CS 5600/6600: Intelligent Systems
Assignment 4

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Learning Objectives

1. Training and Testing ANN Architecture
2. Learning Slow Down
3. Overfitting
4. Regularization

Introduction

This assignment will give you a flavor of training larger ANNs systematically and dealing with overfitting, learning slowdown, through cross-entropy, normalization and systematic evaluation of different ANN architectures. I hope you can use the tools and insights you'll develop in this assignment when you work on Project 1.

Problem 1 (5 pts)

The class `Network` defined in `ai_f18_hw04.py` is another implementation of ANN that allows us to experiment with two different cost functions: MSE and cross-entropy. The MSE is implemented in the class `QuadraticCost`. The cross-entropy function is defined in `CrossEntropyCost`. The method `__init__()` has a keyword parameter that allows you to set a cost function to be used in training and testing a specific ANN.

To begin with, let's load the MNIST training, validation, and testing data using the function `load_data_wrapper` from `mnist_loader.py`, as we did in the previous assignment.

```
train_d, valid_d, test_d = load_data_wrapper()
>>> len(train_d)
50000
>>> len(valid_d)
10000
>>> len(test_d)
10000
```

The member function `SGD` of the `Network` class trains its network object using stochastic gradient descent. Unlike the `SGD` function in the previous version of the network class you worked with in the previous assignment, this method allows you to pass the value of the λ argument used in *L2 regularization*. Regularizatsion is a way to reduce overfitting. A *regularization term* is added to the cost function as follows.

$$C = C_0 + \frac{\lambda}{2n} \sum_w w^2,$$

where $\lambda > 0$ is the regularization parameter and n is the size of the training data set. Note that the regularization term does not include the biases. When λ is small, preference is given to minimizing the cost function, when λ is larger, preference is given to finding smaller weights.

The `SGD` function takes the training data given in `training_data`, the number of epochs specified by `epochs`, the size of the mini-batch specified by `mini_batch_size`, and the learning rate given by `eta`. If the evaluation data is given by the keyword argument `evaluation_data`, then these data are used estimate the network's accuracy after each epoch.

The function returns a 4-tuple of lists. The first list is a list of evaluation cost values: one value per epoch. The second list is a list of evaluation accuracy values: one value per epoch. The third list is a list of training cost values: one value per epoch. The fourth list is a list of training accuracy values: one value per epoch.

Let's create a 784x30x10 network with the cross-entropy cost function and train it on `train_d` for just 2 epochs with a mini-batch size of 10, a η of 0.5, a λ of 5.0. We will use `valid_d` as our evaluation data. In other words, these data are used to evaluate the evaluation cost and accuracy after each epoch.

```
>>> import network2
>>> net = network2.Network([784, 30, 10], cost=network2.CrossEntropyCost)
>>> net_stats = net.SGD(train_d, 2, 10, 0.5, lmbda=5.0,
                        evaluation_data=valid_d,
                        monitor_evaluation_cost=True,
                        monitor_evaluation_accuracy=True,
                        monitor_training_cost=True,
                        monitor_training_accuracy=True)
Epoch 0 training complete
Cost on training data: 1.32348339529
Accuracy on training data: 45866 / 50000
Cost on evaluation data: 4.33465283079
Accuracy on evaluation data: 9190 / 10000

Epoch 1 training complete
Cost on training data: 0.920509073127
Accuracy on training data: 46807 / 50000
Cost on evaluation data: 2.90920686448
Accuracy on evaluation data: 9358 / 10000
```

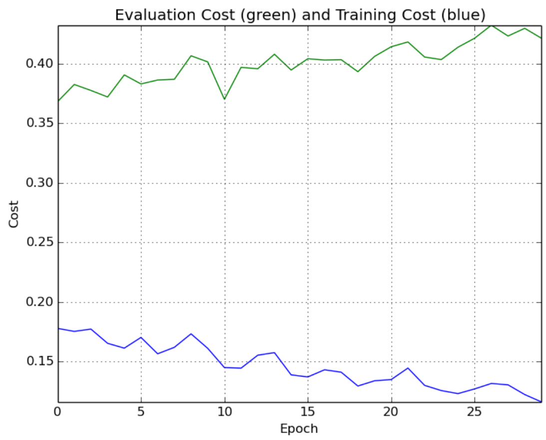
Let's inspect the contents of `net_stats`.

```
>>> net_stats
([4.3346528307886603, 2.9092068644832847],
 [0.919, 0.9358],
```

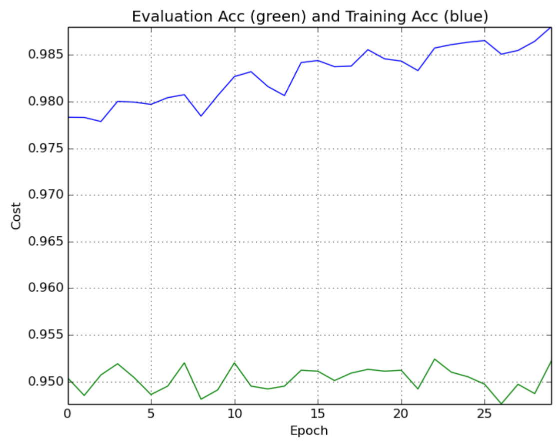
```
[1.3234833952859093, 0.92050907312657826],
[0.91732, 0.93614])
```

The first list contains 2 evaluation cost values (i.e., 1 value for each of the two epochs). The second list has 2 evaluation accuracy values (i.e., 1 value for each of the two epochs). The third list has 2 training cost values (i.e., 1 value for each of the two epochs). The fourth list has 2 training accuracy values (i.e., 1 value for each of the two epochs).

Let's develop two graphing tools you can use to estimate how well your net is training. The first tool is the function `plot_costs(eval_costs, train_costs, num_epochs)` that takes the first and third elements of the 4-tuple returned by `net.SGD()` and the number of epochs over which the ANN was trained with `SGD()` and returns a graph where the evaluation and training costs are plotted as the green and blue lines, respectively. Here is a graph generated by this function for an ANN trained over 30 epochs.



The second tool is a function `plot_accuracies(eval_accs, train_accs, num_epochs)` that takes the second and fourth elements of the 4-tuple returned by `net.SGD()` and the number of epochs over which the ANN was trained with `SGD()` and returns a graph where the evaluation and training accuracies are plotted as the green and blue lines, respectively. Here is a graph generated by this function for the same ANN trained over 30 epochs.



Let's now develop a few functions that will allow us to do systematic training of ANNs on MNIST. Write the function

```
collect_1_hidden_layer_net_stats(lower_num_hidden_nodes,
                                 upper_num_hidden_nodes,
                                 cost_function,
                                 num_epochs,
                                 mbs,
                                 eta,
                                 lmbda,
                                 train_data,
                                 eval_data)
```

Let the first two parameters have the values of l and u, respectively. The first two parameters specify the lower and upper bounds of the parameter n in a $784 \times n \times 10$ ANN for MNIST, where $l \leq n \leq u$. The values specified by the other parameters are evident from their names. In particular, `mbs` specifies the minimum batch size, `eta` is η, and `lmbda` is λ. We cannot use `lambda` as a parameter, because it's a Py keyword. This function returns a dictionary where the keys are the values of n and the values are the 4-tuples returned after training a created $784 \times n \times 10$ ANN with the values specified by parameters 3 through 9.

Let's go through an example. Let's train two networks ($784 \times 10 \times 10$ and $784 \times 11 \times 10$). The training stats for the first network are filed away under key `10` in the returned dictionary. The training stats for the second network are saved under key `11`.

```
>>> d = collect_1_hidden_layer_net_stats(10, 11,
                                         network2.CrossEntropyCost,
                                         2, 10, 0.1, 0.0,
                                         train_d, test_d)
>>> d
{10: ([1.359573119521807, 0.99817263661946443],
      [0.15252, 0.16918],
      [1.3932450994786085, 1.0170595802578979],
      [0.75024, 0.83708]),
 11: ([1.370898305189044, 0.95761536737711583],
      [0.14944, 0.16774],
      [1.3981990807392013, 0.96290041988665143],
      [0.73996, 0.84036])}
```

After you implement this function, use it to find and pickle the best $784 \times n \times 10$ ANN for MNIST, where $30 \leq n \leq 100$ and the training is done over 30 epochs. Pickle this ANN as `net1.pck`. Use your graphing functions to detect overfitting. Experiment with various values of λ and η. You may want to be systematic with λ and η values and write an auxiliary function that loops over λ and η ranges and calls `collect_1_hidden_layer_net_stats` for each possible combination of λ and η from the specified ranges.

Let's now write the same tool for 2-layer ANNs. Write the function

```
collect_2_hidden_layer_net_stats(lower_num_hidden_nodes,
                                 upper_num_hidden_nodes,
                                 cost_function,
                                 num_epochs,
                                 mbs,
                                 eta,
                                 lmbda,
                                 train_data,
                                 eval_data)
```

This function returns the same data structure as `collect_1_hidden_layer_net_stats` by creating and testing all $784 \times n_1 \times n_2 \times 10$ MNIST ANNs where $l \leq n_1, n_2 \leq u$. The first two parameters specify the values of l and u, respectively. This function constructs a dictionary where the keys are strings of the form `'x_y'`, where x and y are specific values of n_1 and n_2. Here is a sample test.

```
>>> d = collect_2_hidden_layer_net_stats(2, 3,
                                          network2.CrossEntropyCost,
                                          2, 10, 0.1, 0.0, train_d, valid_d)
>>> d['2_2']
([2.8617112909670452, 2.761559872222534],
 [0.2331, 0.2189], [2.860738768965704, 2.7646677631818055],
 [0.24584, 0.22082])
>>> d['2_3']
([2.5370767499523663, 2.3592176429391434],
 [0.3564, 0.3937],
 [2.5490976212297496, 2.372676815181264],
 [0.35792, 0.39044])
>>> d['3_2']
([2.819447702054708, 2.7555993049376193],
 [0.1979, 0.2588],
 [2.833450300330617, 2.7643062314776534],
 [0.20744, 0.2695])
>>> d['3_3']
([2.4301053165922357, 2.031793281662343],
 [0.3853, 0.5305],
 [2.429567226198757, 2.0365565490753634],
 [0.38722, 0.53174])
```

Use this function to find and pickle the best $784 \times n_1 \times n_2 \times 10$ ANN for MNIST, where $30 \leq n_1, n_2 \leq 100$ and the training is done over 30 epochs. Pickle this ANN as `net2.pck`. Use your graphing functions to detect overfitting. Experiment with various values of λ and η. Again, you may want to be systematic with λ and η values and write an auxiliary function that loops over λ and η ranges and calls `collect_1_hidden_layer_net_stats` for each possible combination of λ and η from the specified ranges.

Finally, let's systematically tackle 3-layer ANNs. Write the function

```
collect_3_hidden_layer_net_stats(lower_num_hidden_nodes,
                                 upper_num_hidden_nodes,
                                 cost_function,
                                 num_epochs,
                                 mbs,
                                 eta,
                                 lmbda,
                                 train_data,
                                 eval_data)
```

This function returns the same data structure as its two above counterparts. This function creates and tests all $784 \times n_1 \times n_2 \times n_3 \times 10$ MNIST ANNs where $l \leq n_1, n_2, n_3 \leq u$. The first two parameters have the values of l and u, respectively. This function constructs a dictionary where the keys are strings of the form `'x_y_z'`, where x, y, and z are legitimate values of n_1, n_2, and n_3, respectively. Here is a sample test.

```
>>> d = collect_3_hidden_layer_net_stats(2, 3,
                                        network2.CrossEntropyCost,
                                        2, 10, 0.1, 0.0, train_d, valid_d)
>>> d['2_2_2']
([2.616874650046163, 2.317136038701688],
 [0.3807, 0.4025],
 [2.633321254651256, 2.324513198743874],
 [0.3772, 0.40258])
>>> d['2_3_3']
([2.441494271658661, 2.2311953726499225],
 [0.3835, 0.3855],
 [2.438624182307029, 2.2245470998628067],
 [0.38602, 0.39346])
>>> d['3_3_3']
([2.1731375457234785, 1.7794279308657845],
 [0.5595, 0.5871],
 [2.201013781357794, 1.8243996186268596],
 [0.5458, 0.57372])
```

Use this function to find and pickle the best $784 \times n_1 \times n_2 \times n_3 \times 10$ ANN for MNIST, where $30 \leq n_1, n_2, n_3 \leq 100$ and the training is done over 30 epochs. Pickle this ANN as `net3.pck`. Use your graphing functions to detect overfitting. Experiment with various values of λ and η. Again, you may want to be systematic with λ and η values and write an auxiliary function that loops over λ and η ranges and calls `collect_1_hidden_layer_net_stats` for each possible combination of λ and η from the specified ranges.

What to Submit

1. `cs5600_6600_f19_hw04.py` with your code at the end: there are five function stubs at the end of this file that you need to complete. Write in your comments the best performance of your `net1.pck`, `net2.pck`, and `net3.pck`.

2. zipped directory `pck_nets` with `net1.pck`, `net2.pck`, and `net3.pck`.

Happy Hacking!