

SUMMER PROJECT

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Abstract.

1. Introduction

We will focus on Research Project 11 in [GK20]. The following are more specific directions that we plan to pursue.

Goal (8/1/2022).

- (1) Prove/disprove: for an oriented graph G , one always has $\text{Pic}(G) = \mathbb{Z} \times \text{Jac}(G)$, i.e., as a finitely generated abelian group, the rank of $\text{Pic}(G)$ is 1.
- (2) Prove/disprove: for C_n , and $0 \leq m \leq n$, one can always find an orientation of C_n so that $\text{Jac}(C_n) = \mathbb{Z}_m$ (with the orientation).
- (3) Prove/disprove: for an oriented graph G , if $v_0 \in V(G)$ is a sink (or a source) and G' is the graph obtained by reversing the direction for all arrows adjacent to v_0 from G , then $\text{Jac}(G) = \text{Jac}(G')$. (Note: we believe that this should be true for at least some classes of graphs such as cyclic graphs.)
- (4) Prove/disprove: for an oriented planar graph G and its planar dual (should be defined) \hat{G} , one has $\text{Jac}(G) = \text{Jac}(\hat{G})$.
- (5) Prove/disprove: for oriented graphs G_1, G_2 , let G be the graph obtained by gluing G_1 and G_2 along one vertex. Then $\text{Jac}(G) = \text{Jac}(G_1) \times \text{Jac}(G_2)$.
- (6)

2. Preliminaries

3. Propositions

References

[GK20] Darren Glass and Nathan Kaplan. Chip-firing games and critical groups. In *A Project-Based Guide to Undergraduate Research in Mathematics*, pages 107–152. Springer, 2020.

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