This exam contains 8 pages (including cover pages) and 8 problems. Check to see if any pages are missing.

This semester your UMTYMP tests are take home exams. Please read the guidelines on the next page carefully. In short: you can use your book, notes, calculators, and any online resources, but you are not allowed to discuss the exam or collaborate with any person other than your instructors. This includes posting for help on Canvas, or any other webpage or online forum.

Page	Points	Score
3	18	
4	17	
5	18	
6	20	
7	12	
8	15	
Total:	100	

Submit your exam by 6:00pm on Gradescope by 5/13/21.

You will scan and upload your exam to Gradescope, similar to handing in written homework.

If you have a question about the wording of an exam problem, send an email to `umtymp-c1-exam@umn.edu`. It may take 5-10 minutes to get a response.

Suggestion: Upload what you have at 5:40pm to avoid any technical issues. Then continue to check answers and, if you find and fix a mistake, upload a new version of your solutions. If you resubmit, make sure to submit all of your solutions, not just the updated ones.

If your exam is not uploaded by 6:00pm, one point will be deducted for every three minutesit is late. (So you don't have to worry if the network is slow and your upload doesn't finish until 6:05pm. You'll only lose two points!)

When you submit your solutions, sign the following statement. If you are writing solutions on your own paper, copy the statement and sign it on your paper. (If you cannot sign the statement, make sure to tell us why!)

My work on this exam is my own. I have cited any resources I used. I did not discuss the exam with anybody other than my instructors. I did not receive any outside help, and did not provide help to any other student.

Signature: ______________________________

Take Home Exam Guidelines

The problems on the exam can all be answered using ideas and methods in your textbook! If you spend time searching for ideas or help online, you won't have enough time to finish the exam.

Resources

This is an open-book, open-notes, open-internet take home exam. However, standard academic honesty rules apply. If you make use of an outside resource, keep the following in mind if you want to receive credit for your solution:

- You must still write the solution in your words. We're interested in whether you understand the ideas, not whether you can transcribe somebody else's work.
- Cite the resource as part of your solution. Even if you have written a solution in your own words, you must still give credit where credit is due. *Don't claim somebody else's work or ideas as your own.*
- Your solution must be consistent with the definitions and methods of this course. Solutions using methods and ideas which are not covered in your UMTYMP course will not receive credit.

Exceptions: If you model your solution after something from lecture, groupwork, homework or the textbook, you do not need to cite the source.

You are encouraged to use technology where possible to check that your answers are correct, but they must still be supported by work and explanations.

Collaboration

You **may not** discuss the exam or collaborate with anybody else, including (but not limited to) other students, parents, siblings, friends, and people online; "open-internet" means you can make use of resources which are already out there – **not** posting on Stack Exchange (Quora, Chegg, Slader, Yahoo Answers, etc.) for help. (Yes, your instructors are aware of those sites, and are capable of using Google themselves.)

Please follow these guidelines closely. Cheating on exams is a serious offense at the University of Minnesota. Consequences range from a score of 0 on the exam to an F in the course.

Warning: A few students in other University courses have already experienced these consequences after posting their take home exams on Chegg and other sites. Please don't join them.

Standard Guidelines

The following rules from your previous exams still apply:

- **Organize your work** in a reasonable, tidy, and coherent way. Work that is disorganized, hard to follow, or lacks clear reasoning will receive little or no credit.
- **Unsupported answers will not receive full credit.** An answer must be supported by calculations, explanation, and/or algebraic work to receive full credit. Partial credit may be given to well-argued incorrect answers as well.
- When possible give exact answers, not decimal approximations. For example, if the answer to a problem is $\sqrt{2}$, leave the answer in that form rather than 1.414.

1. (8 points) **Examples!** Give examples which satisfy each of the following statements. *For this problem*, you do not need to justify your answers! (2 points each)

   (a) $\lim_{n\to\infty} a_n = 0$ but $\sum a_n$ diverges.

   (b) $\sum a_n$ diverges but $\sum a_n^2$ converges.

   (c) $\sum a_n$ diverges but $\sum (-1)^{n+1} a_n$ converges.

   (d) $\sum a_n$ and $\sum b_n$ diverge, but $\sum (a_n - b_n)$ converges.

2. Suppose $\sum_{n=0}^{\infty} c_n (x-3)^n$ converges when $x = 7$ and diverges when $x = -6$.

   (a) (2 points) Let R be the radius of convergence for this power series. Using the given information, fill in the blanks:

   $$\_\_\_\_\_ \leq R \leq \_\_\_\_\_$$

   (b) (8 points) If possible, determine whether this power series would converge or diverge at each value of x. If not possible, state why.

   $x = 0$ $x = 12$

   $x = 8$ $x = 20$

3. Determine which of the following series converge. If a series converges, find its sum. If a series diverges, explain why. Justify your answers using methods from class.

   (a) (6 points) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$

   (b) (6 points) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n+2}2^n$

   (c) (5 points) $\displaystyle\sum_{n=1}^{\infty}\left(\ln(6n-5)-\ln(2n+1)\right)$

4. Let $f(x) = \cos x$.

   (a) (4 points) Write down the first four derivatives of $f(x)$.

   $f'(x) = $ ________ $f''(x) = $ ________ $f^{(3)}(x) = $ ________ $f^{(4)}(x) = $ ________

   (b) (6 points) Find $p_4(x)$, the fourth-degree Taylor polynomial of $f(x)$ centered at $a = \pi$.
   You do not need to expand or simplify your polynomial!

5. The Maclaurin series for e^x converges for all $x \in (-\infty, \infty)$: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

   (a) (4 points) Find the Macluarin series for $g(x) = e^{x^3}$.

   (b) (4 points) Use series to compute $\lim_{x \to 0} \frac{e^{x^3} - 1 - x^3}{x^6}$.

6. Let $f(x) = \dfrac{1}{2 - 8x}$.

   (a) (8 points) Find a power series representation for $f(x)$. Write your answer using summation notation, and also write out the first four terms of the series. What is the radius of convergence for your power series?

   (b) (6 points) Use your answer to (a) to find a power series representation for $\int f(x)\,dx$. Write your answer using summation notation, and also write out the first four terms of the series.

   (c) (6 points) What is the interval of convergence for your power series in part (b)?

7. **Short Proofs!** Write a short proof – just a few lines! – of each statement below.

(a) (6 points) Suppose $\sum_{n=1}^{\infty}\left(a_n + \frac{1}{n^4}\right)$ converges. Then $\sum a_n$ must converge.

(b) (6 points) Suppose $|b_n| \leq a_n$ and $\sum a_n$ converges. Then $\sum b_n$ converges.

8. (15 points) Suppose $\sum a_n$ **converges absolutely**, $\sum b_n$ **converges conditionally**, and $\sum c_n$ **diverges** Determine whether the following series converge, diverge, or if there is not enough information to decide. Circle your answer and justify your choice with a brief explanation. (5 Points each)

   (a) $\sum a_n^2$ **Converges** **Diverges** **Not enough Info**

   (b) $\sum b_n c_n$ **Converges** **Diverges** **Not enough Info**

   (c) $\sum (-1)^n c_n$ **Converges** **Diverges** **Not enough Info**