

1. Theorem 9: For any positive integer n , $(x + y)^n = \sum_{r=0}^n \binom{n}{r} x^{n-r} y^r$.

Use Theorem 9 to show that:

a. $\sum_{r=0}^n \binom{n}{r} = 2^n$

b. $\sum_{r=0}^n (-1)^r \binom{n}{r} = 0$

c. $\sum_{r=0}^n (a - 1)^r \binom{n}{r} = a^n$

2. Theorem 11: For any positive integer n and $r=1,2,\dots,n-1$, then $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.

Prove Theorem 11 by expressing all the binomial coefficients in terms of factorials and then simplifying algebraically.