Informatics II, Spring 2024, Exercise 10

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Learning Goal

- Understanding hash tables, hash functions and how to deal with collisions.
- Knowing how to use hash tables to solve problems.

Task 1 [Easy]

What is the resulting hashtable `HT` after running the script?

Assume `HTInsert()` is called with `value_array = {3, 14, 33, 27, 9, 30, -2, 0, 2, 16, 8}`, thus also `n = 11`. You can also assume that `HT` is an array of type pointers to `struct Node` and has a length of `n = 11`. It is initialized with `NULL` values.

```
Algorithm: HTInsert(*HT, *value_array, n)
for i=0 to n do
    k = (value_array[i] * 3) mod n;
    new_node = malloc(sizeof(struct Node));
    new_node->val = value_array[i];
    new_node->next = HT[k];
    HT[k] = new_node;
```

Task 2 [Medium]

You are provided the script `hash_functions.c`. This script already sets up the following functions:

- `main()`: Declares some variables needed for the hash functions and run time calculations. You do not need to change something here.
- `initHT(int *HT)`: Initializes the hash table `HT`, i.e, fills it with `-1` which mark unused slots. You do not need to change something here.
- `printHT(int *HT)`: Prints the content of the hash table `HT`. You do not need to change something here.
- `insertKeyH1(int *HT, int k)`: See taks **(a)**.
- `insertKeyH2(int *HT, int k)`: See task **(b)**.

The hash table *HT* is initialized in the `main()` function. It has a size of `N` slots. Implement the tasks **(a)** and **(b)** in order to insert the 100 keys given in `keys`. In the end we want to compare the relative runtime, which is why the `start_time` and `end_time` are declared. Additionally, we want to compare the number of collisions.

Hint: For this task, do not use compiler optimizations, in order to better see the effect on the runtime.

(a) Implement a Hash Function

Consider the following **hash function**: $h_1(k) = k \mod 7$. Your task is now to implement this hash function to implement the `insertKeyH1()` function. This function takes a pointer tho the hash table `HT` array and the key `k`, which should be inserted into the hash table. Use **linear probing** to resolve conflicts and return the number of conflicts occured in the insertion for the key `k`. (These are then added up for the 100 keys to get the total number of conflicts, which is already implemented.)

Compile and run the script. **Answer the following questions**

- What is the runtime?
- What is the number of collisions?
- How are the 100 given keys distributed in the hash table?
- Is this a good hash function?

(b) Implement a Second Hash Function

Implement a second hash function $h_2(k)$, also with **linear probing**, in the `insertKeyH2()`. You can design this hash function.

- Find a hash function that is better than $h_1(k)$. *Hint: Try different values and methods for the hash function.*

Compare the two implementations regarding clock time and number of collisions.

- What can you conclude?

Task 3 [Medium/Hard]

A hash table of length 10 uses open addressing with hash function $h(k) = (k + 4) \mod 10$, and linear probing (probing goes to the right). After inserting five values into an empty hash table, the table is as shown below.

slot	0	1	2	3	4	5	6	7	8	9
key	96						32	52	33	74

(a) Find Insertion Order

Which of the following choice(s) give possible order(s) in which the key values could have been inserted in the table?

1. 32, 33, 52, 96, 74
2. 32, 52, 33, 74, 96
3. 32, 52, 74, 96, 33
4. 96, 32, 52, 33, 74

(b) Delete Keys

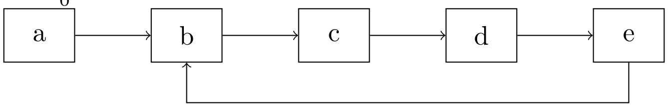
Implement a function `HTDelete()` which takes a key `k` to be deleted. Upon the deletion the succeding keys are moved to the right place, as if `k` was never inserted in the first place.

For example on the above table, we call `HTDelete(33)`, we then get the following table:

slot	0	1	2	3	4	5	6	7	8	9
key	96						32	52	**74**	

Task 4 [Easy]

Consider the following linked list. For simplicity assume that the values, a, b, c, d, e, in the list are distinct and $\in \mathbb{N}_0^+$



As one can see, there is a cycle in it. Implement a function `detectCycle()` which detects such cycles in a linked list. This function should return `0` if there is no cycle, and `1` if there is. The worst case runtime for `detectCycle()` should be $O(n)$, where n is the number of distinct elements.. *Hint: Think about how hashing can help here.*

Task 5 [Medium]

Cuckoo hashing has two hash tables each with n elements. There are two hash functions h_1 and h_2 to map the domain into n. Every element x in the domain will either be at position $h_1(x)$ in the first table or $h_2(x)$ in the second table.

To insert an element x, start by inserting it into the first table. If $h_1(x)$ is empty, x is placed there. Otherwise, place x there by pushing the previous element y into the table 2. Repeat this process, bouncing between tables, until all elements stabilize.

(a) Questions

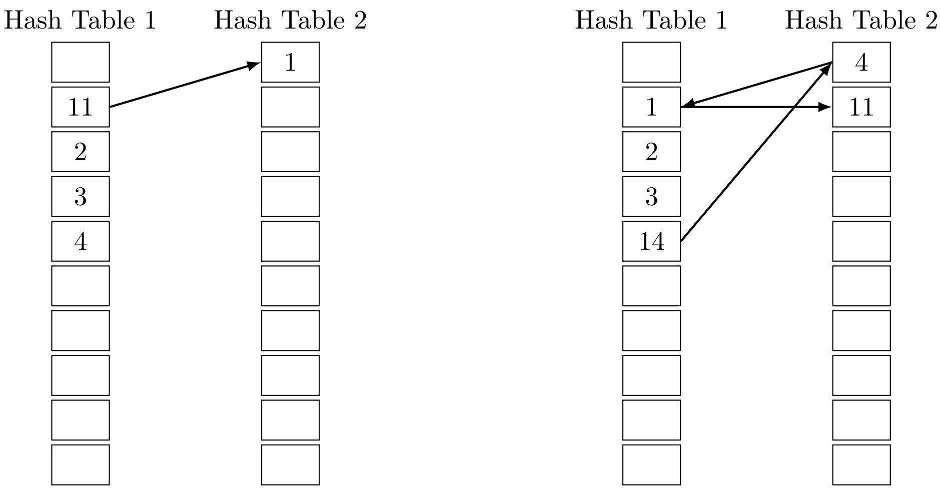
Answer the following:

- What is the worst case lookup time for a key k?
- What is the worst case deletion time for a key k?

(b) Cuckoo Insert

Write a `cuckooInsert()` function, which takes a key `k` and inserts it into the table 1. This function should also handle collisions. It returns the table number and which index in that table. Use the following hash function $h_1(x) = x \mod 10$ and $h_2(x) = \lfloor x/10 \rfloor \mod 10$

Additionally, find an example of insert keys which result in an infinite cycle.



(a) (b)

Figure 1: Hash tables for cuckoo hashing. **Situation (a)** after inserting $1, 2, 3, 4, 11$. Note how the 11 pushes the 1 into the second hash table (marked by arrow). **Situation (b)** after inserting 14. Note how the 14 pushes the 4 into the second hash table, which pushes the 1 back to the first hash table, which pushes the 11 into the second hash table.

(c) Cuckoo Search

Write a `cuckooSearch()` function, which takes a key `k` and searchs for it. It prints the table number and which index in that table. Keep in mind that the asymptotic runtime should adhere to your answer of **(a)**.