Vibration Suppression in 2- and 3-Mass System Based on the Feedback of Imperfect Derivative of the Estimated Torsional Torque

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***Abstract*—In this paper, a new vibration-suppression control method for 2- and 3-mass system is proposed, which uses imperfect derivative feedback of the estimated torsional torque. This controller consists of three simple elements: the disturbance observer, the imperfect derivative filter, and the feedback gain. By adequately adjusting this feedback gain, the damping factor of original flexible system can be controlled so that the vibration caused by the mechanical resonance can be effectively suppressed. Due to the simplicity of the proposed controller, it can be easily applied to various flexible systems with other regulators. The combination of this method with P&I regulator shows good performances in vibration suppression and disturbance rejection which are shown in both simulations and experiments.**

I. Introduction

Vibration suppression and disturbance rejection in multimass system originates in the steel rolling mill system, where the load is coupled to the driving motor by a long shaft. Due to this, the finite but small elasticity of the shaft gets magnified and has a vibrational effect on the load speed. This vibration is not only undesirable but also the origin of the instability of the system in some cases.

This problem is quite a recent problem since before this the solution was always thought of from the viewpoint of the one-mass system. Renewed interest has been there due to the new requirements in the quality of control, i.e., 1) faster speed control response, 2) disturbance rejection on the load speed, and 3) robustness to parameter variations including gear backlash. As the newly required speed response is very close to the resonant frequency of such systems, only the conventional techniques based on P&I control are not effective enough.

A few years ago, one of the authors listed up some promising methods proposed up to then [1], i.e., simulator following control [2], 2-stage model following control [3], speed differentiation feedback [4], some observer-based approaches [5], load-side acceleration control, state-feedback methods [6], and their combinations [7], etc. They are compared from the viewpoints mentioned above. One of the important notices is that we should consider not only the resonant frequency of the system but also the resonance ratio given by $R_j = \omega_L$(antiresonant frequency)/ω_M(resonant frequency) [8], [9]. It is difficult to develop a general method which can be applicable to a wide variation of R_j.

Manuscript received July 29, 1994; revised January 5, 1995.
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Publisher Item Identifier S 0278-0046(96)01384-6.

In this paper, we propose a novel controller design method for 2- and 3-mass system using imperfect derivative feedback of the estimated torsional torque. This controller consists of three simple elements: the disturbance observer, the imperfect derivative filter, and the feedback gain. By the adjustment of the feedback gain, the damping factor of the original undamped flexible system can be designed to an arbitrary value and this damping effect suppresses the vibration caused by mechanical resonance. The speed control system is easily designed by using a normal P&I regulator with this vibration suppressing controller simultaneously . Simulation and experimental results show that the proposed controller suppresses the vibration and rejects the disturbance effectively both on 2-mass and 3-mass systems within a wide range of R_j.

II. Resonance Ratio of 2-Mass System

The model of 2-mass system is illustrated in Fig. 1(a). The block diagram of the system is represented in Fig. 1(b).

The state equations of the 2-mass system is given by (1)

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + BT_m + CT_L \tag{1}$$

$$A = \begin{pmatrix} 0 & -K_{12}/J_1 & 0 \\ 1 & 0 & -1 \\ 0 & K_{12}/J_2 & 0 \end{pmatrix} \tag{2}$$

$$B = (1/J_1 \quad 0 \quad 0)^T \tag{3}$$

$$C = (0 \quad 0 \quad -1/J_2)^T \tag{4}$$

where J_1 and J_2 are the inertia moments of the motor and the load, and K_{12} is the stiffness of the shaft.

The state variable vector is $\boldsymbol{x} = (\omega_1 \ \theta_{12} \ \omega_2)^T$. T_m, the motor torque, is the control input and T_L is the disturbance input. ω_1 is the measurable motor speed. ω_2 is the load speed to be controlled, but we have no sensors to measure ω_2.

The mechanical resonant frequency ω_m and the anti-resonant frequency ω_L are the important characteristics of the 2-mass system. They are calculated from the poles and zeros of the transfer function $T(s) = \omega_1/T_m$ given by (5).

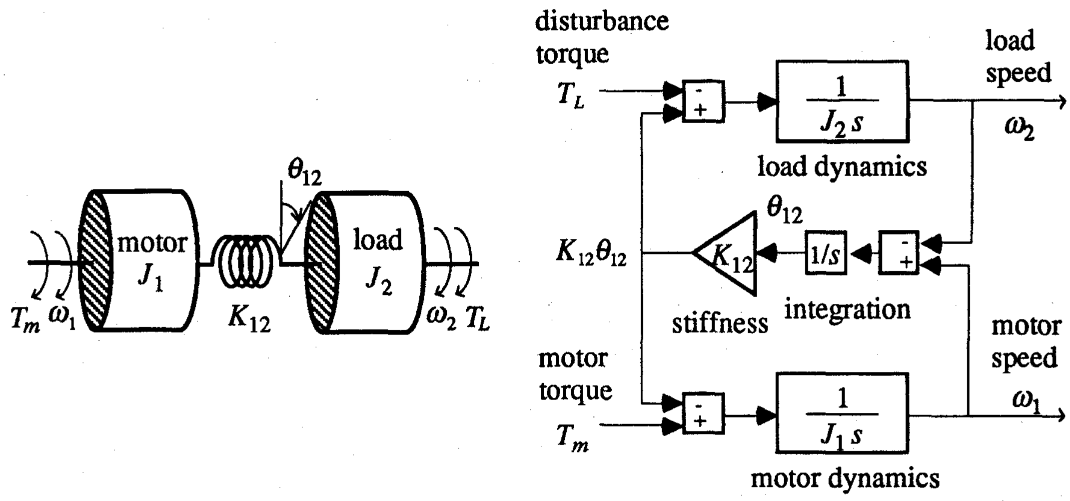
$$T(s) = \omega_1/T_m = \frac{s^2 + \omega_L^2}{J_1 s(s^2 + \omega_m^2)} \tag{5}$$

$$\omega_L = \sqrt{K_{12}/J_2} \tag{6}$$

$$\omega_m = \sqrt{K_{12}/J_1 + K_{12}/J_2}. \tag{7}$$

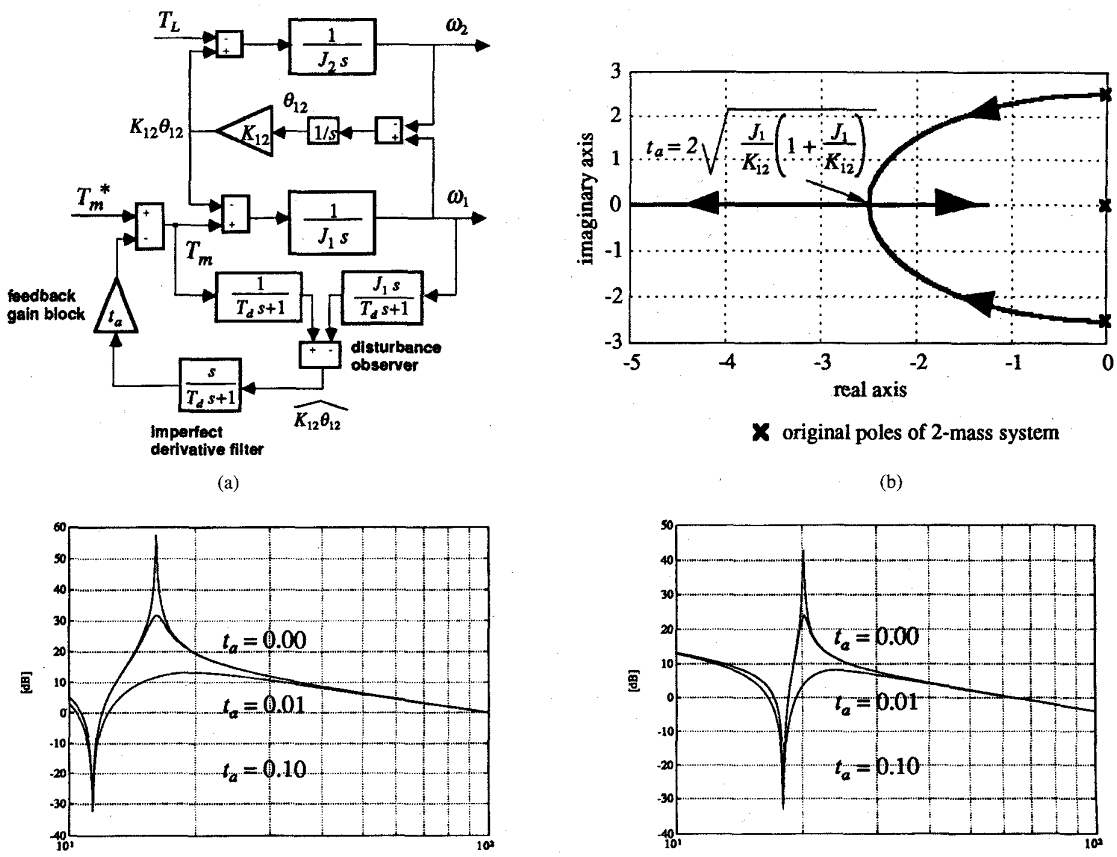
The ratio of these two frequencies $R = \omega_m/\omega_L$ is called the resonance ratio given by (8). It is difficult to suppress the vibration caused by mechanical resonance when R becomes close to 1 [9]

$$R = \sqrt{1 + J_2/J_1}. \tag{8}$$



(a) (b)

Fig. 1. 2-mass system. (a) 2-mass system model. (b) Block diagram of 2-mass system.



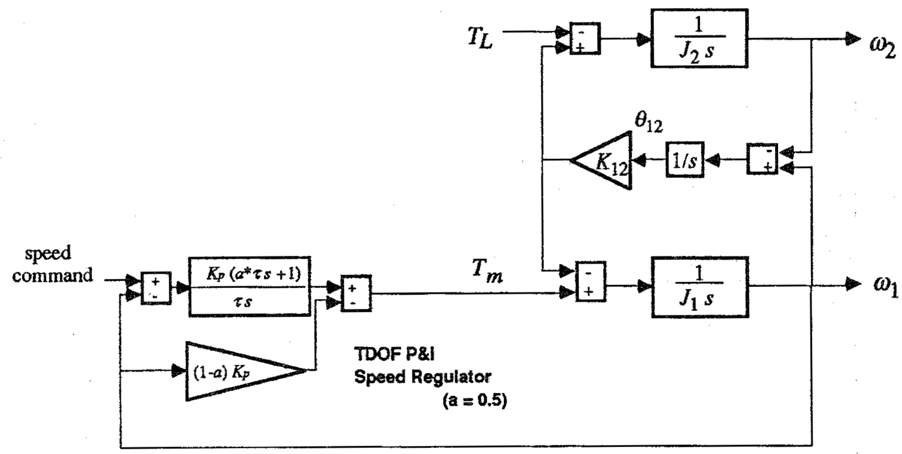
(a) (b)

(c) (d)

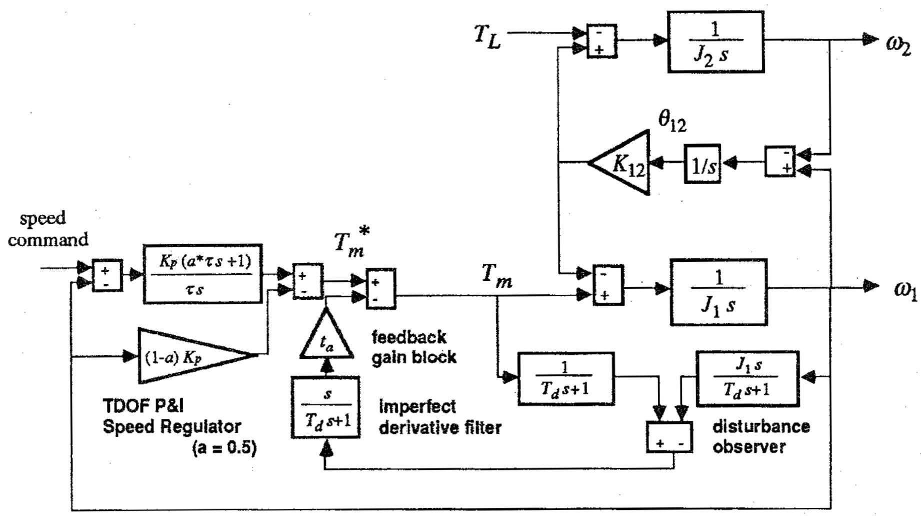
Fig. 2. Block diagram and effect of the imperfect derivative feedback of estimated torque. (a) Feedback of the derivative of the estimated torque through a low-pass filter. (b) Root locus of the 2-mass control system with increasing t_a. (c) Frequency response of $T'(s) = \omega_1/T_m^*$ when $R = \sqrt{2}$. (d) Frequency response of $T'(s) = \omega_1/T_m^*$ when $R = \sqrt{1.25}$.

III. Controller Design for 2-Mass System

The disturbance observer applied to the 2-mass system estimates the torsional torque $\widehat{K_{12}\theta_{12}}$, which is reacting to the motor. Fig. 2(a) shows the structure of the feedback of the time derivative of the estimated torsional torque through a low pass filter (imperfect derivative). This feedback has the damping



(a)



(b)

Fig. 3. Block diagram of the speed control system for 2-mass system. (a) Speed control system using conventional P&I method. (b) Speed control system P&I speed controller with proposed method.

effect on the 2-mass system and suppresses the vibration of mechanical resonance.

By the feedback of imperfect derivative of the torsional torque, the transfer function $T(s)$ in (5) changes to $T'(s) = \omega_1/T_m^*$ in (9). As the time constant of the low pass filter is smaller than the time period of the mechanical vibration, the dynamics of the lowpass filter can be neglected here

$$T'(s) = \omega_1/T_m^* = \frac{(s^2+\omega_L^2)}{J_1 s\left(s^2 + \frac{K_{12}}{J_1} t_a s + \omega_m^2\right)}. \tag{9}$$

Since the coefficient of the second term of the denominator represents the damping factor of the system, (9) shows that this system can be well damped. Fig. 2(c) and (d) also shows that the effective vibration suppression can be expected for the 2-mass system even when $R = \sqrt{1.25}$, the small resonance ratio case.

Root locus of the closed loop system with increasing the gain t_a is drawn in Fig. 2(b). We can know the upper limit of t_a, but which depends on the plant parameters. By increasing the feedback gain, two of the three poles gradually moves away from the imaginary axis and the system becomes more damped one until it satisfies the condition given by (10). When t_a exceeds t_a^{limit}, the response to the motor torque becomes slower since one of the dominant poles returns toward the imaginary axis and the time constant of this closed loop system has a larger value

$$t_a \le t_a^{\mathrm{limit}} = 2\sqrt{\frac{J_1}{K_{12}}\left(1+\frac{J_1}{J_2}\right)}. \tag{10}$$

IV. Simulation Result of 2-Mass System

The structure of the speed control system using conventional P&I regulator is illustrated in Fig. 3(a). When the two masses are coupled by a rigid shaft and can be modeled as one mass

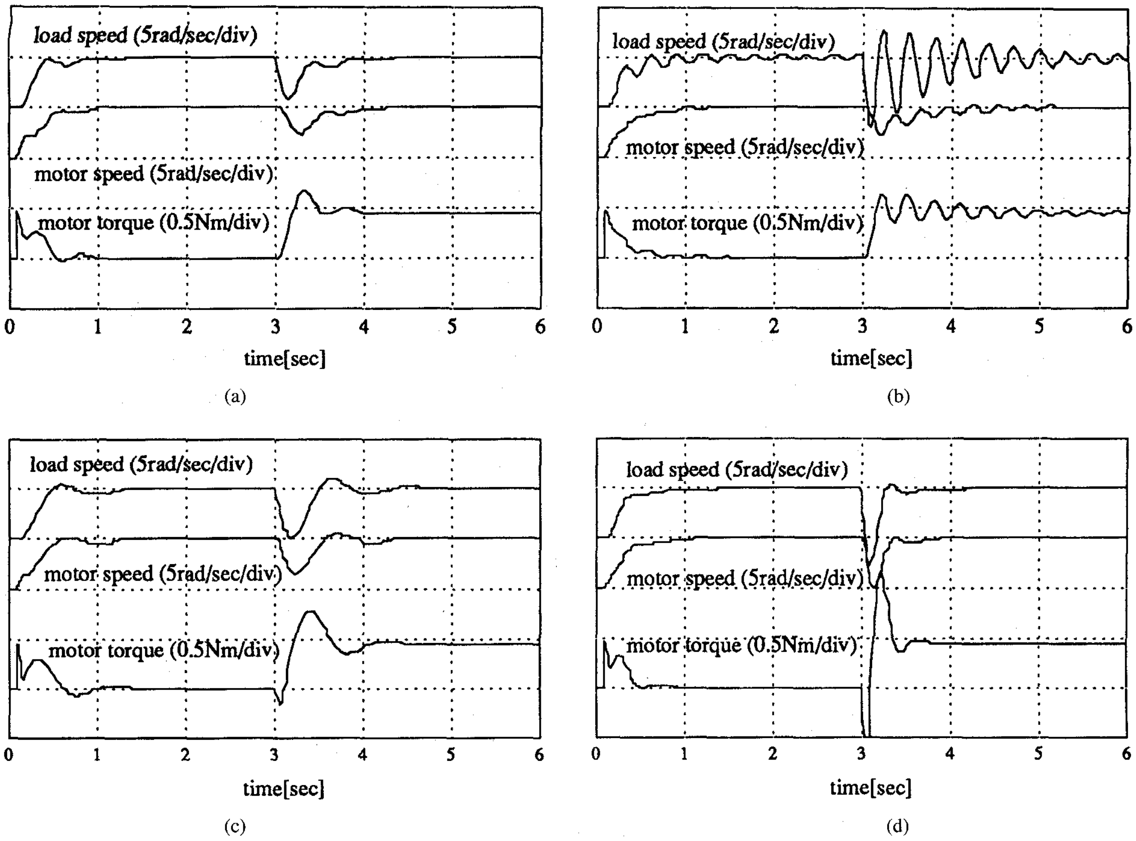


Fig. 4. Simulation results of 2-mass system with (a) P&I regulator only when $R = \sqrt{2}$; (b) P&I regulator only when $R = \sqrt{1.25}$; (c) P&I regulator with the proposed controller when $R = \sqrt{2}$; (d) P&I regulator with the proposed controller when $R = \sqrt{1.25}$.

system, the transfer function of the closed loop system $G_{yr}(s)$ can be easily designed by choosing the gains of P&I regulator as in (11).

$$G_{yr}(s) = \omega/(s+\omega) \tag{11}$$

where

$$K_p = (J_1 + J_2)^* \omega, \quad \tau \gg 1/\omega.$$

Simulation results when $R = \sqrt{2}$ and $R = \sqrt{1.25}$ are shown in Figs. 4(a) and 3(b), respectively. The reference speed of 5 rad/s is commanded at 0.1 s, and 0.5 Nm disturbance torque is added at 3 s.

The controller gains are selected as $K_p = 0.02$ and $\tau = 0.5$ so that $\omega = 10$ in (11). $a = 0.5$ is selected to reduce the overshoot in the response to the reference input. The results show that P&I regulator is not effective enough to control 2-mass systems, especially when the resonance ratio of the system is small.

Fig. 3(b) illustrates the block diagram of the proposed control system combined with the conventional speed control system mentioned above. The parameters used in the simulations are given in Table I. The reference speed of 5 rad/s is commanded at 0.01 s, and the 0.5 Nm disturbance torque is added at 3 s. The results of the proposed controller when $R = \sqrt{2}$ and $R = \sqrt{1.25}$ are illustrated in Fig. 4(c) and (d), respectively. The proposed method is superior to the conventional P&I method both in its command response and disturbance rejection.

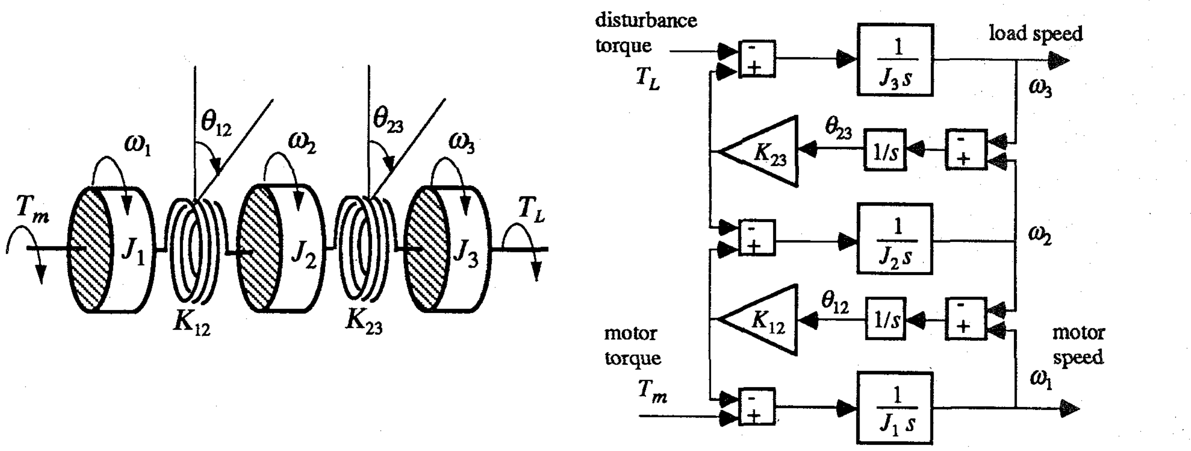
TABLE I
PARAMETERS USED IN THE SIMULATIONS OF THE 2-MASS CONTROL SYSTEM

plant parameters		controller gains	
J_1 [kgm^2]	0.01 *1 0.016 *2	K_p	0.200
J_2 [kgm^2]	0.01 *1 0.004 *2	τ	0.500
		t_a	0.15
K_{12} [Nm/rad]	1.2938	T_d	0.01

*1: when $R = \sqrt{2}$.
*2: when $R = \sqrt{1.25}$

V. APPLICATION TO 3-MASS SYSTEM

As a 3-mass system has two resonant frequencies, we need the higher order system model to represent it than in the 2-mass system. The model of 3-mass system is illustrated in Fig. 5(a) consisting of three rigid masses connected with two flexible shafts. Its block diagram is shown in Fig. 5(b).



(a) (b)

Fig. 5. 3-mass system. (a) 3-mass system model. (b) Block diagram of 3-mass system.

The state equations of the 3-mass system are given in (12)

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + BT_m + CT_L \tag{12}$$

$$A = \begin{pmatrix} 0 & -K_{12}/J_1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & K_{12}/J_2 & 0 & -K_{23}/J_2 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & K_{23}/J_3 & 0 \end{pmatrix} \tag{13}$$

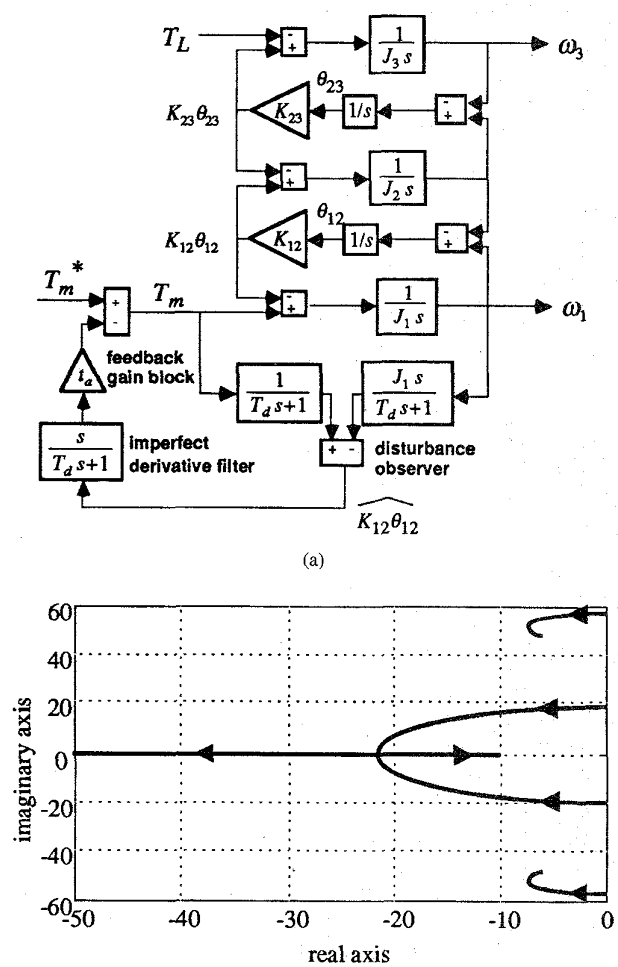
$$B = (1/J_1 \quad 0 \quad 0 \quad 0 \quad 0)^T \tag{14}$$

$$C = (0 \quad 0 \quad 0 \quad 0 \quad -1/J_3)^T \tag{15}$$

where J_1 represents the inertia moment of the motor, and J_2 and J_3 represent inertia moments of the loads. K_{12} and K_{23} are the stiffnesses of the shaft. There are five state variables: $\boldsymbol{x} = (\omega_1\ \theta_{12}\ \omega_2\ \theta_{23}\ \omega_3)^T$. T_m, the motor torque, is the control input and T_L is the disturbance input. ω_1 is the measurable motor speed, and ω_3 is the load speed to be controlled but is not measurable.

The block diagram of the feedback of imperfect derivative of the estimated torsional torque applied to 3-mass system is illustrated in Fig. 6(a). The root locus of this system with increasing t_a is drawn in Fig. 6(b). The effect of the vibration suppression can also be observed here.

The whole structure of speed control system with P&I regulator is illustrated in Fig. 7(a). The parameters used in the simulations are given in Table II. The reference input of 5 rad/s is commanded at 0.1 s, and the 0.5 Nm disturbance torque is given at 3 s. We compared the results with/without the proposed controller, and they are shown in Fig. 7(c) and (b). Also in 3-mass system, the proposed method shows distinct vibration suppression effect both in its command response and disturbance rejection performances.

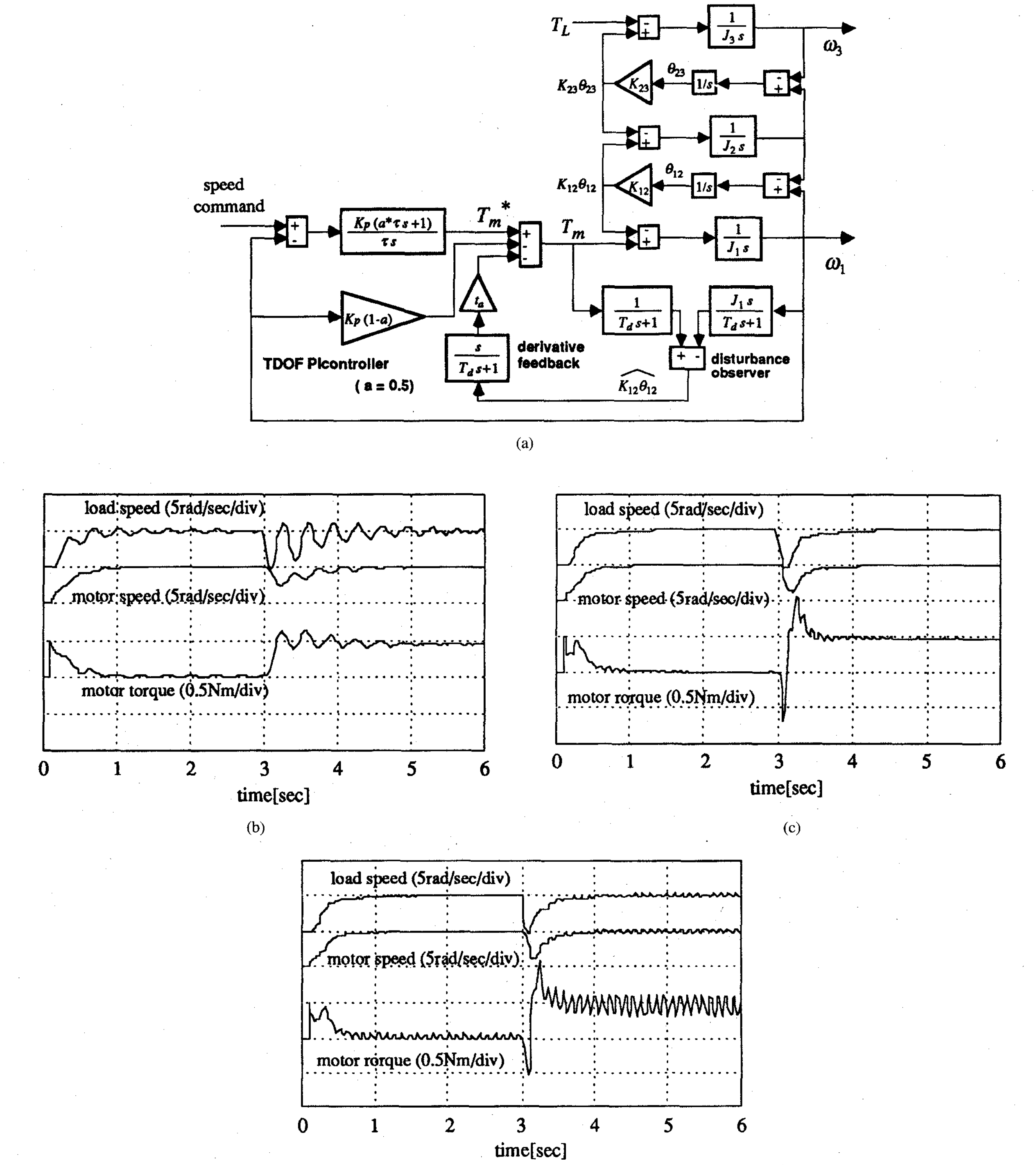


(b)

Fig. 6. Block diagram and effect of the imperfect derivative feedback control applied to the 3-mass system. (a) Block diagram of proposed control method applied to 3-mass system. (b) Root locus of 3-mass system with increasing t_a.

VI. Vibration Suppression and Noise Reduction

There is a trade-off relation between the suppression of high-frequency vibration and the reduction of sensor noise. As the 3-mass system contains the higher resonant frequency, the time constant T_d of the disturbance observer and that of the low pass filter are required to be much smaller than the time period of the higher frequency mechanical resonant vibration.



(a)

(b)

(c)

(d)

Fig. 7. Simulation result of speed control system for 3-mass system. (a) Whole structure of control system for the 3-mass system. (b) Simulation result when $t_a = 0$ (P&I regulator only). (c) Simulation result when $t_a = 0.15$, $T_d = 0.01$. (d) Simulation result when $t_a = 0.15$, $T_d = 0.02$.

Compared with the simulation result when $T_d = 0.01$ s shown in Fig. 7(c), the vibration in high frequency cannot be suppressed so well when $T_d = 0.02$ s as is shown in Fig. 7(d). Since the time period of the vibration caused by the second resonant frequency is 0.0212 s, 0.02 s is not small enough for T_d to be neglected. In order to suppress the vibration sufficiently, the smaller T_d is the better.

However, because of the sensor noise, it is difficult to implement the controller with too small T_d in actual systems. The amplitude of the high frequency noise can be calculated

TABLE II
PARAMETERS USED IN THE SIMULATIONS OF THE 3-MASS SYSTEM

plant parameters		controller gains	
J_1 [kgm^2]	0.0192	K_p	0.155
J_2 [kgm^2]	0.0019	τ	0.500
J_3 [kgm^2]	0.0045	t_a	0.15
K_{12} [Nm/rad]	2.8		
K_{23} [Nm/rad]	2.7	T_d	0.01

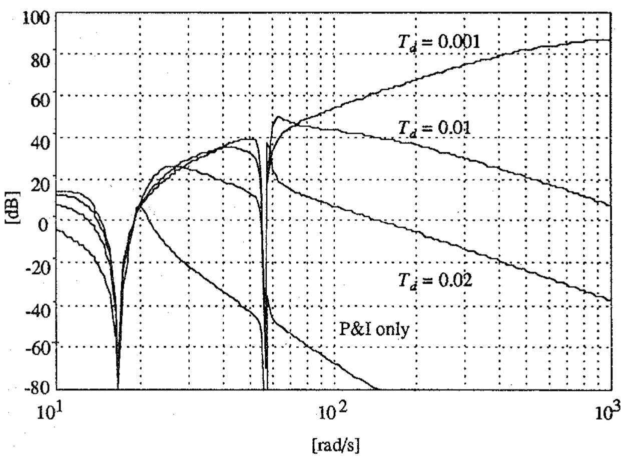
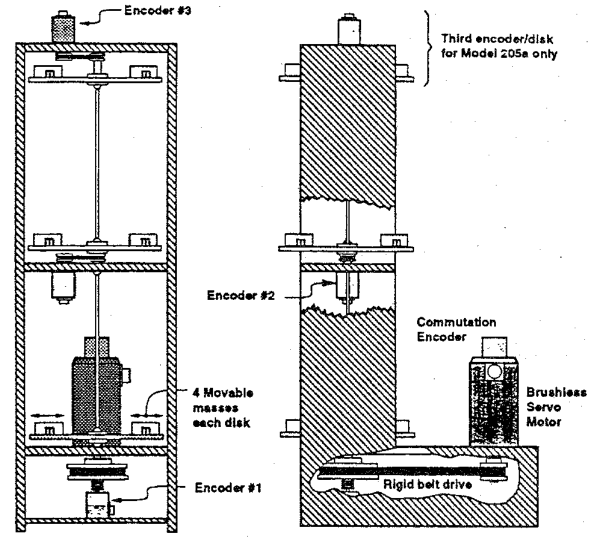
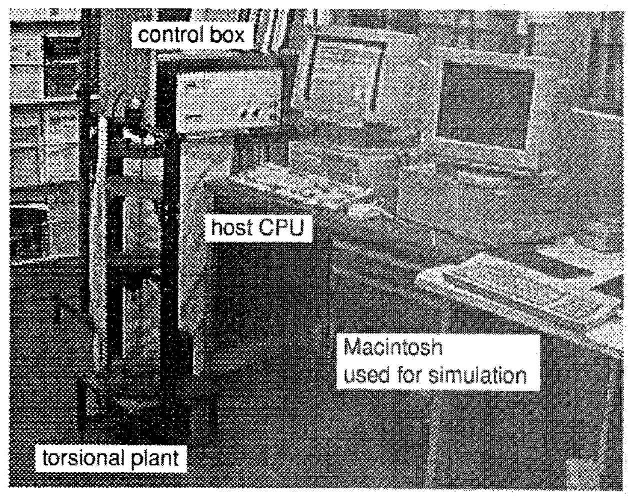


Fig. 8. Frequency response of T_{yn}.

using T_{yn} which is the transfer function from sensor noise to the motor speed of this closed loop system. Frequency response of T_{yn} of the 3-mass system is shown in Fig. 8. When T_d is small, the sensor noise in high frequency region is amplified by the control. The faster estimation of the torsional torque is preferable for the vibration suppression. However, the suppression of the high frequency vibration and the reduction of sensor noise in high frequency region are hard to be obtained at the same time.



(a)



(b)

Fig. 9. The ECP torsional control system used in the experiments. (a) Experimental apparatus from the manual. (b) Picture of the whole system.

VII. EXPERIMENTAL RESULTS

The experimental apparatus is illustrated in Fig. 9(a). Its picture is shown in Fig. 9(b). The mechanical part consists of three disks connected with two flexible shafts. Their inertia moments can be changed by fixing several masses on the disks, and are adjusted to have the same values as those used in previous simulations. The control period of this system is 4.42 ms. The controller used in the experiments also has the same parameters used in the simulations represented in Table II, except that T_d is 0.02 s to decrease the sensitivity to sensor noise.

The command and the disturbance responses using the conventional P&I regulator are shown in Fig. 10(a) and (b), respectively. The results using the proposed controller are shown in Fig. 10(c) and (d). In the experiments for the command response, reference speed of 5 rad/s is commanded at $t = 0$ s and 0 rad/s is commanded at $t = 3$ s. In the experiments of disturbance response, 0.45 Nm is added as the disturbance torque at the third disk.

Although the vibration caused by the first resonance frequency is well suppressed in the controller with imperfect derivative, vibration with high frequency still remains. This is caused by sensor noise. In order to reject the high frequency vibration in steady state, we switched the proposed controller to the conventional P&I regulator only when the deference between the motor speed and the speed command becomes smaller than 0.05 rad/s. By introducing this switching method, the proposed controller shows excellent performances both in command response and in disturbance rejection as is shown in Fig. 10(e) and (f). Since P&I regulator has very good characteristic of the sensor noise reduction as shown in Fig. 8,

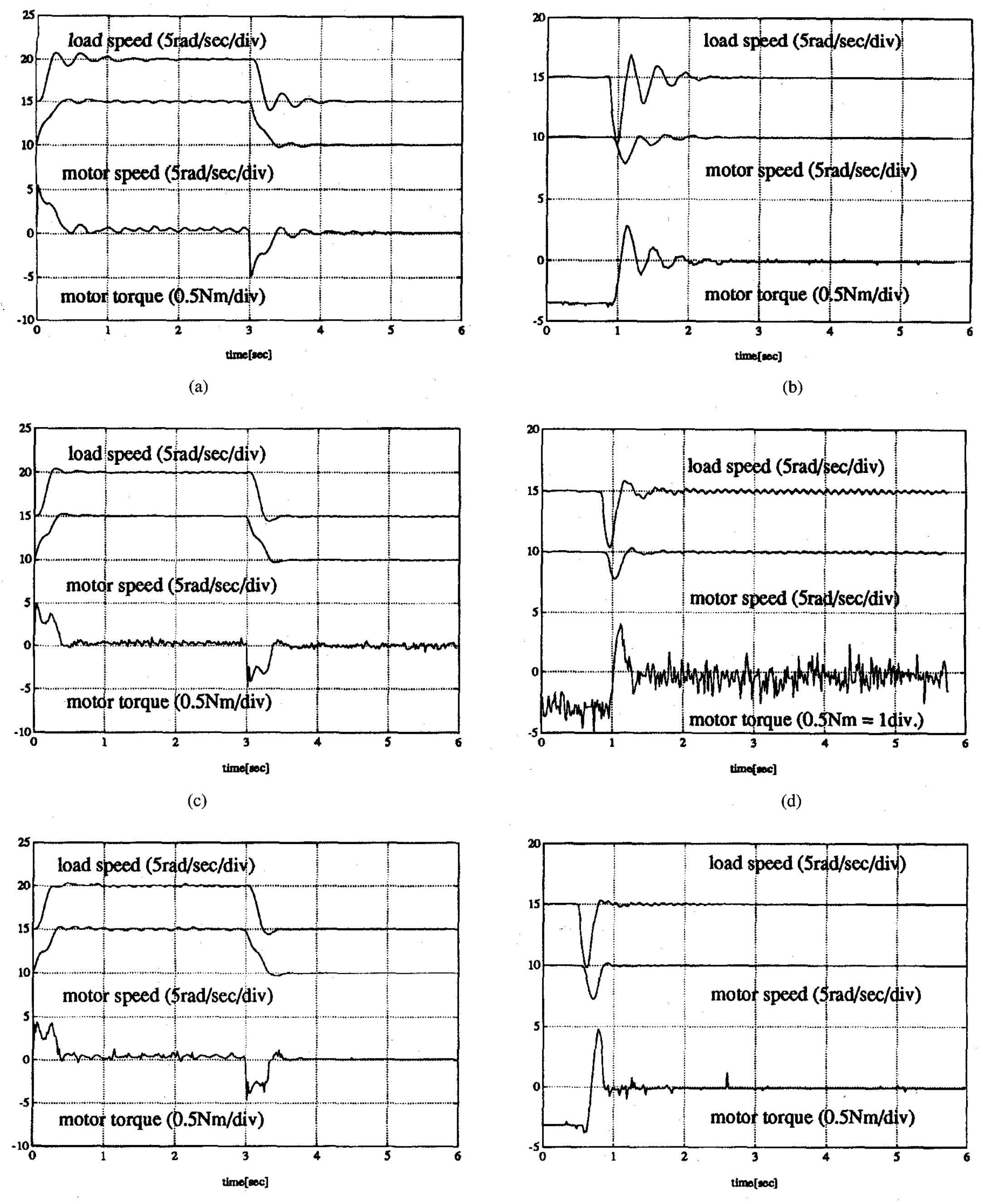


Fig. 10. Experimental results. (a) Command response using P&I only. (b) Disturbance response using P&I only. (c) Command response using P&I with vibration suppression control. (d) Disturbance response using P&I with vibration suppression control. (e) Command response using controller switching from the proposed controller to P&I only. (f) Disturbance response using controller switching from the proposed controller to P&I only.

this controller realizes both effective vibration suppression and enough stability in the steady state.

VIII. Conclusion

In this paper, we proposed a new control method for 2- and 3-mass system using the feedback of imperfect derivative of the estimated torsional torque. The damping factor of the original flexible system can be controlled and the vibration due to the mechanical resonance is suppressed well. The speed control system is implemented by the combination of this simple controller with P&I speed regulator and it shows very good vibration suppression effect on 2- and 3-mass system in simulations. In experiments, due to the sensor noise problem, the time constant of the controller cannot be decreased as sufficiently small as to suppress the vibration of high frequency. However, by switching the controller to the P&I regulator in the steady state, we obtained the stable performance.

ACKNOWLEDGMENT

The authors would like to express our sincere thanks to Yoichi Kaya of the University of Tokyo for his useful discussion, and Shrikrishna Kulkarni and Hiroyuki Iseki for their early contribution as postgraduate students in Hori Laboratory. The authors are also grateful to the Educational Control Products (ECP) for their contribution to this work through the experimental equipment.

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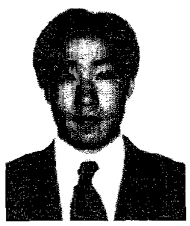

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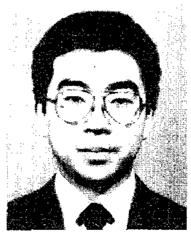

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