Reductionist Philosophy

On this page, we will derive the formula

$$\int \sin^n(x)\ dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\, dx$$

where $n \geq 3$ is an integer. This is called a *reduction formula* because we transform the integral of $\sin^n x$ into a new integral with a reduced power.

1. Rewrite $\sin^n(x)$ as $\sin^{n-1}(x) \cdot \sin(x)$ and use the IBP formula with $u = \sin^{n-1}(x)$ and $dv = \sin(x)\ dx$ and simplify. Don't try to evaluate the $\int v\ du$ term.

2. In the $\int v\ du$ term, rewrite $\cos^2(x)$ as $1 - \sin^2(x)$ and split $\int v\ du$ into two integrals by distributing the $(n-1)\sin^{(n-2)}(x)$.

3. Put all of your work into one equation and combine the two $\int \sin^n(x)\ dx$ terms.

4. Solve the equation for $\int \sin^n(x)\ dx$ and simplify.