

# SUMMER PROJECT

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## Abstract.

### 1. Introduction

We will focus on Research Project 11 in [GK20]. The following are more specific directions that we plan to pursue.

#### Goal (8/1/2022).

- (1) Prove/disprove: for an oriented graph  $G$ , one always has  $\text{Pic}(G) = \mathbb{Z} \times \text{Jac}(G)$ , i.e., as a finitely generated abelian group, the rank of  $\text{Pic}(G)$  is 1.
- (2) Prove/disprove: for  $C_n$ , and  $0 \leq m \leq n$ , one can always find an orientation of  $C_n$  so that  $\text{Jac}(C_n) = \mathbb{Z}_m$  (with the orientation).
- (3) Prove/disprove: for an oriented graph  $G$ , if  $v_0 \in V(G)$  is a sink (or a source) and  $G'$  is the graph obtained by reversing the direction for all arrows adjacent to  $v_0$  from  $G$ , then  $\text{Jac}(G) = \text{Jac}(G')$ . (Note: we believe that this should be true for at least some classes of graphs such as cyclic graphs.)
- (4) Prove/disprove: for an oriented planar graph  $G$  and its planar dual (should be defined)  $\hat{G}$ , one has  $\text{Jac}(G) = \text{Jac}(\hat{G})$ .
- (5) Prove/disprove: for oriented graphs  $G_1, G_2$ , let  $G$  be the graph obtained by gluing  $G_1$  and  $G_2$  along one vertex. Then  $\text{Jac}(G) = \text{Jac}(G_1) \times \text{Jac}(G_2)$ .
- (6)

### 2. Preliminaries

### 3. Propositions

### References

- [CP18] Scott Corry and David Perkinson. *Divisors and sandpiles*, volume 114. American Mathematical Soc., 2018.
- [GK20] Darren Glass and Nathan Kaplan. Chip-firing games and critical groups. In *A Project-Based Guide to Undergraduate Research in Mathematics*, pages 107–152. Springer, 2020.

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