ST420 Assignment 2

1. **(3 marks)** Implement this Metropolis-Hastings algorithm, and show trace plots, on the same figure, of the chain for 5000 iterations started from a) $\theta_0 = -20$ and b) $\theta_0 = 0$. Comment on the mixing of the chain.

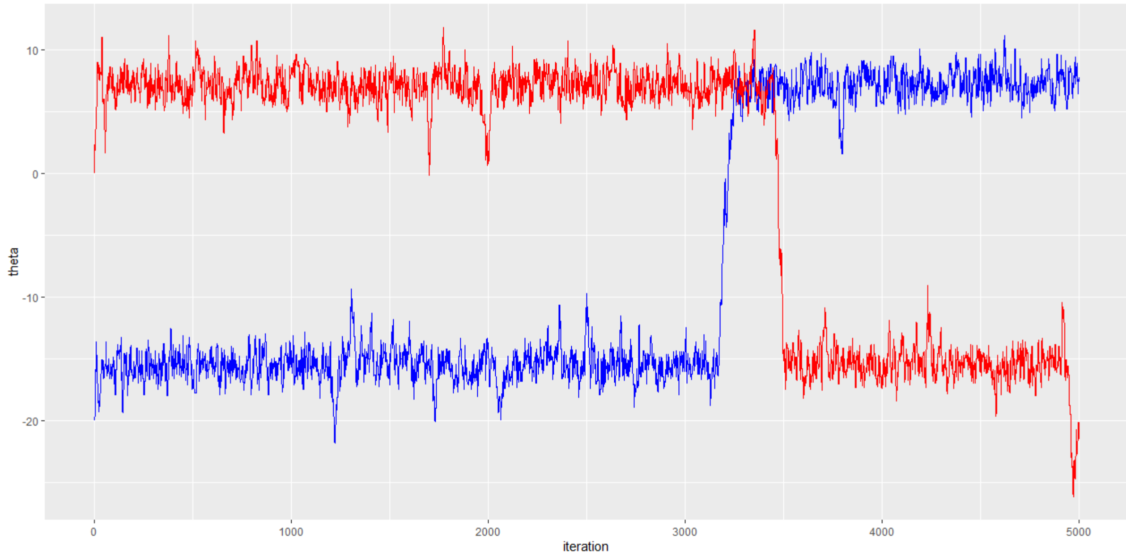

Figure 1: Trace plot for the Metropolis-Hastings algorithm with $\theta_0 = -20$ (in blue) and $\theta_0 = 0$ (in red)

In this trace plot we see poor mixing as evidenced on the significant effect the initial θ_0 has on the outcome of the trace plot as we see that each iteration tends to be close to the initial θ_0. We also see that sometimes there is a large jump in the value of theta for an iteration as seen after iteration 3000. This indicates that the shape of the posterior density makes it difficult for the Metropolis-Hastings algorithm to sample from π; this is due to the multimodal nature of the posterior density as we see the iterations of θ usually having a value that is in between y_1 and y_2 or y_3 and y_4 which are around the peaks of the posterior density. This large jump in value of θ that can happen on an iteration alongside the very infrequent overlapping of the two trace plots implies poor mixing of the chain.

2. **(3 marks)** Devise and implement a rejection sampler to draw a uniform point from any given super-level-set G_s.

```
y_1 = -16.6
y_2 = -14.7
y_3 = 6.3
y_4 = 8.4


pi = function(theta) {
  (1/(1+(y_1-theta)^2))*(1/(1+(y_2-theta)^2))*(1/(1+(y_3-theta)^2))*(1/(1+(y_4-theta)^2))
}
G_s = function(theta) {
  pi(theta) >= s
}
```

```
rejection_sampler = function(G_s) {
  accept = 0
  while (accept == 0) {
    theta = runif(1, y_1-sqrt((1-s^(1/4))/s^(1/4)), y_4+sqrt((1-s^(1/4))/s^(1/4)))
    if (G_s(theta)) {
      accept = 1
    }
  }
  return(theta)
}
```

3. **(3 marks)** Now implement the full Slice Sampling algorithm to sample from π. Show trace plots for the chain for 5000 iterations started from a) $\theta_0 = -20$ and b) $\theta_0 = 0$. Compare these with the previous trace plots obtained from Metropolis–Hastings.

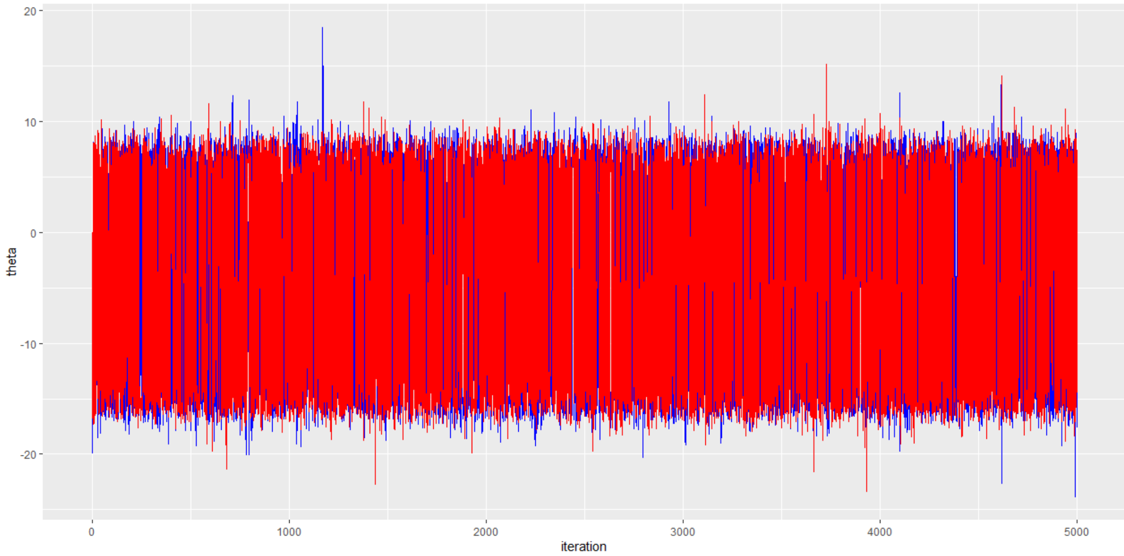

Figure 2: Trace plot for the Slice Sampling algorithm to sample from π with $\theta_0 = -20$ (in blue) and $\theta_0 = 0$ (in red)

Here we see much better mixing for the Slice Sampling algorithm than the Metropolis-Hastings algorithm. Regardless of our initial θ_0, we see constant overlapping of trace plots a and b which indicates good mixing as the initial θ_0 has no effect on outcome of the trace plot. The value of theta after each iteration usually is between the values y_1 and y_4 due to the nature of the Slice Sampling algorithm. Hence, due to constant overlapping of trace plots a and b as well as general coverage of values in between y_1 and y_4 we can say that the chain for the Slice Sampling algorithm mixes quite well. Therefore the trace plots we have for the Slice Sampling algorithm demonstrate much better sampling than what was implied by the trace plots of the Metropolis-Hastings algorithm sampling from π.

4. **(1 mark)** Now use your slice sampler to estimate to give point estimates of the tail probabilities under the posterior that $P(\theta > 8|y)$ and $P(\theta < -15|y)$.

```
[1] "Estimate of probability that theta is more than 8 is:"
[1] 0.1128
[1] "Estimate of probability that theta is less than -15 is:"
[1] 0.334
```