The Long and Winding Road

The goal of this worksheet is to find the value of $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

1. Find a closed form for $\sum_{n=1}^{\infty} nx^{n-1}$. When is it valid?
   "Closed form" means a simple function, not an infinite series.

2. Find a closed form for $\sum_{n=1}^{\infty} nx^{n}$. When is it valid?

3. Find $\sum_{n=1}^{\infty} \frac{n}{2^n}$. How do you know it converges?

4. Find a closed form for $\sum_{n=2}^{\infty} n(n-1)x^{n}$. When is it valid?

5. Find $\sum_{n=2}^{\infty} \frac{n^2-n}{2^n}$. How do you know it converges?

6. Finally, find $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. How do you know it converges? *Watch the indices carefully!*