The following written homework problems are due at the beginning of your next class.

(5.1) Use the substitution $u = \sec x$ to evaluate the integral $\displaystyle\int \tan^7 x \sec^3 x \, dx$.

(5.2) Evaluate the integral $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}} \, dx$ in the following two ways. Make sure your answers are consistent!

(a) Use the substitution $u = x^2 + 1$.

(b) Use the substitution $x = \tan\theta$. (Draw a triangle corresponding to your substitution.)

(5.3) Use an appropriate trig substitution to evaluate the integral $\displaystyle\int \frac{x^2}{x^2+9} \, dx$. Draw a triangle corresponding to your substitution.

(5.4) **Professional Problem.** Define $G(n) = \displaystyle\int_0^\infty x^n e^{-x} \, dx$.

(a) Show $G(n) = nG(n-1)$ for $n \geq 1$.

(b) Let n be a positive integer. Explain why $G(n) = n!$.
(A formal proof by induction is not required.)

*Note (not part of the problem statement): this seems like the world's most complicated way to define a factorial, but this is actually *really* cool: it gives you a way to define "factorial" for non-integers. For example,* $G(3.5) = \int_0^\infty x^{3.5} e^{-x} \, dx \approx 11.6317$, *which is between* $3! = 6$ *and* $4! = 24$.

You may use the groupwork from this week to support your work. As always, refer to the "Professional Problem information" handout to create a *professionally written* solution. This week, you should especially focus on:

Mathematical Details: Pay careful attention to the details and notation involved in the integrals and limits. You can use ∞ as an upper bound in an improper integral, but you should not treat ∞ as a number or write expressions such as e^∞.

Explanation: Explain your work, but *don't confuse length with clarity.* Your solution to this professional problem should fit on one sheet unless your handwriting is particularly large. Solutions that are too long make your work harder to follow and will not receive full credit.

Organization: With improper integrals, integration by parts, and limits, there are a lot of moving pieces in this problem. Put some effort into organizing your work and making it easy for the reader to understand.

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor specific questions about it.

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