

1. Let $f(x) = \frac{4}{4+x^2}$.

(a) Find a power series representation of $f(x)$. What is its radius of convergence?

(b) Find a power series representation of $f'(x)$. What is its *interval* of convergence?

2. If $\sum c_n(x-1)^n$ converges at $x = -1$ and diverges at $x = 5$, what can you say about the convergence of the power series when:

(a) $x = 6$?

(b) $x = 2$?

(c) $x = 4$?

3. Find the 3rd-degree Taylor polynomial centered at $x = 2$ for the function $g(x) = \sin(x-2) + \ln x$.

4. Determine whether the following series converge conditionally, converge absolutely, or diverge. Explain.

(a) $\sum_{k=1}^{\infty} \frac{\sin(k) + 2}{k^3}$

(b) $\sum_{n=1}^{\infty} n e^{-n^2}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 7^{n+1}}{(n+3)!}$

5. True or false?

(a) If $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = 2$, then it also converges for $x = -1$.

(b) If $\sum_{n=1}^{\infty} \left(a_n + \frac{1}{n^2} \right)$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

(c) If $\sum (a_n - b_n)$ is convergent, then $\sum a_n$ and $\sum b_n$ both converge.