Get a Headstart on Series

1. We know $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} = 1+\frac{1}{2}+\frac{1}{4}+\cdots = 2$. Does $\sum_{n=2}^{\infty}\left(\frac{1}{2}\right)^{n-1}$ converge? To what?

2. In general, if $\sum_{n=1}^{\infty} a_n$ converges, does $\sum_{n=2}^{\infty} a_n$ converge? To what? See if you can prove this fact using the limit definition of convergent series.

3. In general, if $\sum_{n=1}^{\infty} a_n$ converges, prove $\sum_{n=k}^{\infty} a_n$ converges for any natural number k.

(In other words: removing numbers from the beginning of an infinite sum does not affect the convergence of the sum.)