

### Can You?

Your goal on this sheet is not just to think about answers, but to explain them to the rest of your group. Some of the questions have many possible correct answers.

1. Can you find a sequence  $\{a_k\}$  such that  $\{a_k\}$  converges and tends to zero, but  $\sum_{k=1}^{\infty} a_k$  diverges?
2. Can you find a sequence  $\{a_k\}$  such that  $\{a_k\}$  converges to a number other than zero, but  $\sum_{k=1}^{\infty} a_k$  converges?
3. Can you find a sequence  $\{a_k\}$  such that  $\{a_k\}$  diverges, and  $\sum_{k=1}^{\infty} a_k$  converges?
4. Can you find a sequence  $\{a_k\}$  such that  $\frac{a_{k+1}}{a_k} < 1$  for all  $k$ , and  $\sum_{k=1}^{\infty} a_k$  diverges?
5. Can you find a sequence  $\{a_k\}$  such that  $\left| \frac{a_{k+1}}{a_k} \right| < 1$  for all  $k$ , and  $\sum_{k=1}^{\infty} a_k$  diverges?
6. Can you find a sequence  $\{a_k\}$  such that  $\frac{a_{k+1}}{a_k} > 9$  for all  $k \leq 10$ , and  $\sum_{k=1}^{\infty} a_k$  converges?
7. Can you find a function  $f(x)$  such that  $\lim_{x \rightarrow \infty} f(x)$  does not exist, yet the series  $\sum_{k=1}^{\infty} a_k$  (with  $a_k = f(k)$ ) converges?

8. What can you say about  $\sum_{n=1}^{\infty} a_n$ , in each of the following cases? *Prove it!*

(a)  $\sum_{n=1}^{\infty} \left( a_n + \frac{1}{n^2} \right)$  converges

(b)  $\sum_{n=1}^{\infty} \left( a_n + \frac{1}{n} \right)$  converges

(c)  $\sum_{n=1}^{\infty} \left( a_n + \frac{1}{n^2} \right)$  diverges

(d)  $\sum_{n=1}^{\infty} \left( a_n + \frac{1}{n} \right)$  diverges

9. Prove that if  $\sum_{n=1}^{\infty} a_n$  is absolutely convergent, then  $\left| \sum_{n=1}^{\infty} a_n \right| \leq \sum_{n=1}^{\infty} |a_n|$ . (Why is absolute convergence needed?)