

The following written homework problems are due on 9/22 (Wednesday) or 9/23 (Thursday). *You also have a WebWork assignment due two days before the written homework.* **Tip:** Not all numbers will be “nice.” For some problems, you may need to use WolframAlpha (or other software) for the final calculations.

- (2.1) The turkey population in Downtown Minneapolis is getting out of control! Let $P(t)$ represent the turkey population after t weeks. The turkey population satisfies the differential equation

$$\frac{dP}{dt} = (0.5)P^{1.5}.$$

- (a) Suppose that there are initially 9 turkeys. Determine the solution to this initial value problem.
- (b) Show that there is a number, D (known as the *takeover number*), such that after D weeks, $P(t) \rightarrow \infty$. In other words, you want to find D such that $\lim_{t \rightarrow D^-} P(t) = \infty$.

- (2.2) Your dad makes you another cup of chai. At 10:00 AM, the chai is 98°C. The room is held at a constant temperature of 22°C. When you take your first sip of chai, it is 82°C and the cooling rate is 1°C per minute. What time did you take your first sip?

- (2.3) **Professional Problem Skills Practice.** Instead of a full professional problem this week, you are going to practice formatting long computations. We are going to consider the following mixing problem:

The country of Mathland currently has \$60 million in paper currency. Each day, the treasury department will add \$2 million worth of newly designed paper currency and remove \$2 million worth of **all** paper currency (new and old) at the same rate, so that the total amount of currency in circulation is always \$60 million. Then we can model this by $\frac{dy}{dt} = [\text{rate in}] - [\text{rate out}]$, as in the lecture notes to obtain:

$$\frac{dy}{dt} = 2 - \frac{y}{30}.$$

As we know that Mathland starts with only **old** currency, we have that $y(0) = 0$.

- (a) Solve the initial value problem.
- (b) How long will it take for the new bills to account for at least half of the currency in circulation?

Instructions: You do not need to write an introduction or re-write the full problem above. However, on the next page is correct scratch work for the problem. Format these computations neatly (and double-check your work, just in case!). You should briefly explain why the constants are changing, and give a brief explanation of a few sentences why the final answer is on the 21st day for part (b).

If you need a refresher on specific professional problem expectations, please read back through the [Professional Problem Checklist](#) on Canvas.

$$\frac{dy}{dt} = 2 - \frac{y}{30} = \frac{60-y}{30} \text{ is a separable DE.}$$

$$\text{Thus } \int \frac{30}{60-y} dy = \int dt$$

$$-30 \cdot \ln|60-y| + C = t + D$$

$$\ln|60-y| = -\frac{t}{30} + E$$

$$|60-y| = e^{-\frac{t}{30} + E} = F e^{-\frac{t}{30}}$$

$$60-y = G e^{-\frac{t}{30}}$$

$$\text{Finally } y = H e^{-\frac{t}{30}} + 60 \text{ is general sol}^n.$$

Since $t=0$, $y=0$, we can find H :

$$0 = 60 - H e^0 = 60 - H \Rightarrow H = 60, \text{ so}$$

$$\text{part. sol}^n \text{ is } 60 - 60e^{-t/30}$$

Solve for t if

$$30 \geq 60(1 - e^{-t/30})$$

$$\frac{1}{2} \geq 1 - e^{-t/30}$$

$$e^{-t/30} \geq \frac{1}{2}$$

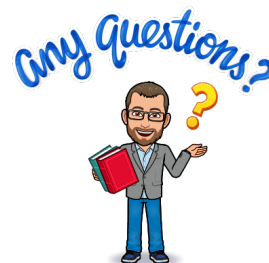
$$-t/30 \geq \log(1/2)$$

$$t \leq 30 \log(2)$$

$$\Rightarrow 21^{st} \text{ day}$$

You should have questions! When you do, here's what to do:

1. Post your question on Canvas: <http://canvas.umn.edu/>
The answers you get will help everyone in the class!
2. Email *all* of the instructors with your question.
3. Write your solution (even if you're unsure about it) and bring it to the study session. Ask an instructor *specific questions* about it.
4. Start an impromptu study session on Google Chat or similar service!



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