The following written homework problems are due on 12/1 or 12/2. *Don't forget to also finish WeBWorK!*.

(12.1) Let $f(x)$ be a function satisfying $1 \le f'(x) \le 4$ for all x. Prove that $5 \le f(10) - f(5) \le 20$.

(12.2) If possible, find the following limit using l'Hôpital's rule. If not possible, explain why.

$$\lim_{x \to 0^+} \ln(x)\sin(x)$$

(12.3) If possible, find the following limit using l'Hôpital's rule. If not possible, explain why.

$$\lim_{x \to 0^+} \left(\frac{x}{x+1}\right)^x$$

(12.4) Professional Problem. In September, we gave you a lot of information about how to write professional problems, much of which was summarized on a Professional Problem Checklist, which is posted on Canvas.

In the last month we've found we're writing lots of comments about issues we covered on that checklist, but many students aren't reading the feedback and adjusting their write-ups. That's unfortunate in both directions: students lose unnecessary points, and it takes us longer to grade!

Hence this week's professional problem is a little different. You can earn a quick 8/8 by practicing a few skills. The catch: from here on out, we'll assume you know these guidelines and expect you to follow all of them, or else lose writing points. That means it's very important to pay attention to the guidelines below, and to ask any questions if you're unsure about something.

Before starting this problem, you should review the Professional Problem Checklist and keep it handy.

> This week only: You do not need to write out the problem statements. For the first part you can simply write (a) followed by the solution to that part; write the other parts in a similar way. However, on every other professional problem you should write the problem statement. If the problem is particularly long, you can paraphrase it, but make sure you keep the essential information.
>
> Remember your target audience: a student in a different calculus course, and not a math professor who knows the assignment and can fill in any missing details on their own. If you don't state the problem, the student in the other course won't know what you're doing.

(a) **Formatting Long Computations**. If you are writing a long chain of equalities, we generally write each equality on its own, and align the equals signs with each other. If the left hand side does not change, you do not need to rewrite it on each line.

No

$xy - 5y - 2x + 10$
$= y(x-5) - 2x +$
$10 = y(x-5) - 2(x-5) =$
$(y-2)(x-5)$

Yes

$$\begin{aligned} xy - 5y - 2x + 10 &= y(x-5) - 2x + 10 \\ &= y(x-5) - 2(x-5) \\ &= (y-2)(x-5) \end{aligned}$$

Previous UMTYMP students dubbed this the "Oklahoma Rule," because, as mentioned on the Checklist, the shape of the expressions in the box on the right resembles the shape of the state of Oklahoma.

Use the Oklahoma Rule to rewrite the following messy computation of the derivative of $f(x) = x^2$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x + h =$$
$$2x$$

(b) **Inline Mathematics, Symbols and Grammar**. Less important equations, or intermediate calculations, can be written inline. "Inline" means as part of the sentence, as in "If $x = 2$, then $f(x) = 5$," as opposed to putting an equation on its own line:

$$f(x) = 5.$$

Generally we put the most important equations on their own lines, or blocks of equations, as in the "Oklahoma Rule" example above.

When writing mathematics inline, avoid a line break in the middle of a mathematical expression or equation. Also, if an equation is important enough to get its own line, you shouldn't have other words on that same line. For example:

No

First, let's compute the derivative of $f(x) =$
$x \sin x$.

$$f'(x) = (x)' \sin x + x(\sin x)' \qquad \text{(use product rule)}$$
so $f'(x) = \sin x + x\cos x$

Yes

We can use the Product Rule to compute the derivative of $f(x) = x\sin x$:

$$\begin{aligned} f'(x) &= (x')\sin x + x(\sin x)' \\ &= \sin x + x\cos x. \end{aligned}$$

Although mathematicians frequently use abbreviations and symbols such as &, $\Rightarrow$, $\Leftrightarrow$, $\therefore$, and $\because$, we tend to avoid them in formal writing. Because professional problems are your chance to learn formal mathematical writing, you should write those symbols out in words. Conversely, don't replace mathematical expressions with words or phrases. Write "Let $f(x) = x^2$," and not "Let $f(x)$ be the function whose output is the square of the number you plug in to the function."

Finally, make sure your sentences are grammatically correct, even if they contain mathematics. For example, writing "Thus $x = 5$." is fine; we'd read it out loud as "Thus x equals 5." That's a full sentence, with a subject (x) and a verb (equals). Remember to punctuate your sentences!

No	Yes
Thus $x = 5$ & $f(x) = 25$	Thus $x = 5$, and $f(x) = 25$.

Rewrite the following, with correct grammar, punctuation, and nicely formatted mathematics, making use of inline expressions as appropriate.

> Since $f'(2) = 3$ & $f(2) = 4 \Rightarrow$ the point-slope equation of the tangent line to $y = f(x)$ @ $x = 2$ is $y - 4 = 3(x - 2)$. To find the slope intercept form we can solve for y: $y = 4 + 3(x - 2) = 3x - 6 + 4 = 3x - 2$

(c) **Ambiguous references**? Avoid using words such as 'it" or "that", especially when referring to mathematical objects, because in many cases, it is not clear what "it" refers to, and the sentence can be misinterpreted.

No	Yes
Let $f(x) = x^2 - 5$. Then $f'(x) = 2x$. Then, whenever it is positive, it is increasing.	Let $f(x) = x^2 - 5$. Then $f'(x) = 2x$. Then, whenever x is positive, $f(x)$ is increasing.

Rewrite the following by getting rid of any ambiguous references.

> Suppose $g(x)$ is continuous on $[a, b]$ and differentiable on (a, b). If $g(1) = 4$ and $g(3) = 8$, then there is a value c in between them such that the derivative at c is 2.

(d) Go to Gradescope and look over the professional problems on your previous assignments. Don't just look at the scores; make sure to look at any comments, annotations, and rubric items which were selected on solutions. Are there common themes, or consistent reasons that you've lost points from week to week? Write one or two sentences describing those reasons. Then make sure to avoid those issues in the future!

(If you've earned 8/8 on every professional problem, you can still look through and report on any comments from the graders, even if they didn't take off any points.)

(e) **Pledge**. Do the following, and take it seriously, but you do <u>not</u> need to hand anything in for this part!

Now that you've practiced these skills, and have looked over the feedback on your previous assignments, raise your right hand and read aloud:

> *In order to promote a more harmonious universe, preserve the sanity of my graders, and earn higher scores on my homework, I hereby pledge (1) to follow the guidelines in this problem on all future professional problems, and (2) read all feedback on future homework, and do my best to avoid the mathematical and writing issues that are pointed out by the graders.*

Furthermore, I acknowledge that, with rare exceptions, if I have written more than two pages for my professional problem, I have likely written too much. Hence I will contact the lecturers in my course to discuss my solution with them before submitting such a long solution.

You should have questions! When you do, here's what to do:

1. Post your question on Canvas: `http://canvas.umn.edu/`
   The answers you get will help everyone in the class!

2. Email *all* of the instructors with your question.

3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor *specific questions* about it.

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