(9.1) Find $\frac{dy}{dx}\sin^{-1}xy = y^2 - x$. Once you have found $\frac{dy}{dx}$, do not simplify your answer.

$$\frac{dy}{dx}\sin^{-1}xy = y^2 - x$$

$$\frac{y + xy\prime}{\sqrt{1 - x^2y^2}} = 2yy\prime - 1$$

$$y + xy\prime = \sqrt{1 - x^2y^2}(2yy\prime - 1)$$

$$y + \sqrt{1 - x^2y^2} = (2yy\prime)\sqrt{1 - x^2y^2} - xy\prime =$$

$$(2y\sqrt{1 - x^2y^2} - x)y\prime$$

$$y\prime(x) = \frac{y + \sqrt{1 - x^2y^2}}{2y\sqrt{1 - x^2y^2 - x}}$$

(9.2) Find $\frac{dy}{dx}$ if $x^y = y^x$.

$$\frac{dy}{dx}x^y = y^x$$

$$\frac{dy}{dx}y\ln x = x\ln y$$

$$y\prime\ln x + \frac{y}{x} = \ln y + \frac{xy\prime}{y}$$

$$\frac{y}{x} - \ln y = \frac{xy\prime}{y} - y\prime\ln x =$$

$$(\frac{x}{y} - \ln x)y\prime$$

$$y\prime(x) = \frac{\frac{y}{x} - \ln y}{\frac{x}{y} - \ln x}$$

(9.3) Let $f(x) = \tan^{-1}x^2$.

(a) Find $f'(x)$ and $f''(x)$.

$$\frac{dy}{dx}y = \tan^{-1}x^2$$

$$\boxed{y\prime(x) = \frac{2x}{x^4 + 1}}$$

$$y\prime\prime(x) = \frac{2\cdot(x^4 + 1) - (4x^3)(2x)}{(x^4 + 1)^2}$$

$$\boxed{y\prime\prime(x) = \frac{2 - 6x^4}{(x^4 + 1)^2}}$$

cont. on next page →

(b) Use your answers from part (a) to determine if $f(x)$ is increasing, decreasing, concave up, and/or concave down at the point $(-1, \frac{\pi}{4})$.

$$y\prime(-1) = \frac{2 \cdot (-1)}{(-1)^4 + 1} =$$

$$-1$$

$$\boxed{f(x) \text{ is decreasing at } (-1, \tfrac{\pi}{4}).}$$

$$y\prime\prime(-1) = \frac{2 - 6(-1)^4}{((-1)^4 + 1)^2} =$$

$$\frac{-4}{4} = -1$$

$$\boxed{f(x) \text{ is concave down at } (-1, \tfrac{\pi}{4}).}$$

(9.4) Professional Problem

(a) Use implicit differentiation to show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

provided that the denominator isn't zero.
(b) Sketch $f(x) = \sqrt{x + e^x}$, then explain why it's plausible to believe $f^{-1}(x)$ exists.
(c) What do you need to show to prove $f^{-1}(x)$ exists.
(d) Find $f^{-1}(1)$.
(e) Use your formula from (a) to evaluate $(f^{-1})'(1)$.

We know that

$$f(f^{-1}(x)) = x.$$

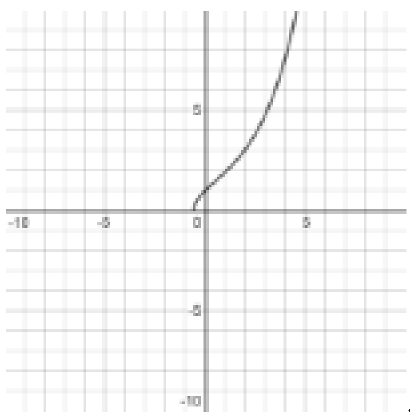
If we differentiate both sides using the Chain Rule, we get

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1.$$

After isolating $(f^{-1})'(x)$, the equation becomes

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x)}.$$

The graph of $\sqrt{x + e^x}$ is



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It is plausible to believe that $f^{-1}(x)$ exists from the graph because the function looks like a one-to-one function because the function seems to be ever-increasing. To prove that $f^{-1}(x)$ exists, the function f must be one-to-one. i=In other words, f' must never change sign or the function will start increasing/decreasing meaning the function won't be one-to-one anymore. To solve for $f^{-1}(1)$, we simply take the fact that

$$f(0) = 1,$$

therefore

$$f^{-1}(1) = 0.$$

Using the equation from (a), we get

$$\frac{1}{f'(0)} =$$

$$\frac{1}{\frac{1+e^0}{2\sqrt{e^0+0}}} = \frac{1}{\frac{2}{2}} = 1.$$

Thus the function $(f^{-1})'(x) = 1$