TORQUE RIPPLE MINIMIZATION IN PM SYNCHRONOUS MOTOR USING ITERATIVE LEARNING CONTROL

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Abstract – Permanent magnet synchronous motor (PMSM) drives are increasingly used in high-performance motion control applications where smooth torque is an essential requirement. However, presence of the air gap flux harmonics gives rise to undesirable torque pulsations. These torque pulsations result in performance deterioration in high-performance drive applications. In this paper, we present a systematic way to design and implement a new instantaneous torque controller using discrete iterative learning control (ILC) scheme with the objective of achieving torque ripple minimization. In the proposed scheme, the ILC-based dynamic torque controller compares the desired and the instantaneous motor torques and generates the reference *q*-axis current (i_{qs_ref}) that is required to produce ripple-free torque. The instantaneous motor torque is estimated by using an estimator based on MRAS technique. The proposed ILC torque controller is simple to implement from computational point of view. The effectiveness of the proposed torque controller is demonstrated through computer simulation studies and results obtained verify the effectiveness of the proposed scheme.

I. INTRODUCTION

For high-performance motion control systems such as the direct drive of industrial servo systems, robotics and machine tool applications permanent magnet synchronous motors (PMSM) have been an attractive choice because of its high power density, high efficiency and high torque-to-inertia ratio [1]. In direct drive systems, the rotor is directly coupled to the load and hence, the use of mechanical gearing is completely eliminated. Therefore, friction, backlash and other mechanical problems due to the gear-box are completely eliminated. However, with the absence of a gear-box, direct drives have to operate at relatively low speeds [2]. At low speeds, the effect of torque harmonics on motor speed that is normally filtered out at high speeds by the system inertia becomes significant. In particular for position servo applications, accuracy and repeatability cannot be guaranteed. One of the principal sources of torque harmonics in the PMSM is the harmonic contents in the air gap flux. In real PMSMs perfect sinusoidal flux density distribution around the air gap periphery is difficult to achieve. Thus, it results in a non-perfect sinusoidal flux linkage distribution which when interacts with the stator currents gives rise to torque pulsations [3].

In recent years, many control techniques have been proposed to minimize torque ripples, with varying degree of success. Broadly these techniques can be divided into two different groups: one on the motor design aspects and the other on active control of stator current excitations. It is the second approach, which has received much attention in recent years. Under the second approach, one way is to use pre-programmed stator current excitation to cancel the torque harmonic components [4], [5]. In [6] a high bandwidth current controller with an on-line adaptation scheme for identification of motor parameters is employed to produce high-frequency current components to suppress torque pulsations. An alternative approach is to use an instantaneous torque controller, which would produce the desired current signals on-line to minimize torque ripples [7]. Most of the approaches assume that sufficient accurate information of the PM synchronous motor parameters, in particular the ripple torque characteristics, are available. However, imperfect knowledge as well as variations in motor parameters limit the effectiveness of such control schemes.

In this paper, a new instantaneous torque control scheme has been proposed with the objective of minimizing torque ripples so as to improve the steady-state torque response of the drive system. The machine model for the PMSM in the synchronous rotating *d-q* reference frame is used. In an earlier paper [8] the authors have already reported the successful application of the ILC scheme for open-loop torque control of PMSM drive i.e. with the assumption that instantaneous torque feedback signal is readily available. In this paper, an extension of the earlier work i.e. complete closed-loop speed control of the PMSM drive is presented. The desired torque signal is obtained from a simple PI speed controller in the outer loop while the instantaneous torque feedback signal is obtained by using a torque estimator based on the model reference adaptive system (MRAS) technique as originally proposed in [7]. In steady-state, the torque is instantaneously controlled via the *proportional (P) - type discrete iterative learning controller* using the *previous cycle feedback (PCF)* algorithm. In the PCF, the previous

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cycle tracking error between the desired and the actual motor torque is stored in the dynamic memory of the controller, and is subsequently used to generate the reference q-axis current ($i_{qs\ ref}$) for the present cycle. The reference q-axis current together with the reference d-axis current ($i_{ds\ ref}$, which is maintained at zero for vector control of PMSM drive) are both fed to the conventional PI current controllers to generate the respective control voltages v_{ds} and v_{qs}. The performances of the proposed torque controller (ILC) have been evaluated through simulation studies and are presented in this paper. Simulation results obtained demonstrate that excellent steady-state torque performance can be obtained after a few iterations. In addition, a performance comparison of the proposed torque controller and the conventional PI torque controller has been carried out to demonstrate the effectiveness of the proposed controller.

II. Machine Model

A detailed mathematical model of the PMSM describing the torque harmonics has already been presented in [8]. For continuity sake few important equations are reproduced here. With assumptions that the PMSM is unsaturated and that eddy currents and hysteresis losses are negligible, the stator d, q axes voltage equations of the PMSM in the synchronous rotating reference frame are given by [7], [9]:

$$v_{ds} = Ri_{ds} + \frac{d}{dt}\lambda_{ds} - \omega_r \lambda_{qs} \qquad (1)$$

$$v_{qs} = Ri_{qs} + \frac{d}{dt}\lambda_{qs} + \omega_r \lambda_{ds} \qquad (2)$$

where

$$\lambda_{ds} = L_d i_{ds} + \psi_{dm} \qquad (3)$$

and

$$\lambda_{qs} = L_q i_{qs} \qquad (4)$$

v_{ds} and v_{qs} are the d, q-axes voltages, i_{ds} and i_{qs} are the d, q-axes stator currents, L_d and L_q are the d, q-axes inductances, λ_{ds} and λ_{qs} are the d, q-axes stator flux linkages, while R and ω_r are the stator resistance and rotor electrical angular velocity, respectively. ψ_{dm} is the flux linkage due to the rotor magnets linking the stator. In (3) and (4), it has been further assumed that the effects of the inductance harmonics and the mutual inductance between the d and q axes are negligible [3]. Also, as the surface mounted PMSM is non-salient, L_d and L_q are equal and are taken as L.

Due to the flux harmonics in the air gap, the resultant flux linkage between the permanent magnets and the stator currents contains harmonics of the order of 5, 7, 11, ... in the *abc* frame (triplen harmonics are absent in Y-connected 3-phase systems). In the synchronous rotating reference frame, the corresponding harmonics are of the order of 6, 12, ... and can be expressed as:

$$\Psi_{dm} = \Psi_{d0} + \Psi_{d6}\cos 6\theta_r + \Psi_{d12}\cos 12\theta_r + \ldots \qquad (5)$$

Ψ_{d0}, Ψ_{d6} and Ψ_{d12} are the fundamental, 6th and 12th harmonic terms of the d-axis flux linkage respectively while θ_r is the rotor electrical angle.

Using the method of field oriented control of the PMSM, the d-axis current is controlled to be zero to maximize the output torque. The motor torque is given by the following:

$$T_m = k_t i_{qs} = \frac{3}{2}\frac{P}{2}\psi_{dm} i_{qs} \qquad (6)$$

k_t is the torque constant and P is the number of poles in the motor. In terms of its harmonics, it can be expressed as [8]:

$$T_m = T_0 + T_6\cos 6\omega_r t + T_{12}\cos 12\omega_r t + \ldots \qquad (7)$$

T_0, T_6 and T_{12} are the fundamental, 6th and 12th harmonic terms of the output torque respectively. The equation of the motor dynamics is:

$$T_m = T_L + B\frac{\omega_r}{P} + \frac{J}{P}\frac{d}{dt}\omega_r \qquad (8)$$

T_L is the load torque, B is the damping coefficient and J is the moment of inertia. For dynamic simulations of the PMSM, (1) and (2) are expressed in their state-space forms.

III. The Discrete ILC Scheme

Iterative learning control is a relatively new approach to the problem of improving the tracking performance using previous experience in the face of design or modeling uncertainties in response to periodic disturbances in inputs. Much work has been done in recent years on the application of the ILC in servomechanism systems for industrial applications [10], [11]. For torque ripple minimization of electric motors, this technique can be applied to determine the reference current that is required to drive the error between the desired and the actual motor torque to a small value. The ILC scheme has been successfully applied for torque ripple minimization in switched reluctance motor [12].

In this paper, the P-type ILC employing the PCF algorithm is used. Using this algorithm in *continuous-time domain*, the learning law for generating the present control input $i_{qs\ ref}^{(i+1)}(t)$ at the $(i+1)^{th}$ trial is:

$$i_{qs\ ref}^{(i+1)}(t) = i_{qs\ ref}^{(i)}(t) + \beta e^{(i)}(t),\ 0 \leq t < T \qquad (9)$$

where

$$e^{(i)}(t) = T_{ref}^{(i)}(t) - T_m^{(i)}(t) \qquad (10)$$

$i_{qs\ ref}^{(i)}(t)$ is the reference current at the i^{th} trial, $T_{ref}^{(i)}(t)$ is the reference torque at the i^{th} trial, β is the learning gain, and T is the period of the torque ripple. However, in practice, the implementation of the ILC is in *discrete time domain.* Data sampling in digital signal processing inherently gives rise to one-step sampling delay that can lead to a potentially unstable

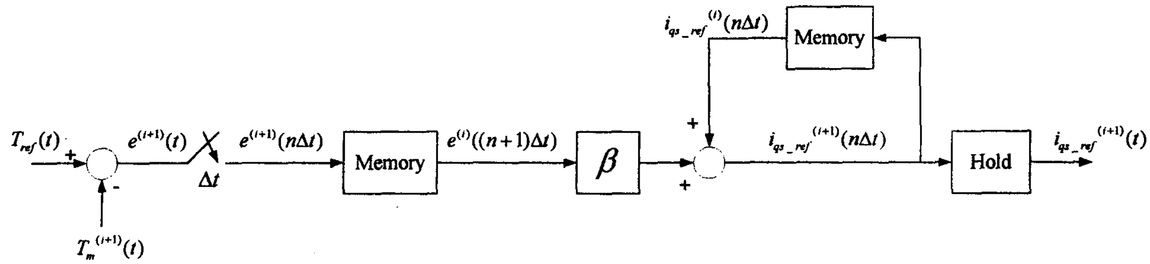


Fig. 1. Discrete - time ILC system configuration

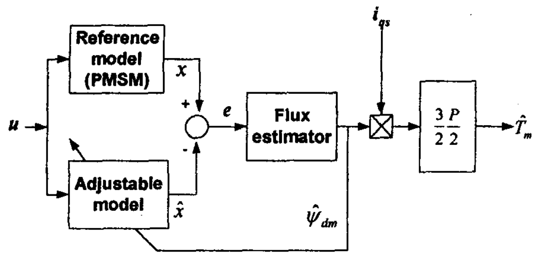


Fig. 2. MRAS torque estimator configuration

system. To make up for the one-step delay due to the digital computation and sampling, 'one-step ahead' compensation is adapted in the learning law [13]:

$$i_{qs\ ref}^{(i+1)}(n\Delta t) = i_{qs\ ref}^{(i)}(n\Delta t) + \beta e^{(i)}((n+1)\Delta t)) \tag{11a}$$

where

$$i_{qs\ ref}^{(i+1)}(t) = i_{qs\ ref}^{(i+1)}(n\Delta t),\ n\Delta t \le t < (n+1)\Delta t \tag{11b}$$

Δt is the sampling time, $n \in \{0, 1, 2, \ldots, N\text{-}1\}$ while N is the number of sampling intervals in one period of the torque ripple with $N\Delta t = T$. The learning controller "learns" the desired q-axis current profile so that $T_m(t)$ tracks $T_{ref}(t)$ closely. Fig. 1 shows the basic configuration of the discrete-time ILC scheme. To ensure convergence, β is selected from the inequality [8]

$$0 < \beta < \frac{2}{b_{\max}} \tag{12a}$$

where $0 < b(t) < b_{max}$. From (6),

$$T_m(t) = b(t) i_{qs}(t) \tag{12b}$$

$$b(t) = \frac{3}{2}\frac{P}{2}\psi_{dm}(t) \tag{12c}$$

Another requirement for convergence is that for every period of the torque ripple, at the beginning of each trial, the system is initialized such that

$$|\,T_{ref}(0) - T_m^{(i)}(0)\,| \le \epsilon \quad \text{for } i = 1, 2, \ldots \tag{13}$$

where $\epsilon > 0$ is a small but bounded error. Therefore, the initial value of the reference q-axis current can be determined with the knowledge of the load torque as shown below

$$i_{qs\_ref}^{(i)}(0) = \frac{T_{ref}(0)}{k_t}, \forall i \tag{14}$$

Such choice of the initial q-axis current reduces the number of iterations required to minimize the torque ripples.

IV. TORQUE ESTIMATOR

The torque transducer, apart from having a very limited bandwidth, is expensive for use in the industrial environments. Therefore, the instantaneous torque signal is obtained by means of estimation using the MRAS technique [7]. Fig. 2 shows the MRAS torque estimator configuration. The reference and adjustable models can be expressed, respectively, by:

$$\dot{x} = Ax + Bu + D\psi \tag{15}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + D\hat{\psi} - Fe \tag{16}$$

where

$$x = \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}, u = \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix}, \psi = \begin{bmatrix} \psi_{dm} \\ \psi_{qm} \end{bmatrix}, e = x - \hat{x}$$

$$A = \begin{bmatrix} -\frac{R}{L} & \omega_r \\ -\omega_r & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, D = \begin{bmatrix} 0 & -\frac{\omega_r}{L} \\ -\frac{\omega_r}{L} & 0 \end{bmatrix}$$

and F is the feedback gain matrix. ψ_{qm} is negligible and is taken to be zero. The adaptation rule to estimate the flux linkage ψ can be given as:

$$\dot{\hat{\psi}} = \gamma D^T G e \tag{17}$$

γ is the adaptation gain and G is a solution of the Lyapunov equation:

$$(A+F)^T G + G(A+F) = -Q \tag{18}$$

where Q is a positive definite matrix. Using the estimated flux linkage, the estimated torque is:

$$\hat{T}_m = \hat{k}_t i_{qs} = \frac{3}{2}\frac{P}{2}\hat{\psi}_{dm} i_{qs} \tag{19}$$

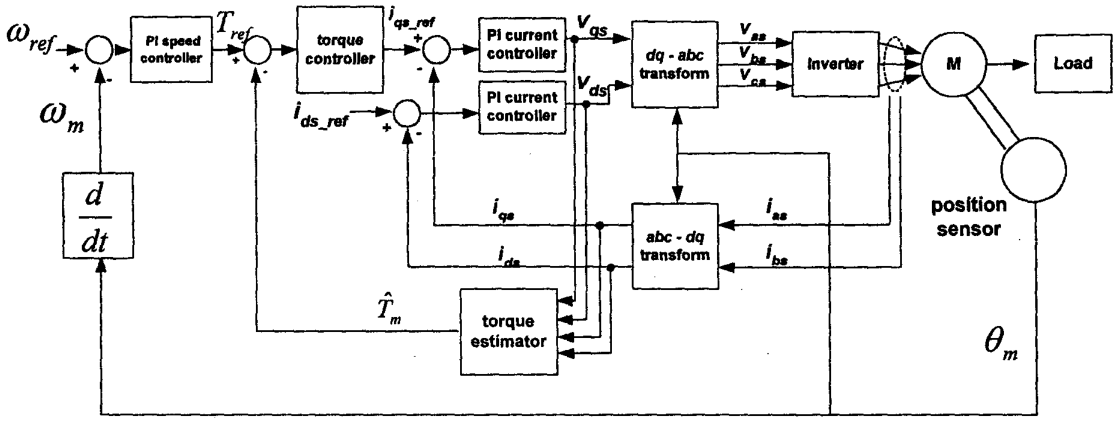


Fig. 3. Closed-loop speed control scheme for the PMSM drive system

TABLE 1: SPECIFICATIONS OF THE SURFACE MOUNTED TEST PMSM

Rated power	1.64 kW
Rated speed	2000 rpm
Rated torque	7.8 Nm
Stator resistance	2.125 Ω
Stator inductance	11.6 mH
Magnet flux	0.387 Wb
Number of poles	6
Inertia	0.00289 kgm^2

V. Implementation of the ILC Based PMSM Drive System

In [8], the open-loop torque control of the PMSM drive system using the torque iterative learning control scheme has been presented. In this paper, the speed loop is included. The speed controller in the outer loop provides the desired torque signal for the torque controller. Fig. 3 shows the closed-loop speed control scheme for the PMSM drive system. The sampling times are: current controller 250 μs, torque controller 500 μs and speed controller 2 ms. The torque controller can be either a conventional PI controller or the discrete-time ILC. The PI torque controller is used here just for the purpose of comparing with the performance of the ILC. For the simulation studies, it has been assumed that the flux linkage ψ_{dm} has 6th harmonic components, Ψ_{d6}, of 5% of the fundamental and the 12th harmonic component is negligible. However, if necessary the 12th harmonic can also be included (we have examined the ILC together with the 12th harmonic component). The ratio of the peak-to-peak torque to the average torque is called the torque ripple factor, *TRF* = T_{pp}/T_{ave}. The *TRF* is used as a performance criterion to evaluate the effectiveness of the proposed scheme for torque ripple minimization. The motor parameters of the surface-mounted PMSM are given in Table 1.

VI. Simulation

A. Determination of the Learning and Estimator Gains

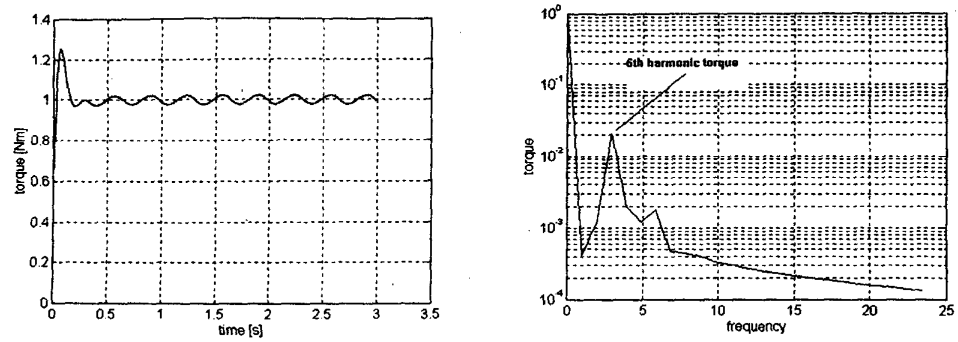
From a-priori knowledge of the range of the flux linkage, b_{max} corresponds to when $\psi_{dm}(t)$ is at its maximum. Therefore, from (12) the controller gain is taken to be as β = 1.1. The torque estimator gains are also determined from the motor parameters. The estimator feedback gain matrix in (16) and the solution of (18) are chosen as:

$$F=\begin{bmatrix}-817 & -\omega_r\\ \omega_r & -817\end{bmatrix},\; G=\begin{bmatrix}-\frac{1}{2000} & 0\\ 0 & -\frac{1}{2000}\end{bmatrix}$$

so that the resulting matrix $(A+F)$ is time-invariant and has stable poles at −1000 in the left-half of the s-plane. Q is taken as the 2×2 identity matrix. The adaptation gain γ is chosen as 1000.

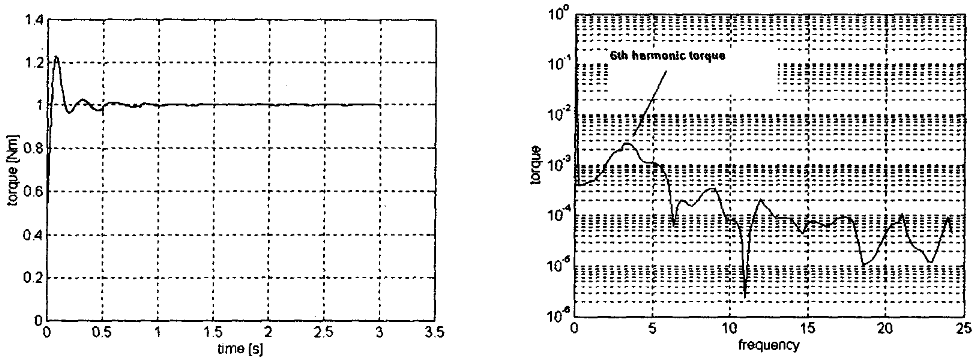
B. Simulation Results

Fig. 4 shows the motor torque responses using the PI torque controller at 10 rpm. Fig. 4(a) shows the motor instantaneous torque as a function of time whereas Fig. 4(b) shows the corresponding frequency spectrum of the torque signal. The PI torque controller generates a *TRF* of 4.2% and contains torque harmonics predominantly the 6th harmonic component. Fig. 5 shows the motor torque responses using the proposed iterative learning torque control scheme for a speed of 10 rpm. It can be seen from Fig. 5(a) that the corresponding torque response is much better than that shown in Fig. 4(a) and the corresponding *TRF* has been reduced to 0.1 %. Fig. 6 shows the speed responses of the PI and ILC schemes. It can be seen that the speed response with the proposed ILC torque controller has negligible speed ripples. Fig. 7(a) shows the actual and estimated torque at steady-state and the corresponding normalized torque estimation error at 10 rpm. It can be seen that at steady-state, the



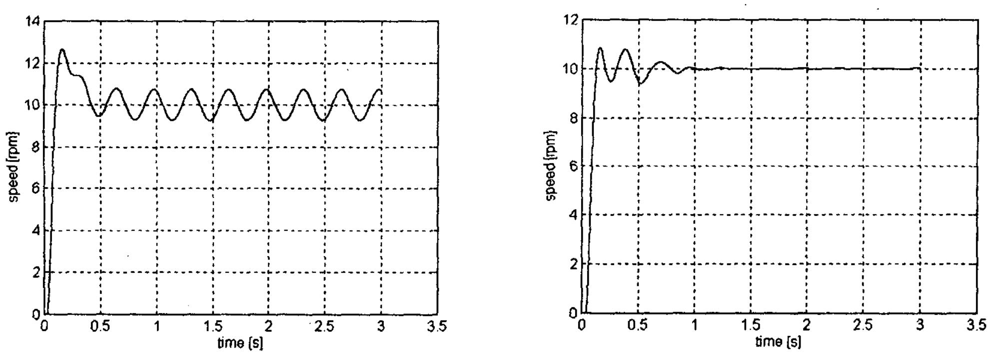
(a) (b)

Fig. 4. Torque responses using the PI torque control scheme



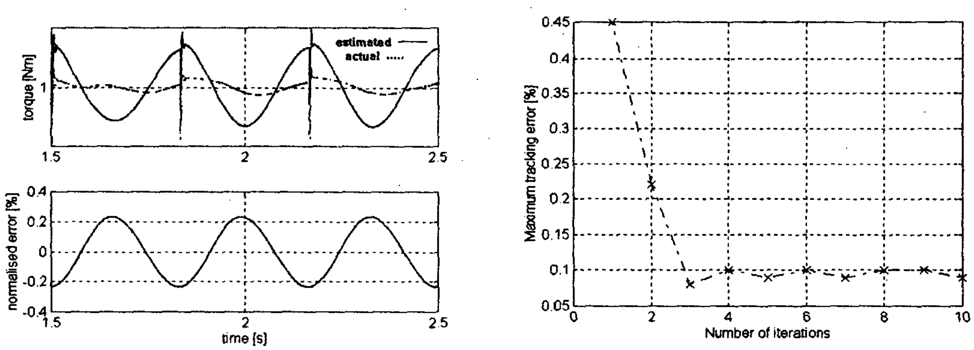
(a) (b)

Fig. 5. Torque responses using the proposed ILC scheme



(a) (b)

Fig. 6. Speed responses. (a) Using the PI torque control scheme. (b) Using the proposed ILC scheme



(a) (b)

Fig. 7. (a) Steady-state torque estimation error. (b) Maximum tracking error of T_m vs number of iterations.

corresponding normalized torque estimation error is less than 0.3 %. Fig. 7 (b) shows how the convergence of the ILC scheme takes place iteration by iteration. By the 3rd iteration, convergence to a certain degree of accuracy has been achieved. Fig. 8 shows the *TRF* of the two controllers at different

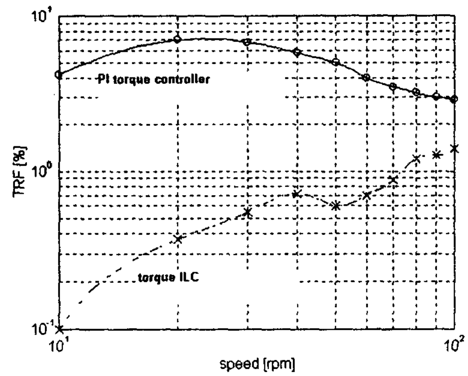


Fig. 8. *TRF* at different motor speeds (T_L=1Nm)

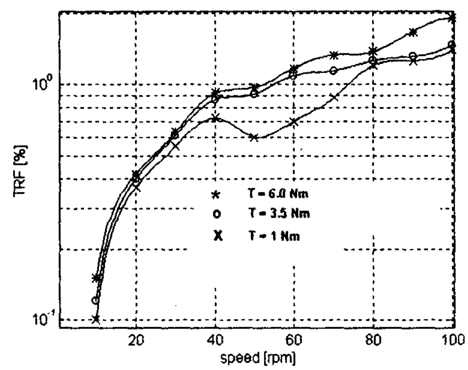


Fig. 9. *TRF* at different loads

speeds for a load torque of 1 N.m. It can be seen that at low motor speeds, the proposed scheme improves the *TRF* by at least one order of magnitude better. However, at higher speeds the ILC torque controller fails to provide better response than a conventional PI controller. This is because for a given ILC sampling period, the number of samples that the ILC 'learns' for each period of torque ripple decreases as the speed increases. Fig. 9 shows the TRF at different loads. It can be seen that the proposed ILC torque controller works satisfactorily over a wide range of load torques.

VII. CONCLUSIONS

The closed-loop control of the PMSM drive based on the iterative learning control scheme for torque ripple minimization has been presented and implemented through digital computer simulations. The proposed scheme guarantees convergence of the motor torque to the desired value and at the same time minimizes torque ripples. The torque ripples are significantly reduced by using the learning controller that repeatedly "learns" the required optimal reference current profile cycle-by-cycle by using previous "experience" of the reference torque and actual motor torque signals. The advantages of the proposed scheme are that it requires only minimal knowledge of the plant, minimal computational power and is easy to implement. The satisfactory simulation test results obtained validate the effectiveness of the proposed scheme in minimizing torque ripples. Real-time implementation of the proposed scheme is under progress.

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