Preliminary Exams: Dr. Kaiser

Revised Submission

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# The generalized Gamma Distribution

Suppose that has a standard gamma distribution with the probability density function, for and ,

And a generalized gamma distribution results from the transformation

where is a known constant such that . Larger values of result in more extreme right tails in the resulting density function.

## 1.) Derive the P.D.F. of using the Jacobian method

The general formula for the Jacobian transformation is

where the pdf of is given above. And if , then . The Jacobian determinant of this term is , and can be substituted in the distribution function for , thus our new equation is:

Which, for positive nonzero , , and , simplifies to:

for all values of , 0 elsewhere.

## 2.) Sample Statistics

Simulate a set of data of size from a generalized gamma distribution with , , and . Note that this is most easily accomplished by first simulating the set of data from a gamma distribution with and .

# Simulating Data  
  
## Original Gamma Distribution  
n<-100 # sample size of data  
alpha<-10 # shape parameter  
beta<-3 # scale parameter  
gamma.sim<-data.frame(rgamma(n,shape=alpha,scale=beta))  
colnames(gamma.sim)<-c("Value")# set column name for our 100 simulated values.

### For this example, first use the *original sample of the gamma distribution* to compute:

* The sample mean
* Sample variance
* Skewness

# Sample Mean  
mean(gamma.sim$Value)

## [1] 29.97514

# Variance  
var(gamma.sim$Value)

## [1] 62.87705

# Skewness  
s<-skewness(gamma.sim$Value)  
s[[1]]

## [1] 0.5493858

### Second, use the *generalized gamma function* to compute:

* The sample mean
* Sample variance
* Skewness

# Mew simulated data - Generalized gamma distribution  
f<-function(n,c.gen=3.25,beta.gen=3,alpha.gen=9){  
 (rgamma(n,shape=alpha.gen,rate=beta.gen))^c.gen  
}  
n=100  
gen.gamma.sim<-data.frame(f(n))  
  
# Sample Mean  
mean(gen.gamma.sim$f.n.)

## [1] 56.12882

# Variance  
var(gen.gamma.sim$f.n.)

## [1] 3135.799

# Skewness  
s2<-skewness(gen.gamma.sim$f.n.)  
s2[[1]]

## [1] 1.478707

## 3.) Plot Empirical vs. Theoretical densities

Graph the empirical density of the generalized gamma sample (histogram on a probability scale) and overlay a curve giving the theoretical probability density function that was derived in part 1.

**The area shaded green is the density curve for the simulated (empirical) data, the red line (without very light shading) is the density curve for the theoretical density from the derived formula.**

# paramaters  
c.gen<-3.25  
alpha.gen<-9  
beta.gen<-3  
# max value  
n<-700  
# sequence of values  
values<-seq(0.1,n,.01)   
#values<-runif(100,0,700) ---- ended up providing the same curve  
  
# break down PDF(y) into parts  
prt1<-(beta.gen^alpha.gen)\*values^((alpha.gen-c.gen)/c.gen)  
prt2<-exp(beta.gen\*(-values^(1/c.gen)))  
prt3<-c.gen\*gamma(alpha.gen)  
pdfy<-data.frame(values,((prt1\*prt2)/prt3))  
  
# plot both transformed distributions  
gen.gamma.sim %>%  
 ggplot( aes(x=f.n.)) +  
 geom\_density(fill="#69b3a2", color="#e9ecef", alpha=0.9)+  
 theme(legend.position = "bottom",  
 panel.grid.major = element\_blank(),   
 panel.grid.minor = element\_blank(),  
 panel.background = element\_rect(fill="white",  
 colour="grey50"),   
 axis.line = element\_line(colour = "black"))+  
 labs(title="Densities of Transformed Gamma R.V.s",  
 x="Value",  
 y="Density")+  
 geom\_ribbon(data=pdfy,  
 aes(x=values,ymin=0,ymax=X..prt1...prt2..prt3.),  
 color="red",fill="red",alpha=0.1)

