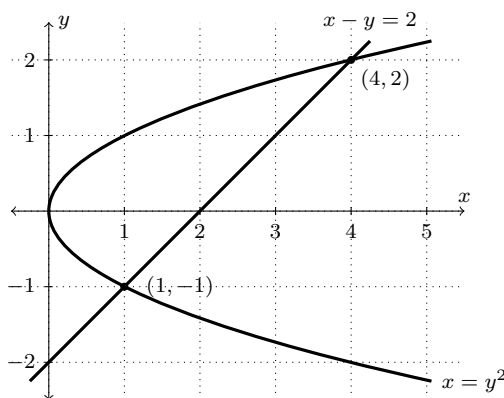


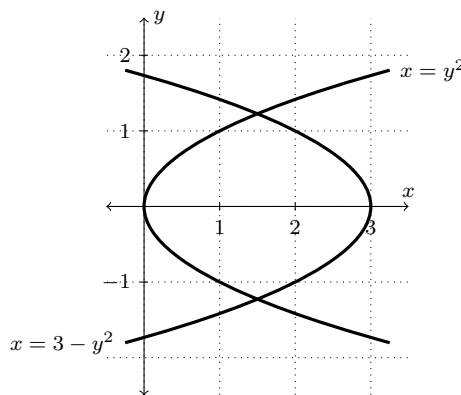
This exam covers the material covered since the last exam, in §5.7, §5.10, §§6.1–6.5, and §11.1 from Stewart's CCC 4th Custom Ed. Also look over your lecture notes, homeworks, and groupworks.

1. Given the functions below, set up and evaluate an integral to find the area between the two curves. Try each with both  $dx$  and  $dy$  slices.

(a)  $x = y^2$ ;  $x - y = 2$



(b)  $x = y^2$ ;  $3 - y^2$



2. Consider the two curves  $f(x) = x^4 - 2x^2 - 8$  and  $g(x) = -3x - 6$ .
- (a) Sketch the graph of the two functions. Label the two intersection points. Be as specific as possible.
- (b) Set up and evaluate an integral to find the area enclosed by the two curves  $f(x)$  and  $g(x)$ .
3. Evaluate the following integrals. If you do a trig substitution (such as  $x = a \sin \theta$ ), make sure to draw any triangles you use in the evaluation of the integral.

(a)  $\int \cos^2 x \, dx$

(e)  $\int \frac{dx}{\sqrt{x^2 - 4}}$

(b)  $\int \frac{y}{\sqrt{4 - y^2}} \, dy$

(f)  $\int_0^3 \frac{t^3}{\sqrt{t^2 + 9}} \, dt$

(c)  $\int \sin^5 \theta \cos \theta \, d\theta$

(g)  $\int \sin^5 \theta \, d\theta$

(d)  $\int \frac{x^3}{\sqrt{9 - x^2}} \, dx$

(h)  $\int \tan^4 \frac{x}{2} \sec^4 \frac{x}{2} \, dx$

4. Determine whether the following improper integrals diverge or converge. If it converges, evaluate the integral.

(a)  $\int_0^\infty \frac{dx}{1 + x^2}$

(c)  $\int_{-3}^3 \frac{dx}{x(x + 1)}$

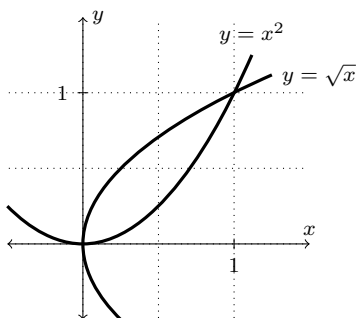
(b)  $\int_0^1 \frac{dx}{\sqrt{1 - x}}$

(d)  $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

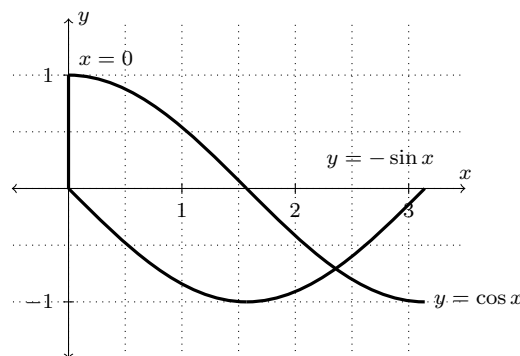
5. Do Ch. 5 Review Exercises 24, 29, 56, 61 on pp. 425–426 in Stewart's CCC 4th Custom Ed.
6. When slicing a solid perpendicular to the  $x$ -axis at point  $x$ , the area of each slice is  $A(x) = x \sin(x^2) \cos(x^2)$ , where  $0 \leq x \leq \sqrt{\frac{\pi}{2}}$ . Find the volume of the solid from  $x = 0$  to  $x = \sqrt{\frac{\pi}{2}}$ .
7. Do Ch. 6 Review Exercises 9, 10, 11, 14, 17, 19, 25, 34 on pp. 488–489 in Stewart's CCC 4th Custom Ed.

8. Find the volume of the solid obtained by rotating the region bounded by the given curves about the given line. Do each problem with *both* a  $dx$  slice and a  $dy$  slice. State the name for the method used.

(a)  $y = \sqrt{x}$  and  $y = x^2$ ; about  $x$ -axis



(b)  $y = -\sin x$ ,  $y = \cos x$ , and  $y$ -axis; about  $y$ -axis



9. Does  $\int_{-2}^{\infty} \frac{\sin^{30}(x)}{e^x} dx$  converge or diverge? Carefully justify any theorems which you use.
10. Let  $f(x) = x^2$ . Find a value  $c$  such that  $f(c)$  is equal to the average value of  $f(x)$  on  $[0, 2]$ .
11. Determine whether the following sequences converge or diverge. If it converges, find its limit.
- (a)  $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \dots\}$
- (b)  $\{-5, \frac{5}{6}, -\frac{5}{36}, \frac{5}{216}, \dots\}$
- (c)  $a_n = \frac{n!}{2^n}$
- (d)  $a_n = \frac{1+n}{\sqrt{1+n}}$
- (e)  $a_n = \cos(\pi n)$
- (f)  $a_n = \frac{3n^2 - 5n + 1}{\sqrt{2n^4 - 1}}$
12. Consider the recursive sequence given by  $a_1 = 1$  and  $a_{n+1} = 1 + \frac{1}{2}a_n$ . Answer the following questions.
- (a) List the first four terms of the sequence.
- (b) Is the sequence monotonically increasing, monotonically decreasing, or neither? Prove your answer.
- (c) Is the sequence bounded? Prove your answer.
- (d) Does the sequence converge? Explain why or why not. If so, find the limit.

### Advice for exam studying

Here are a short bits of advice as you study for the exam :

- *Study frequently.* Studying for one hour every day is better than studying for 7 hours in one day.
- *Work with friends.* Then, try to write up your solutions later on your own.
- *Also, practice without help.* Put away calculators, textbooks, notes, siblings... anything which might help you. Force yourself to complete the problem without help. This mimics the exam.
- *Put away distractions.* Turn off the TV, put away computers, hide the phone.

What should you study? We often find that students don't focus on the most useful study resources.

- *The textbook:* Briefly review the textbook only for basic understanding. Make a list of theorems, test, and definitions relevant to the material. Especially those not covered in class. Don't spend much time here.
- *Lecture notes:* Reread your lecture notes briefly to clarify anything confusing in the textbook.
- *Review groupworks:* Spend an hour or two reviewing and re-doing worksheets. Finish any problems which you didn't complete.
- *Review and redo homework:* This is extremely important! Pay close attention to grader comments. Review Moodle threads and email instructors with questions. Re-work homework problems from scratch, addressing any mistakes you made.
- *Practice problems:* Also very important. Spend a *lot* of time doing problems from the review sheet, practice exams, and chapter reviews. Do these without notes, books, or computers.