The following written homework problems are due at 6pm on Gradescope, the night before your class. You also have a WebWork assignment due at 11pm two days before your class.

(4.1) Evaluate the integral: $\int p^5 \ln p \, dp$.

(4.2) Consider the integral $\displaystyle\int \frac{1}{\sqrt{x+2}+x}\,dx$. Use substitution to express the integrand as a rational function, then evaluate the integral.

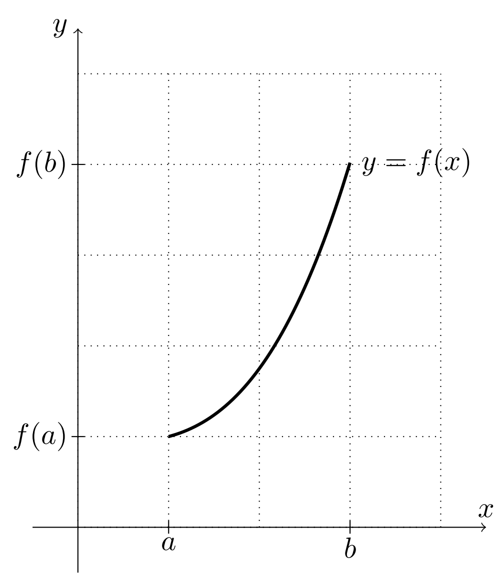
(4.3) Let f be a continuous, increasing function, and let g be the inverse of f. As a reminder, that means if $y = f(x)$, then $x = g(y)$, and $x = g(f(x))$.

(a) Use IBP to show $\displaystyle\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$.

(b) Show that $\displaystyle\int_a^b f(x)\,dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\,dy$.

Hint: Use part (a); in the last integral, rewrite x as in the problem statement and do a "y-substitution."

(c) Suppose $f(x) > 0$ and $0 < a < b$, as shown in the diagram below. Reproduce the diagram; then shade (and label) various areas on the diagram to give a geometric interpretation of part (b).



(4.4) (a) Use the reduction formula in Example 6 of Section 5.6 to show that

$$\int_{-\pi}^{\pi} \sin^n x \, dx = \frac{n-1}{n} \int_{-\pi}^{\pi} \sin^{n-2} x \, dx,$$

where $n \geq 2$ is an integer.

(b) Use part (a) to evaluate $\int_{-\pi}^{\pi} \sin^2 x \, dx$.

(c) Use proof by induction to prove the following formula for every integer $n \geq 1$.

$$\int_{-\pi}^{\pi} \sin^{2n} x \, dx = \frac{(2n-1)\cdots 5 \cdot 3 \cdot 1}{(2n)\cdots 6 \cdot 4 \cdot 2}(2\pi)$$

As always, refer to the "Professional Problem Information" handout to create a *professionally written* solution. This week, you should especially focus on:

Methods: You're asked to write proof by induction. Before writing your proof, make sure to review the *UMTYMP Tips for Writing Induction Proofs* on Canvas.

Organization and Structure: A proof by induction has an expected structure. Make sure you do all of the required steps, and in the correct order.

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor specific questions about it.

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