With Great Power Series Comes Great Responsibility

Let $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$.

1. What is $f(0)$?

2. Explain why $f(x)$ is an even function.

3. Show $f''(x) = -f(x)$.

4. Use the Ratio Test to show that $f(x)$ is defined for all x.

5. Make a conjecture about which function is represented by this power series. What function do you know that matches all of these properties?