The following written homework problems are due on Gradescope by 6:00pm the day before your class day.

(13.1) Calculate the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{5^n}(x+2)^n$.

(13.2) Suppose that $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -6$ and diverges when $x = 8$. What, if anything, can be said about the convergence or divergence of the following series? Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} c_n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 3^n$ (c) $\displaystyle\sum_{n=0}^{\infty} c_n(-8)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

(13.3) Define the following power series:

$$f(x) = \sum_{k=0}^{\infty} x^{2k} = 1 + x^2 + x^4 + \cdots \qquad g(x) = \sum_{k=0}^{\infty} 3^k x^{2k+1} = x + 3x^3 + 9x^5 + \cdots$$

(a) Find the interval of convergence of $f(x)$ and find a formula for $f(x)$.

(b) Find the interval of convergence of $g(x)$ and find a formula for $g(x)$.

(c) Use your answers to (a) and (b) to find a formula for $h(x) = 1 + x + x^2 + 3x^3 + x^4 + 9x^5 + \cdots$. What is the interval of convergence for $h(x)$? (Make sure to justify any steps where you add series together!)

Hint: in (a) and (b), don't bother with the Ratio or Root Test to find the radius of convergence, then test both endpoints, and so on. Just use your knowledge of geometric series.

(13.4) Professional Problem.

Consider the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ and suppose $c_n \neq 0$ for all n. Show: if $\displaystyle\lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$ exists and is nonnegative, then it is equal to the radius of convergence of the power series; and, if $\displaystyle\lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \infty$, the series converges for all x.

Hints/comments: the fraction in the limit is not the same as the fraction in the Ratio Test; it's the reciprocal. We don't like to treat ∞ as a number, but informally: if you think of "converges for all x" as "the radius of convergence is ∞," then the two parts of the problem can be combined to say "if the limit exists (or is ∞), the result is the radius of convergence."

There are special cases to deal with, namely when $\lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$ is ∞ or 0. Common mistakes in past years, resulting in a lower math score, include not using limit laws properly, dividing by 0, and treating infinity as a number. Avoid writing anything like "$\frac{1}{\infty} = 0$." You may use the following limit laws:

- If $\displaystyle\lim_{n\to\infty} a_n = +\infty$, then $\displaystyle\lim_{n\to\infty} \frac{1}{a_n} = 0$
- If $a_n > 0$ and $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\lim_{n\to\infty} \frac{1}{a_n} = +\infty$

(An aside for you to think about: in the second limit law, why is it necessary to say $a_n > 0$? And why wouldn't $a_n \geq 0$ work?)

As always, refer to the "Professional Problem information" handout to create a *professionally written* solution. This week, you should especially focus on:

Organization: Organize your computations and explanations clearly. Use complete sentences. A reader should be able to easily follow your argument.

Explanation: If you use a property or theorem, clearly explain why and how you can apply this result.

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and ask about it at our online study session on Monday.

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