Meaningful Integrals

1. Let $f(x) = x^2 + 4$.

   (a) Find the average value f_{ave} of $f(x)$ on the interval $[0, 3]$.

   (b) Find a $c \in (0, 3)$ such that $f(c) = f_{ave}$.

2. Let $g(x) = \dfrac{1}{1 + x^2}$.

   (a) Find the average value g_{ave} of $g(x)$ on the interval $[-1, 1]$.

   (b) Find a $c \in (-1, 1)$ such that $g(c) = g_{ave}$.

(over)

The values of c you found on the previous page were guaranteed to exist because of the following theorem:

The Mean Value Theorem for integrals: If f is continuous on $[a, b]$, then there exists a number c in (a, b) such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Let's prove this theorem!

3. For a continuous function $f(x)$, define $F(x) = \displaystyle\int_a^x f(t)\, dt$.

   (a) What is $F(a)$? What is $F'(x)$?

   (b) Apply the Mean Value Theorem to $F(x)$ on the interval $[a, b]$. Use your answers above to rewrite your equation and prove the Mean Value Theorem for Integrals.