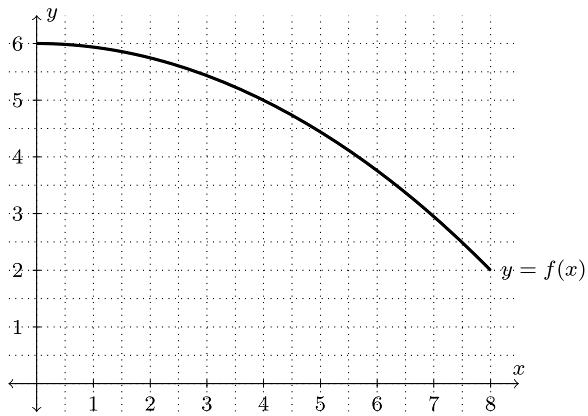
This exam covers the material from Sections 5.1–5.6, plus partial fractions in 5.7. This includes the limit definition of the definite integral, from the winter assignment, but Newton's Method will not be tested. Given the importance of antiderivatives to this material, we also highly suggest reading Section 4.8 before the exam. Be sure to look over your lecture notes, homework, and groupwork!

Start with problems #1–#10 below. Use your book and notes as needed at first, but your goal is to do similar problems without needing those resources. After completing #1-#10, continue on to the later problems or problems from your book to get additional practice.

1. (a) Use eight rectangles ($n = 8$) to find estimates of each type for area under the graph of f from $x = 0$ to $x = 8$. Find L_8 (sample points are the left endpoints), R_8 (sample points are the right endpoints), and M_8 (sample points are the midpoints).



   (b) Is L_8 an underestimate or overestimate of the actual area?

   (c) Is R_8 an underestimate or overestimate of the actual area?

   (d) Which of the values L_8, R_8, and M_8 gives the best estimate? Explain.

2. Use the limit definition of the definition integral to evaluate $\int_0^1 x^2 + 1\,dx$.

3. Evaluate the following indefinite integrals.

   (a) $\displaystyle\int \left[x^{-1/3} - 5x^4 + 3x^{2/5} - 6\right] dx$ (c) $\displaystyle\int \frac{dy}{7-2y}$ (e) $\displaystyle\int \frac{dx}{x\sqrt{\ln x}}$

   (b) $\displaystyle\int \frac{\sin t\; dt}{1-\sin^2 t}$ (d) $\displaystyle\int \frac{2x}{1+x^2}\,dx$ (f) $\displaystyle\int \frac{z\; dz}{\sqrt{z-2}}$

4. If $\displaystyle\int_{-2}^{2} 2f(x)\,dx = 12$, $\displaystyle\int_{-2}^{5} f(x)\,dx = 6$, and $\displaystyle\int_{-2}^{5} g(x)\,dx = 2$, find the following.

   (a) $\displaystyle\int_{-2}^{2} f(x)\,dx$ (c) $\displaystyle\int_{5}^{-2} g(x)\,dx$ (e) $\displaystyle\int_{-2}^{5} \frac{f(x)+g(x)}{2}\,dx$

   (b) $\displaystyle\int_{2}^{5} f(x)\,dx$ (d) $\displaystyle\int_{-2}^{5} (-\pi g(x))\,dx$ (f) $\displaystyle\int_{2}^{2} (2f(x) - g(x))\,dx$

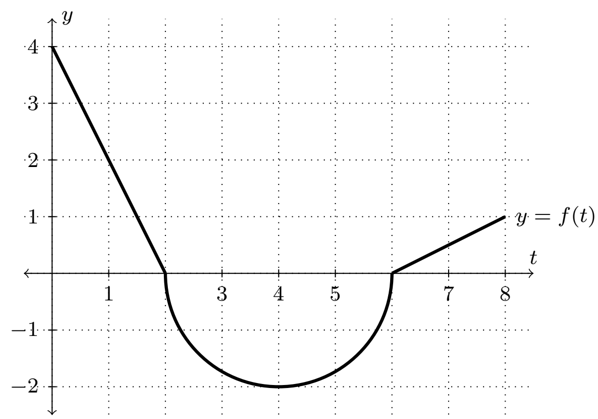
5. Prove that $\displaystyle\pi \le \int_0^{\pi} \sqrt{1+\sin^4 x}\,dx \le \pi\sqrt{2}$.

6. True/False: If f is a continuous function, then f has an antiderivative.

7. Let $f(x)$ be the function given below, and define $g(x) = \int_1^x f(t)\,dt$. Find an explicit formula for $g(x)$ which does not involve integrals.

$$f(x) = \begin{cases} 0, & x \le 1 \\ x-1, & 1 < x \le 2 \\ 1, & 2 < x \end{cases}$$

8. Let $g(x) = \displaystyle\int_0^x f(t)\,dt$, where $y = f(t)$ is graphed below with $0 \le x \le 8$.



(a) Find $g(1)$, $g(2)$, $g(4)$, and $g(8)$.

(b) Where is $g(x)$ increasing? Where is it decreasing?

(c) Where is $g(x)$ concave up? Where is it concave down?

(d) Where does $g(x)$ have local minima and maxima? Find the absolute maximum and minimum for $0 \le x \le 8$.

9. Find $\dfrac{d}{dx}\left(\displaystyle\int_{-5}^{x} \frac{t^3}{t^2+1}\,dt\right)$ and $\dfrac{d}{dx}\left(\displaystyle\int_{-3x}^{x^3} \sqrt{1+\sec(t^8)}\,dt\right)$.

10. Evaluate the following integrals.

(a) $\displaystyle\int x \ln x\,dx$

(b) $\displaystyle\int e^z \sin z\,dz$

(c) $\displaystyle\int \frac{dx}{x^2-2x-3}$

(d) $\displaystyle\int \frac{z^2+5z+2}{(z+1)(z^2+1)}\,dz$

11. Rewrite the following limit using integral notation. Is the resulting integral positive, negative, or zero?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi \sin(\pi + \frac{\pi}{n} i)}{\pi n + \pi i}.$$

12. Find an antiderivative of each function *Check your answer by differentiating.*

(a) $f(x) = x^3 + 3x + 2$

(b) $g(x) = 3\cos x - \sin x$

(c) $h(t) = 5t^3 - t^{-2} - t^{3/5}$

13. Find f.

(a) $f'(x) = \sqrt{x}\cdot(6+5x)$, $f(1) = 10$

(b) $f''(x) = 2 - 12x$, $f(0) = 9$, $f(2) = 15$

14. Use the substitution $u = x/a$ to prove $\displaystyle\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}(x/a) + C$.

15. Express the following integral as a limit by using a Riemann sum: $\int_{-1}^{\pi} e^{2x}\, dx$.

16. **Additional review from your textbook** (Stewart CCC 4th Ed.):

    (a) Ch. 5 Concept Check #1–7 and 8a, True-False #1–10, Review Exercises 5, 8, 10, 53.

    (b) §5.1 Exercises 18, 20 on p.342

    (c) §5.2 Exercise 53 on p.355

    (d) §5.3 Exercise 30 on p.364