Shall I compare thee to a summer's day? Or to $\frac{1}{x^2}$?

For each of the following problems, first use your intuition to guess if the integral will converge or diverge. Then, determine the convergence of the integral by the Comparison Theorem. Use your guess to inform your choice – if you think it diverges, find a lesser function whose integral diverges. If you think it converges, find a greater function whose integral converges.

1. $\displaystyle\int_1^\infty \frac{\sqrt{x}}{x^2+1}\,dx$

2. $\displaystyle\int_1^\infty e^{-x^3}\,dx$

3. $\displaystyle\int_0^{\pi/2} \sec^e(x)\,dx$

4. Use p-tests, or comparison to $g(x)=\dfrac{1}{x}$, to show that $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$ is divergent for all p.