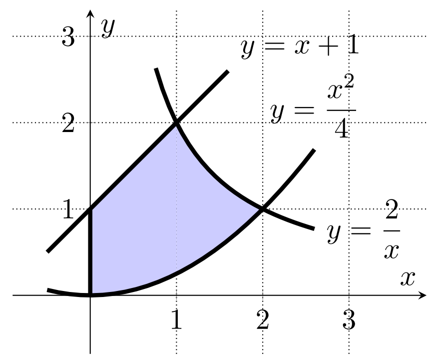
The following written homework problems are due at 6pm on Gradescope, the night before your class. You also have a WebWork assignment due at 11pm two days before your class.

(7.1) Use integrals to find the area of the shaded region below.



(7.2) Find the arc length of the graph of the function $y = \dfrac{e^x + e^{-x}}{2}$ over the interval $[0, 3]$.

(7.3) For the function $f(x) = 1 + \ln x$ on the domain $[a, b]$, where $1 \le a < b$.

(a) Find the average value of f on the given interval.

(b) Find c such that $f_{\text{ave}} = f(c)$.

(7.4) Professional Problem.

Suppose $f(x)$ is a continuous function. Apply the Mean Value Theorem for derivatives to the function $F(x) = \int_a^x f(t)\,dt$. Use this to prove the Mean Value Theorem for Integrals.

Hint: Below are correct statements of three major theorems which are relevant for your solution. The theorems differ in small but important ways from how they are written in your textbook. Use these *versions in your professional problem solution to save you frustration!*

The Mean Value Theorem for Derivatives: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The Fundamental Theorem of Calculus, Part 1: If f is continuous on $[a, b]$ and $g(x) = \int_a^x f(t)\,dt$ on $[a, b]$, then $g(x)$ is continuous on $[a, b]$, and $g'(x) = f(x)$ on (a, b).

The Mean Value Theorem for integrals: If f is continuous on $[a, b]$, then there exists a number c in (a, b) such that

$$f(c) = \frac{1}{b - a}\int_a^b f(x)\,dx.$$

You may notice that you already solved this problem in groupwork. (Or, if you didn't get that far, you were given a groupwork which leads you through the solution.) Hence we expect everybody's mathematics to be correct, and we will focus on your writing when grading this professional problem. You should focus on:

Writing: Review the professional problem checklist, which was handed out in September and is still available on Canvas. We have regularly been making comments about issues which are covered on that list. (For example: formatting chains of equalities using the "Oklahoma Rule," starting sentences with words, not mathematical symbols, or labeling your figures.) This week you can expect to lose points if your solution breaks the guidelines on that checklist. Remember that many of those issues were covered in the professional problem on HW 11 last fall. See this thread on Canvas for a refresher.

Methods: Use the Mean Value Theorem for derivatives properly.

Explanation: Justify your steps briefly but clearly. Cite any theorems you use, and explain briefly why their hypotheses are satisfied, but do *not* re-copy their full statements.

Figure: For full credit, include a well drawn, helpful generic figure of your own creation (i.e. not copied from a book, website or other resource.)

Organization & Structure: Your proof should be concise and clear. A fully justified solution could take half a page or less! Even with a figure, we would generally expect solutions to fit on a page unless your handwriting is large.

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor specific questions about it.

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