Stat 588 | Homework #9 |
Fall 2019 | due Fri 22 Nov @ 2 pm | Name: ____________________

Directions: Print this document and clearly handwrite your solutions to each of the following exercises in the space provided. If your handwriting is not legible, please type your solutions. Partial credit cannot be given unless all work is shown. You may work in groups provided that each person takes responsibility for understanding and writing out the solutions. Additionally, you must provide the names of your groups members on the line below (if you worked alone, write "none"):

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1. If X is a binomial random variable with parameters n and θ, then we have showed that $\frac{X}{n}$ is an unbiased estimator of the mean. Show that the following estimator is a biased estimator of the variance $n\theta(1-\theta)$:
$$n\left(\frac{X}{n}\right)\left(1-\frac{X}{n}\right).$$

2. Consider a random sample of size n from a binomial population with parameter θ. We want to estimate θ by

$$\frac{\overline{X}+1}{n+2}.$$

(a) Show that this estimator is a biased estimator of θ.

(b) Is this estimator asymptotically unbiased?

3. Suppose X_1, X_2, X_3 is a random sample from a normal population with mean μ and variance σ^2.

   (a) Find the variance of $Y = \frac{X_1+2X_2+X_3}{4}$ as an estimator of μ.

   (b) Find the variance of $Z = \frac{X_1+X_2+X_3}{3}$ as an estimator of μ.

   (c) Which estimator is more efficient (i.e. has the smallest variance)?

4. Consider a random sample of size n from a normal population with known mean μ and unknown variance σ^2. Let the following be an estimator of σ^2:

$$\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2.$$

(a) Show that the estimator above is an unbiased estimator of σ^2.

(b) Show that the estimator above is a sufficient estimator of σ^2.

5. Consider a random sample of size n from a Poisson population with parameter λ.

   (a) Show that $\overline{X}$ is a minimum variance unbiased estimator of λ.

   (b) Show that $\overline{X}$ is a sufficient estimator for λ.

6. Consider a random sample of size n from an infinite population with mean μ and variance σ^2.

   (a) Find the method of moments estimator for μ in terms of the sample moments.

   (b) Find the method of moments estimator for σ^2 in terms of the sample moments.

7. Consider a random sample of size n from a beta population with parameters α and $\beta = 1$.

   (a) Find the method of moments estimator for α.

   (b) Find the maximum likelihood estimator for α.

8. Consider a random sample of size n from a *two-parameter exponential* population whose PDF is given by

$$f(x;\theta,\delta) = \begin{cases} \dfrac{1}{\theta}e^{-\frac{(x-\delta)}{\theta}}, & x > \delta \\ 0, & x \leq \delta \end{cases}.$$

You may use the fact that the two-parameter exponential has mean $\delta + \theta$ and variance θ^2.

(a) Find the method of moments estimators for θ and δ.

(b) Find the maximum likelihood estimators for θ and δ.

9. Consider a random sample of size n from a Poisson population with parameter λ.

   (a) Find the method of moments estimator for λ.

   (b) Find the maximum likelihood estimator for λ.

10. Suppose X has a Poisson distribution and the prior distribution for its parameter Λ is a gamma distribution with parameters α and β.

   (a) Show that the posterior distribution of Λ given $X = x$ is a gamma distribution with parameters $\alpha + x$ and $\frac{\beta}{\beta+1}$.

   (b) Find the Bayesian estimator of λ by finding the mean of the posterior distribution of Λ.

Total: 90 points **# correct:** ____________ **%:** ____________