It's Increasingly Clear

Recall that a sequence s_n is *increasing* if $s_{n+1} \geq s_n$ for all n. Similarly, s_n is *decreasing* if $s_{n+1} \leq s_n$ for all n.

1. Let $a_n = \dfrac{n}{n+1}$ for $n = 1, 2, 3, \ldots$. Show a_n is increasing by demonstrating that $\dfrac{a_{n+1}}{a_n} \geq 1$ for all n.

2. Let $b_n = \dfrac{n-2}{n+1}$ for $n = 1, 2, 3, \ldots$. Explain the danger in using the method of the previous problem to show that b_n is increasing. Instead, prove b_n is increasing by demonstrating that $b_{n+1} - b_n \geq 0$ for all n.

3. Let $c_n = \dfrac{n^2+1}{n^3}$. Use calculus to prove c_n is decreasing. Hint: $c_n = f(n)$ for $f(x) = \dfrac{x^2+1}{x^3}$.