This exam contains 10 pages (including cover pages) and 12 problems. Check to see if any pages are missing.

This semester your UMTYMP tests are take home exams. Please read the guidelines on the next page carefully. In short: you can use your book, notes, calculators, and any online resources, but you are not allowed to discuss the exam or collaborate with any person other than your instructors. This includes posting for help on Canvas, or any other webpage or online forum.

Page	Points	Score
4	14	
5	16	
6	14	
7	10	
8	16	
9	18	
10	12	
Total:	100	

Submit your exam by 6:00pm on Gradescope by 3/25/21.

You will scan and upload your exam to Gradescope, similar to handing in written homework.

If you have a question about the wording of an exam problem, send an email to `umtymp-c1-exam@umn.edu`. It may take 5-10 minutes to get a response.

Suggestion: Upload what you have at 5:40pm to avoid any technical issues. Then continue to check answers and, if you find and fix a mistake, upload a new version of your solutions. If you resubmit, make sure to submit all of your solutions, not just the updated ones.

If your exam is not uploaded by 6:00pm, one point will be deducted for every minute it is late. (So you don't have to worry if the network is slow and your upload doesn't finish until 6:01pm. Your exam will still be accepted!)

When you submit your solutions, sign the following statement. If you are writing solutions on your own paper, copy the statement and sign it on your paper. (If you cannot sign the statement, make sure to tell us why!)

My work on this exam is my own. I have cited any resources I used. I did not discuss the exam with anybody other than my instructors. I did not receive any outside help, and did not provide help to any other student.

Signature: ______________________________

Take Home Exam Guidelines

The problems on the exam can all be answered using ideas and methods in your textbook! If you spend time searching for ideas or help online, you won't have enough time to finish the exam.

Resources

This is an open-book, open-notes, open-internet take home exam. However, standard academic honesty rules apply. If you make use of an outside resource, keep the following in mind if you want to receive credit for your solution:

- You must still write the solution in your words. We're interested in whether you understand the ideas, not whether you can transcribe somebody else's work.
- Cite the resource as part of your solution. Even if you have written a solution in your own words, you must still give credit where credit is due. *Don't claim somebody else's work or ideas as your own.*
- Your solution must be consistent with the definitions and methods of this course. Solutions using methods and ideas which are not covered in your UMTYMP course will not receive credit.

Exceptions: If you model your solution after something from lecture, groupwork, homework or the textbook, you do not need to cite the source.

You are encouraged to use technology where possible to check that your answers are correct, but they must still be supported by work and explanations.

Collaboration

You **may not** discuss the exam or collaborate with anybody else, including (but not limited to) other students, parents, siblings, friends, and people online; "open-internet" means you can make use of resources which are already out there – **not** posting on Stack Exchange (Quora, Chegg, Slader, Yahoo Answers, etc.) for help. (Yes, your instructors are aware of those sites, and are capable of using Google themselves.)

Please follow these guidelines closely. Cheating on exams is a serious offense at the University of Minnesota. Consequences range from a score of 0 on the exam to an F in the course.

Warning: A few students in other University courses have already experienced these consequences after posting their take home exams on Chegg and other sites. Please don't join them.

Standard Guidelines

The following rules from your previous exams still apply:

- **Organize your work** in a reasonable, tidy, and coherent way. Work that is disorganized, hard to follow, or lacks clear reasoning will receive little or no credit.
- **Unsupported answers will not receive full credit.** An answer must be supported by calculations, explanation, and/or algebraic work to receive full credit. Partial credit may be given to well-argued incorrect answers as well.
- When possible give exact answers, not decimal approximations. For example, if the answer to a problem is $\sqrt{2}$, leave the answer in that form rather than 1.414.

You may use the following formulas on this exam, without justification.

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\csc x)' = -\cot x \csc x$

$(\sec x)' = \sec x \tan x$

$(\cot x)' = -\csc^2 x$

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \cot x \csc x \, dx = -\csc x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \tan x \, dx = -\ln|\cos x| + C$

$\int \csc x \, dx = \ln|\csc x - \cot x| + C$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$\int \cot x \, dx = \ln|\sin x| + C$

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

1. (6 points) Compute $\int \cos^5(x)\ dx$.

2. (8 points) Use trigonometric substitution to evaluate $\int \frac{1}{x^2\sqrt{9-x^2}}\ dx$. In your justification, include a drawing of any triangles you are using.

3. Determine whether the following improper integrals converge or diverge. Circle the appropriate answer and justify your reasoning. **If the integral converges, evaluate it using limits**.

   (a) (6 points) $\displaystyle\int_{-1}^{\infty} e^{-3t}\,dt$ **Converges** **Diverges**

   (b) (6 points) $\displaystyle\int_{2}^{3} \frac{1}{\sqrt{3-x}}\,dx$ **Converges** **Diverges**

4. (4 points) Write **a single sentence** citing a theorem to determine whether $\displaystyle\int_{1}^{\infty} \frac{1}{\sqrt[3]{x^2}}\,dx$ converges or diverges.

5. Let $a_1 = 1$ and define a_{n+1} recursively by $a_{n+1} = \sqrt{a_n} + 2$.

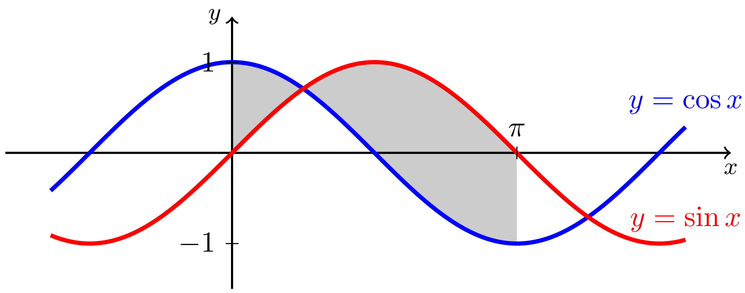
   (a) (4 points) Write out the first 4 terms of $\{a_n\}$.

   $a_1 =$ __________ $a_2 =$ __________ $a_3 =$ __________ $a_4 =$ __________

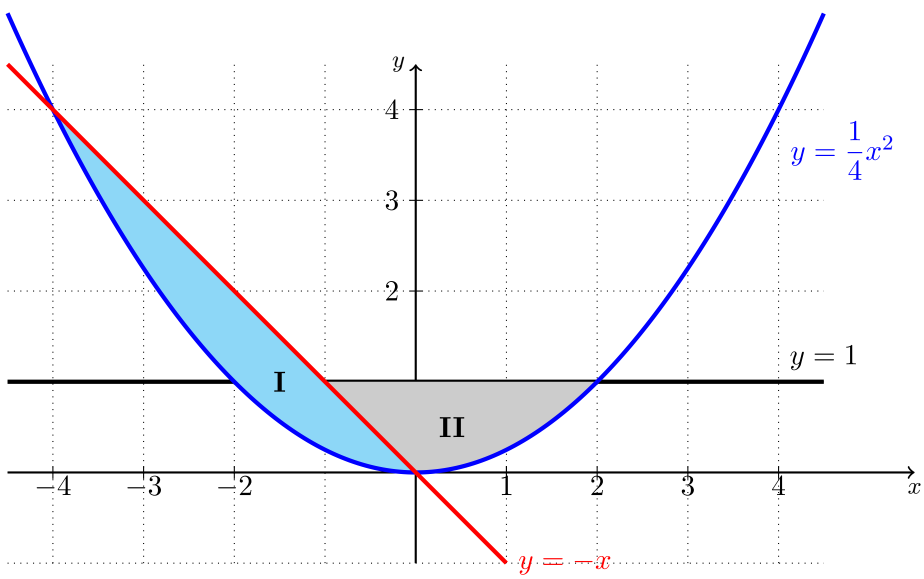
   (b) (6 points) Is $\{a_n\}$ increasing or decreasing? Use induction to prove your claim.

6. (4 points) Prove or give a counterexample: If $\{a_n^2\}$ converges, then $\{a_n\}$ converges.

7. (6 points) Set up (but do not solve) one or more integrals for the total area of the region shaded below. **Do not include absolute values** in any integrand.



8. (4 points) Match each shaded region with the integral that gives you its area.

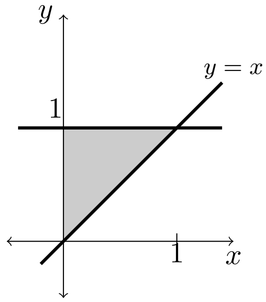


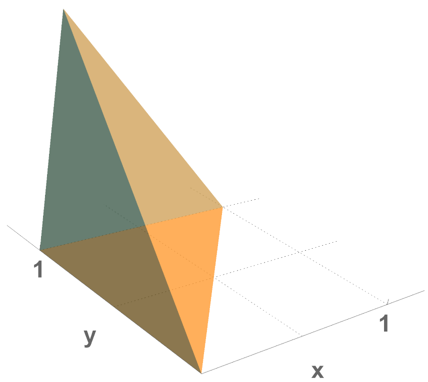
(A) $\displaystyle\int_{-4}^{-1} -x - \frac{1}{4}x^2\ dx + \int_{-1}^{0} 1 - \frac{1}{4}x^2\ dx$ **(B)** $\displaystyle\int_{0}^{1} 2\sqrt{y} + y\ dy$ **(C)** $\displaystyle\int_{0}^{4} -y + 2\sqrt{y}\ dy$

The area of **I** is represented by (A, B, or C): ______________

The area of **II** is represented by (A, B, or C): ______________

9. (8 points) Calculate the volume of S, where S is the solid with a base formed by the region enclosed by $y = x,\ y = 1$ and the y-axis as shown below. The cross sections of the solid perpendicular to the $x-$axis are isoceles right triangles.



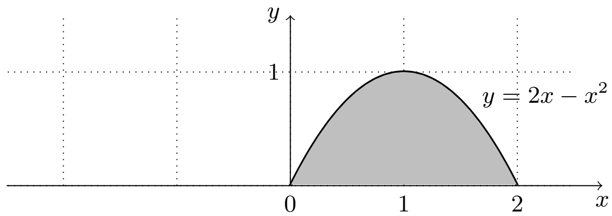


10. (8 points) Calculate the arc length of the curve $y = 4x^{3/2}$ from $x = 0$ to $x = 1$. (You must actually evaluate your integral, not just set it up. Give an exact answer, not a decimal approximation.)

11. Set up, but **do not evaluate**, an integral to find the volume of the solid S described in each part. (Note that the shaded region is the same in each part.)

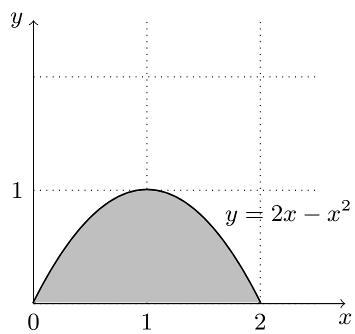
(a) (7 points) S is obtained by rotating the shaded region around the y-axis. Indicate whether you used the washer method or method of cylindrical shells.

Washer method **Shell method**

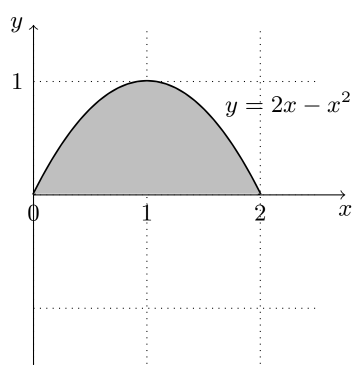


(b) (7 points) S is obtained by rotating the shaded region around the line $y = 1$. Indicate whether you used the washer method or method of cylindrical shells.

Washer method **Shell method**



(c) (4 points) S is the solid obtained by rotating the shaded region below about the x-axis.



12. (12 points) For each statement below, determine whether it is True or False, and circle the appropriate answer. If it is true, **use calculus** to explain why it is true. If it is false, explain why.

(a) If $f(x)$ is continuous for $x \geq 0$ and $\int_7^\infty f(x)\ dx$ converges, then $\int_1^\infty f(x)\ dx$ also converges.

True **False**

(b) If $f(x)$ is continuous everywhere, then there exists $c \in \mathbb{R}$ such that $f(c) = \int_{-1}^1 f(x)\, dx$.

True **False**

(c) Let $\{a_n\}$ be a decreasing sequence such that $a_n \leq 1$ for every $n \geq 1$. Then $\{a_n\}$ converges.

True **False**