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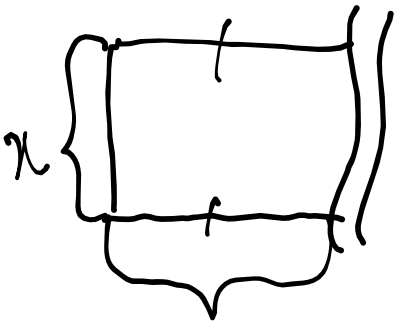
UMTYMP Calculus I Stewart CCC 4th Custom Ed.

Optimization

Complete this groupwork and submit it to Gradescope by 4:00pm on your class day. You can print this sheet, or write on your own paper. Contact us if internet connections or other issues require alternate arrangements.

1. Farmer Allez is building a pen for his Alpacas along a river. He wants to enclose a rectangular area, one side of which will be the river (so will not require fence). What is the largest area he can enclose with 750 ft of fencing?



y - length of pen
x - width of pen
$A = xy$

$A(0) = 0, A(375) = 0$
$A(750/4) = 70312.5$

$2y + x = 750$ $\quad y \in [0, 375]$

$x = 750 - 2y$

$A = (750 - 2y)y$

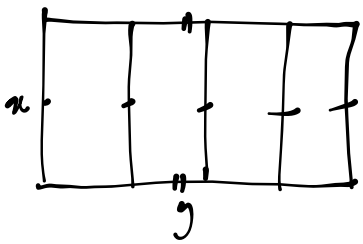
$A(y) = 750y - 2y^2$

$A'(y) = 750 - 4y$

Critical Point: $y = \frac{750}{4}$, Ep: $y = 0, 375$

$70312.5\ ft^2$

2. Oh no! Some of the Alpacas have started swimming away, and the ones that are left are fighting with each other. Farmer Allez is rethinking his pen design.
Now, he would like to enclose a rectangular area (away from the river) and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens that he can build with 750 ft of fencing?



x - width of pen
y - length of pen
$A = xy$
$L = 5x + 2y = 950$

$y = 475 - \frac{5}{2}x$

$A = x\left(475 - \frac{5}{2}x\right) = 475x - \frac{5}{2}x^2 = A(x)$

$A'(x) = 475 - 5x = 975$

$x = 95$ - Critical Number

$A''(x) = -5$ for all x. x is a maximum

$A(95) = 475(95) - \frac{5}{2}(95)^2 =$

$\left(475 - \frac{5 \cdot 95}{2}\right)95 =$

$22562.5\ ft^2$