Getting Even with Odd Functions

Recall that a function is *even* if $f(-x) = f(x)$, and *odd* if $f(-x) = -f(x)$.

1. Determine whether each function is even, odd or neither. Then compute $\int_{-a}^{a} f(x)\,dx$.

   (a) $f(x) = \dfrac{\sin x}{\cos x}$, $a = \pi/4$

   (b) $f(x) = x^5 - x^3 + x$, $a = 1$.

2. Let $f(x) = x^5 - x^3 + x$.

   (a) Integrate $\int_{-1}^{2} f(x)\,dx$.

   (b) Without evaluating any further integrals (i.e. without using the Evaluation Theorem), find $\int_{1}^{2} f(x)\,dx$.

   Hint: Can you split $\int_{-1}^{2} f(x)\,dx$ *up so that you can use your work from earlier problems?*

3. (a) Let $f(x)$ be an even function. Show $\displaystyle\int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(x)\,dx$. (*Hint*: Try the substitution $u = -x$.)

(b) Use part (a) to explain why $\displaystyle 2\int_{0}^{a} f(x)\,dx = \int_{-a}^{a} f(x)\,dx$.