Compare and Be Square

1. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series with positive terms.

   (a) Why must it be true that for sufficiently large n, $a_n^2 \leq a_n$? Justify!

   (b) Use part (a) and the Comparison Test to show that $\sum_{n=1}^{\infty} a_n^2$ converges.

2. Find the error in the following (incorrect) proof that $\sum_{n=1}^{\infty} a_n^2$ converges.

> Because $\sum_{n=1}^{\infty} a_n$ is convergent, it follows that $\lim_{n\to\infty} a_n$ exists. Therefore the limit
>
> $$\lim_{n\to\infty} \frac{a_n^2}{a_n} = \lim_{n\to\infty} a_n$$
>
> also exists. Since both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} a_n^2$ are series with positive terms, the series $\sum_{n=1}^{\infty} a_n^2$ also converges by Limit Comparison Theorem.