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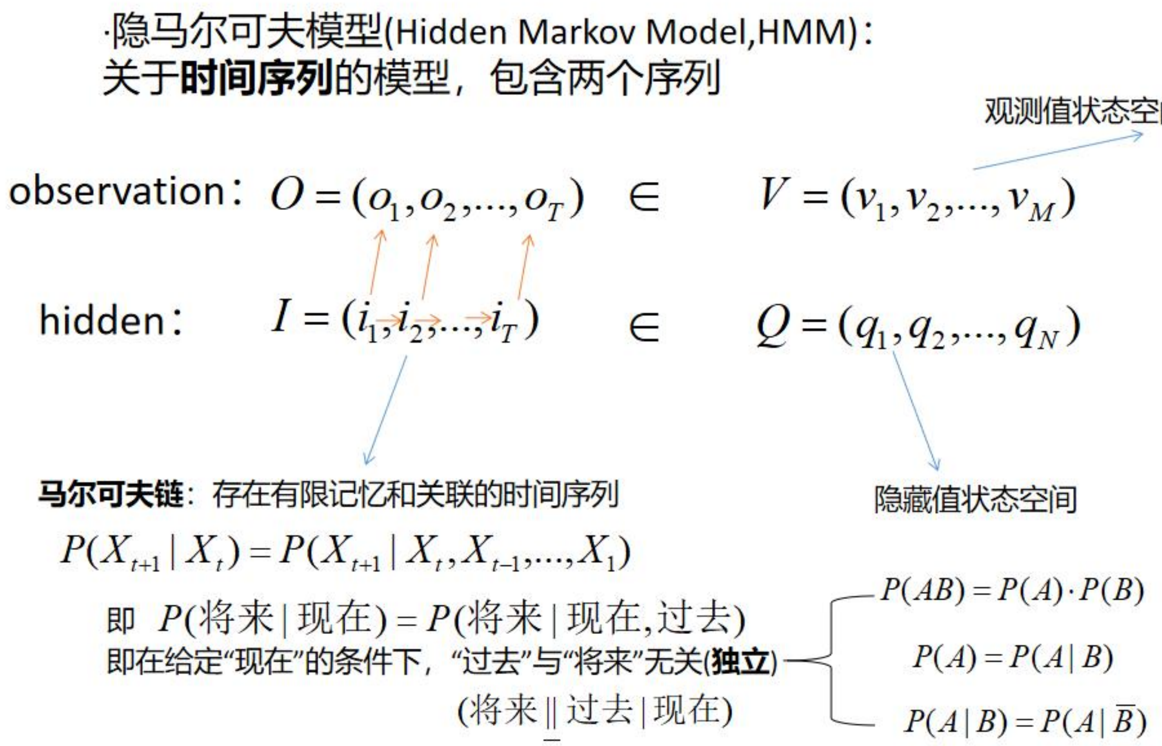
1 基本概念



2 隐马尔可夫模型的两个基本假设

1. 齐次马尔可夫性:

隐藏的马尔可夫链在**任意时刻**t的状态只依赖于其**前一时刻的状态**，与其他时刻的状态及观测无关，也与时刻t无关

(将来 || 过去 | 现在) 即在给定“现在”的条件下，“过去”与“将来”无关(**独立**)

$$P(i_t \mid i_{t-1}, o_{t-1}, \dots, i_1, o_1) = P(i_t \mid i_{t-1}), t = 1,2,\dots,T$$

2. 观测独立性:

即假设任意时刻的观测**只依赖于该时刻的马尔可夫链的状态**，与其他观测及状态无关

$$P(o_t \mid i_T, o_T, \dots, i_1, o_1) = P(o_t \mid i_t)$$

3 隐马尔可夫链三要素

隐马尔可夫模型 $\lambda = (\pi, A, B)$

初始概率分布

$$\pi = \pi_i$$

$$\pi_i = P(i_1 = q_i),$$
$$i = 1,2,\dots,N$$

即在时刻t=1时处于状态qi的概率

状态转移概率分布

$$A = [a_{ij}]_{N \times N}$$

$$a_{ij} = P(i_{t+1} = q_j \mid i_t = q_i),$$
$$i = 1,2,\dots,N,$$
$$j = 1,2,\dots,N$$

即在时刻t时处于状态qi的条件下在时刻t+1转移到状态qj的概率

矩阵都是行和为1

观测概率分布

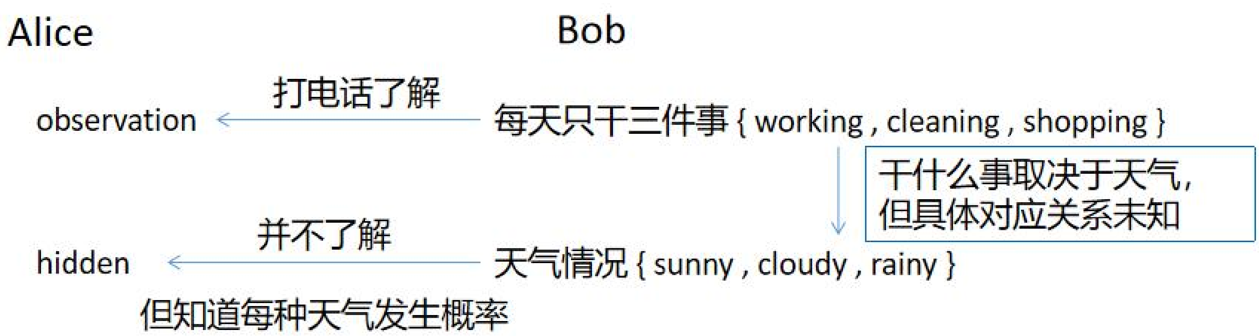
$$B = [b_j(k)]_{N \times M}$$

$$b_j(k) = P(o_t = v_k \mid i_t = q_j),$$
$$k = 1,2,\dots,M,$$
$$j = 1,2,\dots,N$$

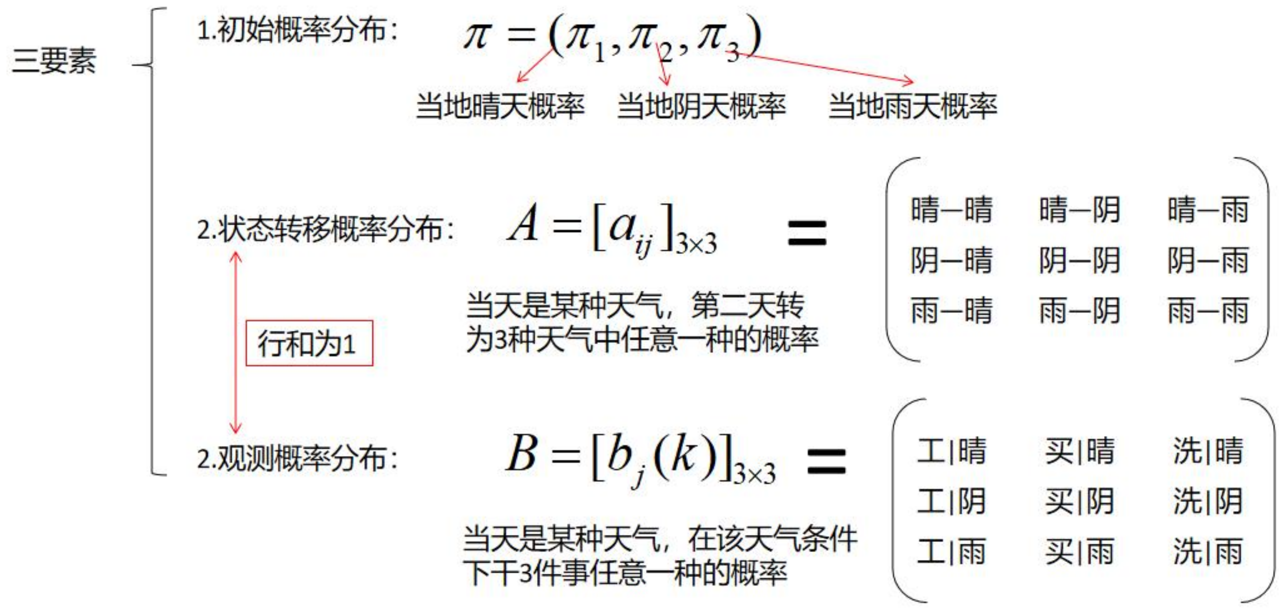
即在时刻t时处于状态qj的条件下生成观测vk的概率

- ## 3.1 实际问题理解三要素

　　Alice和Bob是好朋友，虽然住在两个城市，却还是每天打电话联系。Alice每天从Bob那了解到Bob当天干的事情，那Alice如何用Bob做的事来预测当天天气呢？



分析



4 隐马尔可夫链三个基本问题

1. 观测概率计算： $P(O|\lambda)$

2. 参数学习： $\hat{\lambda}_{MLE}=\arg\max_{\lambda} P(O|\lambda)$

3. 状态预测：解码 decoding

5 观测概率计算

- ## 5.1 直接计算法：（计算量大）

目标：$P(O|\lambda)$

$\lambda=(\pi,A,B)$ 离散整体的**边缘分布** $P(O)$

边缘分布用**联合分布**求出 $P(O)=\sum_I P(O,I)$

$$P(O,I|\lambda)=P(I|\lambda)P(O|I,\lambda)$$

$\lambda=(\pi,A,B)$ 离散整体的**联合分布** $P(O,I)=P(I)P(O|I)$

齐次马尔可夫性

$$\begin{aligned}P(I)&=P(i_1)P(i_2|i_1)P(i_3|i_2,i_1)\dots P(i_t|i_{t-1}\dots i_1)\\&=P(i_1)P(i_2|i_1)P(i_3|i_2)\dots P(i_t|i_{t-1})\\&=\pi_{i_1}\cdot a_{i_1i_2}a_{i_2i_3}\dots a_{i_{T-1}i_T}\end{aligned}$$

观测独立性

$$\begin{aligned}P(O|I)&=\prod_{t=1}^{T}P(o_t|i_t)\\&=b_{i_1}(o_1)b_{i_2}(o_2)\dots b_{i_T}(o_T)\end{aligned}$$

$$\boxed{P(O)=\sum_I P(O,I)=\sum_{i_1,\dots,i_T}\prod_{t=1}^{T}\pi_{i_t}a_{i_{t-1}}b_{i_t}(o_t)=\sum_{i_1,\dots,i_T}\pi_{i_1}b_{i_1}(o_1)a_{i_1i_2}b_{i_2}(o_2)\dots a_{i_{T-1}i_T}b_{i_T}(o_T)}$$

- ## 5.2 前向算法

前向概率 $\alpha_t(i)=P(o_1,o_2,\dots,o_t,i_t=q_t|\lambda)$

t时刻隐变量的状态

t=1时， $\alpha_1(i)=P(o_1,i_1=q_i|\lambda)=\pi_i b_i(o_1)$

t=1,2,3..,t-1 ,**递推**

$$\begin{aligned}&\alpha_t(i)\\&=P(o_1,o_2,\dots,o_t,o_{t+1},i_{t+1}=q_i|\lambda)\\&=P(o_1,o_2,\dots,o_t,i_t=q_j,o_{t+1},i_{t+1}=q_i|\lambda)\\&=\sum_{j=1}^{N}P(o_1,\dots,o_t,i_t=q_j)P(o_{t+1},i_{t+1}=q_i|o_1,\dots o_t,i_t)\\&=[\sum_{j=1}^{N}\alpha_t(j)a_{ji}]b_i(o_{t+1})\end{aligned}$$

终止条件， $P(O|\lambda)=\sum_{i=1}^{N}\alpha_T(i)$

6 参数学习

EM 算法

7 状态预测

维特比算法（动态规划）

8 李航习题10.1和python实现

给定盒子和球组成的隐马尔可夫模型$\lambda=(A,B,\pi)$，其中，

$$A=\left[\begin{array}{ccc}0.5 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.3 & 0.5\end{array}\right], \quad B=\left[\begin{array}{cc}0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.7 & 0.3\end{array}\right], \quad \pi=(0.2,0.4,0.4)^{T}$$

设$T=4, O=(\text{红}, \text{白}, \text{红}, \text{白})$，试用后向算法计算$P(O \mid \lambda)$。

In [1]:

```python
import numpy as np

class HiddenMarkov:
    def __init__(self):
        self.alphas = None
        self.forward_P = None
        self.betas = None
        self.backward_P = None

    # 前向算法
    def forward(self, Q, V, A, B, O, PI):
        # 状态序列的大小
        N = len(Q)
        # 观测序列的大小
        M = len(O)
        # 初始化前向概率alpha值
        alphas = np.zeros((N, M))
        # 时刻数=观测序列数
        T = M
        # 遍历每一个时刻，计算前向概率alpha值
        for t in range(T):
            # 得到序列对应的索引
            indexOfO = V.index(O[t])
            # 遍历状态序列
            for i in range(N):
                # 初始化alpha初值
                if t == 0:
                    # P176 公式(10.15)
                    alphas[i][t] = PI[t][i] * B[i][indexOfO]
                    print('alpha1(%d) = p%db%db(o1) = %f' %
                          (i + 1, i, i, alphas[i][t]))
                else:
                    # P176 公式(10.16)
                    alphas[i][t] = np.dot([alpha[t - 1] for alpha in alphas],
                                          [a[i] for a in A]) * B[i][indexOfO]
                    print('alpha%d(%d) = [sigma alpha%d(i)ai%d]b%d(o%d) = %f' %
                          (t + 1, i + 1, t - 1, i, i, t, alphas[i][t]))
        # P176 公式(10.17)
        self.forward_P = np.sum([alpha[M - 1] for alpha in alphas])
        self.alphas = alphas

    # 后向算法
    def backward(self, Q, V, A, B, O, PI):
        # 状态序列的大小
        N = len(Q)
        # 观测序列的大小
        M = len(O)
        # 初始化后向概率beta值，P178 公式(10.19)
        betas = np.ones((N, M))
        #
        for i in range(N):
            print('beta%d(%d) = 1' % (M, i + 1))
        # 对观测序列逆向遍历
        for t in range(M - 2, -1, -1):
            # 得到序列对应的索引
            indexOfO = V.index(O[t + 1])
            # 遍历状态序列
            for i in range(N):
                # P178 公式(10.20)
```

```
                betas[i][t] = np.dot(
                    np.multiply(A[i], [b[indexOfO] for b in B]),
                    [beta[t + 1] for beta in betas])
                realT = t + 1
                realI = i + 1
                print('beta%d(%d) = sigma[a%djbj(o%d)beta%d(j)] = (' %
                      (realT, realI, realI, realT + 1, realT + 1),
                      end='')
                for j in range(N):
                    print("%.2f * %.2f * %.2f + " %
                          (A[i][j], B[j][indexOfO], betas[j][t + 1]),
                          end='')
                print("0) = %.3f" % betas[i][t])
        # 取出第一个值
        indexOfO = V.index(O[0])
        self.betas = betas
        # P178 公式(10.21)
        P = np.dot(np.multiply(PI, [b[indexOfO] for b in B]),
                   [beta[0] for beta in betas])
        self.backward_P = P
        print("P(O|lambda) = ", end="")
        for i in range(N):
            print("%.1f * %.1f * %.5f + " %
                  (PI[0][i], B[i][indexOfO], betas[i][0]),
                  end="")
        print("O = %f" % P)

    # 维特比算法
    def viterbi(self, Q, V, A, B, O, PI):
        # 状态序列的大小
        N = len(Q)
        # 观测序列的大小
        M = len(O)
        # 初始化daltas
        deltas = np.zeros((N, M))
        # 初始化psis
        psis = np.zeros((N, M))
        # 初始化最优路径矩阵，该矩阵维度与观测序列维度相同
        I = np.zeros((1, M))
        # 遍历观测序列
        for t in range(M):
            # 递推从t=2开始
            realT = t + 1
            # 得到序列对应的索引
            indexOfO = V.index(O[t])
            for i in range(N):
                realI = i + 1
                if t == 0:
                    # P185 算法10.5 步骤(1)
                    deltas[i][t] = PI[0][i] * B[i][indexOfO]
                    psis[i][t] = 0
                    print('delta1(%d) = pi%d * b%d(o1) = %.2f * %.2f = %.2f' %
                          (realI, realI, realI, PI[0][i], B[i][indexOfO],
                           deltas[i][t]))
                    print('psis1(%d) = 0' % (realI))
                else:
                    # # P185 算法10.5 步骤(2)
                    deltas[i][t] = np.max(
                        np.multiply([delta[t - 1] for delta in deltas],
                                    [a[i] for a in A])) * B[i][indexOfO]
                    print(
```

```
                        'delta%d(%d) = max[delta%d(j)aj%d]b%d(o%d) = %.2f * %.2f = %.5f'
                        % (realT, realI, realT - 1, realI, realI, realT,
                           np.max(
                               np.multiply([delta[t - 1] for delta in deltas],
                                           [a[i] for a in A])), B[i][indexOfO],
                           deltas[i][t]))
                    psis[i][t] = np.argmax(
                        np.multiply([delta[t - 1] for delta in deltas],
                                    [a[i] for a in A]))
                    print('psis%d(%d) = argmax[delta%d(j)aj%d] = %d' %
                          (realT, realI, realT - 1, realI, psis[i][t]))
        #print(deltas)
        #print(psis)
        # 得到最优路径的终结点
        I[0][M - 1] = np.argmax([delta[M - 1] for delta in deltas])
        print('i%d = argmax[deltaT(i)] = %d' % (M, I[0][M - 1] + 1))
        # 递归由后向前得到其他结点
        for t in range(M - 2, -1, -1):
            I[0][t] = psis[int(I[0][t + 1])][t + 1]
            print('i%d = psis%d(i%d) = %d' %
                  (t + 1, t + 2, t + 2, I[0][t] + 1))
        # 输出最优路径
        print('最优路径是：', "->".join([str(int(i + 1)) for i in I[0]]))
```

In [2]:

```
#习题10.1
Q = [1, 2, 3]
V = ['红', '白']
A = [[0.5, 0.2, 0.3], [0.3, 0.5, 0.2], [0.2, 0.3, 0.5]]
B = [[0.5, 0.5], [0.4, 0.6], [0.7, 0.3]]
# O = ['红', '白', '红', '红', '白', '红', '白', '白']
O = ['红', '白', '红', '白']    #习题10.1的例子
PI = [[0.2, 0.4, 0.4]]

HMM = HiddenMarkov()
# HMM.forward(Q, V, A, B, O, PI)
# HMM.backward(Q, V, A, B, O, PI)
HMM.viterbi(Q, V, A, B, O, PI)
```

```
delta1(1) = pi1 * b1(o1) = 0.20 * 0.50 = 0.10
psis1(1) = 0
delta1(2) = pi2 * b2(o1) = 0.40 * 0.40 = 0.16
psis1(2) = 0
delta1(3) = pi3 * b3(o1) = 0.40 * 0.70 = 0.28
psis1(3) = 0
delta2(1) = max[delta1(j)aj1]b1(o2) = 0.06 * 0.50 = 0.02800
psis2(1) = argmax[delta1(j)aj1] = 2
delta2(2) = max[delta1(j)aj2]b2(o2) = 0.08 * 0.60 = 0.05040
psis2(2) = argmax[delta1(j)aj2] = 2
delta2(3) = max[delta1(j)aj3]b3(o2) = 0.14 * 0.30 = 0.04200
psis2(3) = argmax[delta1(j)aj3] = 2
delta3(1) = max[delta2(j)aj1]b1(o3) = 0.02 * 0.50 = 0.00756
psis3(1) = argmax[delta2(j)aj1] = 1
delta3(2) = max[delta2(j)aj2]b2(o3) = 0.03 * 0.40 = 0.01008
psis3(2) = argmax[delta2(j)aj2] = 1
delta3(3) = max[delta2(j)aj3]b3(o3) = 0.02 * 0.70 = 0.01470
psis3(3) = argmax[delta2(j)aj3] = 2
delta4(1) = max[delta3(j)aj1]b1(o4) = 0.00 * 0.50 = 0.00189
psis4(1) = argmax[delta3(j)aj1] = 0
delta4(2) = max[delta3(j)aj2]b2(o4) = 0.01 * 0.60 = 0.00302
psis4(2) = argmax[delta3(j)aj2] = 1
delta4(3) = max[delta3(j)aj3]b3(o4) = 0.01 * 0.30 = 0.00220
psis4(3) = argmax[delta3(j)aj3] = 2
i4 = argmax[deltaT(i)] = 2
i3 = psis4(i4) = 2
i2 = psis3(i3) = 2
i1 = psis2(i2) = 3
最优路径是： 3->2->2->2
```

9 李航习题10.2和python实现（自编HMM函数同上）

给定盒子和球组成的隐马尔可夫模型$\lambda = (A, B, \pi)$，其中，

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}, \quad \pi = (0.2, 0.3, 0.5)^T$$

设$T = 8, O = (红, 白, 红, 红, 白, 红, 白, 白)$，试用前向后向概率计算$P(i_4 = q_3 | O, \lambda)$

In [3]:

```python
Q = [1, 2, 3]
V = ['红', '白']
A = [[0.5, 0.2, 0.3], [0.3, 0.5, 0.2], [0.2, 0.3, 0.5]]
B = [[0.5, 0.5], [0.4, 0.6], [0.7, 0.3]]
O = ['红', '白', '红', '红', '白', '红', '白', '白']
PI = [[0.2, 0.3, 0.5]]

HMM.forward(Q, V, A, B, O, PI)
HMM.backward(Q, V, A, B, O, PI)
```

```
alpha1(1) = p0b0b(o1) = 0.100000
alpha1(2) = p1b1b(o1) = 0.120000
alpha1(3) = p2b2b(o1) = 0.350000
alpha2(1) = [sigma alpha0(i)ai0]b0(o1) = 0.078000
alpha2(2) = [sigma alpha0(i)ai1]b1(o1) = 0.111000
alpha2(3) = [sigma alpha0(i)ai2]b2(o1) = 0.068700
alpha3(1) = [sigma alpha1(i)ai0]b0(o2) = 0.043020
alpha3(2) = [sigma alpha1(i)ai1]b1(o2) = 0.036684
alpha3(3) = [sigma alpha1(i)ai2]b2(o2) = 0.055965
alpha4(1) = [sigma alpha2(i)ai0]b0(o3) = 0.021854
alpha4(2) = [sigma alpha2(i)ai1]b1(o3) = 0.017494
alpha4(3) = [sigma alpha2(i)ai2]b2(o3) = 0.033758
alpha5(1) = [sigma alpha3(i)ai0]b0(o4) = 0.011463
alpha5(2) = [sigma alpha3(i)ai1]b1(o4) = 0.013947
alpha5(3) = [sigma alpha3(i)ai2]b2(o4) = 0.008080
alpha6(1) = [sigma alpha4(i)ai0]b0(o5) = 0.005766
alpha6(2) = [sigma alpha4(i)ai1]b1(o5) = 0.004676
alpha6(3) = [sigma alpha4(i)ai2]b2(o5) = 0.007188
alpha7(1) = [sigma alpha5(i)ai0]b0(o6) = 0.002862
alpha7(2) = [sigma alpha5(i)ai1]b1(o6) = 0.003389
alpha7(3) = [sigma alpha5(i)ai2]b2(o6) = 0.001878
alpha8(1) = [sigma alpha6(i)ai0]b0(o7) = 0.001411
alpha8(2) = [sigma alpha6(i)ai1]b1(o7) = 0.001698
alpha8(3) = [sigma alpha6(i)ai2]b2(o7) = 0.000743
beta8(1) = 1
beta8(2) = 1
beta8(3) = 1
beta7(1) = sigma[a1jbj(o8)beta8(j)] = (0.50 * 0.50 * 1.00 + 0.20 * 0.60 * 1.00 + 0.3
0 * 0.30 * 1.00 + 0) = 0.460
beta7(2) = sigma[a2jbj(o8)beta8(j)] = (0.30 * 0.50 * 1.00 + 0.50 * 0.60 * 1.00 + 0.2
0 * 0.30 * 1.00 + 0) = 0.510
beta7(3) = sigma[a3jbj(o8)beta8(j)] = (0.20 * 0.50 * 1.00 + 0.30 * 0.60 * 1.00 + 0.5
0 * 0.30 * 1.00 + 0) = 0.430
beta6(1) = sigma[a1jbj(o7)beta7(j)] = (0.50 * 0.50 * 0.46 + 0.20 * 0.60 * 0.51 + 0.3
0 * 0.30 * 0.43 + 0) = 0.215
beta6(2) = sigma[a2jbj(o7)beta7(j)] = (0.30 * 0.50 * 0.46 + 0.50 * 0.60 * 0.51 + 0.2
0 * 0.30 * 0.43 + 0) = 0.248
beta6(3) = sigma[a3jbj(o7)beta7(j)] = (0.20 * 0.50 * 0.46 + 0.30 * 0.60 * 0.51 + 0.5
0 * 0.30 * 0.43 + 0) = 0.202
beta5(1) = sigma[a1jbj(o6)beta6(j)] = (0.50 * 0.50 * 0.21 + 0.20 * 0.40 * 0.25 + 0.3
0 * 0.70 * 0.20 + 0) = 0.116
beta5(2) = sigma[a2jbj(o6)beta6(j)] = (0.30 * 0.50 * 0.21 + 0.50 * 0.40 * 0.25 + 0.2
0 * 0.70 * 0.20 + 0) = 0.110
beta5(3) = sigma[a3jbj(o6)beta6(j)] = (0.20 * 0.50 * 0.21 + 0.30 * 0.40 * 0.25 + 0.5
0 * 0.70 * 0.20 + 0) = 0.122
beta4(1) = sigma[a1jbj(o5)beta5(j)] = (0.50 * 0.50 * 0.12 + 0.20 * 0.60 * 0.11 + 0.3
0 * 0.30 * 0.12 + 0) = 0.053
beta4(2) = sigma[a2jbj(o5)beta5(j)] = (0.30 * 0.50 * 0.12 + 0.50 * 0.60 * 0.11 + 0.2
```

```
0 * 0.30 * 0.12 + 0) = 0.058
beta4(3) = sigma[a3jbj(o5)beta5(j)] = (0.20 * 0.50 * 0.12 + 0.30 * 0.60 * 0.11 + 0.5
0 * 0.30 * 0.12 + 0) = 0.050
beta3(1) = sigma[a1jbj(o4)beta4(j)] = (0.50 * 0.50 * 0.05 + 0.20 * 0.40 * 0.06 + 0.3
0 * 0.70 * 0.05 + 0) = 0.028
beta3(2) = sigma[a2jbj(o4)beta4(j)] = (0.30 * 0.50 * 0.05 + 0.50 * 0.40 * 0.06 + 0.2
0 * 0.70 * 0.05 + 0) = 0.026
beta3(3) = sigma[a3jbj(o4)beta4(j)] = (0.20 * 0.50 * 0.05 + 0.30 * 0.40 * 0.06 + 0.5
0 * 0.70 * 0.05 + 0) = 0.030
beta2(1) = sigma[a1jbj(o3)beta3(j)] = (0.50 * 0.50 * 0.03 + 0.20 * 0.40 * 0.03 + 0.3
0 * 0.70 * 0.03 + 0) = 0.015
beta2(2) = sigma[a2jbj(o3)beta3(j)] = (0.30 * 0.50 * 0.03 + 0.50 * 0.40 * 0.03 + 0.2
0 * 0.70 * 0.03 + 0) = 0.014
beta2(3) = sigma[a3jbj(o3)beta3(j)] = (0.20 * 0.50 * 0.03 + 0.30 * 0.40 * 0.03 + 0.5
0 * 0.70 * 0.03 + 0) = 0.016
beta1(1) = sigma[a1jbj(o2)beta2(j)] = (0.50 * 0.50 * 0.02 + 0.20 * 0.60 * 0.01 + 0.3
0 * 0.30 * 0.02 + 0) = 0.007
beta1(2) = sigma[a2jbj(o2)beta2(j)] = (0.30 * 0.50 * 0.02 + 0.50 * 0.60 * 0.01 + 0.2
0 * 0.30 * 0.02 + 0) = 0.007
beta1(3) = sigma[a3jbj(o2)beta2(j)] = (0.20 * 0.50 * 0.02 + 0.30 * 0.60 * 0.01 + 0.5
0 * 0.30 * 0.02 + 0) = 0.006
P(O|lambda) = 0.2 * 0.5 * 0.00698 + 0.3 * 0.4 * 0.00741 + 0.5 * 0.7 * 0.00647 + 0 =
0.003852
```

可知，$P(i_4=q_3|O,\lambda)=\dfrac{P(i_4=q_3,O|\lambda)}{P(O|\lambda)}=\dfrac{\alpha_4(3)\beta_4(3)}{P(O|\lambda)}$

In [4]:

```python
print("alpha4(3)=", HMM.alphas[3 - 1][4 - 1])
print("beta4(3)=", HMM.betas[3 - 1][4 - 1])
print("P(O|lambda)=", HMM.backward_P[0])
result = (HMM.alphas[3 - 1][4 - 1] *
          HMM.betas[3 - 1][4 - 1]) / HMM.backward_P[0]
print("P(i4=q3|O,lambda) =", result)
```

```
alpha4(3)= 0.033757709999999996
beta4(3)= 0.049728909999999994
P(O|lambda)= 0.0038519735794910986
P(i4=q3|O,lambda) = 0.4358114321796269
```

10 李航习题10.3和python实现

在习题10.1中，试用维特比算法求最优路径$I^*=(i_1^*,i_2^*,i_3^*,i_4^*)$。

In [5]:

```
Q = [1, 2, 3]
V = ['红', '白']
A = [[0.5, 0.2, 0.3], [0.3, 0.5, 0.2], [0.2, 0.3, 0.5]]
B = [[0.5, 0.5], [0.4, 0.6], [0.7, 0.3]]
O = ['红', '白', '红', '白']
PI = [[0.2, 0.4, 0.4]]

HMM = HiddenMarkov()
HMM.viterbi(Q, V, A, B, O, PI)
```

```
delta1(1) = pi1 * b1(o1) = 0.20 * 0.50 = 0.10
psis1(1) = 0
delta1(2) = pi2 * b2(o1) = 0.40 * 0.40 = 0.16
psis1(2) = 0
delta1(3) = pi3 * b3(o1) = 0.40 * 0.70 = 0.28
psis1(3) = 0
delta2(1) = max[delta1(j)aj1]b1(o2) = 0.06 * 0.50 = 0.02800
psis2(1) = argmax[delta1(j)aj1] = 2
delta2(2) = max[delta1(j)aj2]b2(o2) = 0.08 * 0.60 = 0.05040
psis2(2) = argmax[delta1(j)aj2] = 2
delta2(3) = max[delta1(j)aj3]b3(o2) = 0.14 * 0.30 = 0.04200
psis2(3) = argmax[delta1(j)aj3] = 2
delta3(1) = max[delta2(j)aj1]b1(o3) = 0.02 * 0.50 = 0.00756
psis3(1) = argmax[delta2(j)aj1] = 1
delta3(2) = max[delta2(j)aj2]b2(o3) = 0.03 * 0.40 = 0.01008
psis3(2) = argmax[delta2(j)aj2] = 1
delta3(3) = max[delta2(j)aj3]b3(o3) = 0.02 * 0.70 = 0.01470
psis3(3) = argmax[delta2(j)aj3] = 2
delta4(1) = max[delta3(j)aj1]b1(o4) = 0.00 * 0.50 = 0.00189
psis4(1) = argmax[delta3(j)aj1] = 0
delta4(2) = max[delta3(j)aj2]b2(o4) = 0.01 * 0.60 = 0.00302
psis4(2) = argmax[delta3(j)aj2] = 1
delta4(3) = max[delta3(j)aj3]b3(o4) = 0.01 * 0.30 = 0.00220
psis4(3) = argmax[delta3(j)aj3] = 2
i4 = argmax[deltaT(i)] = 2
i3 = psis4(i4) = 2
i2 = psis3(i3) = 2
i1 = psis2(i2) = 3
最优路径是： 3->2->2->2
```

In [ ]:

```

```