(13.1) If a and b are positive numbers, find the maximum value of $f(x) = x^a(1-x)^b$, $0 \leq x \leq 1$.

We will complete the first derivative test to find the Critical Points

$$\begin{aligned}
f'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^a(1-x)^b\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^a\right)(1-x)^b + \frac{\mathrm{d}}{\mathrm{d}x}\left((1-x)^b\right)x^a \\
&= (ax^{a-1})(1-x)^b + b(1-x)^{b-1}\frac{\mathrm{d}}{\mathrm{d}x}(1-x)x^a \\
&= -x^a b(1-x)^{b-1} + a(x-1)^b x^{a-1} \\
&= x^{a-1}(1-x)^{b-1}[(1-x)a - bx] \\
&= x^{a-1}(1-x)^{b-1}[a - x(a+b)]
\end{aligned}$$

Now to find the critical points, we must solve for $f' = 0$.

$$x^{a-1}(1-x)^{b-1}[a - x(a+b) = 0$$

For $f'(x) = 0$ to be true, either x^{a-1}, $(x-1)^{b-1}$, or $a - x(a+b)$ must equal 0.

$$\therefore x = 0 \text{ or } x = 1 \text{ or } x = \frac{a}{a+b}, a+b \neq 0$$