Are you Serious about Series?

Recall that $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + x^4 + \cdots$ and $\displaystyle\int \dfrac{1}{1-x}\, dx = -\ln|1-x| + C.$

1. Is it true that $\dfrac{1}{1-2} = 1 + 2 + 4 + 8 + 16 + \cdots$?

   What do we *actually* mean when we say that $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + x^4 + \cdots$?

2. Express $\ln(1-x)$ as a power series. What is its radius of convergence? Is it ok to write $\ln(1-x)$ instead of $\ln|1-x|$ here?

3. Express $\ln(1+x^2)$ as a power series. What is its radius of convergence?

4. In #2 you wrote a power series for $\ln(1-x)$, centered at $x = 0$. Why can't we write a power series for $\ln(x)$ centered at $x = 0$? Instead, write a power series for $\ln x$ centered at $x = 1$.