

The following written homework problems are due on Gradescope at 6pm the day before class. You also have a WebWork assignment due at 11pm two days before your class.

(12.1) Consider the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$.

(a) Show that the root test is inconclusive about the convergence or divergence of this series.

(b) Use another test to determine the convergence or divergence of this series.

(12.2) The terms of a series are defined recursively by the equations $a_1 = 7$ and $a_{n+1} = \frac{5n-4}{3n+2}a_n$. Determine whether $\sum a_n$ converges or diverges.

(12.3) (a) Show that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all x .

(b) Deduce that $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$ for all x .

Notes: In (a) you must show the series converges no matter what the value of x is. (By convention, $x^0 = 1$ for all x , and $0! = 1$.) You can then use (a) to prove (b), which shows that $n!$ (eventually) grows faster than x^n , no matter how large x is.

(12.4) **Professional Problem.** Consider the series $\sum (-1)^{n-1} a_n$, where $a_n = 1/n^2$ if n is odd and $a_n = 1/n$ if n is even.

(a) Why does the Alternating Series Test not apply?

(b) Determine whether the series made up of the even terms, $a_2 + a_4 + a_6 + \cdots$, converges or diverges.

(c) Determine whether the series made up of the odd terms, $a_1 + a_3 + a_5 + \cdots$, converges or diverges.

(d) Use your results from parts (b) and (c) and the result you proved in Problem 10.4 to show this series diverges.

Hints & Reminders: Remember that you can rearrange *finite* sums, but you should not rearrange infinite sums. Any answer to (d) which relies on rearranging infinitely many numbers will receive little credit. Instead, use Problem 10.4. You can use that result even if you didn't complete that homework problem.

As always, refer to previous guidelines and handouts to create a *professionally written* solution. This week, you should especially focus on:

Methods: Be careful about the distinction between sequences and series. Be precise with your notation. In (b), for example, the series of even terms is *not* $\sum_{n=1}^{\infty} \frac{1}{n}$. (Write out some terms if you're not sure why.) To avoid confusion with the original series you might want to switch from n to k or some other indexing variable in (b) and (c).

Organization & Structure: This week, state each solution immediately after the question (don't write all of the questions at the top of the page). Also,

- Avoid beginning a sentence with a mathematical symbol.
- Use blank lines to make it clear where each part of the problem statement and solution begins and ends.
- Avoid the possessive form with math expressions. For example, do not write " a_n 's limit." Instead, write "the limit of a_n ."

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor specific questions about it.

<i>Instructor</i>	<i>Email</i>
Alexis Johnson	akjohns@umn.edu
Julie Leifeld	leif0020@umn.edu
Jonathan Rogness	rogness@umn.edu
Anila Yadavalli	anilayad@umn.edu
Eric Erdmann (Duluth)	erdm0063@d.umn.edu
Paul Kinion (Rochester)	paulkinion@gmail.com