Intervals of Convergence (Lecture Assignment)

Complete this assignment and submit it to Gradescope by 4:00pm on your class day. You can print this sheet, or write on your own paper. Contact us if internet connections or other issues require alternate arrangements.

1. Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$.

   (a) Use the Ratio Test to test when the series converges. What is the radius of convergence?

   $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x\sqrt{n}}{\sqrt{n+1}}\right| = \lim_{n\to\infty} |x| \Rightarrow |x|<1$

   $R = 2$

   (b) Determine whether the series converges at the two endpoints of the interval of convergence.

   $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \int_1^{\infty} \frac{1}{\sqrt{n}}\,dn = \infty$

   $\sum_{n=1}^{\infty} -\frac{1}{\sqrt{n}} = -\int_1^{\infty} \frac{1}{\sqrt{n}} = -\infty$

   → The series doesn't converge at the endpoints of the interval of convergence

   (c) What is the interval of convergence? $(-1,1)$

2. Consider the power series $\sum_{n=1}^{\infty} n^n(x-5)^n$.

   (a) Use the Root Test to find the radius of convergence.

   $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |n(x-5)| < 1 \rightarrow x = 5$, $R = 0$

   (b) What is the interval of convergence? $\{5\}$

One-Minute Questions: Write a sentence for each.

A. What's one mathematical question you have after watching the videos?

Just to be clear, the series of a Taylor is the original function?

B. What's one interesting thing you learned from the book or videos?

I learned that Taylor's are how the calculator can give a value for any sin(x)