Analysis of Direct Torque Control in Permanent Magnet Synchronous Motor Drives

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***Abstract*— This paper describes an investigation of direct torque control (DTC) for permanent magnet synchronous motor (PMSM) drives. It is mathematically proven that the increase of electromagnetic torque in a permanent magnet motor is proportional to the increase of the angle between the stator and rotor flux linkages, and, therefore, the fast torque response can be obtained by adjusting the rotating speed of the stator flux linkage as fast as possible. It is also shown that the zero voltage vectors should not be used, and stator flux linkage should be kept moving with respect to the rotor flux linkage all the time. The implementation of DTC in the permanent magnet motor is discussed, and it is found that for DTC using currently available digital signal processors (DSP's), it is advantageous to have a motor with a high ratio of the rated stator flux linkage to stator voltage. The simulation results verify the proposed control and also show that the torque response under DTC is much faster than the one under current control.**

***Index Terms*— Direct torque control, permanent magnet synchronous motor, saliency, sensorless control, stator flux linkage.**

I. Introduction

PERMANENT MAGNET synchronous motors (PMSM's) are used in many applications that require rapid torque response and high-performance operation. The torque in PMSM's is usually controlled by controlling the armature current based on the fact that the electromagnetic torque is proportional to the armature current. For high performance, the current control is normally executed in the rotor dq reference frame that rotates with the synchronous speed. In this frame, the armature inductances and magnet flux linkage are constant if the back electromotive force (EMF) and variation of inductances are sinusoidal. In addition to the influence of the harmonic terms in inductances and back EMF, saturation in flux, and temperature effect on the magnet, the torque response under current control is limited by the time constant of the armature winding. Recently, with the appearance of high-speed digital signal processors (DSP's), a control method called direct torque control (DTC) has become popular in induction motor drives [1], [2]. The basic principle of DTC is to directly select stator voltage vectors according to the differences between the reference and actual torque and stator flux linkage. The current controller followed by a pulse width modulation (PWM) comparator is not used in DTC systems, and the parameters of the motor are also not used, except the stator resistance. Therefore, the DTC possesses advantages such as lesser parameter dependence and fast torque response when compared with the torque control via PWM current control. Many papers on the torque control in PMSM's have appeared in recent years. Some use a torque controller in addition to current controllers [3], [4], while others use a torque controller to replace one of the current controllers [5], [6]. These, therefore, still suffer from the problems mentioned above. The application of DTC in permanent magnet synchronous motor (PMSM) drives has not yet been reported in the literature. This paper analyzes the torque production of PMSM's in the stationary flux linkage xy reference frame and finds that the concept of DTC can also be applied in the PMSM drive. The implementation aspects are discussed, and the simulation of a proposed DTC has been carried out. The simulation results show that the torque response of DTC is much faster than the one of vector control, as is the case for induction motors. Since only the stator resistance is used in the controller, it is expected that the effect of the nonsinusoidal back EMF and saturation will be decreased. Sensorless operation of PMSM's under DTC is possible if the initial position of the rotor is known approximately.

In this paper, the torque expression in terms of the stator flux linkage and its angle, with respect to the rotor flux linkage, is derived first. Then, the control of the amplitude and rotating speed of the stator flux linkage are analyzed. Finally, a PMSM drive with DTC is proposed and demonstrated by simulation.

II. Motor Equations in the Stator Flux Reference Frame

The stator flux linkage vector $\boldsymbol{\varphi_s}$ and rotor (magnet) flux linkage vector $\boldsymbol{\varphi_f}$ can be drawn in the rotor flux (dq), stator flux (xy), and stationary (dq) reference frames, as in Fig. 1.

The angle between the stator and rotor flux linkages δ is the load angle when the stator resistance is neglected. In the steady state, δ is constant corresponding to a load torque, and both stator and rotor flux rotate at the synchronous speed. In transient operation, δ varies and the stator and rotor flux rotate at different speeds. Since the electrical time constant is normally much smaller than the mechanical time constant, the rotating speed of stator flux, with respect to the rotor flux, can be easily changed. It is shown in this section that the increase of torque can be controlled by controlling the change of δ or the rotating speed of the stator flux.

The well-known stator flux linkage, voltage, and electromagnetic torque equations in the dq reference frame are as

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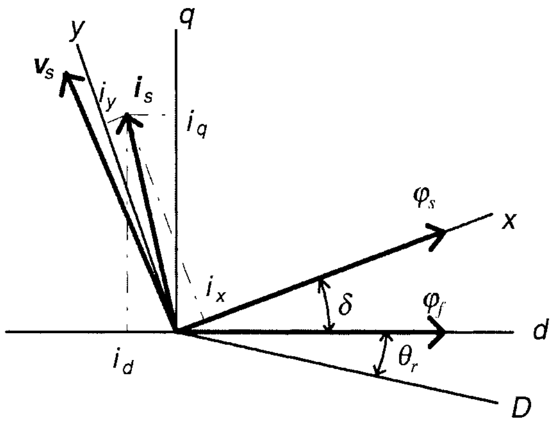


Fig. 1. The stator and rotor flux linkages in different reference frames.

follows:

$$\begin{aligned} \varphi_d &= L_d i_d + \varphi_f \\ \varphi_d &= L_q i_q \end{aligned} \tag{1}$$

$$\begin{aligned} v_d &= R_s i_d + p\varphi_d - \omega_r \varphi_q \\ v_q &= R_s i_q + p\varphi_q + \omega_r \varphi_d \end{aligned} \tag{2}$$

$$T = \tfrac{3}{2} p(\varphi_d i_q - \varphi_q i_d) \tag{3}$$

where φ_f, L_d, and L_q are the armature (or stator) back EMF constant and inductances, respectively, when the back EMF and the variation of the stator inductances are sinusoidal. Otherwise, these are the fundamental quantities of these variables. With the transformation in (4) and (5), (1)–(3) can be transformed to the xy reference frame:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} F_d \\ F_q \end{bmatrix}. \tag{4}$$

The inverse transformation is

$$\begin{bmatrix} F_d \\ F_q \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{5}$$

where F represents the voltage, current, and flux linkage.

A. The Torque Equation in xy Reference Frame

From Fig. 1, it can be found that

$$\begin{aligned} \sin\delta &= \frac{\varphi_q}{|\varphi_s|} \\ \cos\delta &= \frac{\varphi_d}{|\varphi_s|} \end{aligned} \tag{6}$$

where $|\varphi_s|$ represents the amplitude of the stator flux linkage.

Substituting (5) and (6) for current into (2) gives

$$\begin{aligned} T &= \frac{3}{2} p[\varphi_d(i_x \sin\delta + i_y \cos\delta) - \varphi_q(i_x \cos\delta - i_y \sin\delta)] \\ &= \frac{3}{2} p\left[i_x \frac{\varphi_d \varphi_q}{|\varphi_s|} + i_y \frac{\varphi_d^2}{|\varphi_s|} - i_x \frac{\varphi_d \varphi_q}{|\varphi_s|} + i_y \frac{\varphi_d^2}{|\varphi_s|}\right] = \frac{3}{2} p|\varphi_s| i_y. \end{aligned} \tag{7}$$

Equation (7) means that the torque is directly proportional to the y-axis component of the stator current if the amplitude of the stator flux linkage is constant.

B. The Flux Linkage Equations in the xy Reference Frame

Equation (3) can be rewritten into matrix form as follows:

$$\begin{bmatrix} \varphi_d \\ \varphi_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \varphi_f \\ 0 \end{bmatrix}. \tag{8}$$

Substituting (5) into (8) gives

$$\begin{aligned} &\begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} \\ &\quad = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \begin{bmatrix} \varphi_f \\ 0 \end{bmatrix}. \end{aligned} \tag{9}$$

Premultiplying (9) with

$$\begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix}^{-1} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \tag{10}$$

gives (11), shown at the bottom of the page.

1) PMSM's with Uniform Airgap: For this type of PMSM, $L_d = L_q = L_s$, (11) can be simplified as in (12)

$$\begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix} \tag{12}$$

or

$$\begin{aligned} \varphi_x &= L_s i_x + \varphi_f \cos\delta \\ \varphi_y &= L_s i_y - \varphi_f \sin\delta. \end{aligned} \tag{13}$$

φ_y is zero since the x-axis is fixed at the stator flux linkage. Then, i_y can be solved from the second equation of (13)

$$i_y = \frac{1}{L_s} \varphi_f \sin\delta. \tag{14}$$

Substituting (14) into torque (7) gives

$$T = \frac{3}{2} \frac{1}{L_s} p|\varphi_s||\varphi_f| \sin\delta = \frac{3}{2} \frac{1}{L_s} p|\varphi_s|\varphi_f \sin\dot{\delta}t \tag{15}$$

where $\dot{\delta}$ is the angular velocity of the stator flux linkage relative to magnet flux linkage.

Equation (15) implies that the torque increases with the increase in δ if the amplitude of the stator flux linkage is kept constant and δ is controlled within the range of $-\pi/2$–$\pi/2$. The maximum torque occurs when δ is $\pi/2$.

$$\begin{aligned} \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} &= \begin{bmatrix} L_d \cos\delta & L_q \sin\delta \\ -L_d \sin\delta & L_q \cos\delta \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix} \\ &= \begin{bmatrix} L_d \cos^2\delta + L_q \sin^2\delta & -L_d \sin\delta\cos\delta + L_q \sin\delta\cos\delta \\ -L_d \sin\delta\cos\delta + L_q \sin\delta\cos\delta & L_d \sin^2\delta + L_q \cos^2\delta \end{bmatrix} \begin{bmatrix} i_x \\ i_y \end{bmatrix} + \varphi_f \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix}. \end{aligned} \tag{11}$$

$\dot{\delta}$ is considered to be a step change corresponding to a change of voltage vector. Then, the derivative of (15) becomes

$$\left.\frac{dT}{dt}\right|_{t=0} = \frac{3}{2}p\frac{|\varphi_s|\varphi_f}{L_s}\dot{\delta}\cos\delta. \tag{16}$$

The right-hand side of (16) is always positive if δ is within the range of $-\pi/2$–$\pi/2$. This equation implies that the increase of torque is proportional to the increase of the angle δ, which is the angle between the stator and magnet flux linkages. In other words, the stator flux linkage should be controlled in such a way that the amplitude is kept constant and the rotating speed is controlled as fast as possible to obtain the maximum change in actual torque.

2) PMSM's with Pole Saliency: For a PMSM with pole saliency, that is, $L_d \neq L_q$, the torque equation in terms of stator flux linkage and angle δ can be obtained by solving i_x from (12), with $\varphi_y = 0$:

$$i_x = \frac{2\varphi_f \sin\delta - [(L_d + L_q) + (L_d - L_q)\cos 2\delta]}{(L_q - L_d)\sin 2\delta} i_y. \tag{17}$$

Substituting (17) into the first equation in (11), one obtains

$$i_y = \frac{1}{2L_d L_q}[2\varphi_f L_q \sin\delta - |\varphi_s|(L_q - L_d)\sin 2\delta]. \tag{18}$$

Then, the torque equation is as follows:

$$T = \frac{3p|\varphi_s|}{4L_d L_q}[2\varphi_f L_q \sin\delta - |\varphi_s|(L_q - L_d)\sin 2\delta]. \tag{19}$$

Equation (19) consists of two terms. The first is the excitation torque, which is produced by the permanent magnet flux, and the second term is the reluctance torque. For each stator flux linkage, there exists the maximum in this equation. It will not be discussed how to control the amplitude of stator flux linkage and load angle to get maximum torque in this paper. However, it is necessary to discuss the relationship between the amplitude of stator flux linkage and the derivative of the torque. Figs. 2–5 show the torque-δ characteristics when the amplitude of stator flux linkage is at $0.7\varphi_f, \varphi_f, 1.5\varphi_f$, and $2\varphi_f$. Note the torque near the zero crossings. In Fig. 5, the derivative of torque near the zero cross is negative with respect to δ, which implies that DTC cannot be applied in this case. Therefore, for a PMSM with pole saliency, the amplitude of the stator flux linkage should be changed, with the change of actual torque even for constant torque operation.

The derivative of torque in (20) is as shown in (21), with constant stator flux and $d\delta/dt$:

$$\frac{dT}{dt} = \frac{3p|\varphi_s|}{4L_d L_q}[2\varphi_f L_q \dot{\delta}\cos\delta - 2|\varphi_s|(L_q - L_d)\dot{\delta}\cos 2\delta]. \tag{20}$$

At $t = 0$:

$$\left.\frac{dT}{dt}\right|_{t=0} = \frac{3p|\varphi_s|}{2L_d L_q}[\varphi_f L_q \dot{\delta} - |\varphi_s|(L_q - L_d)\dot{\delta}]. \tag{21}$$

The condition for dT/dt for positive $d\delta/dt$ is

$$|\varphi_s| < \frac{L_q}{L_q - L_d}\varphi_f. \tag{22}$$

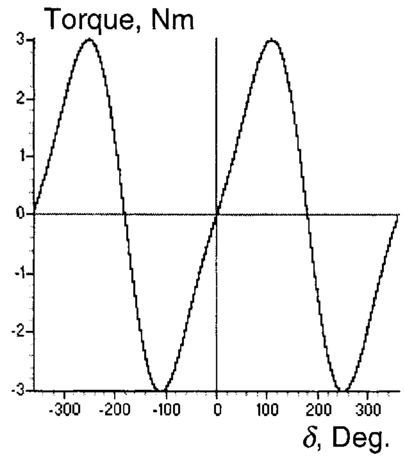


Fig. 2. Torque with respect to δ: $|\varphi_s| = 0.7\varphi_f$.

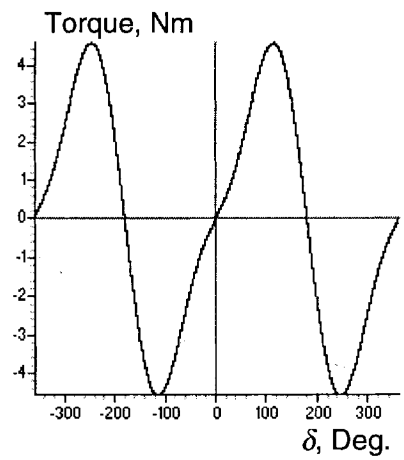


Fig. 3. Torque with respect to δ: $|\varphi_s| = \varphi_f$.

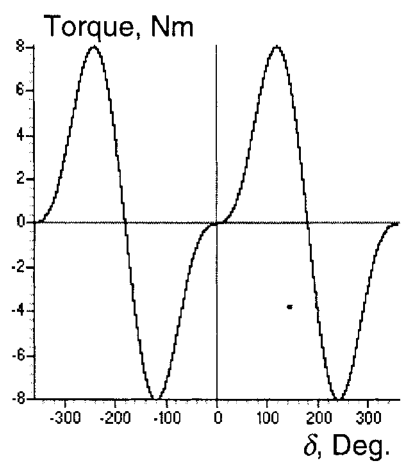


Fig. 4. Torque with respect to δ: $|\varphi_s| = 1.5\varphi_f$.

The amplitude of the stator flux linkage should be chosen according to (23) if fast dynamic response is desired. Otherwise, it should be varied with the change of actual torque if the linearity is more important. It should also be kept in mind that for the same torque, a higher stator current is needed when the amplitude of the stator flux linkage is lower.

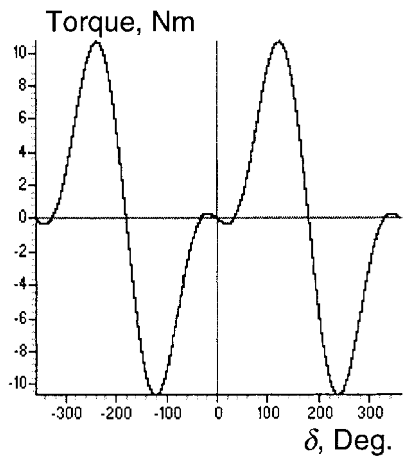


Fig. 5. Torque with respect to δ: $|\varphi_s| = 2\varphi_f$.

III. Control of Stator Flux Linkage by Selecting the Proper Stator Voltage Vector

In the previous section, it has been proven that the change of torque can be controlled by keeping the amplitude of the stator flux linkage constant and increasing the rotating speed of the stator flux linkage as fast as possible. It will be shown in this section that both the amplitude and rotating speed of the stator flux linkage can be controlled by selecting the proper stator voltage vectors.

The primary voltage vector $\boldsymbol{v}_s$ is defined by the following equation:

$$\boldsymbol{v}_s = \tfrac{2}{3}(v_a + v_b e^{j(2/3)\pi} + v_c e^{j(4/3)\pi}) \tag{23}$$

where v_a, v_b, and v_c are the instantaneous values of the primary line-to-neutral voltages. When the primary windings are fed by an inverter, as shown in Fig. 6, the primary voltages v_a, v_b, and v_c are determined by the status of the three switches, S_a, S_b, and S_c. v_a is connected to $V_{\rm dc}$ if S_a is one, otherwise, v_a is connected to zero—similar for v_b and v_c. Therefore, there are six nonzero voltage vectors: $\boldsymbol{V}_1(100), \boldsymbol{V}_2(110), \cdots$, and $\boldsymbol{V}_6(101)$ and two zero voltage vectors: $\boldsymbol{V}_7(000)$ and $\boldsymbol{V}_8(111)$. The six nonzero voltage vectors are 60° apart from each other as in Fig. 7. These eight voltage vectors can be expressed as

$$\boldsymbol{v}_s(S_a, S_b, S_c) = \tfrac{2}{3}V_{\rm dc}(S_a + S_b e^{j(2/3)\pi} + S_c e^{j(4/3)\pi}) \tag{24}$$

where $V_{\rm dc}$ is the dc-link voltage and 2/3 is the factor of Park Transformation.

A. The Control of the Amplitude of Stator Flux Linkage.

The stator flux linkage of a PMSM that can be expressed in the stationary reference frame is

$$\boldsymbol{\varphi_s} = \int (\boldsymbol{v}_s - R\boldsymbol{i}_s)\, dt. \tag{25}$$

During the switching interval, each voltage vector is constant, and (25) is then rewritten as in (26):

$$\boldsymbol{\varphi_s} = \boldsymbol{v}_s t - R\int \boldsymbol{i}_s\, dt + \varphi_{s|t=0}. \tag{26}$$

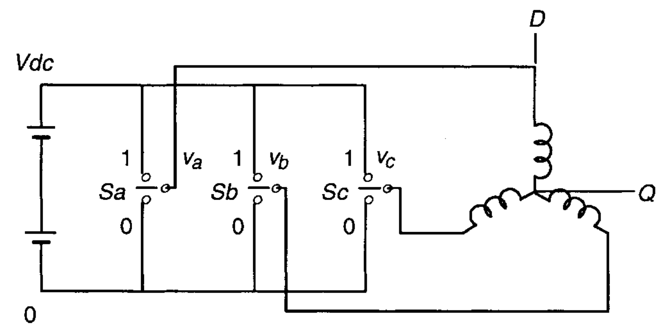


Fig. 6. An inverter-fed PMSM.

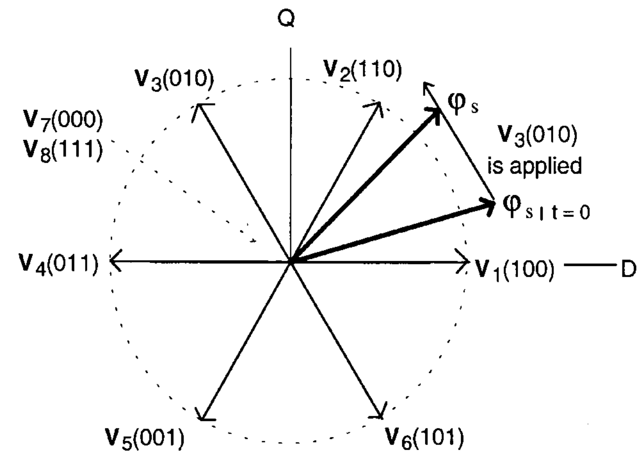


Fig. 7. The movement of the end of stator flux linkage.

Neglecting the stator resistance, (26) implies that the end of the stator flux vector $\boldsymbol{\varphi_s}$ will move in the direction of the applied voltage vector, as shown in Fig. 7. $\varphi_s|t = 0$ is the initial stator flux linkage at the instant of switching. To select the voltage vectors for controlling the amplitude of the stator flux linkage, the voltage vector plane is divided into six regions, as shown in Fig. 8. In each region, two adjacent voltage vectors, which give the minimum switching frequency, are selected to increase or decrease the amplitude of $\boldsymbol{\varphi_s}$, respectively. For instance, vectors $\boldsymbol{V}_2$ and $\boldsymbol{V}_3$ are selected to increase and decrease the amplitude of $\boldsymbol{\varphi_s}$ when $\boldsymbol{\varphi_s}$ is in region one and is rotating in a counter-clockwise direction. In this way, $\boldsymbol{\varphi_s}$ can be controlled at the required value by selecting the proper voltage vectors. Fig. 8 shows how the voltage vectors are selected for keeping $\boldsymbol{\varphi_s}$ within a hysteresis band when $\boldsymbol{\varphi_s}$ is rotating in the counter-clockwise direction.

B. The Control of the Rotation of $\boldsymbol{\varphi_s}$

It is seen from (26) that $\boldsymbol{\varphi_s}$ will stay at its original position when zero voltage vectors are applied. This is true for an induction motor since the stator flux linkage is uniquely determined by the stator voltage. In the case of a PMSM, $\boldsymbol{\varphi_s}$ will change even when the zero voltage vectors are applied since the magnets rotate with the rotor. Therefore, zero voltage vectors are not used for controlling $\boldsymbol{\varphi_s}$ in PMSM. In other words, $\boldsymbol{\varphi_s}$ should always be in motion with respect to the rotor flux linkage.

TABLE I
THE SWITCHING TABLE FOR INVERTER

ϕ	τ	θ					
		$\theta(1)$	$\theta(2)$	$\theta(3)$	$\theta(4)$	$\theta(5)$	$\theta(6)$
$\phi = 1$	$\tau = 1$	$\mathbf{V}_2(110)$	$\mathbf{V}_3(010)$	$\mathbf{V}_4(011)$	$\mathbf{V}_5(001)$	$\mathbf{V}_6(101)$	$\mathbf{V}_1(100)$
	$\tau = 0$	$\mathbf{V}_6(101)$	$\mathbf{V}_1(100)$	$\mathbf{V}_2(110)$	$\mathbf{V}_3(010)$	$\mathbf{V}_4(011)$	$\mathbf{V}_5(001)$
$\phi = 0$	$\tau = 1$	$\mathbf{V}_3(010)$	$\mathbf{V}_4(011)$	$\mathbf{V}_5(001)$	$\mathbf{V}_6(101)$	$\mathbf{V}_1(100)$	$\mathbf{V}_2(110)$
	$\tau = 0$	$\mathbf{V}_5(001)$	$\mathbf{V}_6(101)$	$\mathbf{V}_1(100)$	$\mathbf{V}_2(110)$	$\mathbf{V}_3(010)$	$\mathbf{V}_4(011)$

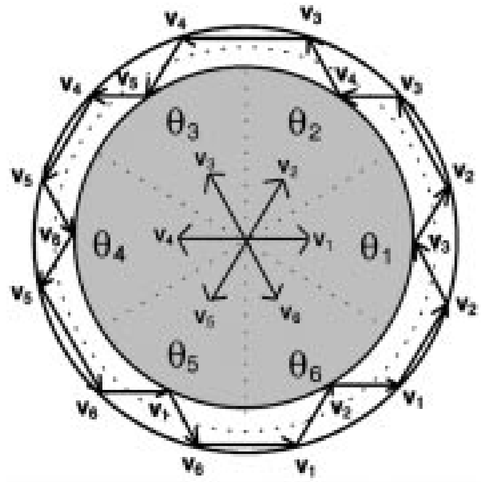


Fig. 8. The control of the stator flux linkage.

According to the torque (17) and (19), the electromagnetic torque can be controlled effectively by controlling the amplitude and rotational speed of $\boldsymbol{\varphi_s}$. For counter-clockwise operation, if the actual torque is smaller than the reference, the voltage vectors that keep $\boldsymbol{\varphi_s}$ rotating in the same direction are selected. The angle δ increases as fast as it can, and the actual torque increases as well. Once the actual torque is greater than the reference, the voltage vectors that keep $\boldsymbol{\varphi_s}$ rotating in the reverse direction are selected instead of the zero voltage vectors. The angle δ decreases, and the torque decreases also. By selecting the voltage vectors in this way, $\boldsymbol{\varphi_s}$ is rotated all the time and its rotational direction is determined by the output of the hysteresis controller for the torque.

The switching table for controlling both the amplitude and rotating direction of $\boldsymbol{\varphi_s}$ is as follows and is used for both directions of operations.

In Table I, ϕ and τ are the outputs of the hysteresis controllers for flux linkage and torque, respectively. If $\phi = 1$, then the actual flux linkage is smaller than the reference value. The same is true for the torque. $\theta(1)$–$\theta(6)$ are the region numbers for the stator flux linkage positions.

IV. IMPLEMENTATION OF DTC FOR A PMSM DRIVE

A. System Configuration

The block diagram of a PMSM drive with DTC may be as shown in Fig. 9, where the shaded part is in hardware and the remaining parts are implemented in software with a DSP. The three-phase variables are transformed into dq axes variables

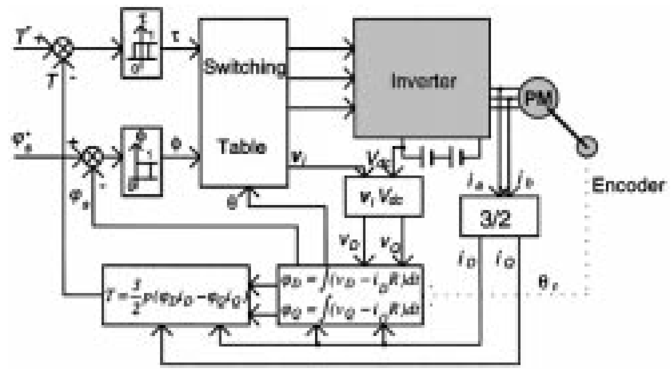


Fig. 9. The block diagram of a PMSM drive with DTC.

with the following transformation:

$$\begin{bmatrix} f_D \\ f_Q \\ f_0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \tag{27}$$

where f represents the stator currents, voltages, and flux linkages. f_0 is zero for symmetrical stator windings.

The DQ axes currents, i_D and i_Q, can be obtained from the measured three-phase currents, and voltages v_D and v_Q are calculated from dc-link voltage since the voltage vectors determined by the switching table are known.

The flux linkages $\varphi_D(k)$ and $\varphi_Q(k)$ at the kth sampling instant are calculated from the integration of the stator voltages as follows:

$$\varphi_D(k) = \varphi_{D/k-1} + (V_{D|k-1} - Ri_D)T_s \tag{28}$$

$$\varphi_Q(k) = \varphi_{Q|k-1} + (V_{Q|k-1} - Ri_D)T_s \tag{29}$$

$$\varphi_s(k) = \sqrt{\varphi_D^2(k) + \varphi_Q^2(k)} \tag{30}$$

where T_s is the sampling interval and the variables with subscript $k-1$ are the previous samples.

The initial values of φ_D and φ_Q are not zero for PMSM's. These may be obtained from an encoder. Subsequently, the encoder is not needed. If the initial values of φ_D and φ_Q can be estimated, the encoder may entirely be eliminated. The synchronism between the stator voltage and rotor position is achieved by keeping the stator flux linkage an angle of δ ahead or behind the rotor flux linkage.

The torque in (3) can be rewritten in the stationary reference frame as

$$T(k) = \tfrac{3}{2}p\{\varphi_D(k)i_Q(k) - \varphi_Q(k)i_D(k)\}. \tag{31}$$

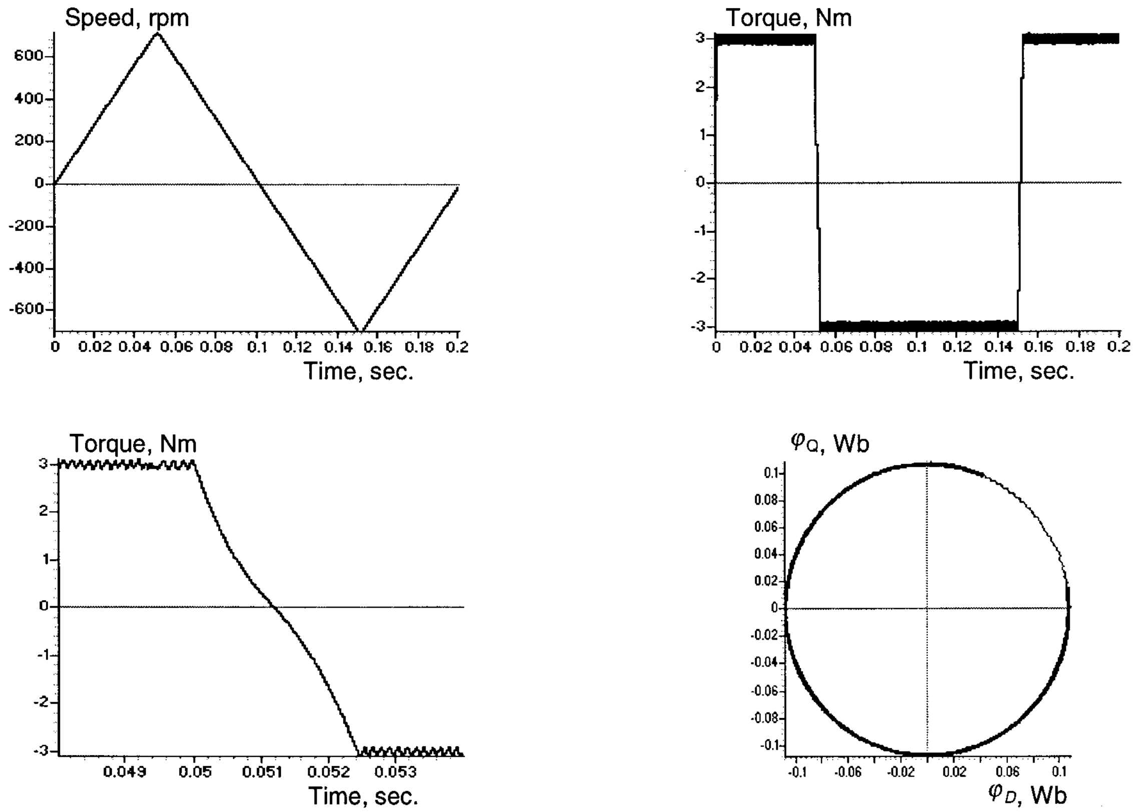


Fig. 10. Dynamic responses of a PMSM drive with DTC: $T_s = 10$ μs.

TABLE II
DQ AXES VOLTAGES

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
v_D	V_d	0.5 V_d	-0.5 V_d	- V_d	-0.5 V_d	0.5 V_d	0	0
v_Q	0	0.866 V_d	0.866 V_d	0	-0.866 V_d	-0.866 V_d	0	0

The reference torque is obtained from the output of the speed controller and is limited at a certain value, with respect to a given reference flux linkage, which guarantees the stator current not to exceed the limit value. For a PMSM with no pole saliency, the stator flux linkage can be kept at its rated value for constant torque operation, while for a PMSM with pole saliency, the reference flux linkage should increase with the actual torque for positive slope with respect to δ and for good linearity, as indicated by (23) and Figs. 2–5.

It is seen from Fig. 9 that the DTC controller is independent of motor parameters except for the stator resistance, which affects only the low-speed performance of the drive and can be compensated. The inductances and back EMF constant, which change with the saturation and temperature, respectively, are not used in the controller, and, therefore, there is no need to compensate for the saturation and back EMF constant variation.

B. Modeling of a PMSM Drive with DTC

The system shown in Fig. 9 has been modeled with the speed loop open. To verify the analysis in the previous section, a PMSM with pole saliency is used in simulation. The amplitude of the stator flux linkage is kept the same as the magnet flux linkage, and the reference torque changes abruptly from 3 to -3 Nm at $t = 0.05$ s and from -3 to 3 Nm at $t = 0.15$ s. In modeling, it is assumed that the rotor starts at its initial position $\theta_r = 0$ [region $\theta(1)$], and thus, no encoder is used in simulation. Figs. 10 and 11 show the dynamic responses of the system for sampling time $T_s = 10$ and 100 μs, respectively. It is seen from Fig. 10 that the stator flux linkage is controlled at its required value quite well. The trajectory of φ_D and φ_Q is a circle as expected. The actual torque is controlled within the bandwidth and follows the reference rapidly when compared with the current control system. However, the change of torque is not linear when the torque approaches zero, which agrees with the analysis in the previous section and Fig. 3. If the linearity has priority to the fast dynamic, the stator flux linkage reference should be changed with the change of torque.

The simulation results reveal one important difficulty in implementing the DTC for the PMSM. If the stator flux linkage

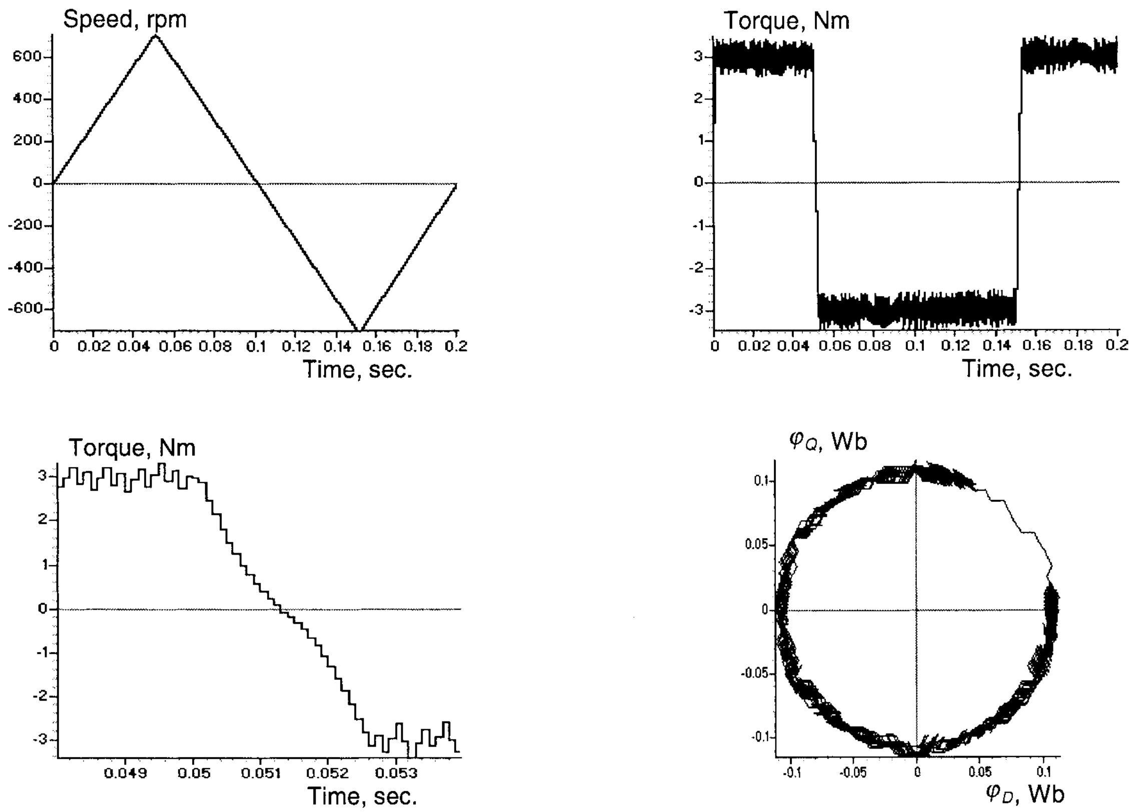


Fig. 11. Dynamic responses of a PMSM drive with DTC: $T_s = 100$ μs.

$\boldsymbol{\varphi_s}$ is small, as is the case with low-voltage, high-speed motors, the low value of stator flux linkage implies that the sampling interval must be very small, on the order of 40 μs or less. For standard induction motors, this problem [40] does not arise because of the sufficiently large value of the flux linkage for the standard voltages and speeds of these motors. With DTC, the stator voltage vector changes every sampling time instead of every switching time as in a PWM current-controlled drive. According to (26), the change of stator flux linkage is equal to the product of dc-link voltage and sampling time. The bandwidth of the flux linkage hysteresis controller is normally set at 5% of the rated value. Therefore, the sampling time should be very small for controlling the flux linkage properly, as shown in the simulation results of Fig. 10 for which the sampling interval is 10 μs. If the sampling interval is not small enough, the change of flux linkage in each sampling time will exceed the bandwidth, as is the case in Fig. 11. It is known that the sampling frequency of 10 kHz is sufficient for an induction motor to control the stator flux linkage. Examining the parameters of the induction motor, it is found that the ratio of stator flux linkage to dc-link voltage is 1.08/678 (1.6×10^{-3}), which is 2 times of that of PMSM I in Table III and 13 times of that of PMSM II in Table IV. For applying the DTC in a PMSM drive, one PMSM, with the desirable ratio of flux linkage to dc-link voltage, has been built in which a standard induction motor stator is used, and the stator flux linkage is designed to have the same rated value as the induction motor. The data for this motor (PMSM III) is in Table V.

TABLE III
DATA OF PMSM I

Number of pole pairs	p	2
Armature resistance	R	0.57 Ω
Magnet flux linkage	φ_f	0.108 Wb
d-axis inductance	L_d	8.72 mH
q-axis inductance	L_q	22.8 mH
Phase voltage	V	50 V
Phase current	I	8.66 A
Base speed	ω_b	1200 rpm

TABLE IV
DATA OF PMSM II

Number of pole pairs	p	4
Armature resistance	R	0.34 Ω
Magnet flux linkage	φ_f	0.022 Wb
d-axis inductance	L_d	2.5 mH
q-axis inductance	L_q	2.5 mH
Phase voltage	V	57 V
Phase current	I	16 A
Base speed	ω_b	4000 rpm

C. The Comparison of the Torque Response Between DTC and PWM Current Control

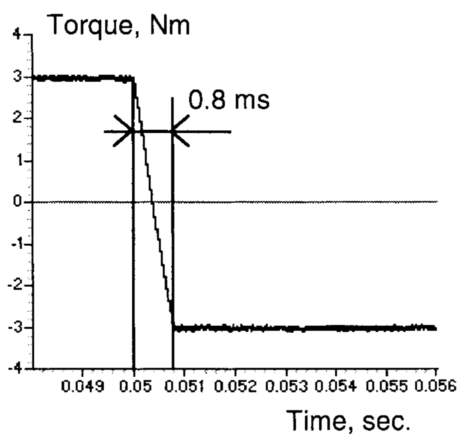
To examine the performance of DTC, simulations on a PMSM with no saliency in Table VI (PMSM IV) under DTC and under PWM current control have been carried out. For DTC, the stator flux linkage is kept at its rated value, while for current control, i_d is kept at zero. In both cases, the reference

TABLE V
DATA OF PMSM III

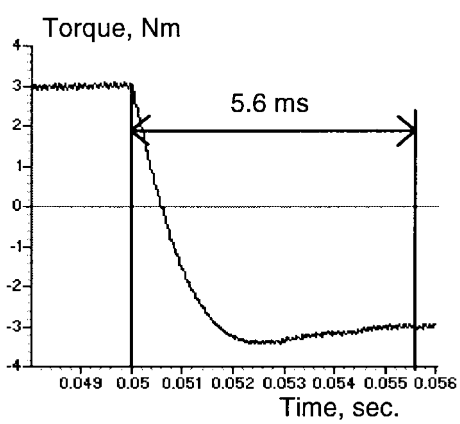
Number of pole pairs p	2
Armature resistance R	18.6 Ω
Magnet flux linkage φ_f	0.447 Wb
d-axis inductance L_d	0.3885 H
q-axis inductance L_q	0.4755 H
Phase voltage V	240 V
Phase current I	1.6 A
Base speed ω_b	1500 rpm

TABLE VI
DATA OF PMSM IV

Number of pole pairs p	4
Armature resistance R	0.9 Ω
Magnet flux linkage φ_f	0.0837 Wb
d-axis inductance L_d	7.2 mH
q-axis inductance L_q	7.2 mH
Phase voltage V	200 V
Phase current I	8 A
Base speed ω_b	4000 rpm



(a)



(b)

Fig. 12. Torque responses with DTC and current control: (a) torque response under DTC and (b) torque response under PWM current control in the rotor reference frame.

torque is changed abruptly from 3.0 to −3.0 Nm. As shown in Fig. 12, the torque response with DTC is seven times faster than with current control.

V. CONCLUSION

The application of DTC in PMSM drives has been explored. It was mathematically proven that the increase of electromagnetic torque in a permanent magnet motor is proportional to the increase of the angle between the stator and rotor flux linkages, and, therefore, the fast torque response can be obtained by adjusting the rotating speed of the stator flux linkage as fast as possible. The differences in the DTC technique for the induction motor and PMSM have been investigated. The implementation of DTC in the permanent magnet motor is also discussed, and it is found that the motor with a high ratio of the rated stator flux linkage-to-stator voltage is required for controlling the flux linkage using currently available DSP's. The simulation results verify the proposed control and also show that the torque response under DTC is much faster than the one under current control.

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