EXCELLENCE AND GODLINESS

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ASSIGNMENT PROBLEM

STA 322 (OPERATIONS RESEARCH) LECTURE NOTE

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LEARNING OUTCOMES

At the end of this class, students should be able to:

- ✓ Understand the concept of Assignment Problem.
- ✓ Understand the need for an assignment problem.
- ✓ List out the various methods of solving an assignment problem.
- ✓ Effectively use the Hungarian method to solve an assignment problem.
- ✓ Solve a maximization type of assignment problem.

INTRODUCTION TO ASSIGNMENT PROBLEMS

WHAT IS ASSIGNMENT PROBLEM ?

- Assignment problem refers to special class of linear programming problems that involves determining the most efficient assignment of people to projects, salespeople to territories, contracts to bidders and so on.

- It is often used to minimize total cost or time of performing task.

- One important characteristic of assignment problems is that only one job (or worker) is assigned to one machine (or project).

INTRODUCTION TO ASSIGNMENT PROBLEMS CONT'D

- **Each Assignment problem has a Matrix associated with it.**
- **The number in the table indicates COST associated with the assignment.**
- **The most efficient linear programming algorithm to find optimum solution to an assignment problem is Hungarian Method (It is also known as Flood's Technique)**

NEED FOR ASSIGNMENT PROBLEM

The assignment problem is actually a special case of transportation problem where $m = n$ and $a_i = b_j = 1$. However, it may be easily noted that any basic feasible solution of an assignment problem contain $2n - 1$ variables of which $n - 1$ variables are zero.

If there are n facilities and n jobs, there are **n! possible assignments**. The simplest way of finding an optimum assignment is to write all the n! Possible arrangements evaluate their total cost and select the assignment with the minimum cost. But that method leads to a calculation problem of formidable size even when the value of n is moderate e.g for n=10, the possible number of arrangement is 3268800, therefore the need for assignment problem.

STRUCTURE OF AN ASSIGNMENT PROBLEM

- The structure of the assignment problem is similar to a transportation problem as follows:

	Jobs					
Workers		1	2	………	n	
	1	C_{11}	C_{12}	……….	C_{1n}	1
	2	C_{21}	C_{22}	……….	C_{2n}	1
	. . .	. . .	. . .	. . .	. . .	. . .
	m	C_{m1}	C_{m1}	………	C_{mn}	1
	1	1	1	……….	1	

METHODS OF SOLVING ASSIGNMENT PROBLEMS

- A) complete enumeration method
- B) Transportation method
- C) Simplex method
- D) Hungarian assignment method

ASSIGNMENT PROBLEM USING HUNGARIAN METHOD

- The Hungarian method is a combinatorial optimization algorithm that solves the assignment problem in polynomial time and which anticipated later primal-dual methods.

- It was developed and published in 1955 by Harold Kuhn, who gave the name "Hungarian method" because the algorithm was largely based on the earlier works of two Hungarian mathematicians: Denes Konig and Jeno Egervary.

HUNGARIAN METHOD CONT'D

An efficient method for solving an assignment problem, known as Hungarian method of assignment based on the following properties

1. In an assignment problem, if a constant quantity is added or subtracted from every element of any row or column in the given cost matrix, an assignment that minimizes the total cost in the matrix also minimize the total cost in the other.

2. In an assignment problem, a solution having zero total cost is an optimum solution. It can be summarized as follows.

***Step 1*:-** In the given matrix, subtract the smallest element in each row from every element of the row.

***Step 2*:-** Reduced matrix obtained from step1, subtract the smallest element in each column from every element of that column.

***Step 3*:-** Make the assignment for the reduced matrix obtained from steps 1 and 2.

Continued ....

HUNGARIAN METHOD CONT'D

***Step 4*:-** Draw the minimum number of horizontal and vertical lines to cover all zeros in the reduced matrix obtained from 3 in the following way.

a) Mark (√) all rows that do not have assignments.

b) Mark (√) all column that have zeros in marked rows (step 4 (a)).

c) Mark (√) all rows that have assignments in marked columns (step 4 (b))

d) Repeat steps 4 (a) to 4 (c) until no more rows or columns can be marked.

e) Draw straight lines through all unmarked rows and marked columns.

***Step 5*:-** If the number of lines drawn (step 4 (c) are equal to the number of columns or rows, then it is an optimal solution, otherwise go to step 6.

***Step 6*:-** Select the smallest elements among all the uncovered elements.

***Step 7*:-** Go to step 3 and repeat the procedure until the number of assignments become equal to the number of rows or column.

EXAMPLE 1:

A workshop contains four persons available for work on four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

	Jobs				
Persons		1	2	3	4
	A	20	25	22	28
	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

SOLUTION

Step 1: Subtract the least cost in each row from the other cost in that same row. In row 1, the least cost is 20, Row 2, 3 and 4 has 15, 17 and 23 as their least cost respectively. This will form the first reduced cost (FRC) table.

Persons	**Jobs**				
		1	2	3	4
	A	20-20=0	25-20=5	22-20=2	28-20=8
	B	15-15=0	18-15=3	23-15=8	17-15=2
	C	19-17=2	17-17=0	21-17=4	24-17=7
	D	25-23=2	23-23=0	24-23=1	24-23=1

Solution cont'd

- Step 2: Find the least in each column cost in the first reduced cost table and subtract it from all the elements of that particular column. This will form the second reduced cost (SRC) table.

	1	2	3	4
A	0-0=0	5-0=5	2-1=1	8-1=7
B	0-0=0	3-0=3	8-1=7	2-1=1
C	2-0=2	0-0=0	4-1=3	7-1=6
D	2-0=2	0-0=0	1-1=0	1-1=0

Remark: Step 1 and 2 creates at least one Zero '0' in each row and Column.

Solution cont'd

- Step 3: We check the row and box the first zero and cross out the other zero in the column and row.

	1	2	3	4
A	$\boxed{0}$	5	1	7
B	~~0~~	3	7	1
C	2	$\boxed{0}$	3	6
D	2	~~0~~	$\boxed{0}$	~~0~~

SO, PERSON A IS ASSIGNED TO JOB I, C TO JOB 2 AND D TO JOB 3. HERE A FEASIBLE ASSIGNMENT IS NOT POSSIBLE BECAUSE IT IS NOT COMPLETE(PERSON B DOES NOT HAVA A JOB). WE THEN APPLY THE RULE IN STEP VI TO THE ASSIGNMENT PROBLEM,

Solution cont'd

Because the assignment is not feasible, apply the rules in step 4 to the assignment problem.

Step 4:

- ✓ First, mark or check(✓) for the rows where assignment has not been made(**where there are no boxed zeros**) as seen below:

	1	2	3	4	
A	$\boxed{0}$	5	1	7	
B	$\cancel{0}$	3	7	1	✓
C	2	$\boxed{0}$	3	6	
D	2	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	

Solution cont'd

- ✓ Also, mark or check(✓) for the columns that has zero in the marked row(**the zero that is crossed in the checked row**). In this case, column 1 contain that crossed zero in the checked row

	1	2	3	4
A	$\boxed{0}$	5	1	7
B	$\cancel{0}$	3	7	1 ✓
C	2	$\boxed{0}$	3	6
D	2 ✓	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

Solution cont'd

- ✓ Then, mark or check(✓) for the row that has an assigned zero in the marked column(**the zero that is boxed in the checked column**). In this case, row I contain that boxed zero in the checked column

	1	2	3	4
A	$\boxed{0}$	5	1	7 ✓
B	0	3	7	1 ✓
C	2	$\boxed{0}$	3	6
D	2 ✓	0	$\boxed{0}$	0

Solution cont'd

- Draw lines through all unmarked row and marked columns. In this case, **row 3 and 4** are not marked, draw line through them . In the same way, **column 1** is marked, so, draw line through it.

	1	2	3	4	
A	$\boxed{0}$	5	1	7	✓
B	0	3	7	1	✓
C	2	$\boxed{0}$	3	6	
D	2	0	$\boxed{0}$	0	
	✓				

Solution cont'd

Now we select the smallest cost out of the ones that are not covered by the lines, in this case it is 1. We do two things with this selected number:

i. Subtract the number from all the elements which are not covered by the lines (as shown in the cost highlighted in red);

ii. Add the number to all those cost at the intersection of the lines drawn through the columns and rows (as shown in the cost highlighted in blue). This gives the following reduced cost matrix:

	1	2	3	4	
A	0	5-1=4	1-1=0	7-1=6	✓
B	0	3-1=2	7-1=6	1-1=0	✓
C	2+1=3	0	3	6	
D	2+1=3	0	0	0	
	✓				

Solution cont'd

Assignment is then done again as in step I

	1	2	3	4
A	$\boxed{0}$	4	0	6
B	0	2	6	$\boxed{0}$
C	3	$\boxed{0}$	3	6
D	3	0	$\boxed{0}$	0

A feasible assignment is now possible and an optimal solution is achieved. In case a feasible solution is not yet possible, one has to repeat the step of drawing lines to cover the zeros and continue until a feasible assignment is obtained.

OPTIMAL SOLUTION TO THE ASSIGNMENT PROBLEM

Assignment to Job	**Cost of assigning person to Job**
Person A to Job 1	20
Person B to Job 4	17
Person C to Job 2	17
Person D to Job 3	24
	Minimum Total Cost= ₦78

MAXIMIZATION PROBLEM

Maximization is just a step different from minimization in assignment problem just like in transportation problem. Here, the cost are minimized before obtaining the FRC and SRC. The Least cost is subtracted from the entire cost including itself.

Example 2: Consider the problem of five different machines which can do any of the required five jobs with different profits resulting from each assignment

	Jobs					
Machines		1	2	3	4	5
	1	104	107	94	106	94
	2	94	110	107	113	98
	3	94	102	101	104	99
	4	109	96	94	98	98
	5	106	72	93	93	95

SOLUTION

- Find out the minimum profit (in this case, the minimum profit is 72) and subtract it from all other profit as shown below:

Machines	Jobs				
		1	2	3	4	5
	1	32	35	22	34	22
	2	22	38	35	41	26
	3	22	30	29	32	27
	4	37	24	22	26	26
	5	34	0	21	21	23

The Hungarian method is then used to obtain the optimal solution.

DRAWBACK OF ASSIGNMENT PROBLEM

- Assignment becomes a problem because each job requires different skills and the capacity or efficiency of each person with respect to these jobs can be different. This gives rise to cost differences. If each person is able to do all jobs equally efficiently then all costs will be the same and each job can be assigned to any person.
- When assignment is a problem it becomes a typical optimization problem it can therefore be compared to a transportation problem. The cost elements are given and is a square matrix and requirement at each destination is one and availability at each origin is also one.
- In addition we have number of origins which equals the number of destinations hence the total demand equals total supply . There is only one assignment in each row and each column .However If we compare this to a transportation problem we find that a general transportation problem does not have the above mentioned limitations. These limitations are peculiar to assignment problem only.

Summary

Assignment problem is a special case of transportation problem. It deals with allocating the various items to various activities on a one to one basis in such a way that the resultant effectiveness is optimised. In this unit, we have solved assignment problem using Hungarian Method. We have also disscussed the special cases in assignment problems.

REFERENCES

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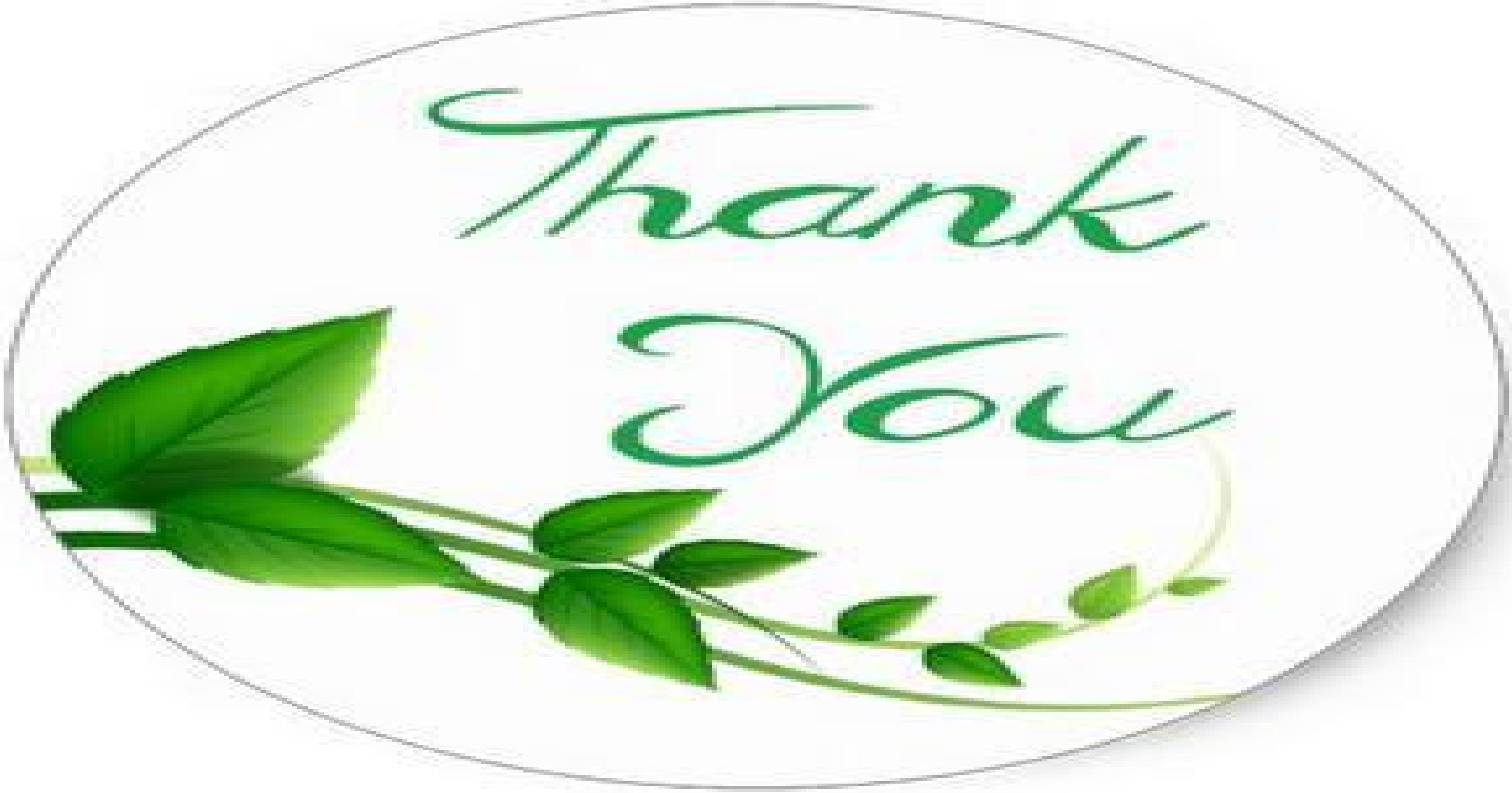
Thank You