ZJU-UIUC INSTITUTE

ECE-210 Analog Signal Processing Spring 2023 **Laboratory Exercise #3**

Student ID	3210111519	**Pre-lab**	**/25**
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Semester/Section	LA4	**Total**	**/83**

Lab 3: Frequency Response and Fourier Series

In this lab you will build an active bandpass filter circuit with two capacitors and an op-amp, and examine the response of the circuit to periodic inputs over a range of frequencies. The same circuit will be used in Lab 4 in your AM radio receiver system as an intermediate frequency (IF) filter, but in this lab our main focus will be on the frequency response $H(\omega)$ of the filter circuit and the Fourier series of its periodic input and output signals. In particular we want to examine and gain experience about the response of linear time-invariant circuits to periodic inputs.

1 Prelab

1. Determine the compact-form Fourier series of the periodic square wave signal, $f(t)$ shown in Figure 1, with a period T and amplitude A. That is, find c_n and θ_n such that

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n), \text{ where } \omega_o = \frac{2\pi}{T}.$$

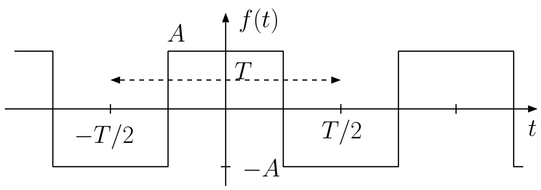


Figure 1: Square wave signal for prelab.

Notice $\frac{C_0}{2} = 0$. How could you have determined that without any calculation? (____/2)

Because the amplitude is A and it's symmetric about x-axis, so $\frac{C_0}{2}$ must equal to zero.

Show your work.(____/3) (____/11)

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt$$

$$= \frac{4}{T}\left[\int_0^{\frac{T}{4}} A\cos\left(\frac{2n\pi}{T}t\right)dt + \int_{\frac{T}{4}}^{\frac{T}{2}} -A\cos\left(\frac{2n\pi}{T}t\right)\right.$$

$$= 4A\sin\left(\frac{n\pi}{2}\right)/n\pi$$

$$f(t) = \sum_{n=1}^{\infty} a_n \cdot \cos\left(\frac{2n\pi}{T}t\right) = \sum_{n=1}^{\infty} \frac{4A}{n\pi}\cos\left(\frac{2n\pi}{T}t + \theta_n\right)$$

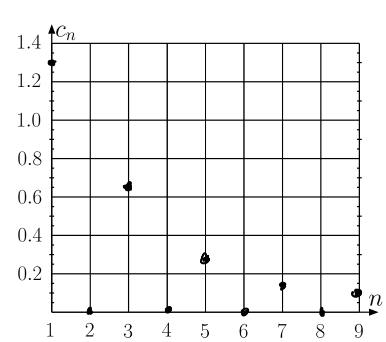
$$\theta_n = \begin{cases} 0 & n \neq 4k+3 \\ \pi & n = 4k+3 \end{cases}$$

$$F_n = \frac{2A\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

$F_n = \dfrac{2A\sin\left(\frac{n\pi}{2}\right)}{n\pi}$ (____/2)

$c_n = \begin{cases} 0 & n \text{ is even} \\ \frac{4A}{n\pi} & n \text{ is odd} \end{cases}$ (____/2)

$\theta_n = \begin{cases} 0 & n \neq 4k+3 \\ \pi & n = 4k+3 \end{cases}$ (____/2)



With $A = 1$, plot c_n over n ($n \in [1, 9]$) (____/2)

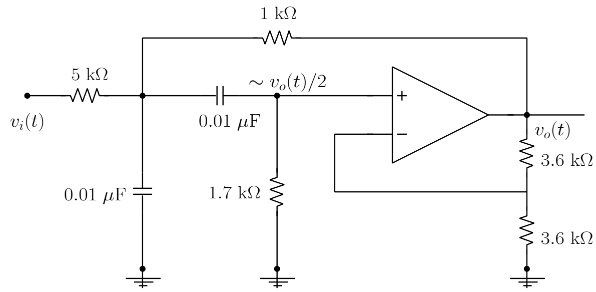


Figure 2: Circuit for analysis in prelab and lab.

2. Consider the circuit in Figure 2 where $v_i(t)$ is a co-sinusoidal input with some radian frequency ω.

   (a) What is the phasor gain $\frac{V_o}{V_i}$ in the circuit as $\omega \to 0$? (Hint: How does one model a capacitor at DC — open or short?)

   Show your work

   open circuit

   $\frac{V_o}{V_i} = 0$

   (____/3)

   (b) What is the gain $\frac{V_o}{V_i}$ as $\omega \to \infty$? (Hint: think of capacitor behavior in $\omega \to \infty$ limit)

   Show your work

   capacitor's $Z \Rightarrow 0$, so $V_o \to 0$. $\frac{V_o}{V_i} \to 0$

   (____/3)

   (c) In view of the answers to part (a) and (b), and the fact that the circuit is 2nd order (it contains two energy storage elements), try to guess what kind of a filter the system frequency response $H(\omega) \equiv \frac{V_o}{V_i}$ implements — lowpass, highpass, or bandpass? The amplitude response $|H(\omega)|$ of the circuit will be measured in the lab.

   Give your answer and explain your reasoning.

   Bandpass, because when $w \to 0$ or $w \to \infty$, $|H(w)| \to 0$

   (____/2)

3. Decibels (dB) is a unit of measurement widely used in science and engineering to compare power or intensity quantities. A decibel (dB) is one-tenth of a Bel (B), which is the name given to $\log_{10}\left(\frac{P_1}{P_0}\right)$, where $\log_{10}$ is the base 10 logarithm, and $\frac{P_1}{P_0}$ is the ratio of two power quantities. The formula for calculating decibels is : $10\log_{10}\left(\frac{P_1}{P_0}\right)$. and for comparing voltages we can use: $20\log_{10}\left(\frac{V_1}{V_0}\right)$, which is derived from $10\log_{10}\left(\frac{V_1^2/R}{V_0^2/R}\right)$. Complete the following table of useful ratios: (use accuracy of 1 decimal in dB row)

P_1/P_0	1000	100	10	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
V_1/V_0	$\sqrt{1000}$	10	$\sqrt{10}$	$2\sqrt{2}$	2	$\sqrt{2}$	1	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{10}}{10}$	$\frac{1}{10}$	$\frac{1}{\sqrt{1000}}$
Decibels (dB)	30	20	10	9	6	3	0	-3	-6	-10	-20	-30

(____/2)

4. In the case of the Fourier analysis, the oscilloscope compares the signals with a reference of $V_{\text{rms}} = 1\,\text{V}$. Recall that a sine wave with rms (root mean square) amplitude of $V_{\text{rms}} = 1\,\text{V}$ corresponds to a sine wave with a peak amplitude of $\sqrt{2}\,\text{V}$, which is to say a peak-to-peak amplitude of $2\sqrt{2}\,\text{V}$. To specify that the reference is $V_{\text{rms}} = 1\,\text{V}$, the decibel symbol is modified with the suffix "V" becoming "dBV". Convert the voltages in the table below to dBV units (use accuracy of 1 decimal).

v	$V_{\text{rms}} = 1\,\text{V}$	$V_{\text{rms}} = 2\,\text{V}$	$V_{\text{rms}} = \sqrt{10}\,\text{V}$	$V_{\text{rms}} = \sqrt{2}\,\text{V}$	$V_{\text{rms}} = 1/10\,\text{V}$	$2\sqrt{2}\,\text{V}$peak-to-peak	$4\sqrt{5}\,\text{V}$peak-to-peak	$2\,\text{V}$peak-to-peak	$\frac{2\sqrt{2}}{\sqrt{10}}\,\text{V}$peak-to-peak
v (dBV)	0.0	6.0	10.0	3.0	-20.0	0.0	10.0	-3.0	-10.0

(____/2)

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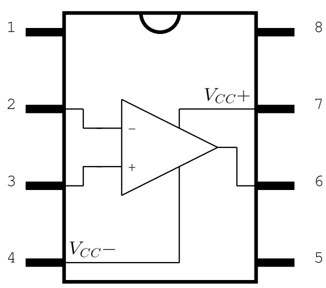
2 Laboratory exercise

- Equipment: Function generator, oscilloscope, protoboard, and wires.
- Components: 741 op-amp, two 0.01 μF capacitors, one 1 kΩ resistor, one 1.7 kΩ resistor, two 3.6 kΩ resistors, and one 5 kΩ resistor.

2.1 Frequency Response $H(\omega)$

The **frequency response** $H(\omega)$ of a linear and dissipative time-invariant circuit contains all the key information about the circuit which is needed to predict the circuit response to arbitrary inputs. Its magnitude $|H(\omega)|$ is known as **amplitude response** and $\angle H(\omega)$ is usually referred to as **phase response**. In this section, you will construct an active bandpass filter circuit and measure its amplitude response over the frequency range 1-20 kHz.

1. Construct the circuit shown in Figure 4 on your protoboard. For now, do not connect it to the three-stage circuit from Lab 2. Remember the rules for wiring and using the 741 op-amp, which are repeated in Figure 3.



Pins 1,5	offset null correction — not used
2	inverting input (-)
3	non-inverting input (+)
4	$V_{CC}-$ supply set to -15 V (DC)
7	$V_{CC}+$ supply set to 15 V (DC)
6	output voltage v_o
8	not connected

Figure 3: Pin-out diagram for the 741 op-am.

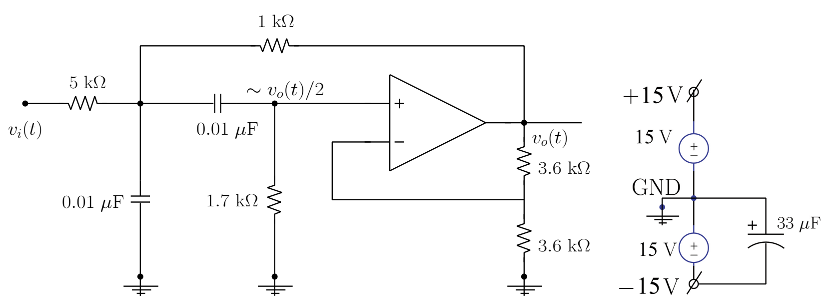


Figure 4: Circuit for analysis in prelab and lab.

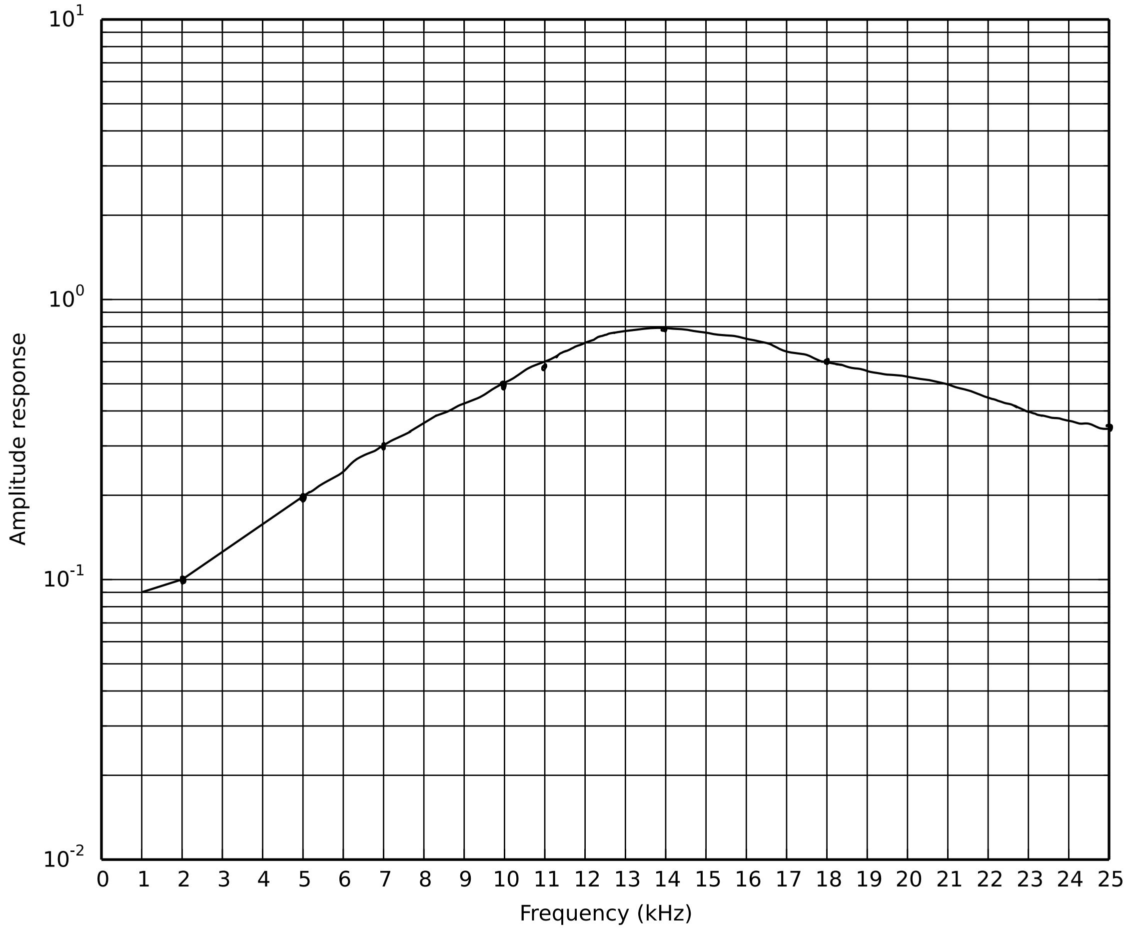
2. Turn on the DC supplies, then connect a 1 kHz sine wave with amplitude 1 V peak-to-peak as the AC input $v_i(t)$ (Don't forget to set the function generator in High Z mode). Display $v_i(t)$ on channel 1 of the oscilloscope and $v_o(t)$ on channel 2. The Trigger should be set to the rising edge of Channel 1, and the time/div around 500 μs. You should obtain an output of approximately of 30 to 50 mV peak-to-peak. To obtain a clear measurement please follow the instructions below:
   - Be sure to have the 33μF electrolytic capacitor in your protoboard, between -15V and ground as shown in Figure 4. This is done to filter out voltage fluctuations produced by the power-supply. Recall that electrolytic capacitors have a voltage polarity requirement. The correct polarity is indicated on the packaging with minus signs and possible arrowheads, denoting the negative terminal. Also the negative terminal lead of radial electrolytic capacitors are shorter. (a reverse-bias voltage above 1 to 1.5 V will destroy the capacitor).
   - Select, in the Oscilloscope, the "High Resolution" acquisition mode. This is done by pressing the "Acquire" button and selecting the "High Res" mode under the "Acq Mode" menu on the display.

- Use the "Meas" button, and select the peak-to-peak measurement "Pk-Pk" applied to the Source 2(Channel 2 $= v_0(t)$). **Recall, that in any measurement performed by the oscilloscope, the entire signal has to fit inside the display, to have a valid value**.

3. Increase the function generator frequency from 1 kHz to 100 kHz according to the table below. At each frequency record the magnitude of the phasor voltage gain $\frac{V_o}{V_i}$ and plot the values in the semi-log axis shown below only from 1 kHz to 20 kHz — $\frac{V_o}{V_i}$ is the system frequency response $H(\omega)$ and its magnitude $\frac{|V_o|}{|V_i|}$ is the system amplitude response $|H(\omega)|$.

f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert} =$	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert} =$	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert} =$	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert} =$	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert} =$
1	0.09	7	0.28	13	0.74	19	0.56	25	0.36
2	0.10	8	0.34	14	0.77	20	0.51	30	0.28
3	0.13	9	0.39	15	0.76	21	0.47	40	0.20
4	0.16	10	0.48	16	0.72	22	0.44	50	0.17
5	0.19	11	0.57	17	0.67	23	0.40	75	0.12
6	0.24	12	0.67	18	0.61	24	0.38	100	0.08

(____/3)



(____/5)

4. The **center frequency** $\omega_o = 2\pi f_o$ of a bandpass $H(\omega)$ is defined as the frequency at which the amplitude response $|H(\omega)|$ is maximized. What is the center frequency f_o in kHz units and what is the maximum amplitude response $|H(\omega_o)|$ of the circuit? Estimate f_o and $|H(\omega_o)|$ from your graph as accurately as you can.

$f_0 =$ 14 kHz	(____/2)	$\lvert H(\omega_0)\rvert =$ 0.77	(____/2)

5. The **3 dB cutoff frequencies** $\omega_u = 2\pi f_u$ and $\omega_l = 2\pi f_l$ are the frequencies above and below $\omega_o = 2\pi f_o$ at which the amplitude response $|H(\omega)|$ is $\frac{1}{\sqrt{2}} \approx 0.707$ times its maximum value $|H(\omega_o)|$. The same frequencies are also known as **half-power** cutoff frequencies since at frequencies ω_u and ω_l the output signal power is one half the value at ω_o, assuming equal input powers at all three frequencies.

$\frac{1}{\sqrt{2}}\|H(\omega_0)\|=$ 0.544	(____/2)	$f_l =$ 11 kHz	(____/2)	$f_u =$ 19 kHz	(____/2)

6. Determine the **3 dB bandwidth** $B \equiv f_u - f_l$ of the bandpass filter in kHz units and calculate the **quality factor** of the circuit defined as $Q \equiv \frac{\omega_o}{2\pi B} = \frac{f_o}{B}$.

$B =$ 8kHz	(____/2)	$Q =$ 0.279	(____/2)

2.2 Displaying Fourier coefficients

In order to display the Fourier coefficients of a periodic signal on the oscilloscope we can use the built in FFT function[1]. On the "FFT screen" of your scope the horizontal axis will represent frequency ω (like in the frequency response plot of the last section) normalized by 2π, and you will see narrow spikes positioned at values $f = \frac{\omega}{2\pi}$ equal to harmonic frequencies $n\frac{\omega_o}{2\pi}$ of the periodic input signal; spike amplitudes will be proportional to compact Fourier coefficients $c_n = 2|F_n|$ in dB. The next set of instructions tells you how to view the single Fourier coefficient (namely c_1) of a co-sinusoidal signal:

1. Connect Channel 1 of your oscilloscope to the input terminal of your circuit from the previous section. Connect the output of the circuit to Channel 2.
2. Set the function generator to a 13 kHz sinusoid with amplitude 500 mV peak-to-peak. In the scope, press the "Auto-Scale" button and adjust the volts/div "Amplitude" knobs, so that the peak-to-peak amplitude of the input and the output are of the order of 1 division in the display (i.e. the signals should occupy approximately 1 division). For better results use the acquisition mode "High Res".
3. Set the oscilloscope to compute the Fourier transform:
   (a) Press "Math"
   (b) Select "FFT" under the "Operator" item menu on the display.
   (c) Set the Operand to Source 2 (The output).
   (d) Press "More FFT". The default "Window" setting "Hanning" should be used.
   (e) Then set the time/div to 2 ms, so that the frequency span(second item in the menu) is 25.0 kHz and the Center frequency is 12.5 kHz.
4. Observe the output signal's Fourier coefficient . How does the FFT display change as you sweep the frequency of the input from 1 kHz to 20 kHz? Describe how the signal changes in frequency domain and explain why the signal changes as a function of the frequency. If necessary turn off the time-domain signals by pressing "1" and/or "2" until the signal disappears.

Describe how the signal changes in the frequency domain and explain why. (____/4)

The peak value frequency become bigger (move to right)

[1] *FFT* stands for *fast Fourier transform* and it is a method for calculating Fourier transforms with sampled signal data — see Example 9.26 in Section 9.3 of Chapter 9 to understand the relation of windowed Fourier transforms to Fourier coefficients.

2.3 Fourier coefficients of a square wave

Now you will introduce a periodic signal with a more interesting set of Fourier coefficients — a square-wave:

1. Change the function generator setting to create a 15 kHz square wave with amplitude 0.5 V peak-to peak as the input to your circuit.

2. Display the FFT of the square wave at the filter input by setting the Operand to Channel 1, and set the time/div to 100μs in order to have a frequency span of 500kHz and centered at 250kHz. Fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power. Keeping in mind the result for Problem 1 in the Prelab, describe the signal in the frequency domain: (____/4)

freq.	Ampl.(dBV)	Ampl.(V)
15k	-18.55	0.2425
45k	-23.71	0.06705
75k	-26.72	0.04623
105k	-28.56	0.03615

It's a sinusoid wave, and peak at 15, 45, 75, 105 kHz
As freq increase, the amplitude decrease.

3. Display the FFT of the filter's output, by setting the FFT Operand to Channel 2. In addition, display on the scope both the input and output signals in the time domain. Describe the output in time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power. (____/4)

freq.	Ampl.(dBV)	Ampl.(V)
15k	-21.23	0.0831
45k	-41.73	0.0081
75k	-50.53	0.0028
105k	-57.02	0.0015

Describe the output signal in the time domain.

It is a sinusoid wave with a shapper peak, and the V_{pp} is about 0.7V. $f \approx 10kHz$

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain: (____/4)

$$THD = \frac{8.1^2 + 2.8^2 + 1.5^2}{83.1^2} = 0.963\%$$

From above, we can say that the peak at 45, 75, 105kHz have only a little influence of the output, and the output is mainly same as 15kHz wave.

4. Repeat for a 10 kHz square wave. Describe the output signal in the time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power. (____/4)

freq.	Ampl.(dBV)	Ampl.(V)
10k	-18.7	0.131
30k	-40.5	0.0082
50k	-46.7	0.0042
70k	-51.6	0.0021

Describe the output signal in the time domain.

It is a sinusoid wave with a shapper peak, and the V_{pp} is about 0.7V. $f \approx 10kHz$

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain:

(____/4)

THD ≃ 0.613%
This THD is smaller than before, so, the output become more close to a sinusoid wave.

5. Repeat for a 5 kHz square wave. Describe the output signal in time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

freq.	Ampl.(dBV)	Ampl.(V)
5k	-26.35	0.048
15k	-28.44	0.037
25k	-42.55	0.0085
35k	-49.12	0.0044

Describe the output signal in the time domain.
The final wave is a periodic wave.
The freq is about 5kHz, and close to combination of a 5kHz and 15kHz sine wave.

(____/4)

Based on the amplitude of the harmonics of the output signal, explain the shape of the output signal seen in time domain:

(____/3)

Because the 5kHz and 15kHz wave have a much bigger amplitude than others, So the output is close to a combination of these two waves.
This is also due to 5kHz and 15kHz is out of 3dB area

6. In terms of the amplitude response of the system ($|H(\omega)|$) explain the change in amplitude of the harmonics from the input to the output (Hint: what frequencies are been attenuated, and what frequencies are inside the bandwidth of the filter):

(____/5)

Because 10kHz is in the 3dB area, and 5kHz, 15kHz are not, so, the 10kHz wave has smallest distortion. And the 5kHz, 15kHz have larger distortion.

Important!
Leave your active filter assembled on your protoboard! You will need it in the next lab session.

The Next Step

The active filter is the last component you will build for the AM radio receiver. In Lab 4, you will combine your components from Labs 1 through 3 to create a working AM radio receiver. The frequency-domain techniques you learned this week will be essential to following the AM signal through each stage of the receiver system. Please make sure you have your circuits ready (i.e. working lab 2 and lab 3) for lab 4 if you want to get **bonus**!