Keerthik M
Calculus I
Hollingsworth
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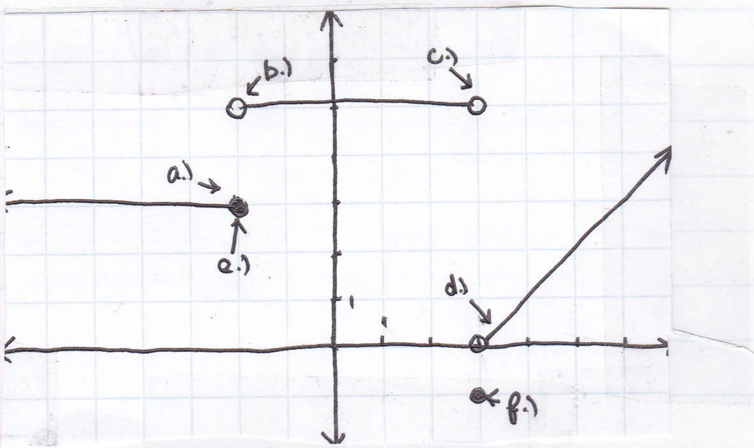
1.1



1.2 $\lim_{x\to 0^+} \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} = 0$

$-1 \le \cos\left(\frac{2\pi}{x}\right) \le 1$:

$\sqrt{x} \cdot \frac{1}{e} \le \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} \le \sqrt{x} \cdot e$:

$\lim_{x\to 0^+} \frac{\sqrt{x}}{e} \le \lim_{x\to 0^+} \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} \le \lim_{x\to 0^+} \sqrt{x}\, e$:

$\frac{0}{e} \le \lim \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} \le 0$:

$0 \le \lim_{x\to 0^+} \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} \le 0$

$\therefore \lim_{x\to 0^+} \sqrt{x}\, e^{\cos\left(\frac{2\pi}{x}\right)} = 0$

1.3 $\lim_{x\to 0} \frac{g(x)}{x^4} = -3$: $\frac{ax^4}{x^4} = -3$; $a = -3$; $g(x) = -3x^4$

a) $\lim_{x\to 0} g(x) = -3 \cdot 0^4 = 0$

b) $\lim_{x\to 0} \frac{g(x)}{x^3} = \lim_{x\to 0} -3x = 0$

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Suppose $\lim_{x \to a} f(x)$ exists, but $\lim_{x \to a} [f(x) + g(x)]$ does not exist.
Prove $\lim_{x \to a} (g)$ does not exist.

This will be a proof by contradiction.

Let $\lim_{x \to a} f(x)$ exist and $\lim_{x \to a} [f(x) + g(x)]$ not exist. By proof of contradiction, we can assume $\lim_{x \to a} g(x)$ equals some real number.

That means, when $\lim_{x \to a} f(x) = \mathcal{L}$ and $\lim_{x \to a} [f(x) + g(x)] = \text{DNE}$, $\lim_{x \to a} g(x) = n$.

Therefore,

$$\lim_{x \to a} f(x) + \lim_{x \to a} g(x) = \lim_{x \to a} [f(x) + g(x)]$$

because of limit laws.

Simplifying, we get:

$$\mathcal{L} \quad + \quad n \quad = \quad \text{DNE}.$$

The left hand side contains two real numbers. Due to limit laws, $\lim_{x \to a} f(x) + \lim_{x \to a} g(x)$ should equal $\lim_{x \to a} [f(x) + g(x)]$. But, this cannot be true, because two continuous functions cannot equal a discontinuous one.

Thus,

$$\lim_{x \to a} g(x) = \text{DNE} \text{ when } \lim_{x \to a} [f(x) + g(x)] = \text{DNE} \text{ and } \lim_{x \to a} f(x) = \mathcal{L}$$