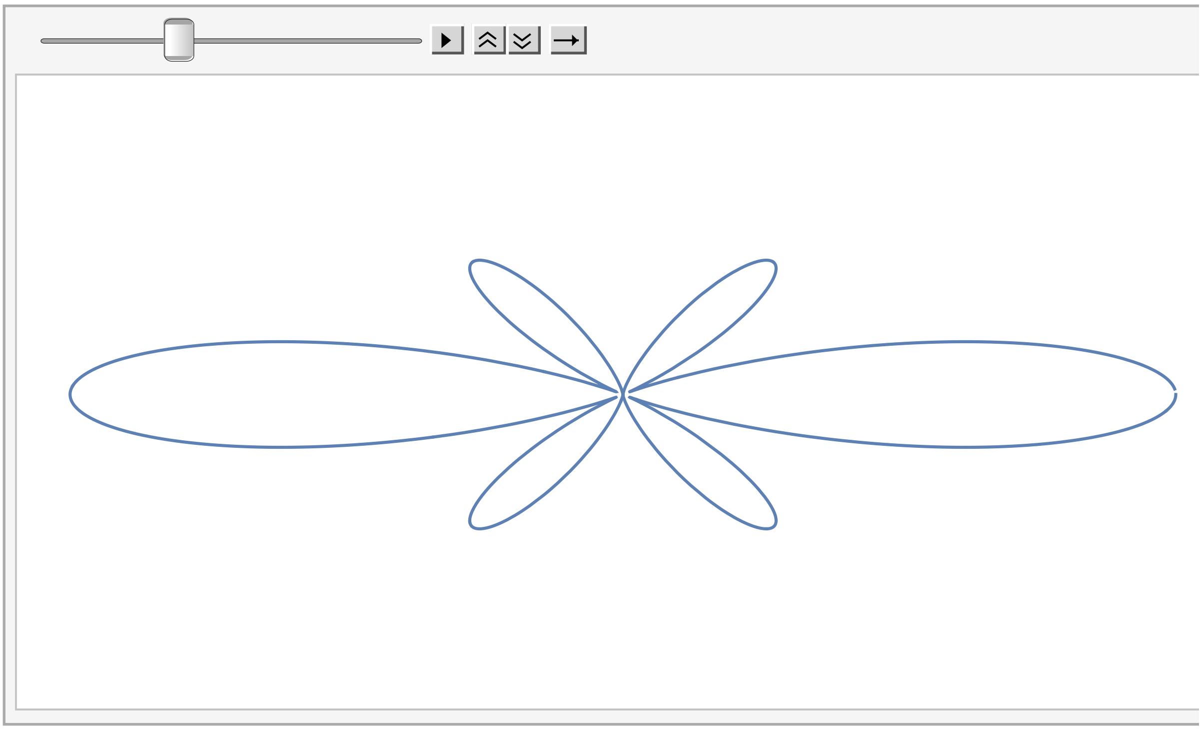
In[•]:=
```
tbl[[20]]
```

Out[•]=



In[•]:= ListAnimate[tbl, Alignment → Center]

Out[•]=



```
Export["firstharmonic.gif", %]
```

Out[•]= test.gif

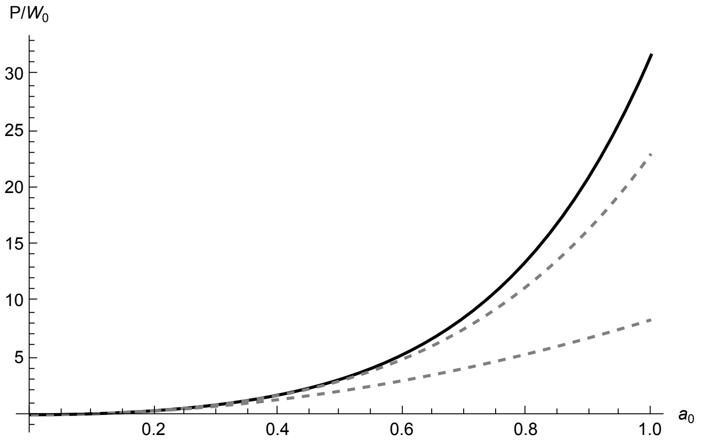
---

lpw power

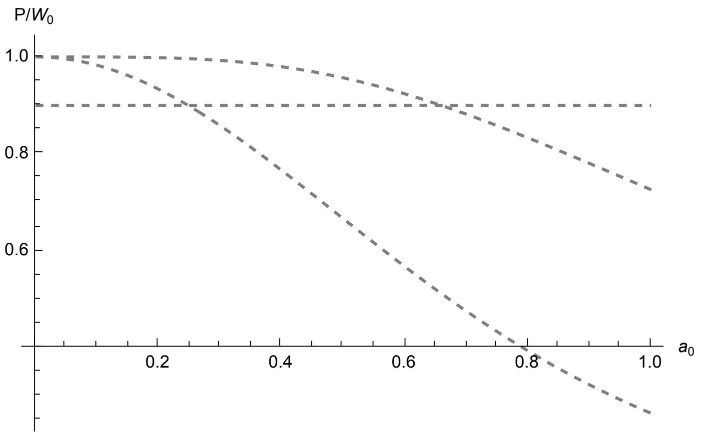
```
(* Mathematica notebook for power analysis of harmonics
 in linear polarized plane wave. Following [Gibbon]'s lecture 4
  date : 20/04/2019
  author: Óscar L. Amaro
*)
```

In[∘]:=
```
(*leading 3 terms*)
f[a0_] := (8π/3) a0^2 + (14π/3) a0^4 + (621π/224) a0^6
(*compare contribution of each term up to a₀~1*)
Plot[{f[a0], (8π/3) a0^2, (8π/3) a0^2 + (14π/3) a0^4},
 {a0, 0, 1}, AxesLabel → {"a₀", "P/W₀"}, PlotRange → All,
 PlotStyle → {Black, Directive[Gray, Dashed], Directive[Gray, Dashed]}]
(*up to which value of a₀ does each incremented term contribute 90% of the
 total power. The first harmonic contributes almost interily up to a₀~0.25,
the first two contribute the most up to a₀~0.65*)
Plot[{((8π/3) a0^2)/f[a0], ((8π/3) a0^2 + (14π/3) a0^4)/f[a0], 0.9}, {a0, 0, 1}, AxesLabel → {"a₀", "P/W₀"},
 PlotRange → All, PlotStyle → {Directive[Gray, Dashed], Directive[Gray, Dashed]}]
```

Out[∘]=

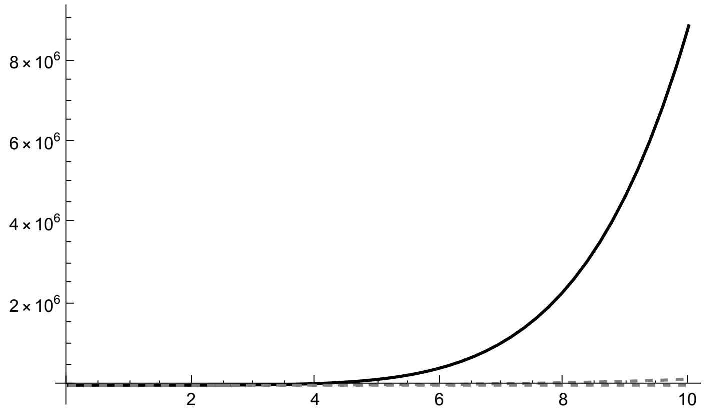


Out[∘]=



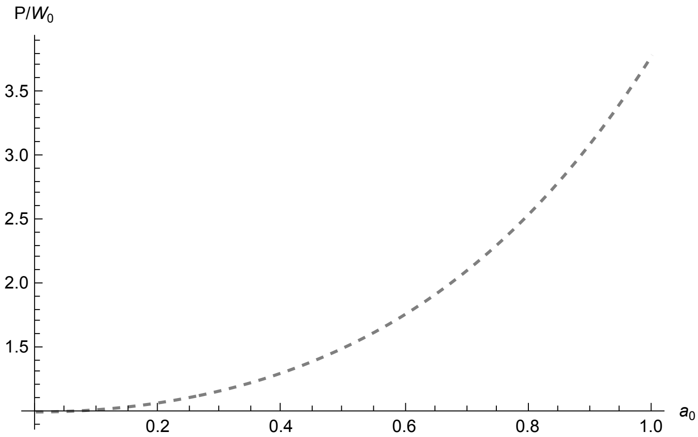
```
(*one cannot expect the total power to only include
 up to the third term when a₀ takes larger values*)
Plot[{f[a0], (8 π)/3 a0^2, (8 π)/3 a0^2 + (14 π)/3 a0^4}, {a0, 0, 10}, PlotRange → All,
 PlotStyle → {Black, Directive[Gray, Dashed], Directive[Gray, Dashed]}]
```

Out[•]=

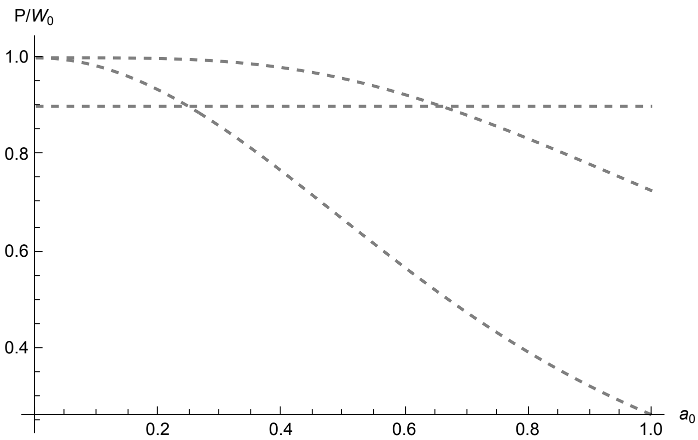


```
(*testing configuration a₀ω₀=cte*)
ω0[a0_, cte_] := cte / a0
g[a0_, cte_] :=
 (a0 ω0[a0, cte]) ^2 + 7/4 (a0 ^2 ω0[a0, cte]) ^2 + 1863/1792 (a0 ^3 ω0[a0, cte]) ^2
cte = 1;
Plot[{g[a0, cte]}, {a0, 0, 1}, AxesLabel → {"a₀", "P/W₀"}, PlotRange → Automatic,
 PlotStyle → {Black, Directive[Gray, Dashed], Directive[Gray, Dashed]}]

Plot[{(a0 ω0[a0, cte]) ^2 / g[a0, cte], ((a0 ω0[a0, cte]) ^2 + 7/4 (a0 ^2 ω0[a0, cte]) ^2) / g[a0, cte], 0.9},
 {a0, 0, 1}, AxesLabel → {"a₀", "P/W₀"}, PlotRange → Automatic,
 PlotStyle → {Directive[Gray, Dashed], Directive[Gray, Dashed]}]
```

Out[∘]=



Out[∘]=



---

lpw spectra

```
(* Mathematica notebook for spectra
 analysis 2d (ω,θ). Following [Gibbon]'s lecture 4
  date : 24/02/2019
  author: Óscar L. Amaro
*)
```

In[∘]:=
```
SetOptions[$FrontEndSession, NotebookAutoSave → True]
NotebookSave[]
```

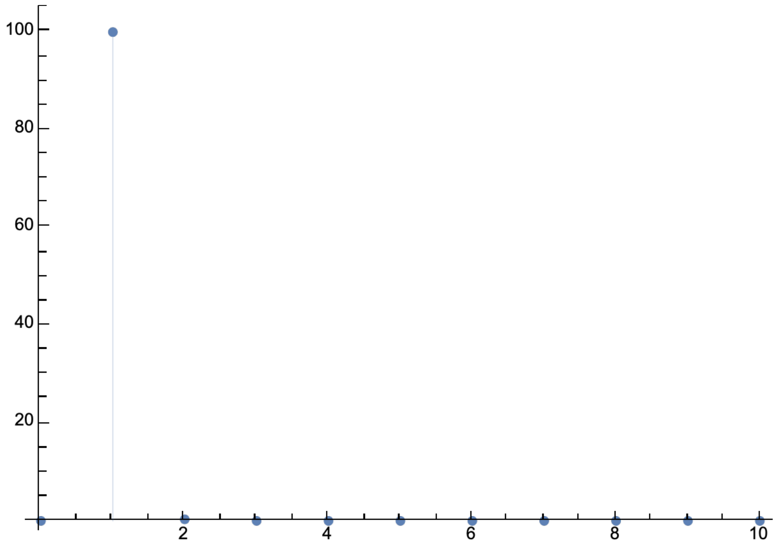
In[∘]:=
```
imgsize = 400; asp = 1; tck = 0.01; (*style*)
```

$$d2I[\theta\_, m\_, \gamma0\_, a0\_] := 0.5 * \left(\frac{m * a0}{\gamma0\,^\wedge 2}\right)^\wedge 2 * \left(\left(\frac{(Cot[\theta])}{Sqrt[2] * (a0 / \gamma0)} * BesselJ\left[m, \sqrt{2} * \frac{a0}{\gamma0} * m * Sin[\theta]\right]\right)^\wedge 2 + \left(Evaluate\left[(D[BesselJ[m, x], x]) /. x \to \left(\sqrt{2} * \frac{a0}{\gamma0} * m * Sin[\theta]\right)\right]\right)^\wedge 2\right)$$

In[∘]:=
```
(*Δ=1*)
θ = 0; θinc = 0.05; θmin = 0.001; θmax = Pi / 2;
mmax = 10; minc = 1;
γ0 = 1; (*gamma factor*)
ω0 = 1; (*fundamental frequency*)
a0 = 20; (*normalised intensity*)
θ = θmin;
mat = Table[d2I[θ, ω, γ0, a0], {ω, 0, mmax, minc}];
```

```
ListPlot[mat, AspectRatio → 0.7, ImageSize → imgsize,
 PlotRange → All, AxesLabel → Automatic, DataRange → {0, har},
 Filling → Axis] (*Simple stem plot, on axis*)
```

Out[∘]=



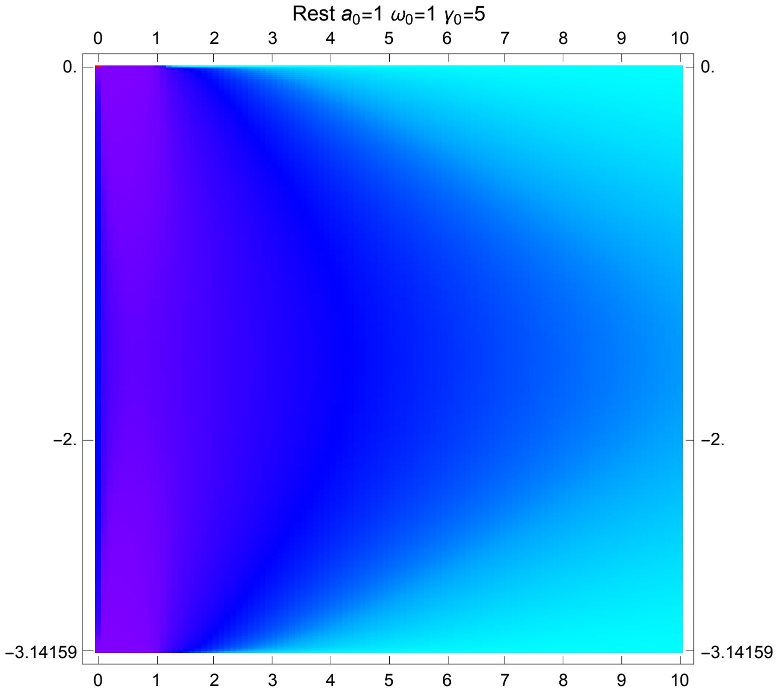
In[∘]:=
```
Thmin = -Pi;
Thmax = 0;
Clear[θ]
mat3 = Table[d2I[Abs[θ], ω, γ0, a0], {θ, Thmin, Thmax, 0.01}, {ω, 0.01, mmax, 0.1}];
```

In[∘]:=

$$mat4 = Table\left[d2I[Abs[\theta], \omega, \gamma0, a0] * \left(\frac{\gamma0}{1 + \frac{a0^\wedge 2}{2} Sin\left[\frac{\theta}{2}\right]^\wedge 2}\right)^\wedge 4, \{\theta, Thmin, Thmax, 0.01\}, \{\omega, 0.01, mmax, 0.1\}\right];$$

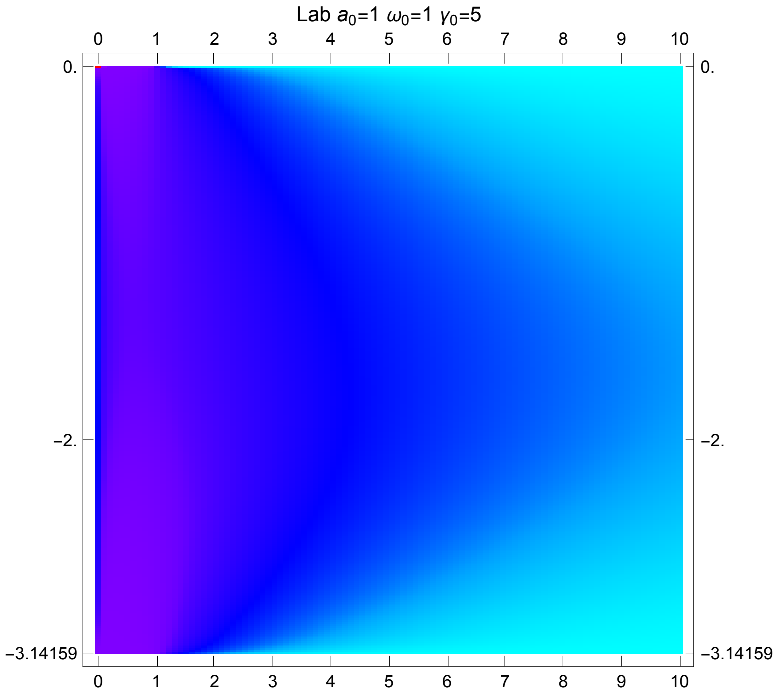
In[•]:=
```
MatrixPlot[mat3, AspectRatio → 1, ColorFunction → Hue, PlotLabel →
  "Rest a₀=" <> ToString[a0] <> " ω₀=" <> ToString[ω0] <> " γ₀=" <> ToString[γ0],
 DataRange → {{0, mmax}, {Thmin, Thmax}}, ImageSize → imgsize]
```

Out[•]=



In[•]:=
```
MatrixPlot[mat4, AspectRatio → 1, ColorFunction → Hue, PlotLabel →
  "Lab a₀=" <> ToString[a0] <> " ω₀=" <> ToString[ω0] <> " γ₀=" <> ToString[γ0],
 DataRange → {{0, mmax}, {Thmin, Thmax}}, ImageSize → imgsize]
```

Out[•]=

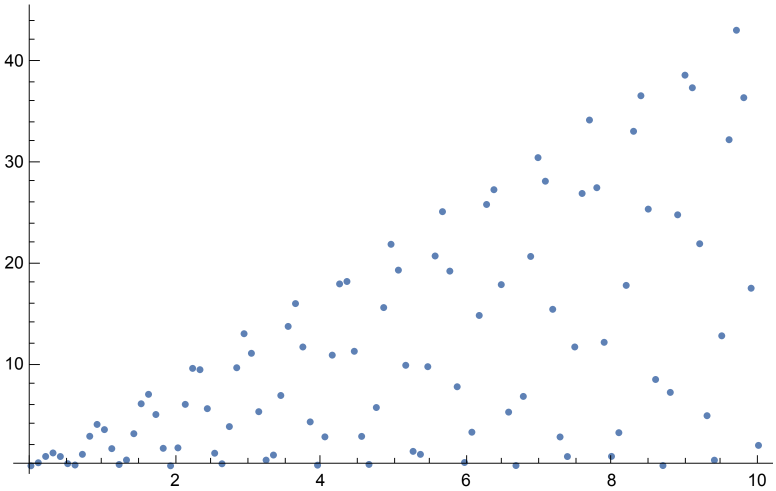


In[•]:=
```
Export[ToString[a0] <> "_" <> ToString[ω0] <> "_" <> ToString[γ0] <> ".png", %]
```

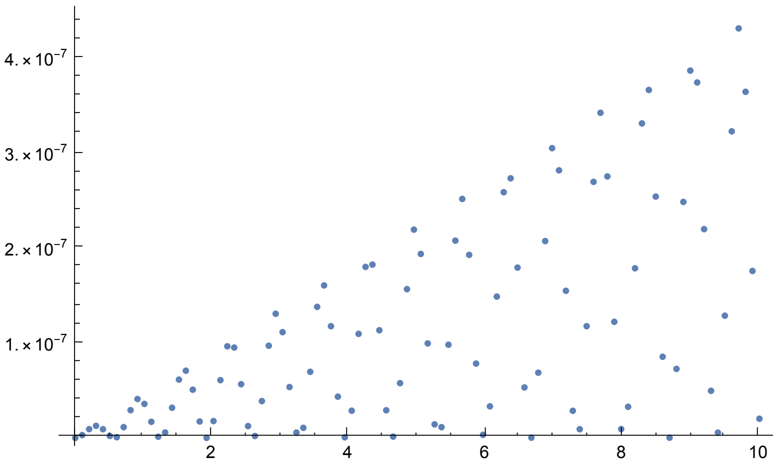
Out[•]=
```
2000_1_1.png
```

In[∘]:=
```
ListPlot[mat3⟦Floor[dm / 2]⟧, PlotRange → All,
 DataRange → {0, mmax}, ImageSize → imgsize]
ListPlot[mat4⟦Floor[dm / 2]⟧, PlotRange → All,
 DataRange → {0, mmax}, ImageSize → imgsize]
```

Out[∘]=



Out[∘]=



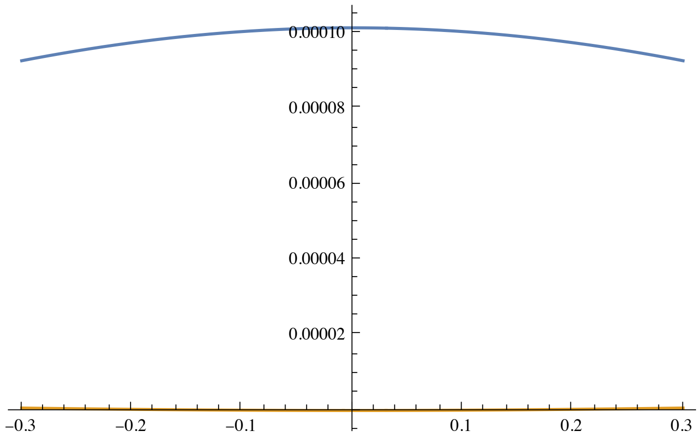
lpw

```
(* Mathematica notebook for spectra
 analysis 1d (ω). Following [Gibbon]'s lecture 4
  date : 10/12/2018
  author: Óscar L. Amaro
*)
```

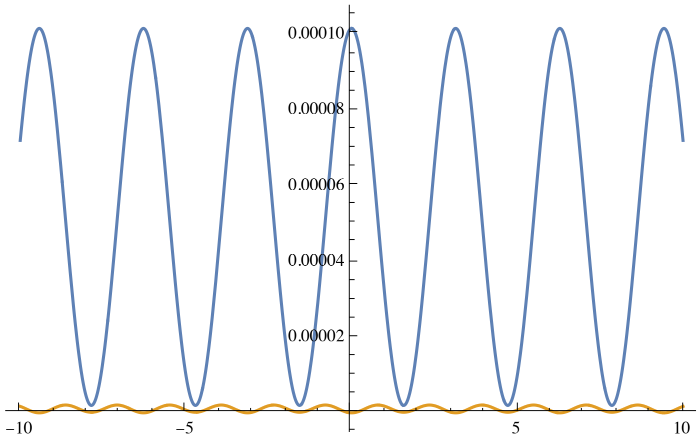
In[∘]:=
```
SetOptions[$FrontEndSession, NotebookAutoSave → True]
NotebookSave[]
```

In[∘]:=
```
a0 = 1;(* low a0→1st mode dominates *)
ω0 = 1;
γ0 = 10;
q = a0 / γ0;
ωR = ω0 / γ0;
AwR = (ωR^2 a0^2)/(8 Pi);

dPdO = Function[m, (2 m^2 AwR)/(γ0^2) ((Cot[θR]/(2 q^2))^2 BesselJ[m, (√2) q m Sin[Abs[θR]]]^2 +
    (D[BesselJ[m, x], x] /. x → ((√2) q m Sin[Abs[θR]]))^2)];
Plot[{dPdO[1], dPdO[2]}, {θR, -3 / γ0, 3 / γ0}]
Plot[{dPdO[1], dPdO[2]}, {θR, -100 / γ0, 100 / γ0}]
```

Out[∘]=

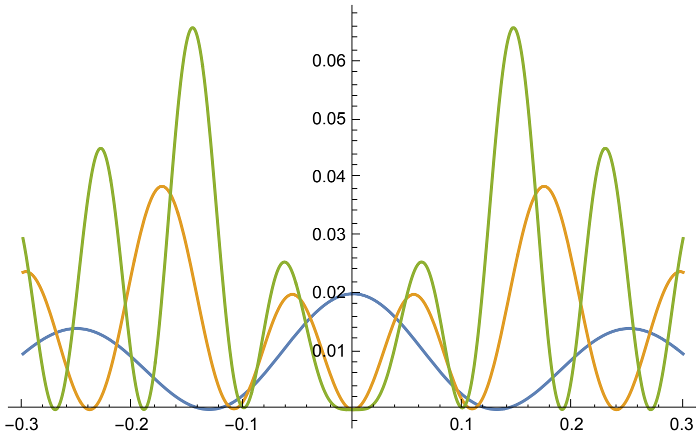


Out[∘]=

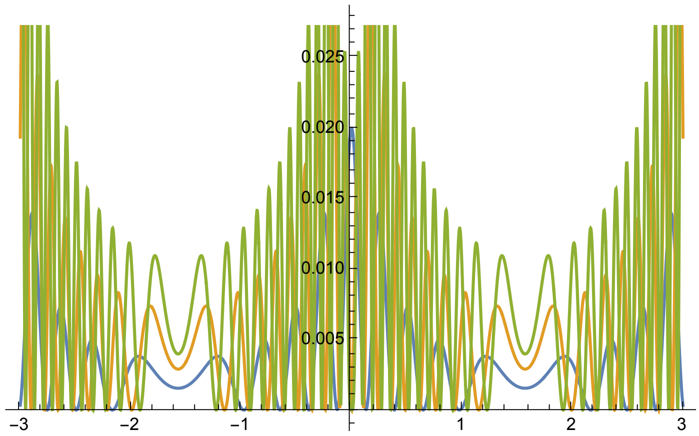


In[∘]:=
```
a0 = 100;(* hogh a0→higher modes dominate *)
ω0 = 1;
γ0 = 10;
q = a0 / γ0;
ωR = ω0 / γ0;
AwR = (ωR^2 a0^2)/(8 Pi);

dPdO = Function[m, (2 m^2 AwR)/(γ0^2) ((Cot[θR]/(2 q^2))^2 BesselJ[m, (√2) q m Sin[Abs[θR]]]^2 +
    (D[BesselJ[m, x], x] /. x → ((√2) q m Sin[Abs[θR]]))^2)];
Plot[{dPdO[1], dPdO[2], dPdO[3]}, {θR, -3 / γ0, 3 / γ0}]
Plot[{dPdO[1], dPdO[2], dPdO[3]}, {θR, -30 / γ0, 30 / γ0}]
```

Out[∘]=

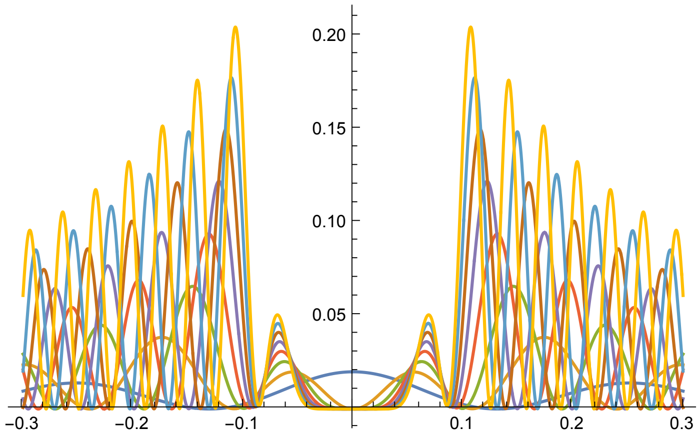


Out[∘]=



In[∘]:=
```
Plot[{dPd0[1], dPd0[2], dPd0[3], dPd0[4], dPd0[5], dPd0[6], dPd0[7], dPd0[8]},
  {θR, -3 / γ0, 3 / γ0}]
```

Out[∘]=




```
a0 = 3(* high a0→higher modes dominate *)
ω0 = 1
γ0 = 3
mat = Table[dPd0[m], {m, 10, 1, -0.1}, {θR, -3 / γ0, 3 / γ0, 0.011}];
```

Out[∘]= 3

Out[∘]= 1

Out[∘]= 3