B-SPLINES-BASED SOLUTIONS OF DIFFERENTIAL EQUATIONS IN BIOENGINEERING: MOTION MONITORING AND IMAGE PROCESSING APPLICATIONS

Bachelor’s thesis

Degree in Biomedical Engineering

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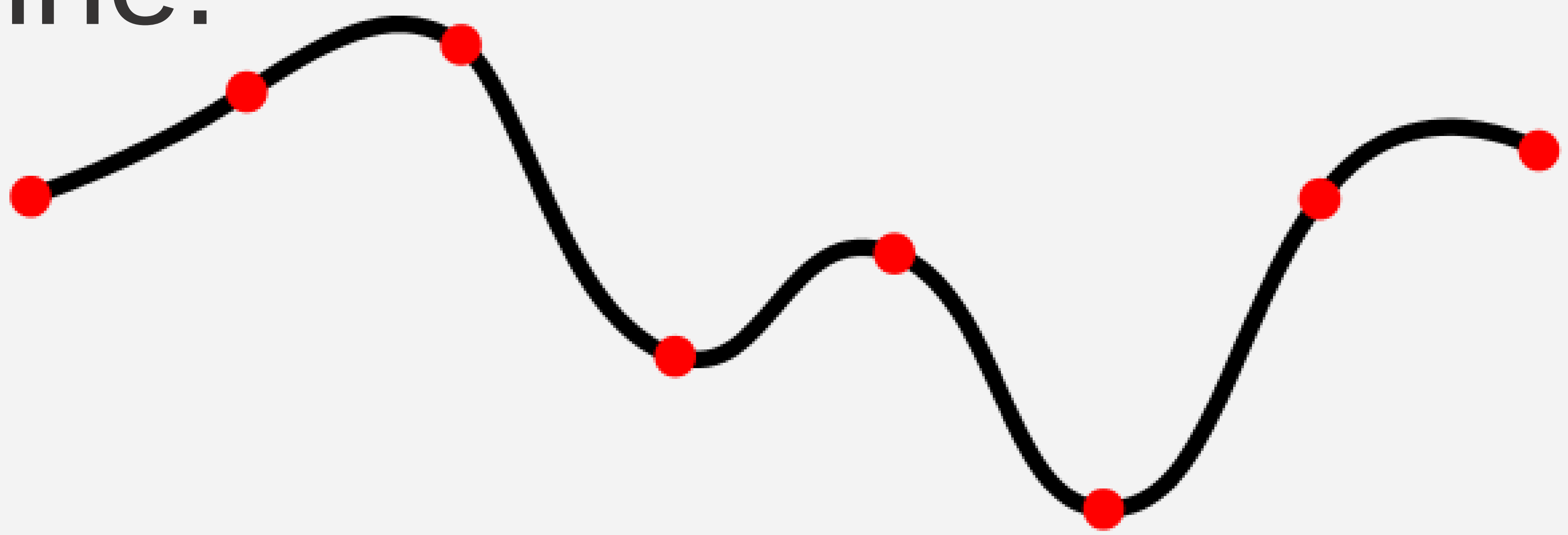
Introduction

Framework to provide solutions to differential equation in the field of bioengineering.

Spline:

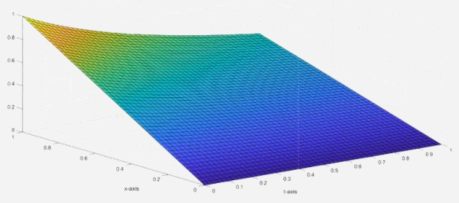


Bioengineering applications:

Estimate human motion in rehabilitation.

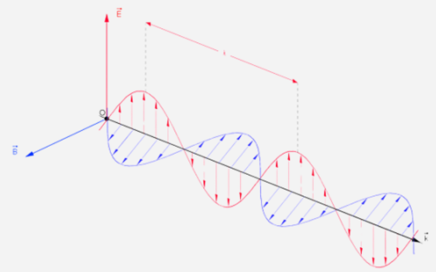
Represent pre-operative models as splines.

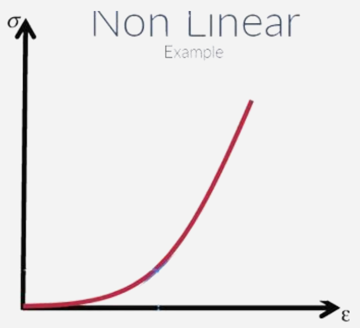
Objectives



1 The methodology is scalable in term of variate nature.

2 The methodology is scalable to systems of equations.



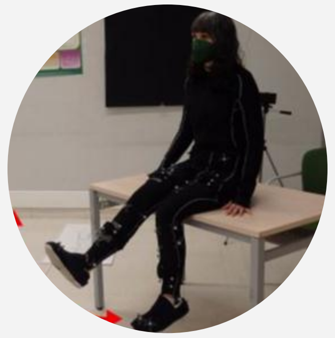


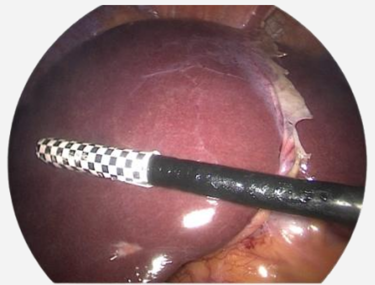
3 The methodology is scalable to non-linear equations.

4 To determine good choice in the process decisions.

5 To estimate tumour cells proliferation.

6 To estimate orientation and human motion from IMUs.





7 To represent a preoperative model as a spline.

About splines

Smoothness

$$B_{j,0} = \begin{cases} 1 & t_j < t < t_{j+1} \\ 0 & otherwise \end{cases}$$

$$B_{j,k}(x) = \frac{x - t_j}{t_{j+k-1} - t_j} B_{j,k-1}(x) + \frac{t_{j+k} - x}{t_{j+k} - t_{j+1}} B_{j+1,k-1}(x)$$

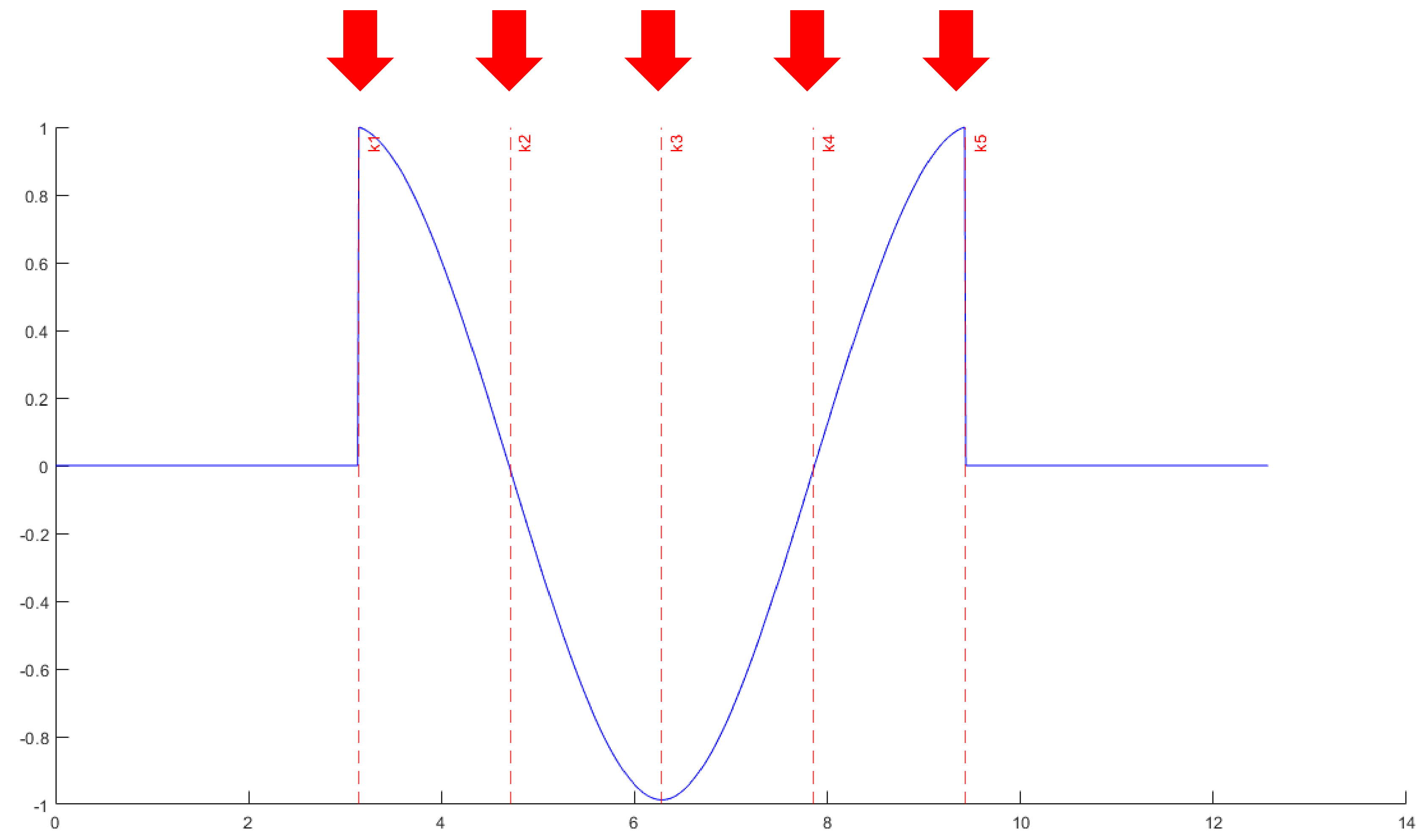
$$f(x) = \sum_{j=1}^{n} B_{j,k} a_j$$

Tensor product splines

$$f(x_1, ..., x_v) = \sum_{i=1}^{m} ... \sum_{j=1}^{n} B_{i,k} ... B_{j,k} a_{i,...,j}$$

$$\mathbb{R}^n \Rightarrow \mathbb{R}^d \qquad dx a_1 x \dots x a_{n+1}$$

Knots: (k1, k1, k1, k1, k2, k3, k4, k5, k5, k5, k5)

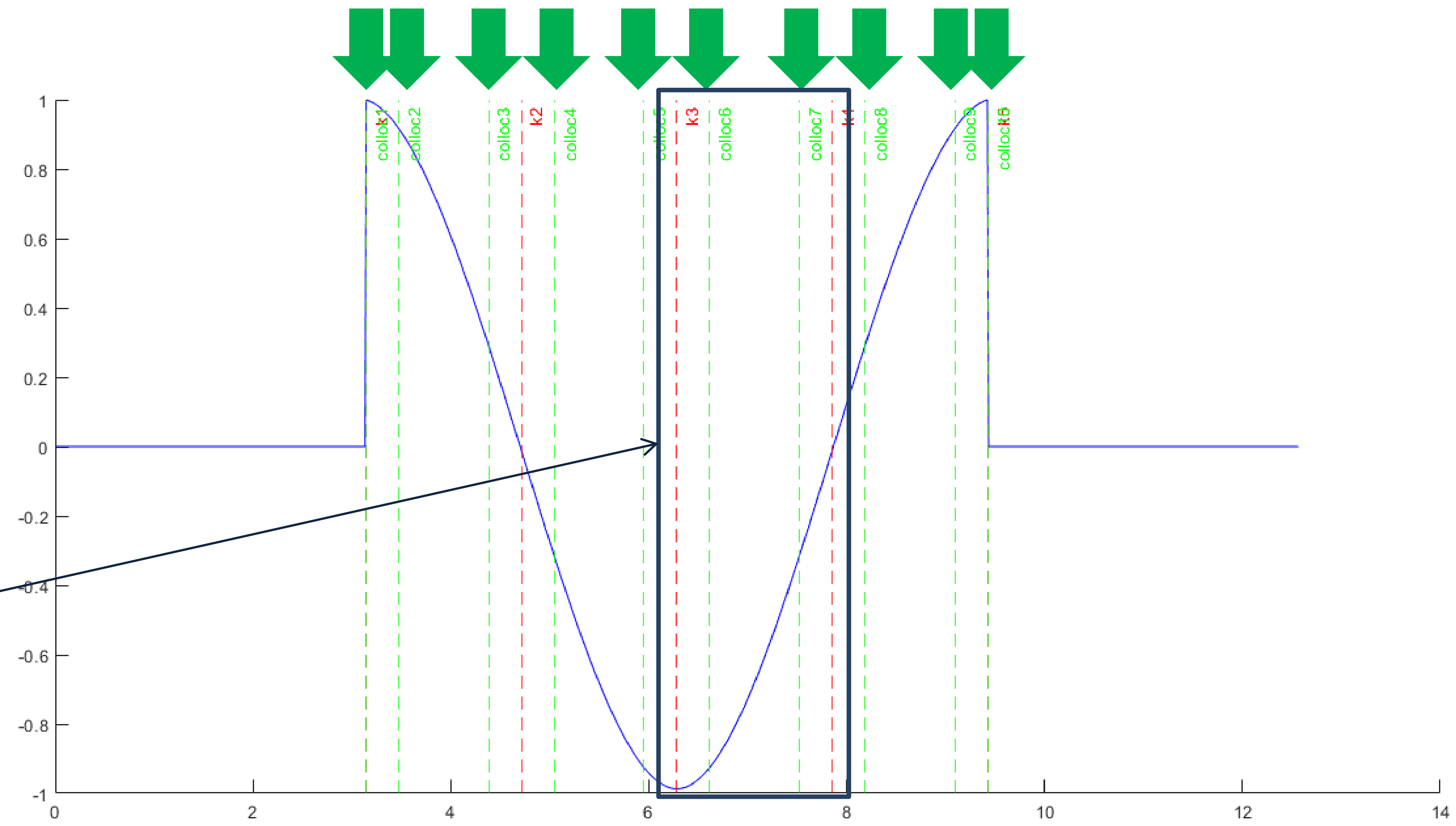


Collocation methods

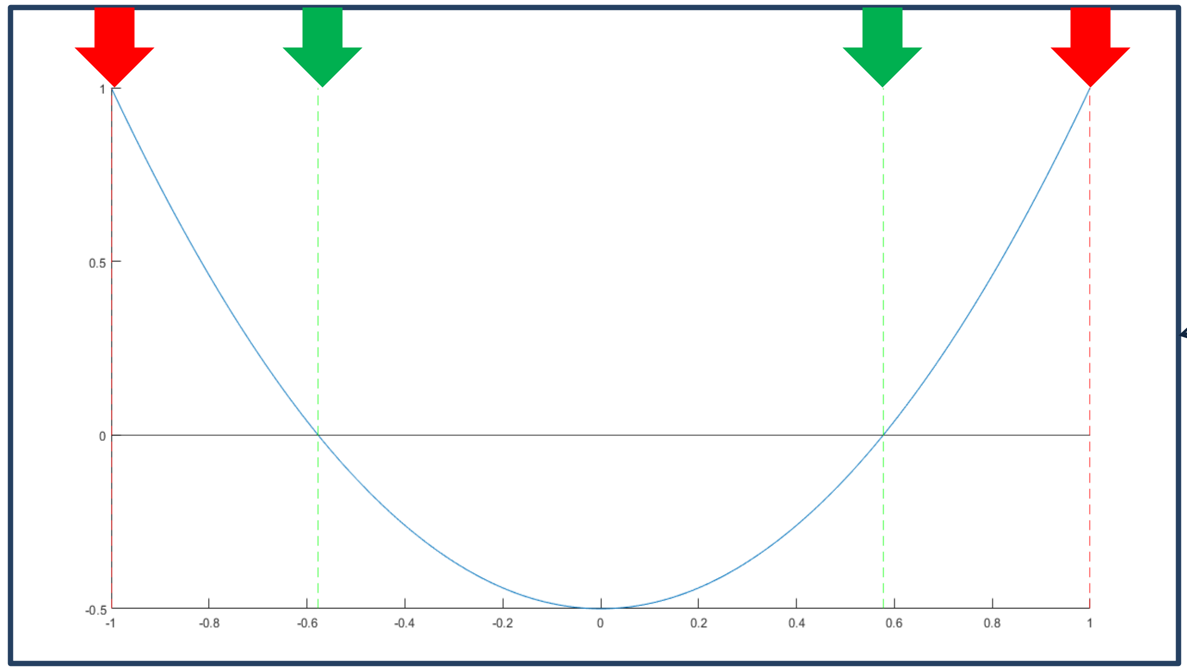
Spline design

- Order
- Knots sequence
    - Domain
    - Smoothness

$$f\left(x_0, y(c_k|x_0), \frac{dy(c_k|x_0)}{dx}, \dots, \frac{d^n y(c_k|x_0)}{dx^n}\right) = 0$$



Legendre polynomial



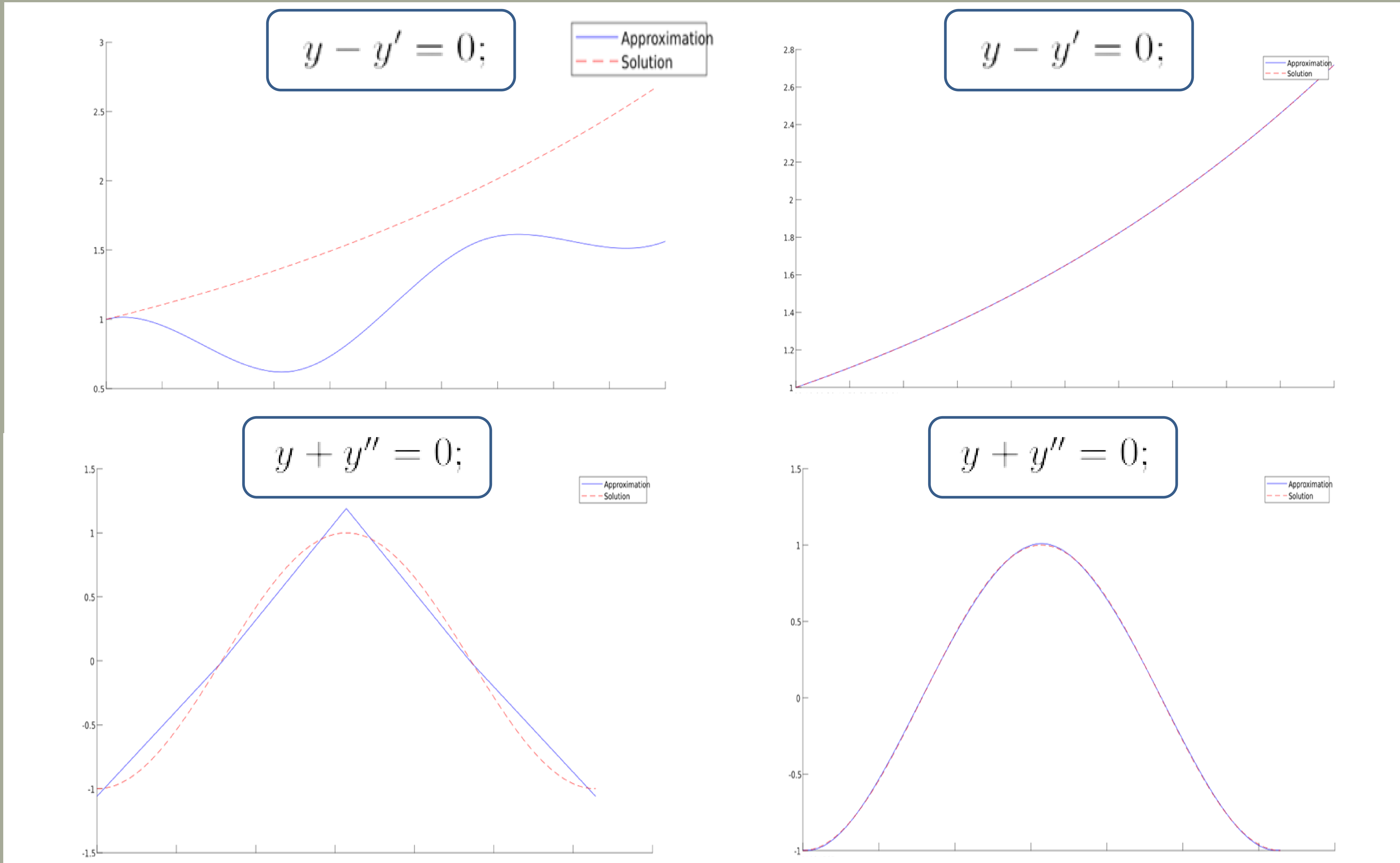
Decisions and impact

$$\begin{cases} y - y' = 0; \\ y(0) = 1 \end{cases}$$

Best choices

•Collocation points at Legendre Quadrature
• 5 breaks per sin cycle
• 4 collocation points per sub polynomial
• Sixth-order spline

$$\begin{cases} y + y'' = 0; \\ y(0) = 1 \\ y(2\pi) = 1 \end{cases}$$



Multivariate splines

Laplace equation

$$\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = f(x,y)$$

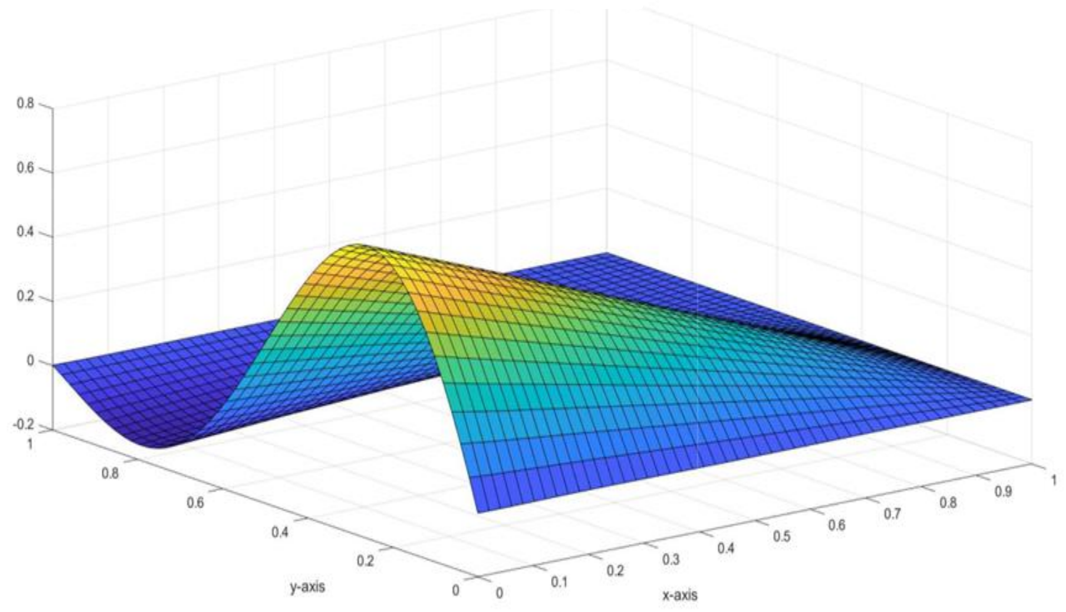
Multivariate collocation

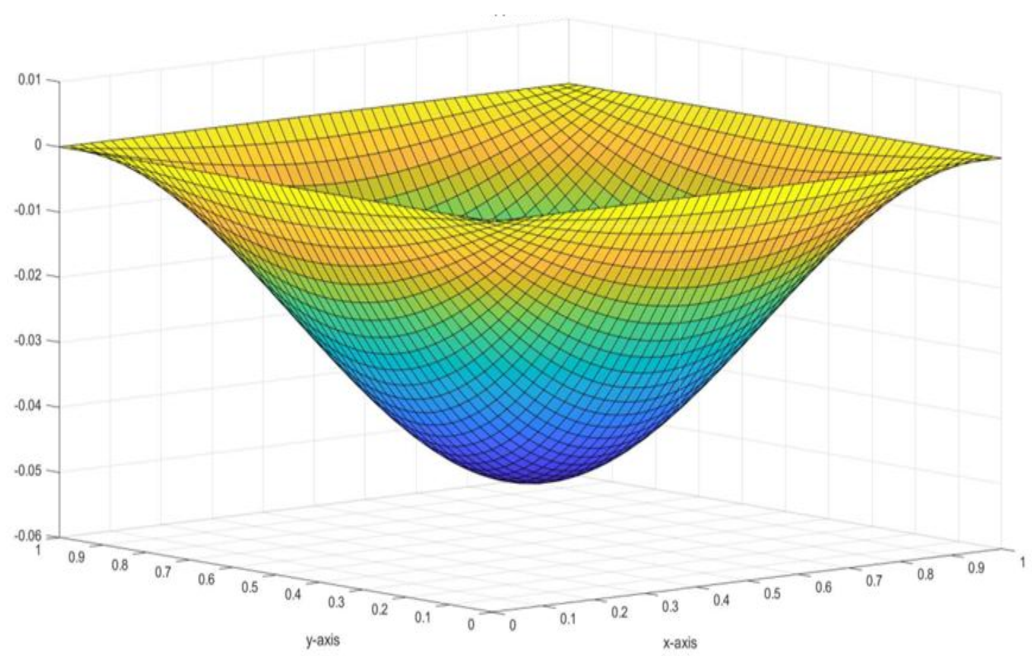
- Cartesian grid of the collocation sites of each variable.
- Collocation as Cartesian product of the univariate collocation.

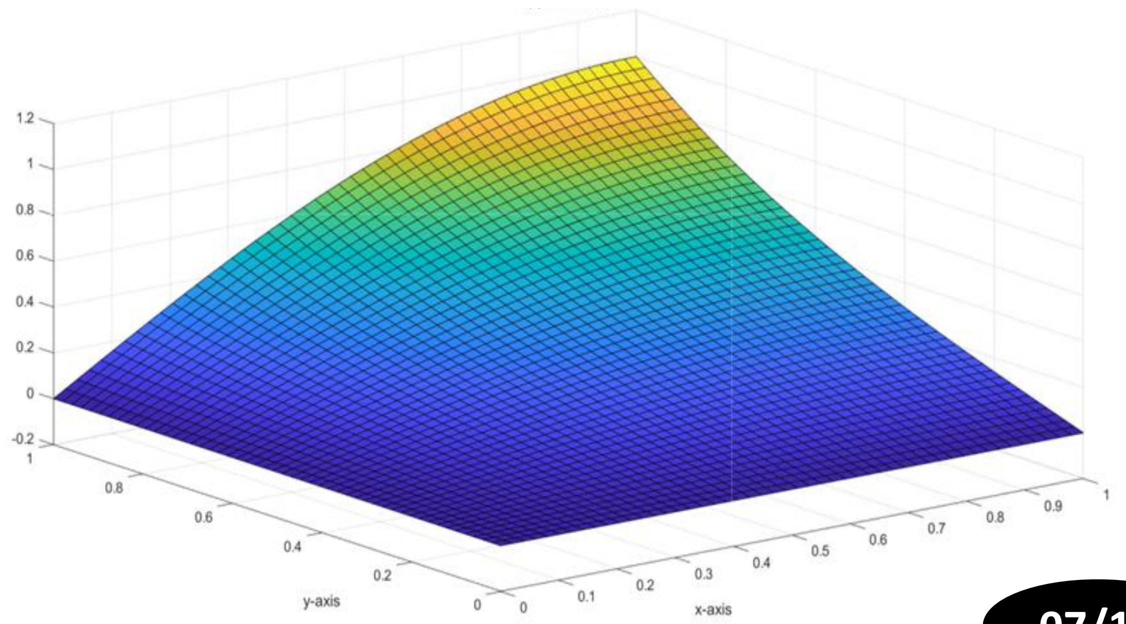
$$\begin{cases} f(x,y) = 10(x-1)\cos(5y) - 25(x-1)(y-1)\sin(5y) \\ z(0,y) = (1-y)5y \\ z(1,y) = 0 \\ z(x,0) = 0 \\ z(x,1) = 0 \end{cases}$$

$$\begin{cases} f(x,y) = f(x,y) = \sin x\pi \sin y\pi \\ z(0,y) = 0 \\ z(1,y) = 0 \\ z(x,0) = 0 \\ z(x,1) = 0 \end{cases}$$

$$\begin{cases} f(x,y) = 0 \\ z(0,y) = 0 \\ \frac{dz}{dx}(1,y) = 0 \\ z(x,0) = 0 \\ z(x,1) = \sin\left(\frac{x\pi}{2}\right) \end{cases}$$





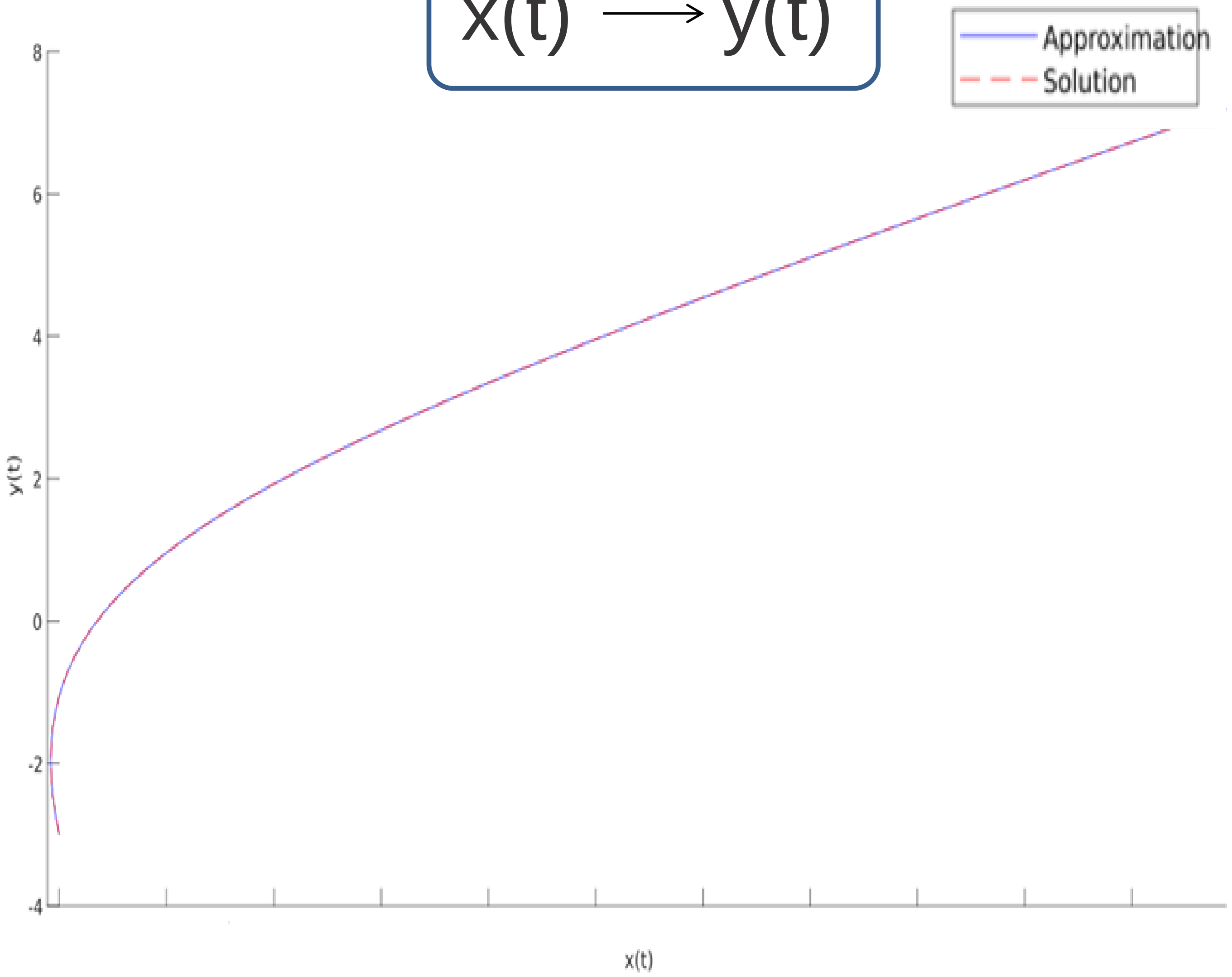


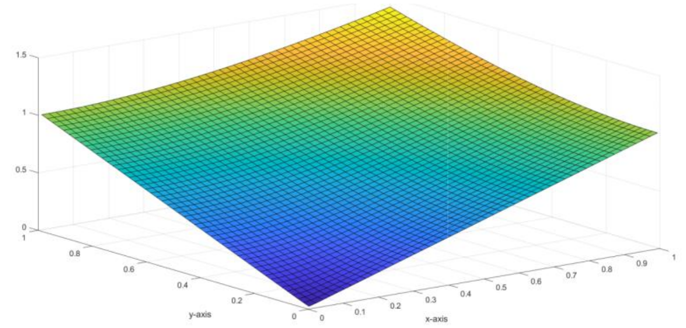
Multidimensional splines

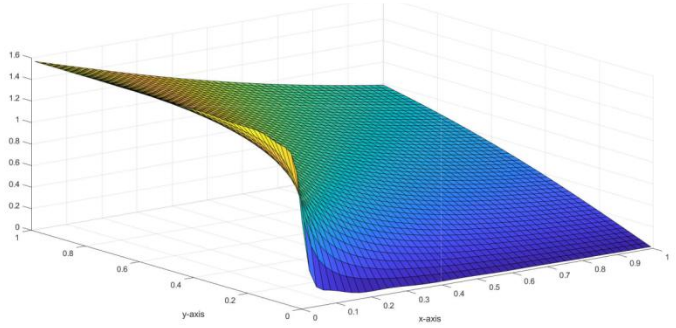
System of equations and domain outcome

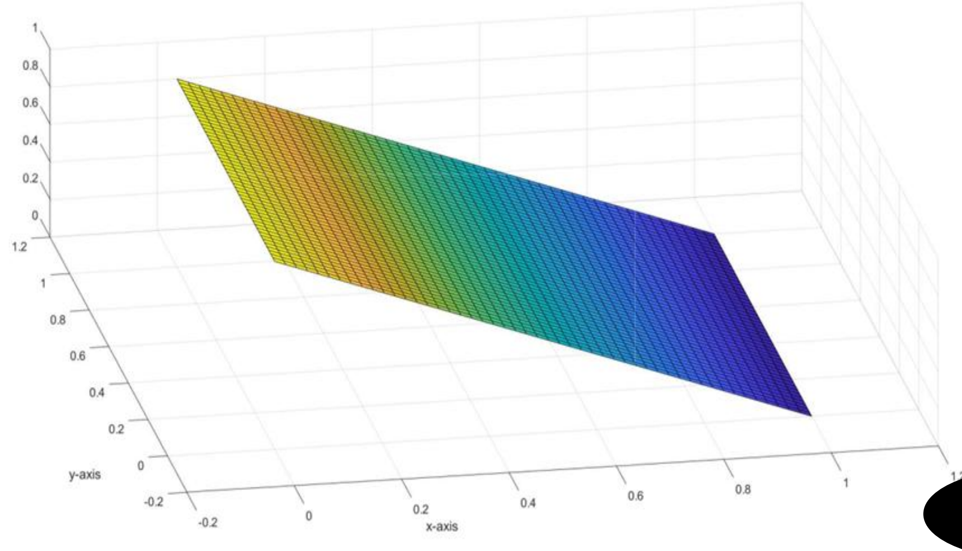
$$\begin{cases} x' = x + y \\ y' = 4x - 2y \\ x(0) = 2 \\ y(0) = -3 \end{cases}$$

x(t) $\longrightarrow$ y(t)









Non-linear DE

- A linear term that is one order higher than the order of the differential equation is introduced to it.
- The method consists on applying the Newton method to that equation.

Non-viscous approach

- The viscosity term is set to zero.

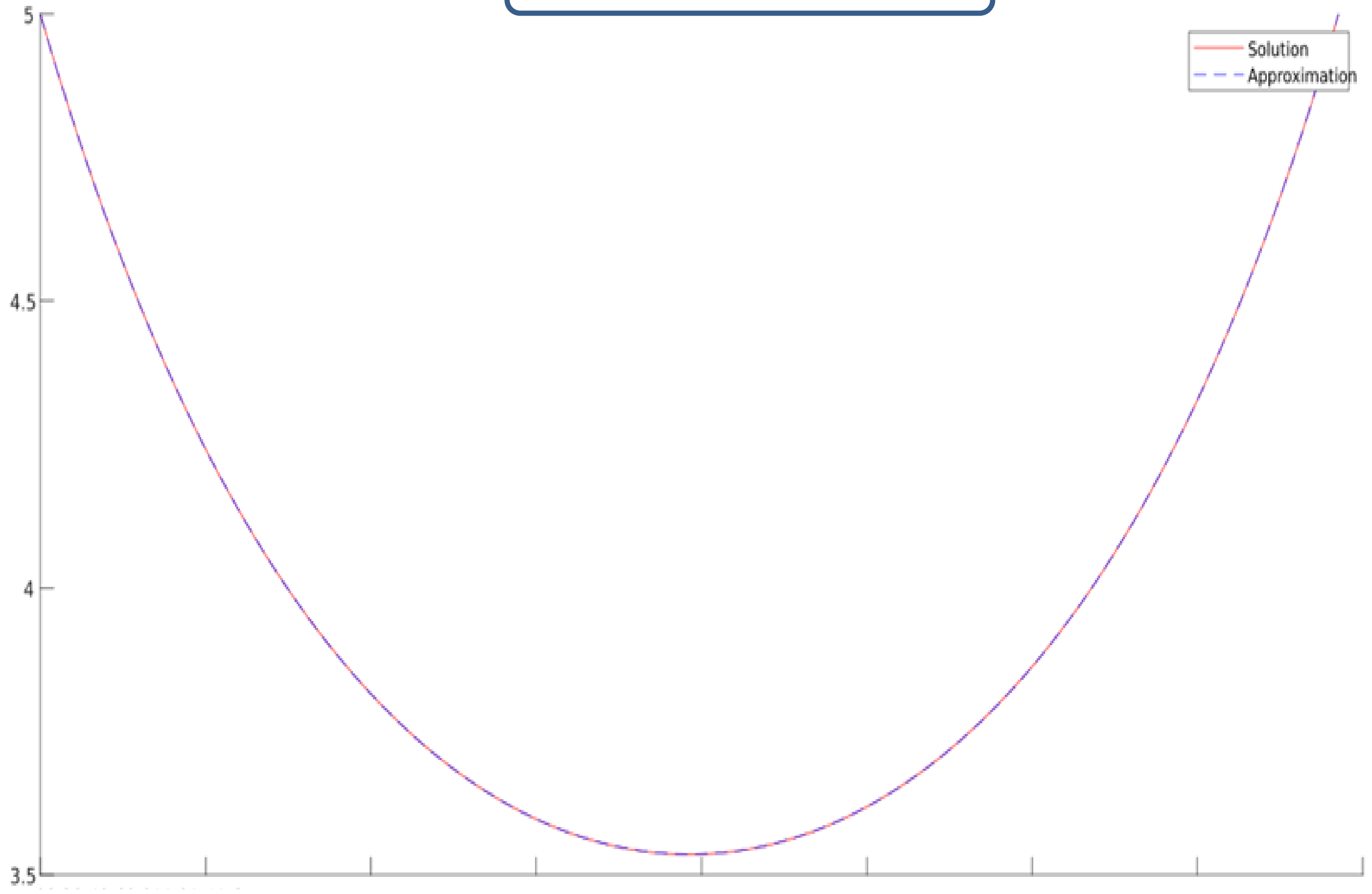
$$\varepsilon y'' = \frac{50}{(1+\sin 2x)^2} - (y')^2 - y^2$$

Viscous approach

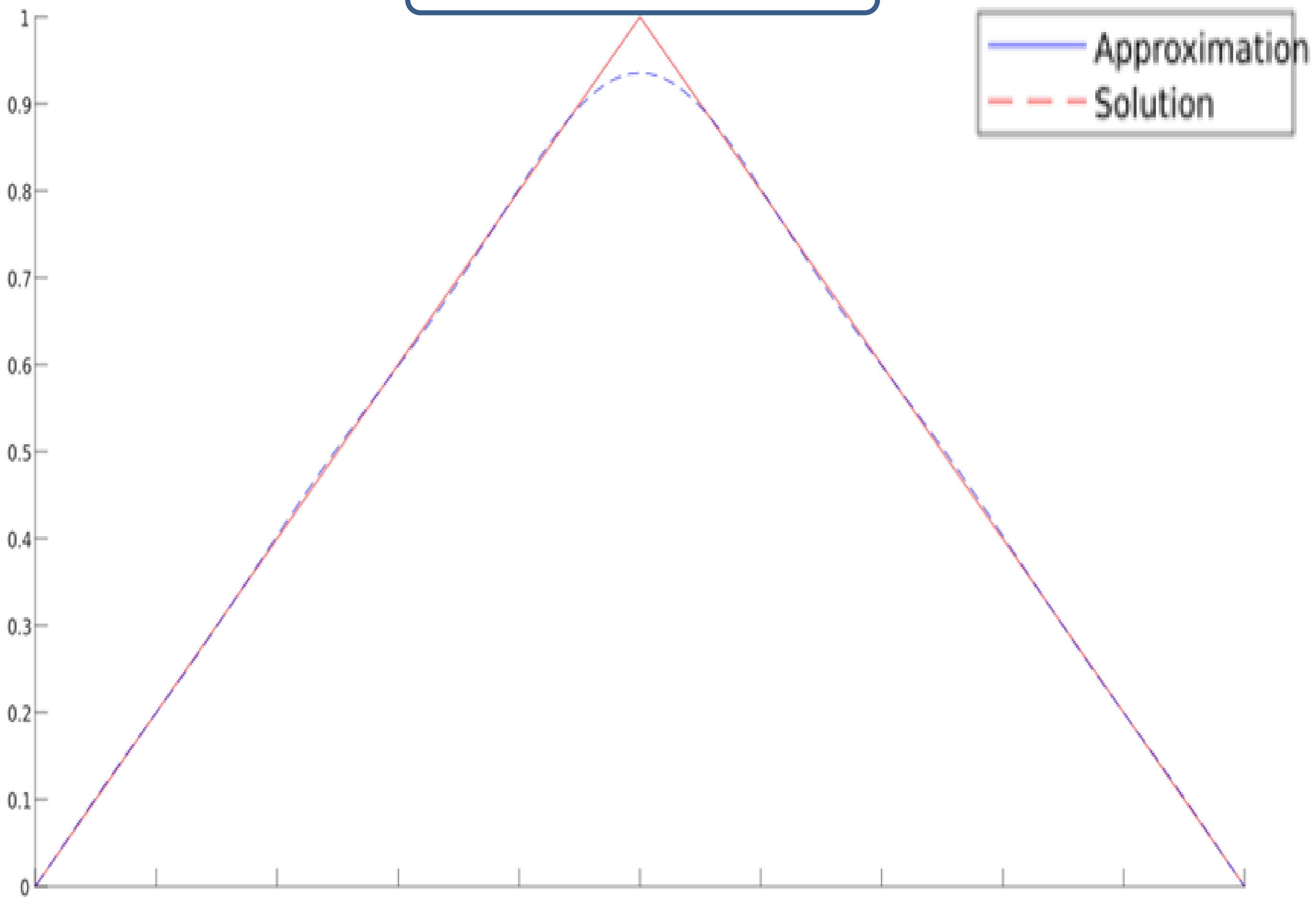
- The viscosity term is set.

$$\varepsilon y'' = (y')^2 - 1$$

Non-linear



Eikonal

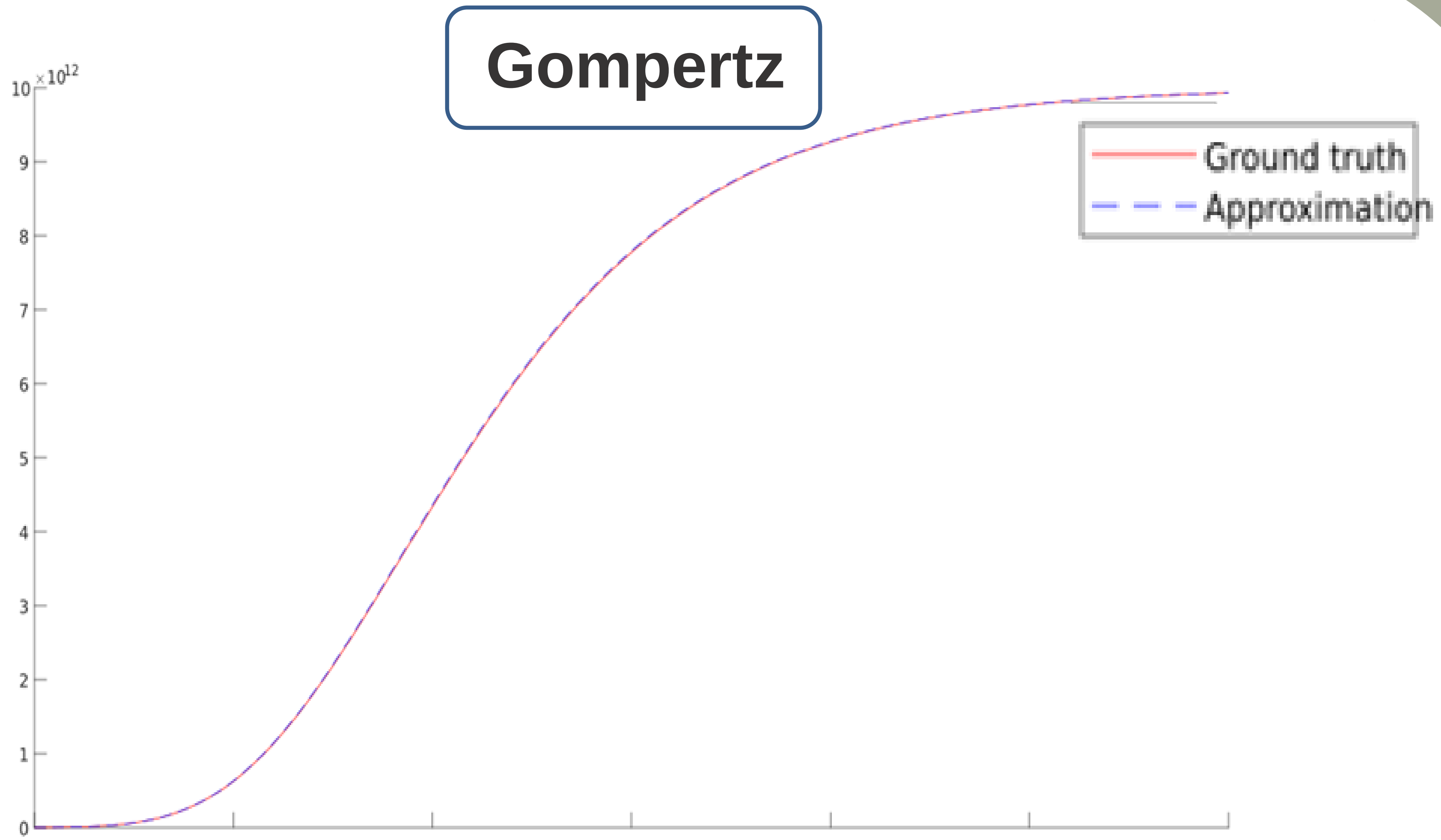


Gompertz model

Estimate the number of cells depending on the time of a tumour in angiogenesis stage.

It depends on the initial number of cells, the intrinsic growth rate and the carrying capacity.

$$N'(t) = rN \log\left(\frac{C}{N}\right)$$

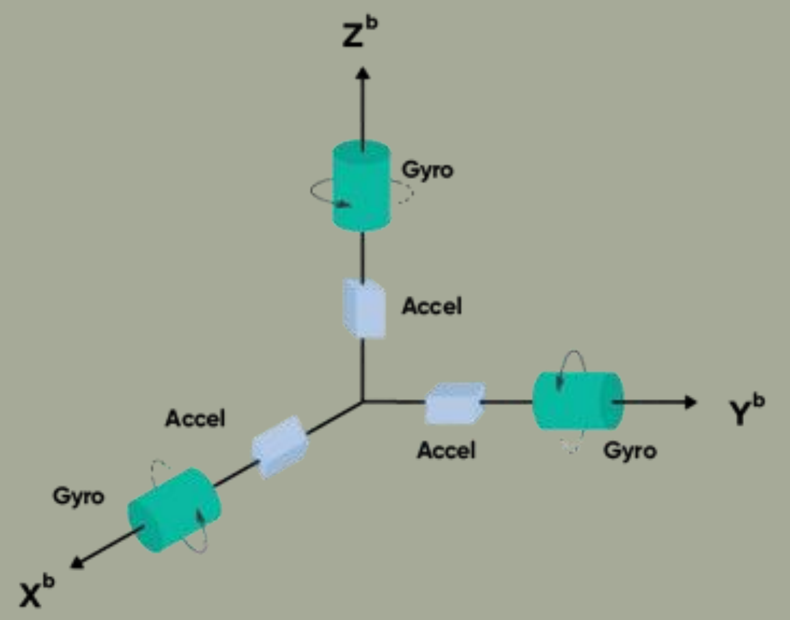


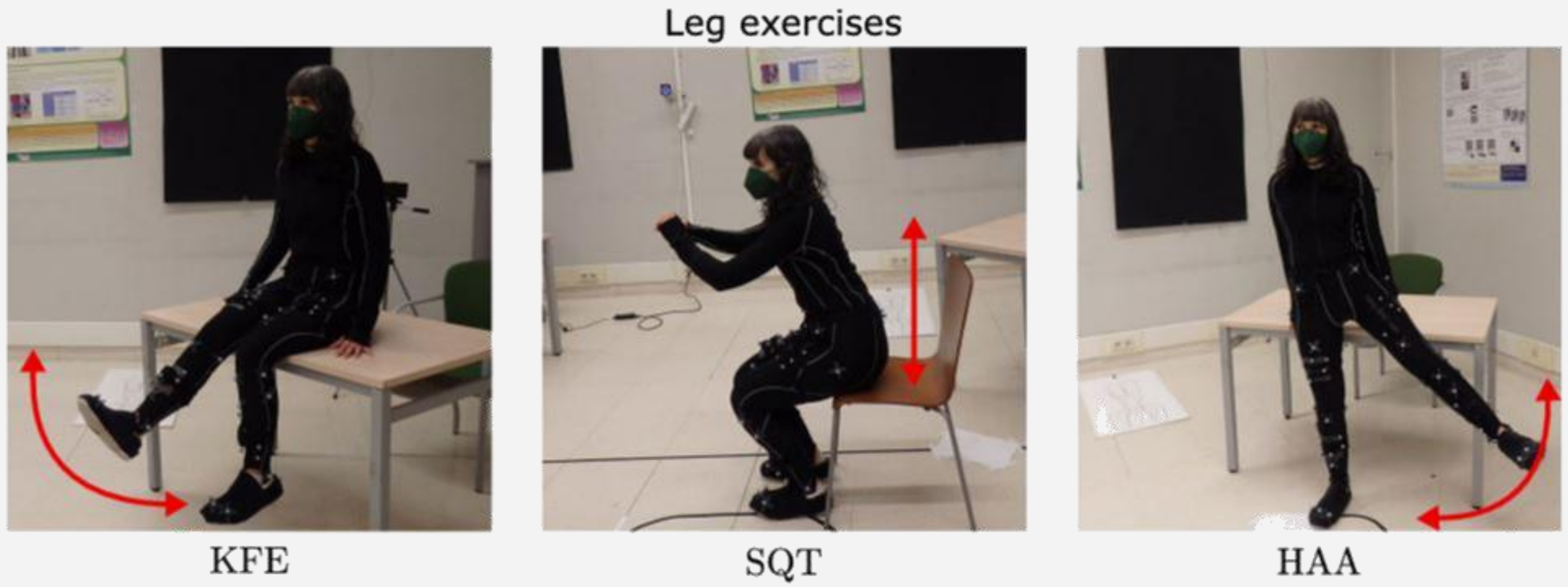
Parameters

- N(0) = $1_{10}9$ cells
- r = 0.006 cells/t
- C = $1_{10}13$ cells

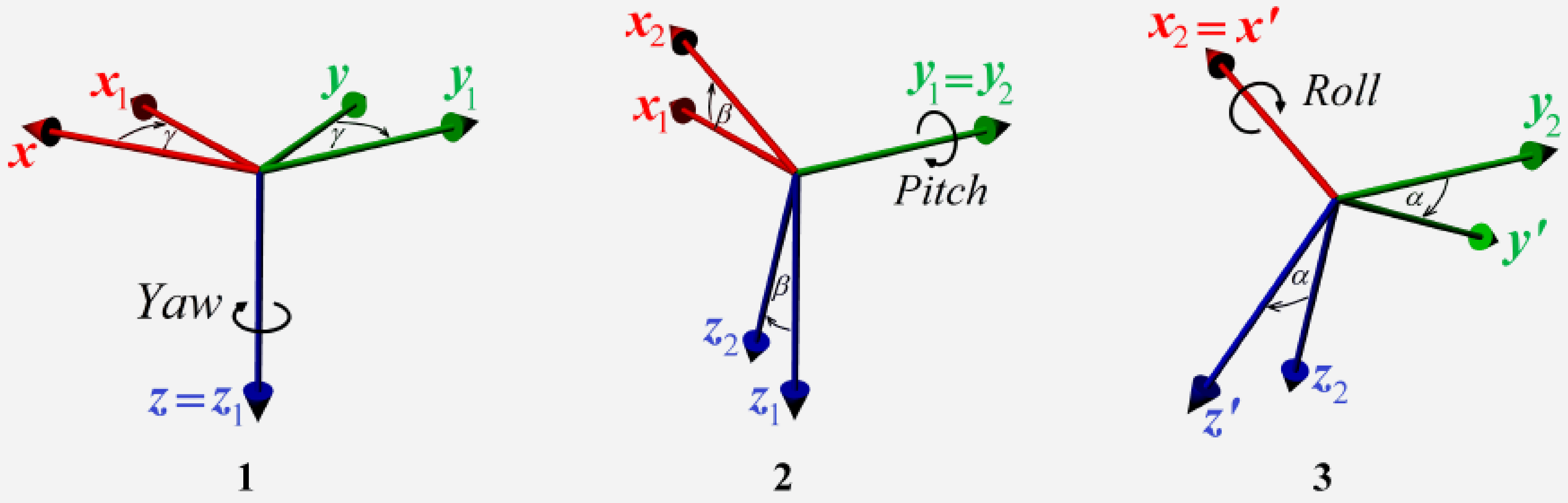
Motion monitoring

Inertial Measurement Unit (IMU):





Euler angles:



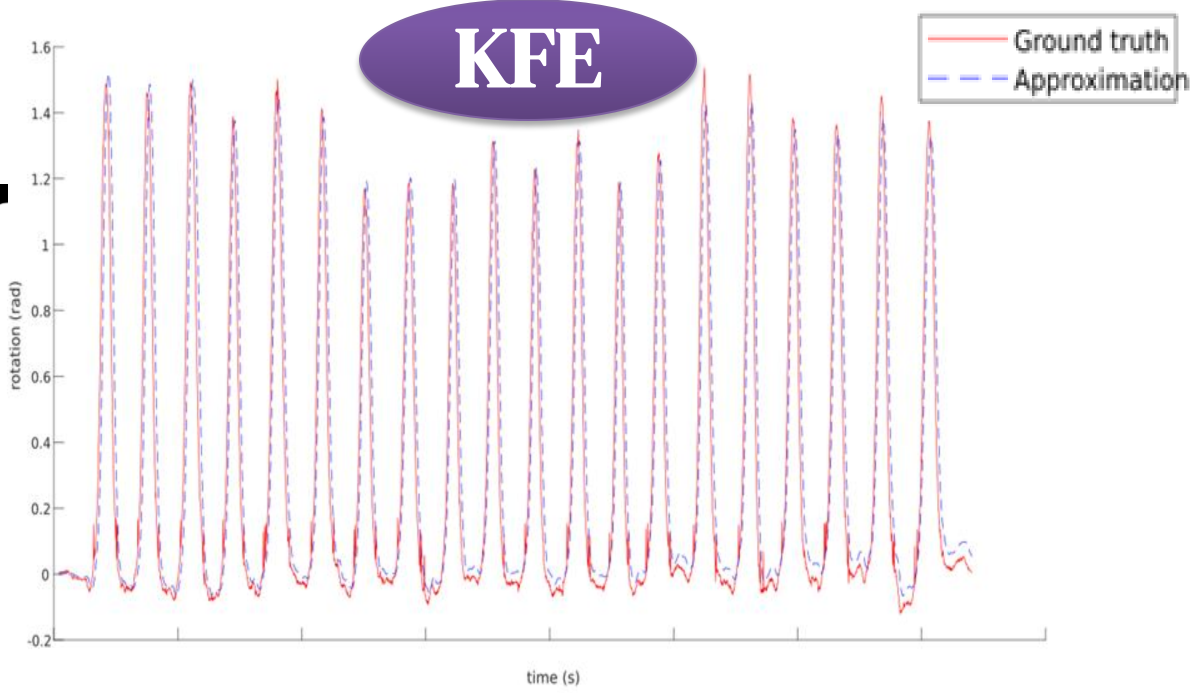
Orientation results

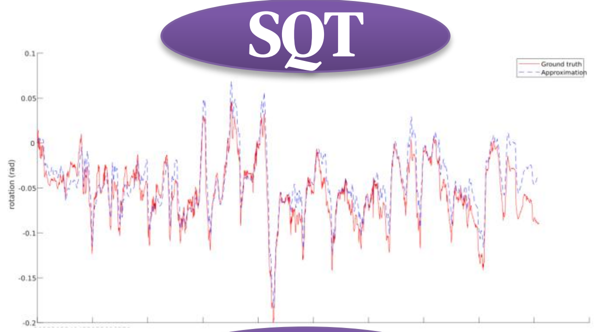
$$\overrightarrow{\omega} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\psi & \sin\psi\cos\theta \\ 0 & -\sin\psi & \cos\psi\cos\theta \end{bmatrix} \begin{pmatrix} \psi' \\ \theta' \\ \phi' \end{pmatrix}$$

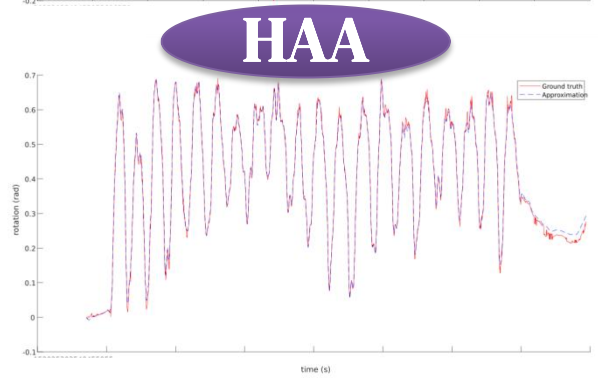
Table 5.18: Error metrics of KFE orientation.

Angle (rad)	Mean error	Max error	Std error
Yaw (ψ)	0.12	0.57	0.12
Pitch (θ)	0.11	0.30	0.07
Roll (ϕ)	0.16	0.55	0.12

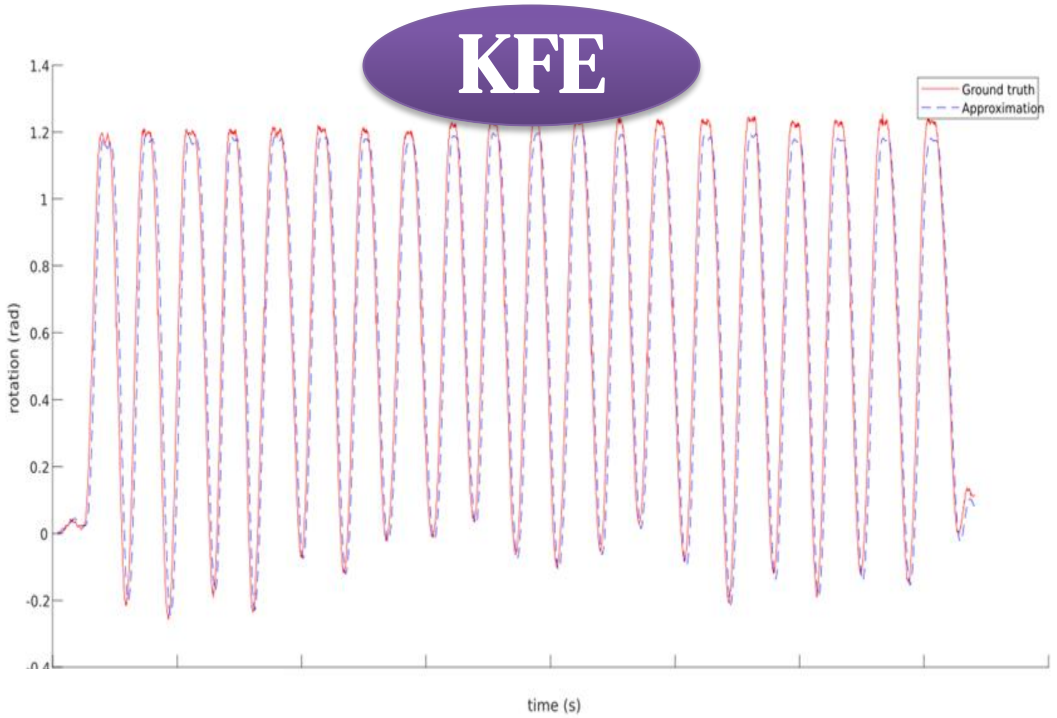
Ψ

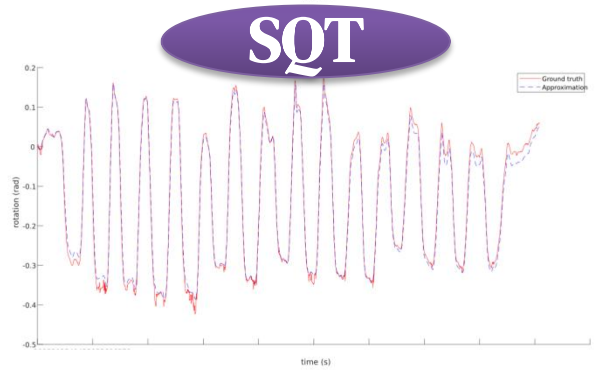


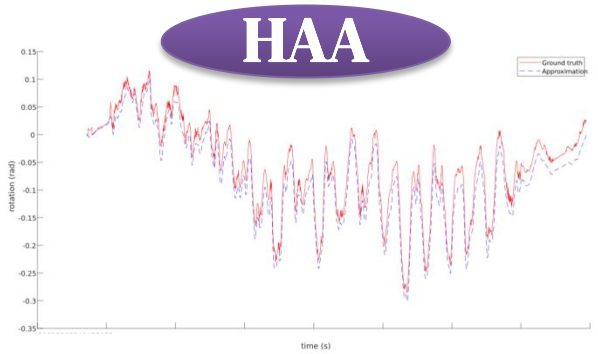




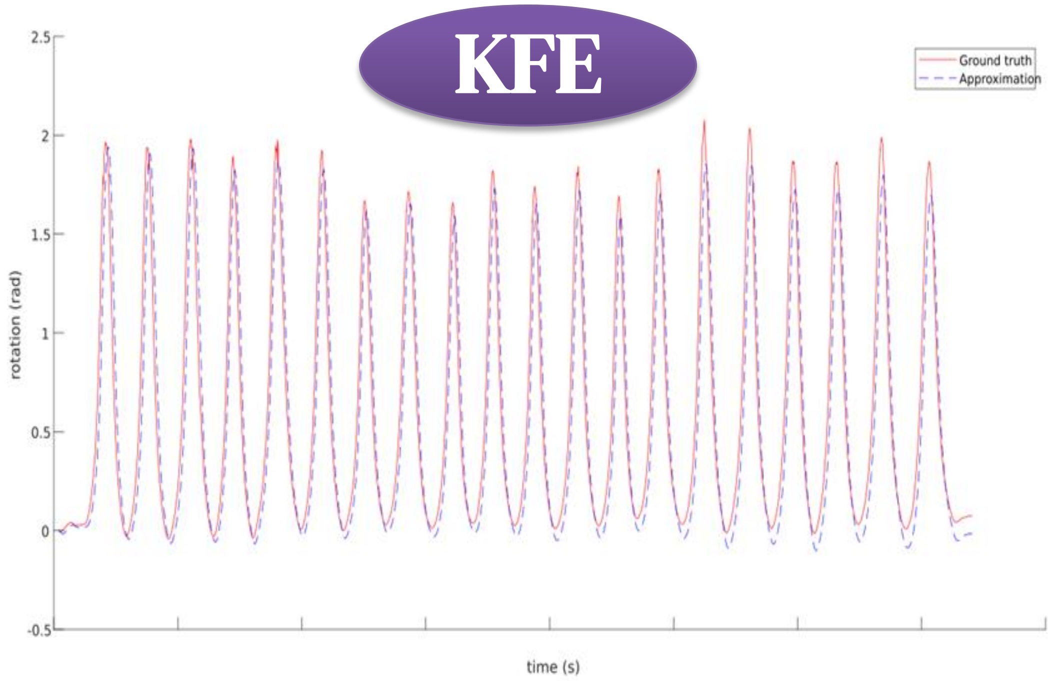
Θ

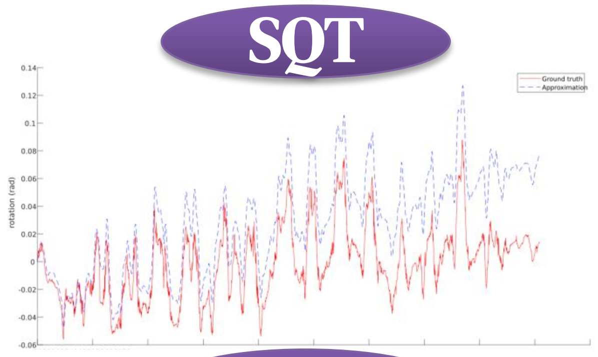


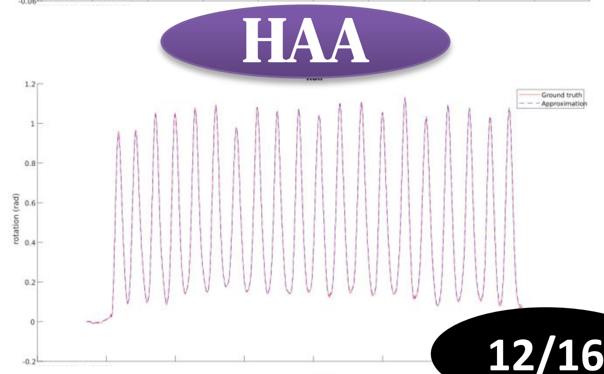




Φ





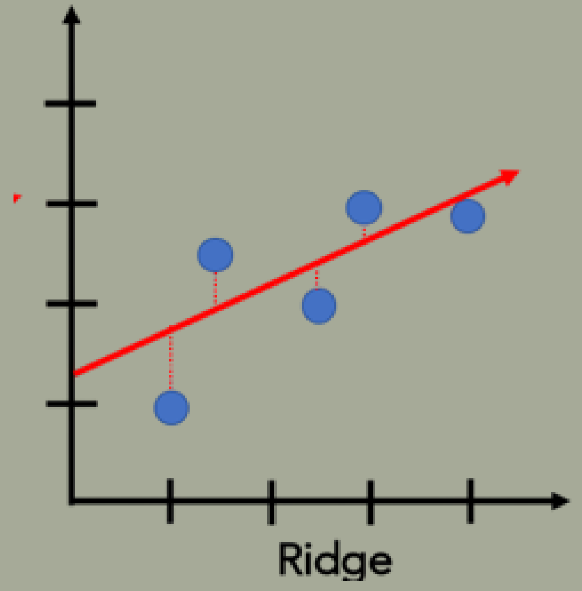


Motion results

$$a(t) \cdot M_z(\psi, \theta, \phi) - \frac{d^2 z(t)}{dt^2} = 0$$

$$a(t) \cdot M_y(\psi, \theta, \phi) - \frac{d^2 y(t)}{dt^2} = 0$$

$$a(t) \cdot M_x(\psi, \theta, \phi) - \frac{d^2 x(t)}{dt^2} = 0$$

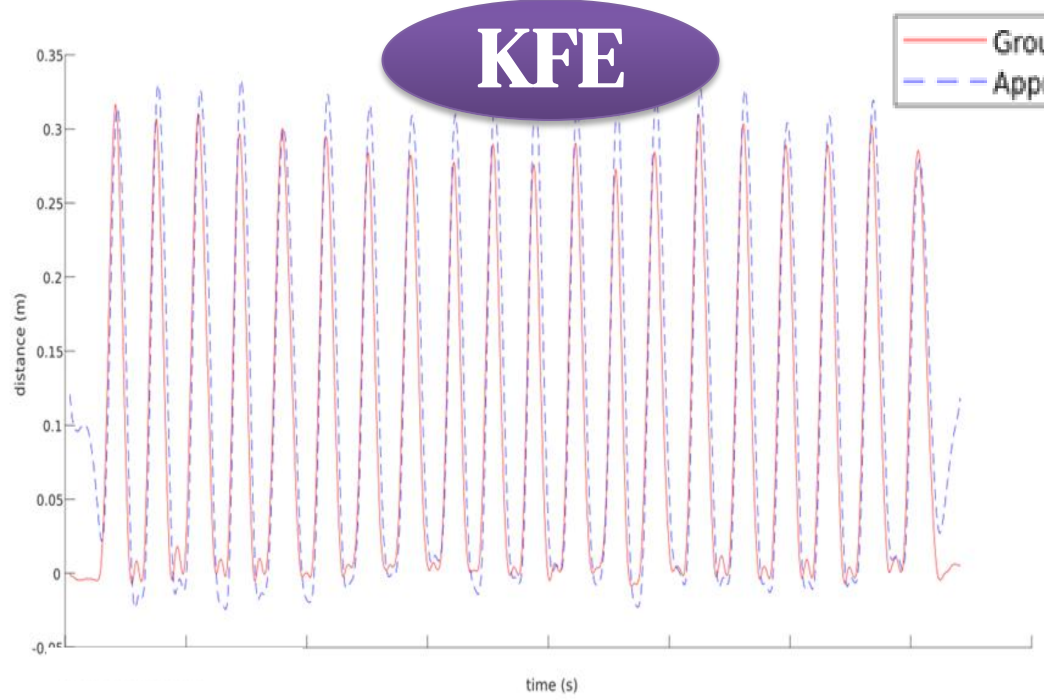


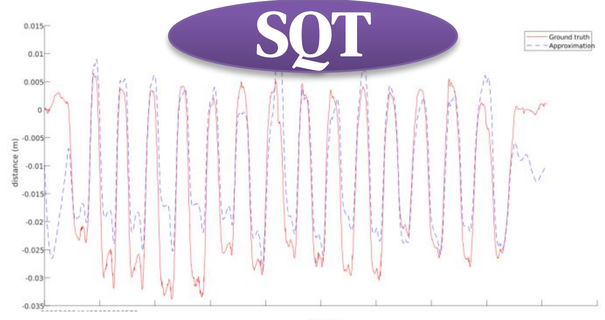
$$M = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ -\sin\psi\cos\theta + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\theta + \sin\psi\sin\theta\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\theta + \cos\psi\sin\theta\cos\phi & -\sin\psi\sin\theta + \sin\psi\sin\theta\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

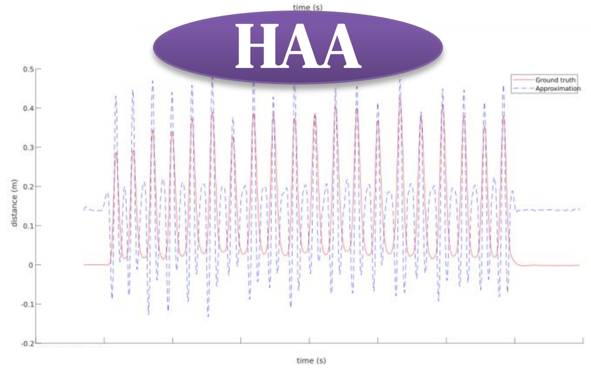
Table 5.19: Error metrics of KFE linear displacement.

Axis (m)	Mean error	Max error	Std error
Z	0.03	0.12	0.02
Y	0.01	0.04	0.01
X	0.05	0.10	0.02

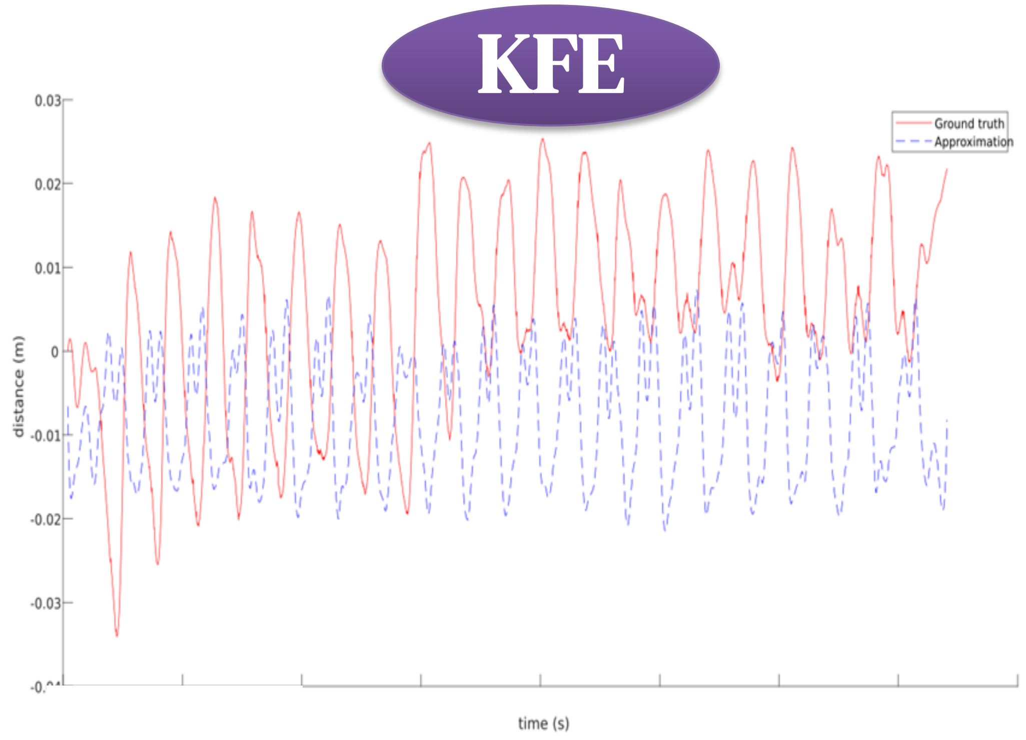
Z

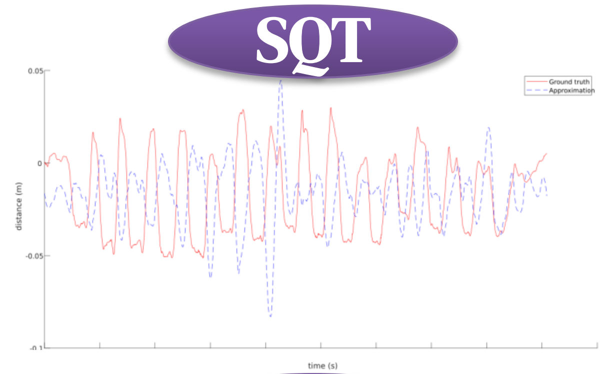


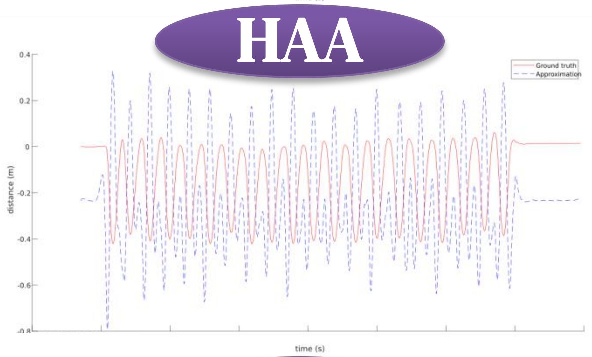




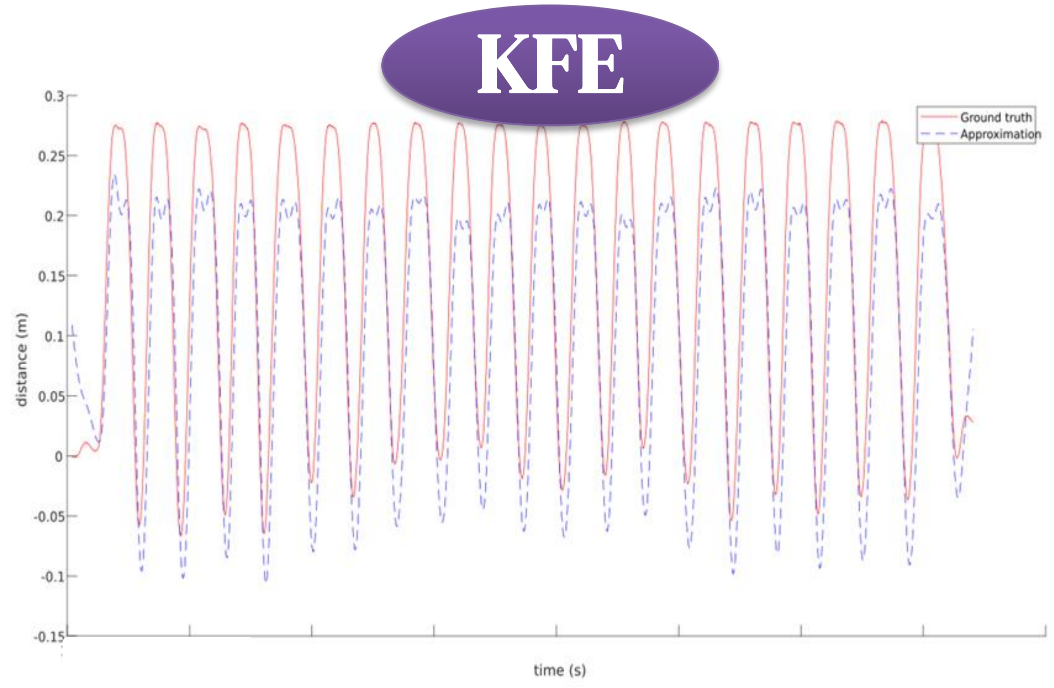
Y

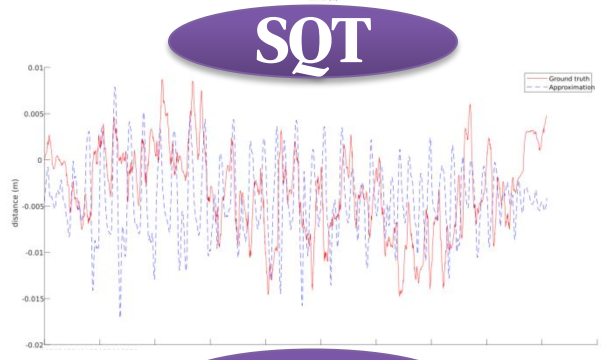






X





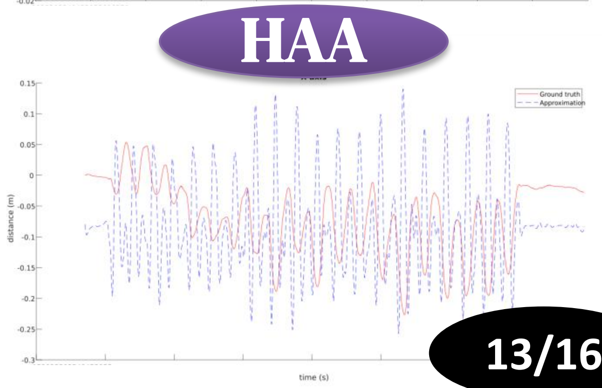
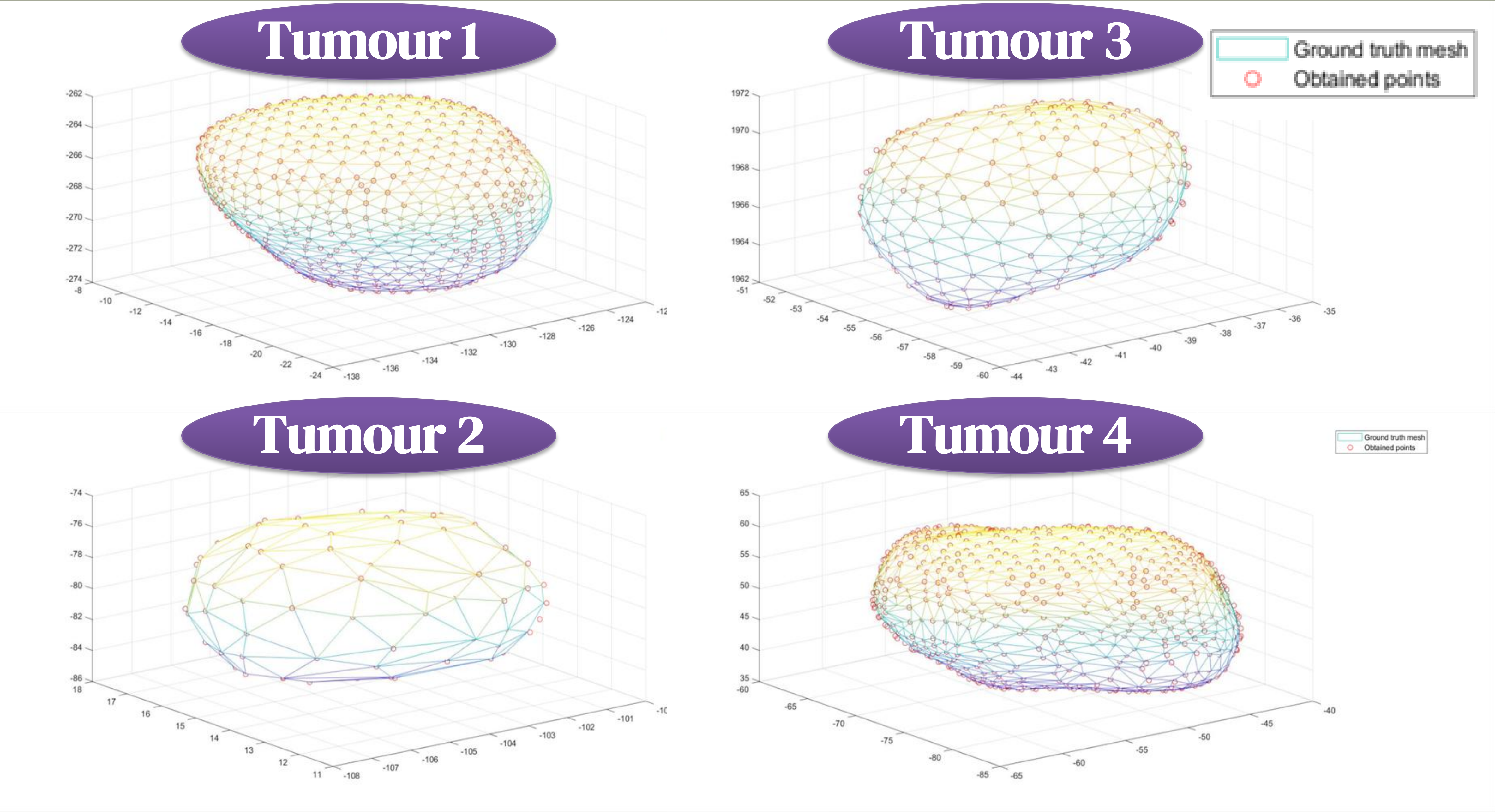
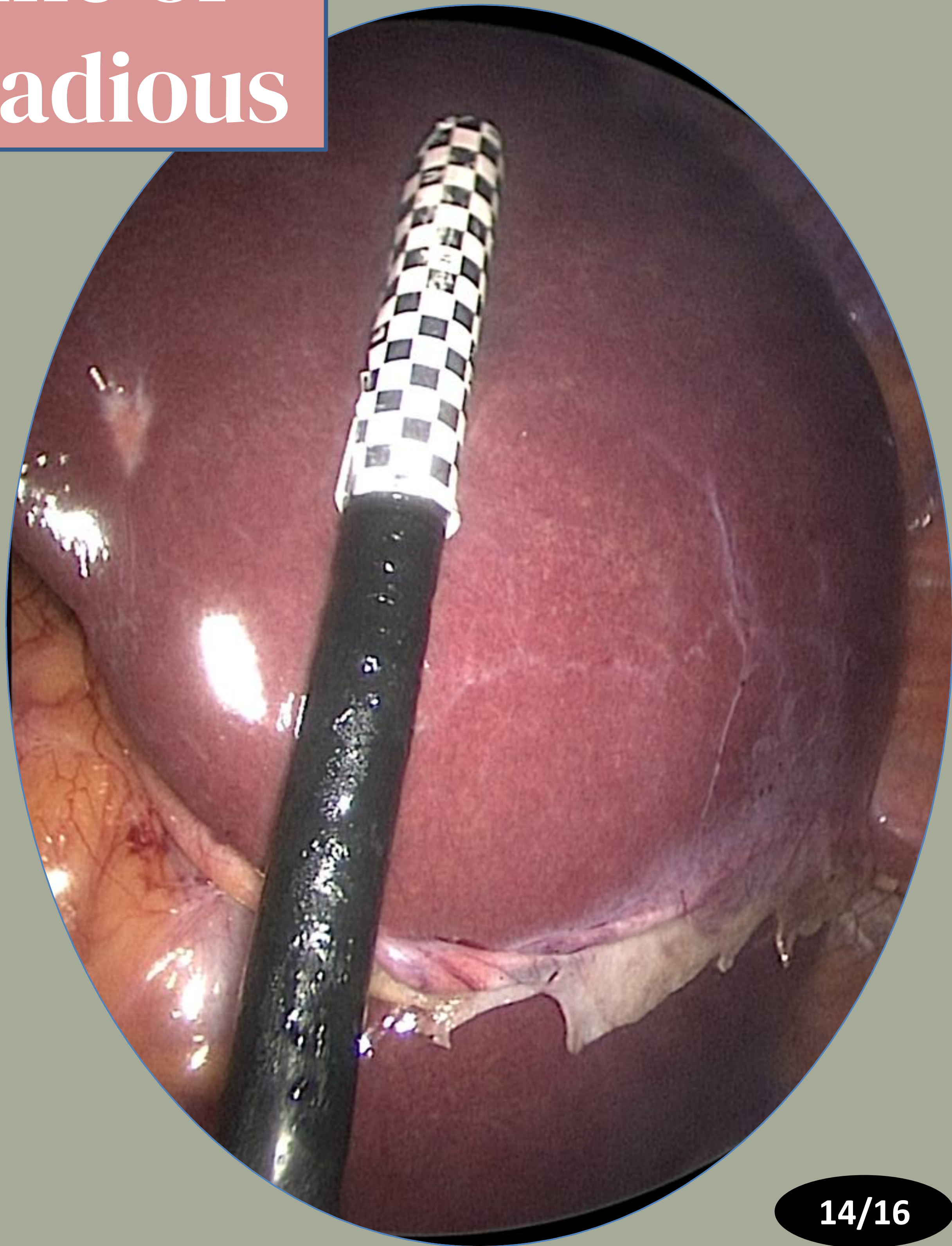


Image processing

Preoperative model → Spherical coordinates → Spline of the radious





Conclusions and SDGs

1 The method is scalable to multivariate function, systems and non-linear equations.

2 It is determined a good criteria for different choices including the order, the collocation sites, the viscosity, etc.

3 The estimation of the orientation is satisfactory and the human motion in the KFE exercise.

4 The preoperative model of tumours are successfully represented as differentiable splines.

SDGs: Goal 3: Good health and well-being

- Non-communicable diseases
- Universal health coverage
- Health risk management

Future lines

A1 We must evaluate the motion estimation including the time requirements.

A2 It is needed to improve the determination of the ridge parameter in the displacement estimation.

B To provide a close path framework is needed for the circular bases domain in the preoperative tumour models.