Kunthik M 2/17/2021

UMTYMP Calculus I CCC 5.10

Proper Evaluation of Improper Integrals (Lecture Assignment)

Complete this assignment and submit it to Gradescope by 4:00pm on your class day. You can print this sheet, or write on your own paper. Contact us if internet connections or other issues require alternate arrangements.

For Problems 1–2, use the limit definition to determine whether the given improper integrals converge or diverge; do not use the Comparison Theorem. If an integral converges, find its value.

1. $\displaystyle\int_0^8 \frac{1}{x^{\frac{2}{3}}}\,dx$

$= \lim_{b\to 0} \int_b^8 x^{-\frac{2}{3}}\,dx$

$= \lim_{b\to 0} \left[3x^{\frac{1}{3}}\right]_b^8 = \lim_{b\to 0} \left(3\cdot 8^{\frac{1}{3}} - 3b^{\frac{1}{3}}\right)$

$= \lim_{b\to 0} \left(6 - 3b^{\frac{1}{3}}\right)$

$= 6 - 3\lim_{b\to 0^+} \sqrt[3]{b}$

$= \boxed{6}$

Converges

2. $\displaystyle\int_0^{\pi/2} \sec x\,dx$ (Recall: $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$. See this thread on Canvas for details.)

$\displaystyle\int_0^{\pi/2} \sec x\,dx = \lim_{b\to \frac{\pi}{2}} \int_0^b \sec(x)\,dx$

$= \lim_{b\to \frac{\pi}{2}} \left[\ln\left|\sec(x) + \tan(x)\right|\right]_0^b = \lim_{b\to \frac{\pi}{2}} \left(\ln|\sec(b) + \tan(b)|\right)$

$= \infty$, Diverges

One-Minute Questions: Write a sentence for each.

A. What's one mathematical question you have after watching the videos?

Nothing's coming to mind

B. What's one interesting thing you learned from the book or videos?

I thought that if $0 \le f(x) \le g(x)$ then $\ln(f(x)) \le \ln(g(x))$ was very interesting

Could be integrals