**(1.1)** The velocity of a moving particle at time $t$ is given by $v(t) = \sin(t)$.

**(a)** Find the *net distance* traveled by the particle from $t = 0$ to $t = \dfrac{3\pi}{2}$. To find the net distance traveled in the time of 0 to $\dfrac{3\pi}{2}$, simply take the integral of $v(t)$.

$$\int_a^b (v(t))\,\mathrm{d} = \int_0^{3\pi/2} (\sin(t))\,\mathrm{d}x$$

Next apply the Evaluation Theorem, which states $\int_a^b (v(t))\,\mathrm{d}x = F(b) - F(a)$ when $f(x)$ is continuous on $[a, b]$ and $F$ is any antiderivative of $f$, to the integral.

$$\int_0^{3\pi/2} (\sin(t))\,\mathrm{d}x = -\cos\left(\frac{3\pi}{2}\right) - (-\cos(0))$$

All that is left is to simplify the expression above to find the integral

$$\begin{aligned}-\cos\left(\frac{3\pi}{2}\right) - (-\cos(0)) &= -\cos\left(\frac{3\pi}{2}\right) + \cos(0)\\ &= 1\end{aligned}$$

Now we know the value of $F(b) - F(a)$ is 1, so

$$\int_0^{3\pi/2} (\sin(t))\,\mathrm{d}x = 1$$

**(b)** Find the *total distance* traveled by the particle from $t = 0$ to $t = \dfrac{3\pi}{2}$. Finding the total distance is similar to finding the net distance, but instead take $|f(x)|$ instead of plain $f(x)$. The integral of $v(t)$ is negative from $\pi \leq t \leq 3\pi/2$ so we take the integral of $-\sin(t)$ on that interval to turn the negative area positive.Thus there are two integrals.

$$\int_0^{\pi} \sin(x)\,\mathrm{d}x + \int_{\pi}^{3\pi/2} (-\sin(x))\ \mathrm{d}x$$

We know that the antiderivative of $\sin(x)$ is $-\cos(x)$ and inversely $-\sin(x)$'s antiderivative is $\cos(x)$, so we can use the Evaluation Theorem we used before to get the expression below.

$$(-\cos(\pi) - (-\cos(0))) + \left(\cos\left(\frac{3\pi}{2}\right) - \cos(\pi)\right)$$

Which simplifies to

$$\begin{aligned}-(-1) - (-(1)) + 0 - (-1) &= 1 + 1 + 0 + 1\\ &= 3\end{aligned}$$

Thus,

$$\int_0^{\pi} \sin(x)\,\mathrm{d}x + \int_{\pi}^{3\pi/2} (-\sin(x))\ \mathrm{d}x = 3$$

Therefore the total distance traveled by the particle from $t = 0$ to $t = \dfrac{3\pi}{2}$ is 3.

**(1.2)** Algebraically evaluate the integral $\int_{-1}^{2} \left(x^2 + |x|\right)\, \mathrm{d}x$.

First we split the integrals into two separate integrals.

$$\int_{-1}^{2} x^2 \,\mathrm{d}x + \int_{-1}^{2} (|x|)\, \mathrm{d}x$$

Then we split the integral of $|x|$ again with two different bounds of $(-1, 0)$ and $(0, 2)$.

$$\int_{-1}^{2} x^2 \,\mathrm{d}x - \int_{-1}^{0} x \,\mathrm{d}x + \int_{0}^{2} x \,\mathrm{d}x$$

The second integral is negative because the function of $|x| = -x$ on (-1,0) and we can take the constant $-1$ out of $f(x)$.

$$|x| = \begin{cases} -x, & \text{if } x \leq 0 \\ x, & \text{if } x > 0 \end{cases}$$

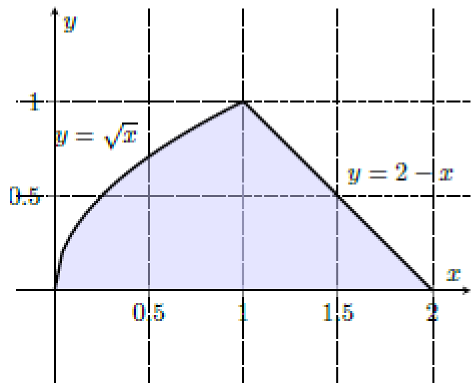
Next we can use the Evaluation Theorem and create a simple expression which we can then substitute into. We need to find the antiderivative of $x$ and $x^2$ first. We know that the antiderivative of $x^n = \dfrac{x^{n+1}}{n+1}$ so the antiderivative of $x$ is $\dfrac{x^2}{2}$ and the antiderivative of $x^2$ is $\dfrac{x^3}{3}$. Thus we compute the expression

$$\begin{aligned}
\left[\frac{x^3}{3}\right]_{-1}^{2} - \left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2} &= \frac{2^3}{3} - \frac{-1^3}{3} - \left(\frac{0^2}{2} - \frac{-1^2}{2}\right) + \frac{2^2}{2} - \frac{0^2}{2} \\
&= \frac{8}{3} + \frac{1}{3} + \frac{1}{2} + \frac{4}{2} \\
&= \frac{9}{3} + \frac{5}{2} \\
&= 5 + \frac{1}{2} \\
&= \frac{11}{2}
\end{aligned}$$

Now that we know what the value of the sum of our three integrals is, we can loop it back to the original problem and say that

$$\int_{-1}^{2} \left(x^2 + |x|\right)\, \mathrm{d}x = \frac{11}{2}$$

**(1.3)** Find the area of the shaded region below.




**(a)** by evaluating two integrals with respect to $x$, We can see from the graph that the integral of $\sqrt{x}$ is what is used from (0,1) and the integral of $2 - x$ has been used from (1,2). Thus we get the integrals

$$\int_0^1 \sqrt{x}\,\mathrm{d}x + \int_1^2 (2 - x)\,\mathrm{d}x$$

Next we find the antiderivatives of $\sqrt{x}$ and $2-x$. The former is equal to $x^{1/2}$ so the antiderivative is $\frac{2x^{3/2}}{3}$ and the anti derivative of $2 - x$ is $2x - \frac{x^2}{2}$. Next we use the Evaluation Theorem to solve.

$$\begin{aligned}\frac{2 \cdot 1^{3/2}}{3} + \left(2 \cdot 2 - \frac{2^2}{2}\right) - \left(2 \cdot 1 - \frac{1^2}{2}\right) &= \frac{2}{3} + 4 - 2 - 2 + \frac{1}{2} \\ &= \frac{2}{3} + \frac{1}{2} \\ &= \frac{7}{6}\end{aligned}$$

The area of the shaded region is 7/6.

**(b)** by evaluating one integral with respect to $y$. To do this, first we must get our functions in terms of $y$. $2-x = y$ becomes $2-y = x$ and $y = \sqrt{x}$ becomes $x = y^2$. New we look back at our graph to see that we can take the integral of $2 - y$ on (0,1) and then subtract the unshaded area, which is conveniently equal to the integral of $y^2$ on (0,1). Thus we have the set up for our original integral expression

$$\int_0^1 (2 - y)\,\mathrm{d}y - \int_0^1 y^2\,\mathrm{d}y$$

Since both are on the same interval, unlike when we calculated with respect to $x$ we can combine their functions to get

$$\int (2 - y - y^2)\,\mathrm{d}x$$

Now we can solve using the Evaluation Theorem as usual. But first we need to find the antiderivative which we can find easily using the property $\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + \mathrm{C}$. Thus the antiderivative is

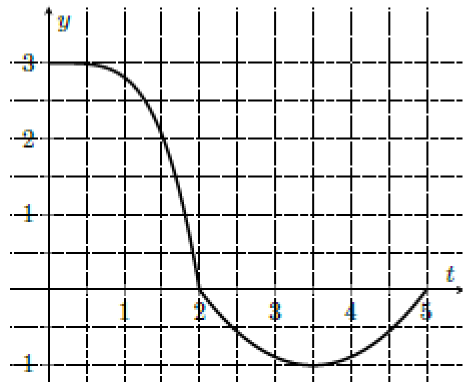
$$2y - \frac{y^2}{2} - \frac{y^3}{3}$$

Now we can solve.

$$\left(2 \cdot 1 - \frac{1^2}{2} - \frac{1^3}{3}\right) - \left(2 \cdot 0 - \frac{0^2}{2} - \frac{0^3}{3}\right) = 2 - \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$$

Checking above, both of our answers match and we have found the shaded area with both one and two integrals.

**(1.4) Professional Problem** Let $F(x) = \int_2^x f(t)\,\mathrm{d}t$. the graph of $f$ is given below. Determine the largest and smallest values of $F(0)$, $F(2)$, $F(3)$, $F(4)$, $F(5)$.




Let's start by analyzing the graph and what it means for our integrals. First of all, we observe that all of the integrals are negative *except* $F(2)$. This is because $F(3)$, $F(4)$, $F(5)$ have the entire graph below $y = 0$ so the integral is negative. For $F(0)$, the integral in $F(x)$ will be from (2,0). Since $a > b$ in the integral $\int_a^b f(t)\,\mathrm{d}t$, the function of 0 will also be negative. This is because when calculating $\Delta x$ in the limit definition of the definite integral, $\frac{b-a}{n} = -\left(\frac{a-b}{n}\right)$. From this we can conclude that $F(2)$ is the greatest because $0 > x$ for all negative $x$.

Now to find the smallest value out of $F(0)$, $F(3)$, $F(4)$, $F(5)$ we will look back at the graph. If we take into account the y-values on the y-axis we see that $f(3)$, $f(4)$, and $f(5)$ are all less than 5. Now if we use the distance 3, 4, 5 are from 2, we notice that their integrals cannot be less than $-1$, $-2$, $-3$. On the other hand, if we try to take $F(0)$ we see that at the minimum it is less than $-3$. How we came to this conclusion is that if we take the line $g(x) = \frac{-3}{2}x + 3$ it cuts directly across $F(0)$. Since $g(x)$ is lower than $f(x)$ at all points on (0,2), $F(x)$ must be less than the integral of $g(x)$ on (2,0)

$$\int_2^0 \left(-\frac{3}{2}x + 3\right)\,\mathrm{d}x = -3$$

Now we have proved that $F(0)$ is less than -3 we know that $F(0)$ must be the least out the given choices because it is least than $-3$ which is the absolute least that $F(5)$ was able to go. Along with the fact $F(5)$ was the had the smallest output out of $F(3)$, $F(4)$, and $F(5)$, we can confidently say that $F(0)$ and $F(2)$ have the smallest and greatest values respectively out of $F(0)$, $F(2)$, $F(3)$, $F(4)$, $F(5)$.