The following written homework problems are due at 6pm on Gradescope, the night before your class. You also have a WebWork assignment due at 11pm two days before your class.

(9.1) Let $a_n = \ln(4n+2) - \ln(n+1)$. Write the first three numbers in the sequence $\{a_n\}$ and calculate its limit. If the limit does not exist, explain why.

(9.2) A sequence $\{b_n\}$ is given by $b_1 = 2$, $b_{n+1} = \sqrt{6 + b_n}$.

(a) Use induction to prove $\{b_n\}$ is increasing.

(b) Use induction to prove $\{b_n\}$ is bounded. *Hint: $\{b_n\}$ is bounded above by 3.*

(c) In one sentence, explain why the Monotonic Sequence Theorem applies to $\{b_n\}$, so that we know $\lim_{n\to\infty} b_n$ exists.

(d) Find $\lim_{n\to\infty} b_n$.

(9.3) Let $c_n = \dfrac{3-2n}{n}$.

(a) Use algebra to prove $\{c_n\}$ is decreasing.

(b) Use algebra to prove $\{c_n\}$ is bounded below by -2 and above by 1. If it helps, you can use the fact that $\{c_n\}$ is decreasing, but you don't have to.

(9.4) **Professional Problem.** For each of the following statements, either prove or give a counterexample.

(a) If $\{|a_n|\}$ is convergent, then $\{a_n\}$ is convergent.

(b) If $\{a_n\}$ and $\{b_n\}$ are divergent sequences, then $\{a_n + b_n\}$ diverges.

(c) If $\{a_n\}$ and $\{b_n\}$ are divergent sequences, then $\{a_n \cdot b_n\}$ diverges.

(d) If $\{a_n\}$ and $\{a_n \cdot b_n\}$ are convergent sequences, then $\{b_n\}$ converges.

As always, refer to the "Professional Problem information" handout to create a *professionally written* solution. This week, you should especially focus on:

Explanation: Justify your proofs or counterexamples using the definitions and theorems in Section 11.1. Don't just write "The sequence $a_n = \ldots$ is a counterexample". Explain *why* it is a counterexample. Your reward for reading this information carefully: none of the above statements are true.

Organization: Organize your computations and explanations clearly. Center important equations but leave less important ones inline. Use complete sentences.

Hint: Don't craft overly elaborate counterexamples. Any counterexamples in this problem can be constructed using fairly simple combinations of 0, 1, $(-1)^n$, $1/n$, etc.

To repeat: Because the sequences involved are not complicated, *make sure your writing and explanations are clear*, but also concise. Unless your handwriting is large, I'd expect your solution to fit on one page.

A counterexample to a statement "If p, then q" is an example where p is true, but q is not. For example, a counterexample to (a) would be a sequence $\{a_n\}$ for which "$\{|a_n|\}$ is convergent" is true, but "$\{a_n\}$ is convergent" is false.

You should have questions!

When you do, here's what to do:

1. Post your question on Canvas.
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor specific questions about it.

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