This exam covers the material from classes 10 through 15 on your syllabus, which correspond to ET §§11.2–11.10 from Stewart's CCC 4th Custom Ed. Also look over your lecture notes, homeworks, and groupworks. All of the problems on this sheet are worth woking on; focus on those topics that you know you have to work on. If you want suggestions: problems 1-9 are a good place to start. Problem 10 is a good test of whether you can take known power series and manipulate them to meet certain conditions. Problem 11 includes many good review problems from the book. Problems 12 and beyond provide some extra practice with Taylor and Maclaurin Series, in case you'd like more practice with those topics after finishing your WebWork assignment.

1. Find the sum of the series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+3}}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right]$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{7^n n!}$

2. Determine whether the following series converge or diverge using any appropriate method. *State which method you used and explain why it can be used.*

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + n\cos^2 n}$ (e) $\displaystyle\sum_{j=1}^{\infty} \frac{j+1}{j(j+2)}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ (d) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{3^k}$ (f) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$

3. Let $\sum a_n$ be a convergent series of positive terms. Prove that $\sum a_n^2$ converges.

4. Let the series $\sum a_n$ and $\sum b_n$ be positive term series for which $\sum a_n$ converges and $\sum b_n$ diverges. Determine whether the following series *converge*, *diverge*, or *not sufficient information.* Explain why.

   (a) $\displaystyle\sum (-1)^n a_n$ (b) $\displaystyle\sum a_n b_n$ (c) $\displaystyle\sum b_n^2$ (d) $\displaystyle\sum \frac{1}{a_n}$

5. Determine the radius and interval of convergence for the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{n!}$

6. Use series to evaluate the integral $\displaystyle\int_0^1 \sin x^2\, dx$. *State your answer as a series.*

7. Find the power series representation centered at $x = 0$ for the function $f(x)$. Write the first five non-zero terms and the general summation formula. Then find the radius of convergence.

   (a) $f(x) = \dfrac{4}{1-x^4}$ (c) $f(x) = e^x + 2e^{-x}$

   (b) $f(x) = \arctan\left(\dfrac{x}{5}\right)$ (d) $f(x) = \dfrac{5x^5}{(3-x)^2}$

8. Find the radius of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$.

9. Write down a series centered at $x = 0$ for $f(x) = \frac{1}{1+x^2}$. Use that answer to find series for $f'(x)$, $\int f(x)\, dx$, and $\arctan x$.

10. Create power series that have the following intervals of convergence.

    (a) $[1, 3]$ (b) $[1, 3)$ (c) $(1, 3)$ (d) $(1, 3]$

11. Do ET Ch. 11 Review True-False Quiz 1,2,4,5, 7–13 and Exercises 16, 18, 19, 22–25, 28, 29, 31, 38, 58(a) on pp.778–780 in Stewart's CCC 4th Custom Ed.

12. Find the Taylor series centered at $x = c$ for the function $f(x)$. Write the first five non-zero terms and the general summation formula. State the radius of convergence.

    (a) $f(x) = e^{2x}, \quad c = 0$

    (b) $f(x) = \cos x, \quad c = \frac{\pi}{4}$

    (c) $f(x) = \ln x, \quad c = 1$

    (d) $f(x) = \ln(x^2 + 1), \quad c = 0$

13. Evaluate the following limits.

    (a) $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

    (b) $\lim_{x \to 0} \dfrac{1 + x + \frac{1}{2}x^2 - e^x}{x^3}$

14. Find the third-degree Taylor polynomial centered at $x = 0$ for the function $f(x) = \tan x$.

15. Find the fourth-degree Taylor polynomial centered at $x = 1$ for the function $g(x) = \sqrt{x}$.

Advice for exam studying

Here are a short bits of advice as you study for the last exam (and next year!)

- *Study frequently.* Studying for one hour every day is better than studying for 7 hours in one day.
- *Work with friends.* Then, try to write up your solutions later on your own.
- *Also, practice without help.* Put away calculators, textbooks, notes, siblings... anything which might help you. Force yourself to complete the problem without help. This mimics the exam.
- *Put away distractions.* Turn off the TV, put away computers, hide the phone.

What should you study? We often find that students don't focus on the most useful study resources.

- *The textbook:* Briefly review the textbook only for basic understanding. Focus on definitions, theorems, and examples. Don't spend much time here.
- *Lecture notes:* Reread your lecture notes briefly to clarify anything confusing in the textbook.
- *Review groupworks:* Spend an hour or two reviewing and re-doing worksheets. Finish any problems which you didn't complete but that you know are similar to other groupwork and homework problems.
- *Review and redo homework:* This is extremely important! Pay close attention to grader comments. Review Canvas threads and email instructors with questions. Re-work homework problems from scratch, addressing any mistakes you made.
- *Practice problems:* Also very important. Spend a *lot* of time doing problems from the review sheet, practice exams, and chapter reviews. Do these without notes, books, or computers.