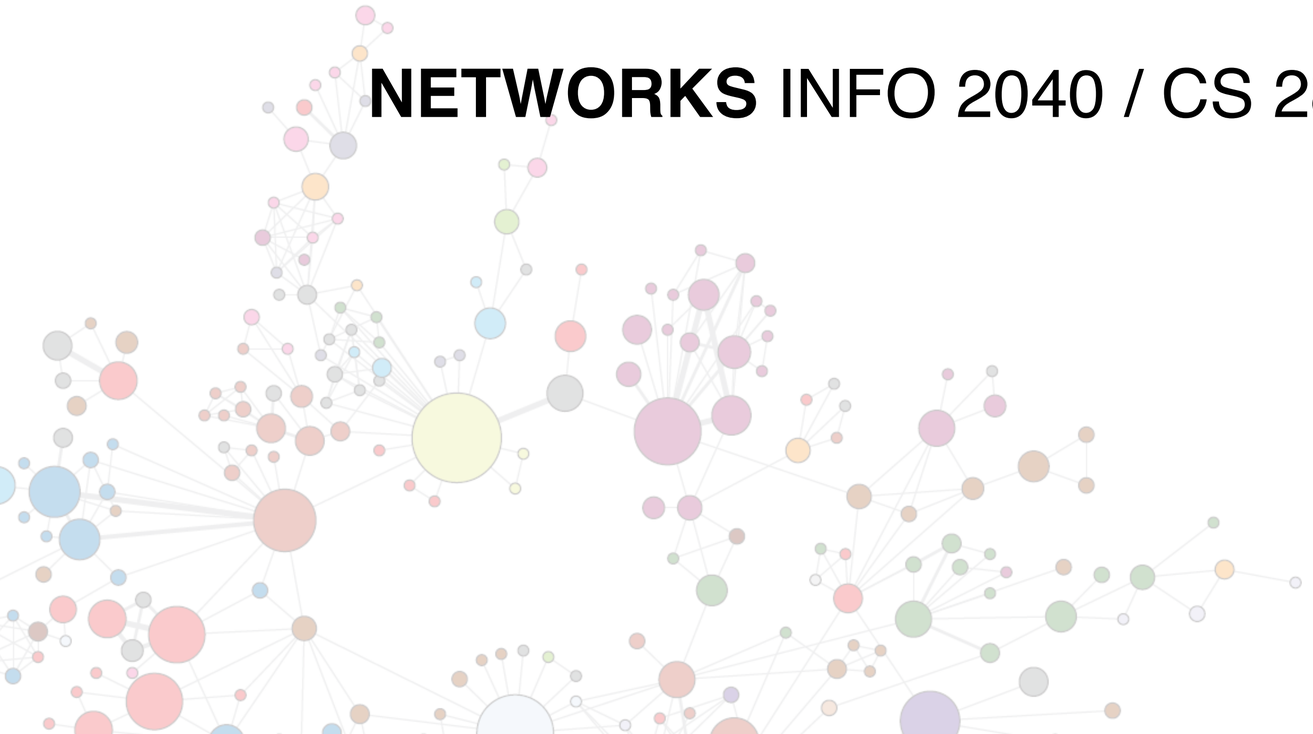
Strong and Weak Ties

NETWORKS INFO 2040 / CS 2850 / ECON 2040 / SOC 2090



Cornell Bowers C·IS
College of Computing
and Information Science

Networks: Fall 2025	**Homework 1**
Rohit Lamba and Yian Yin	**Due 11:59pm, Monday, September 7, 2025**

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope **be sure to assign your answers to the correct question**.

To be eligible for full credit, your homework must come in by 11:59pm Sunday (Sept 7th). We will also accept late homeworks after 11:59pm Sunday until 11:59pm Monday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 11:59pm Monday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and approved by the TA in charge of this homework prior to the due date of the homework. (These include illness, family emergencies, and travel associated with university activities.)

The TA in charge of this homework is Farnoosh Hashemi, sh2574@cornell.edu

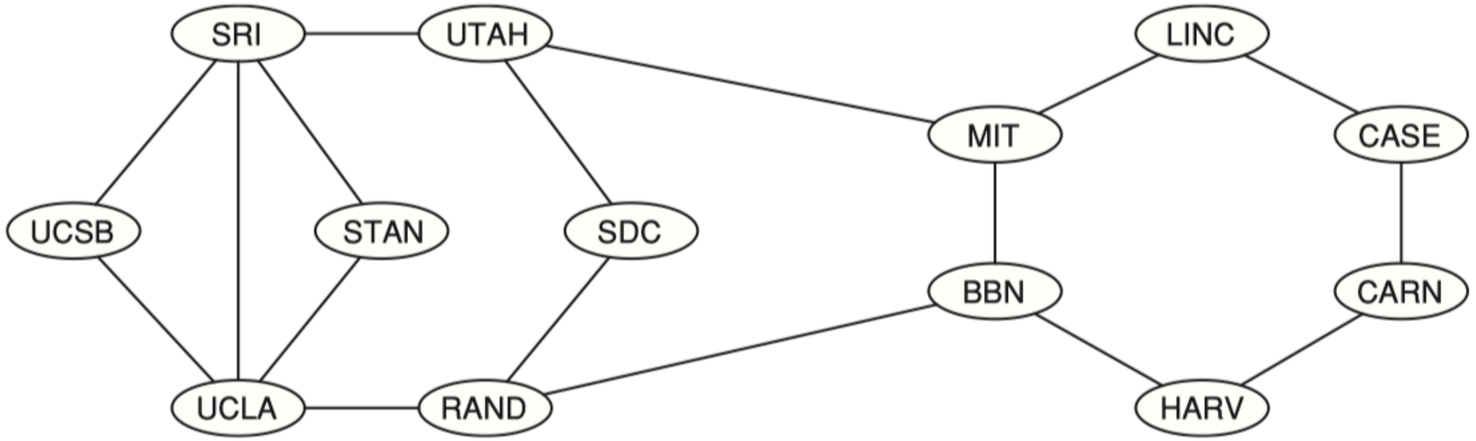
Reading: The questions below are primarily based on the material in Chapters 2 and 3.

Wed: Positive and Negative Relationship (Chapter 5.1-5.2)

Undirected Networks: Path length

What if … we have multiple paths connecting two nodes?

MIT - BBN - HARV vs MIT - LINC - CASE - CARN - HARV

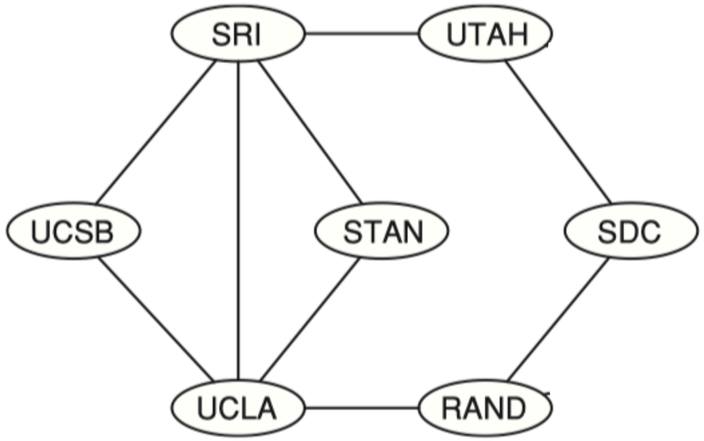


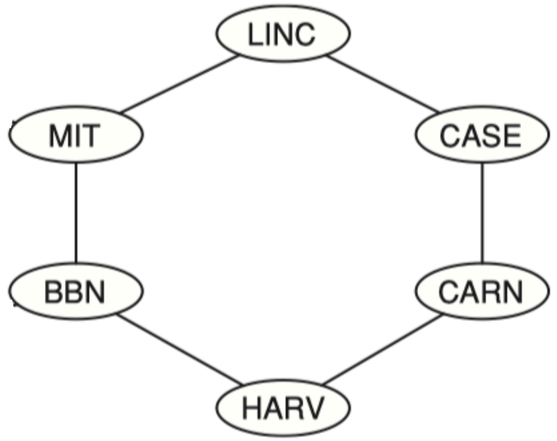
Length of a path: number of steps it contains from beginning to end

We are often interested in the length of shortest path between two nodes (**distance**)

What is the shortest path and distance between MIT and CARN

Undirected Networks: Connectivity





Undirected Networks: Connectivity

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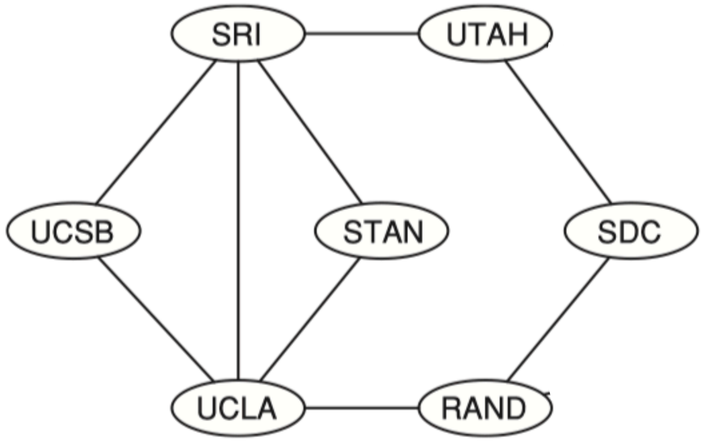
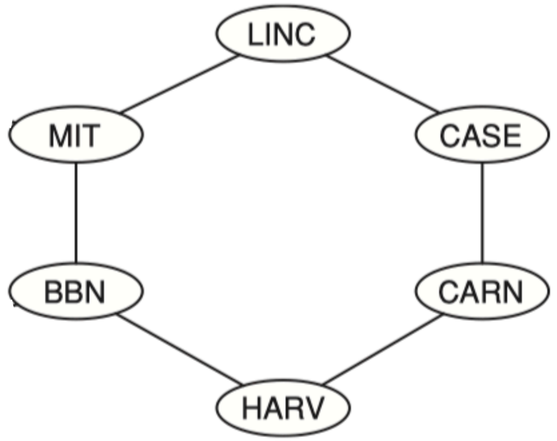
SRI - ? - LINC





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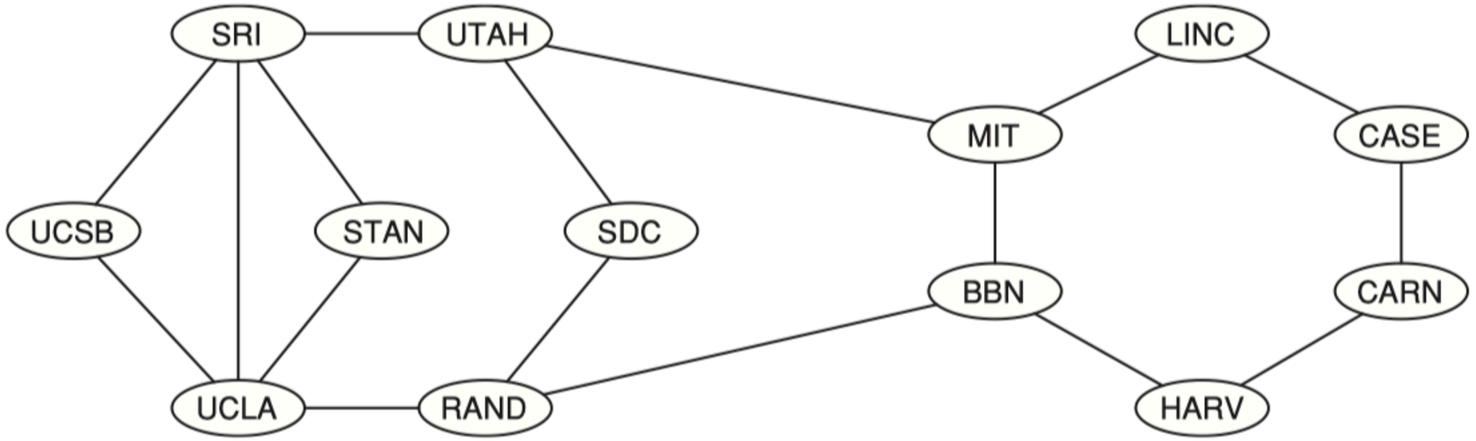

We consider a graph **“connected”** if each pair of nodes can be connected through a path

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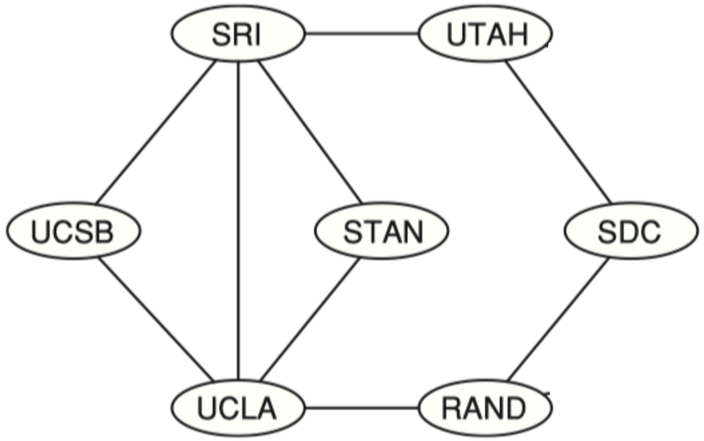
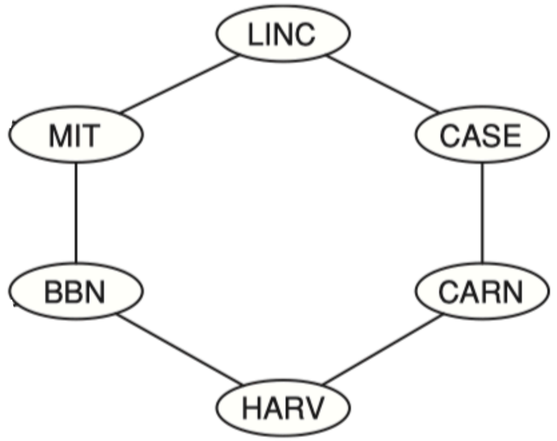




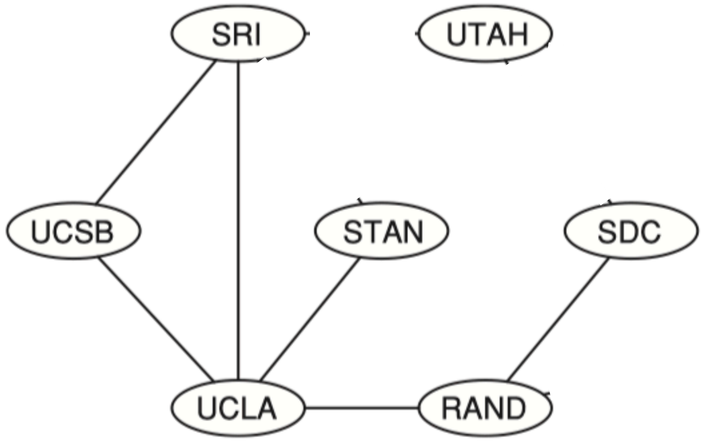
We consider a graph **“connected”** if each pair of nodes can be connected through a path

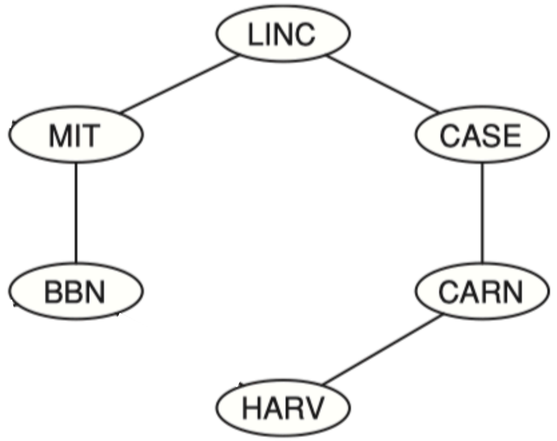
While the graph is not connected, it can still be decomposed into **connected components**

Two nodes are connected through a path **[if and only if]** they are in the same components

Undirected Networks: Connectivity

How many connected components in the network?





A few definitions

Bridge

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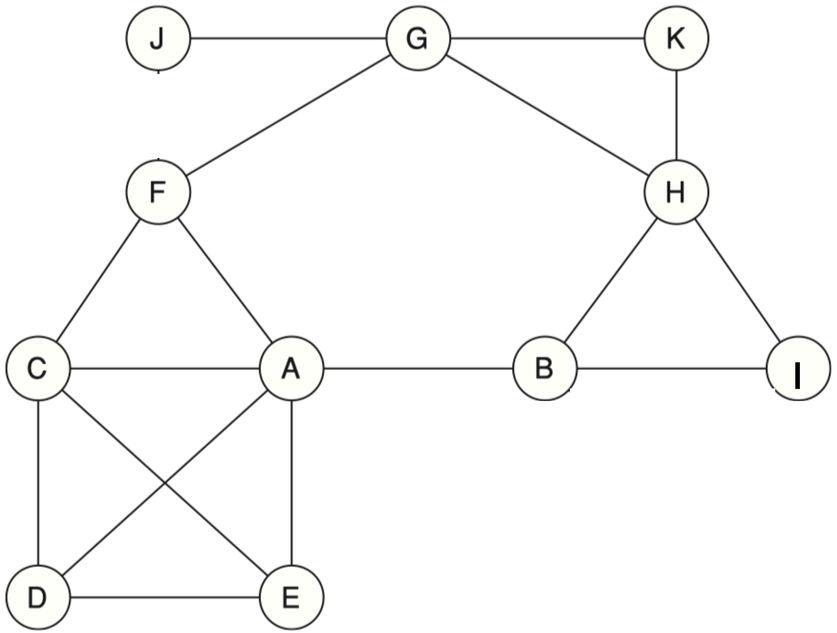

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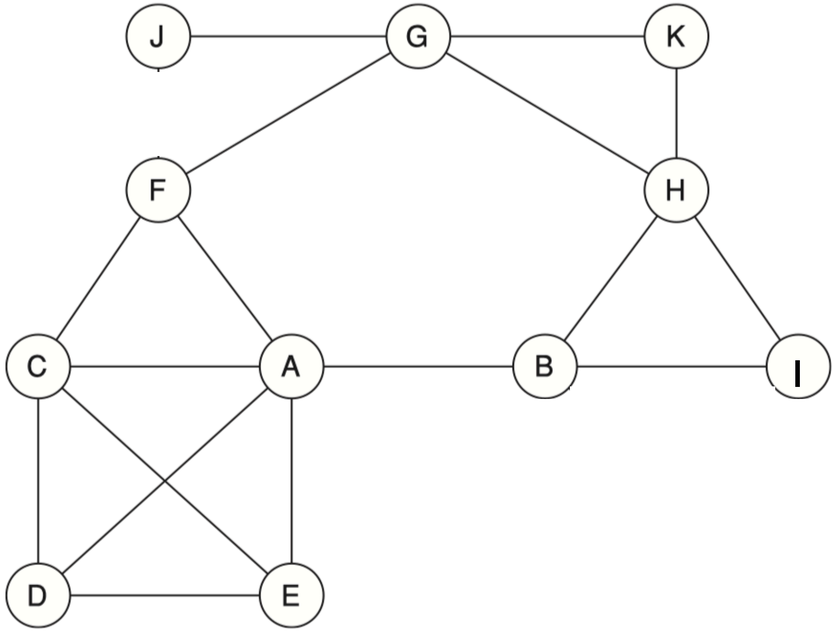

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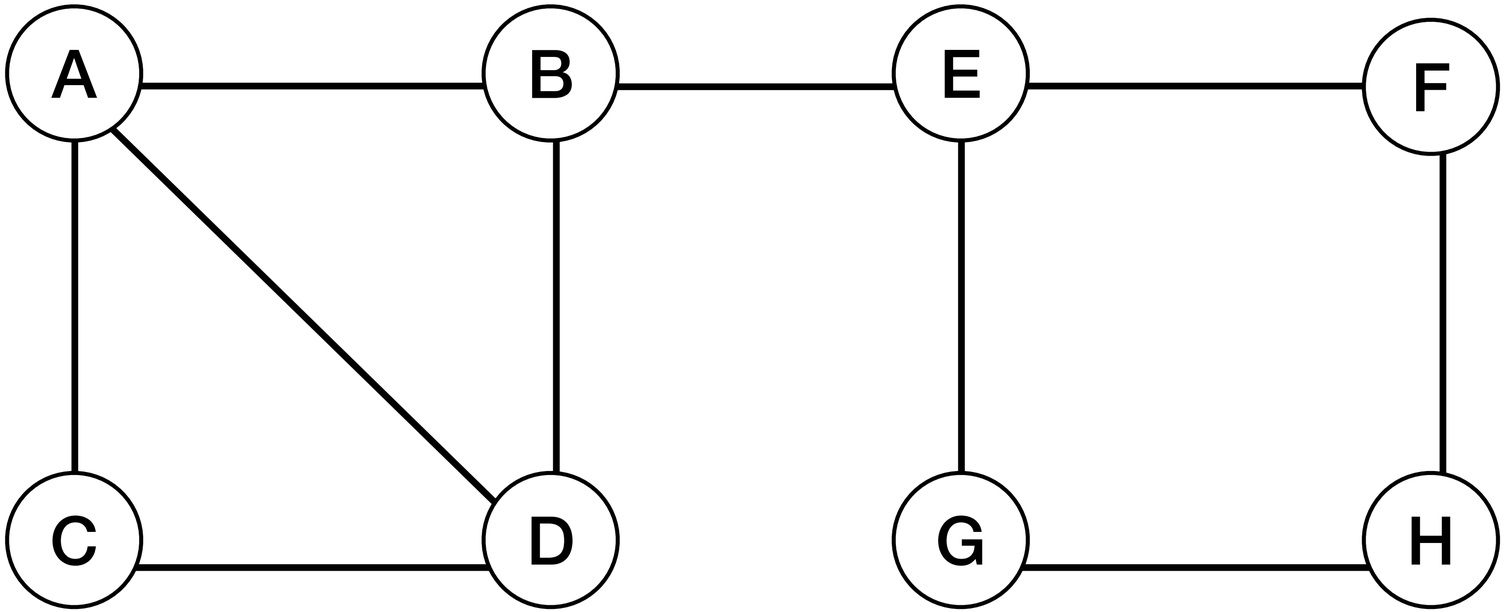

Bridge

Local bridge

Real-world networks

The Anatomy of the Facebook Social Graph

Johan Ugander[1,2*], Brian Karrer[1,3*], Lars Backstrom[1], Cameron Marlow[1†]
1 Facebook, Palo Alto, CA, USA
2 Cornell University, Ithaca, NY, USA
3 University of Michigan, Ann Arbor, MI, USA
∗ These authors contributed equally to this work.
† Corresponding author: cameron@fb.com

Analyzing data on 721 million active Facebook users (2011)

- component, covering users
- average distance between two users is
- The median degree (# friends):

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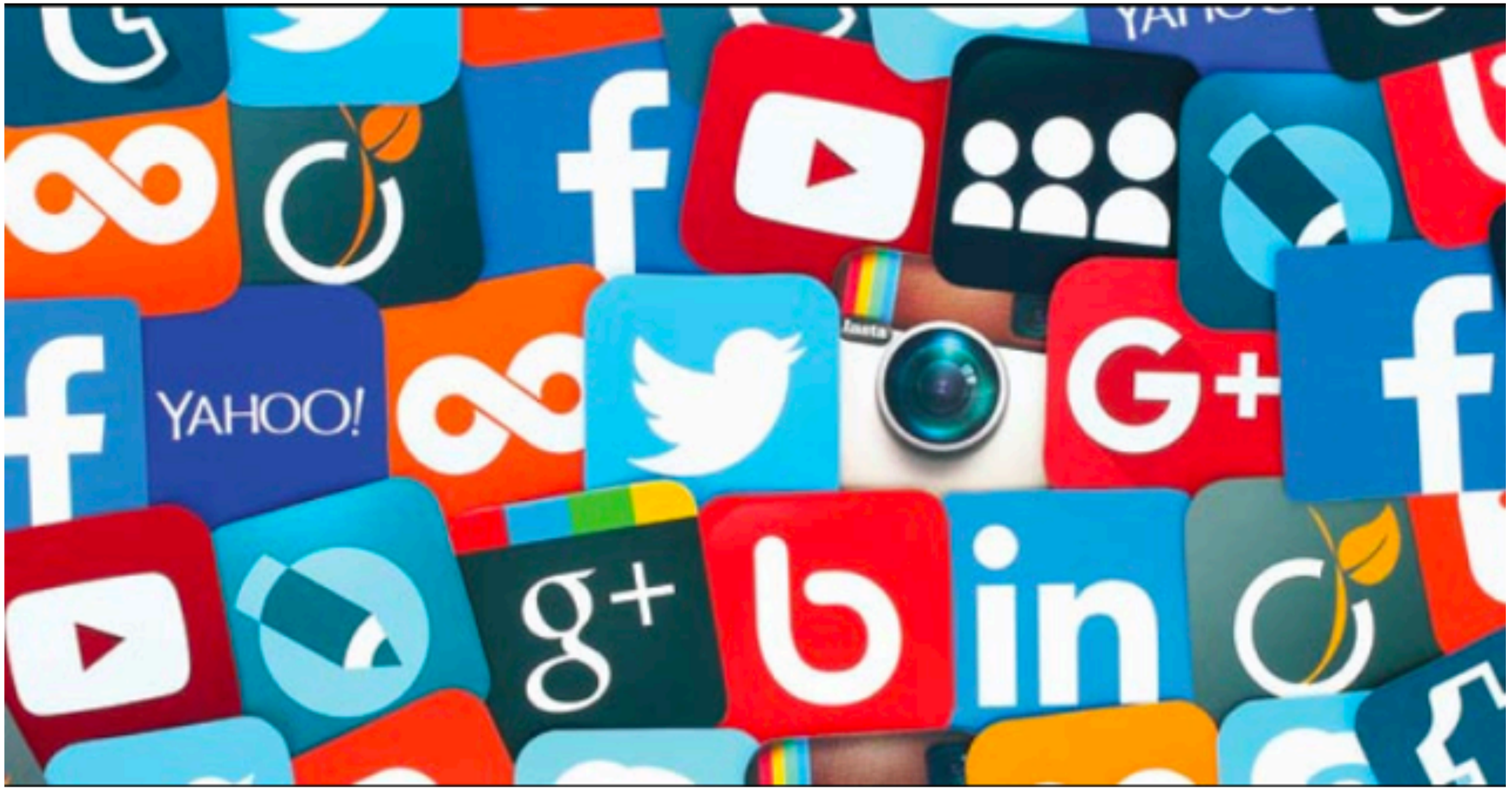
Analyzing data on 721 million active Facebook users (2011)

- A “giant” component, covering 99.91% users
- A “small” world, average distance between two users is 4.7
- The median degree (# friends): 99

Grow a network!

Promote “connections” as a product manager at a social network company



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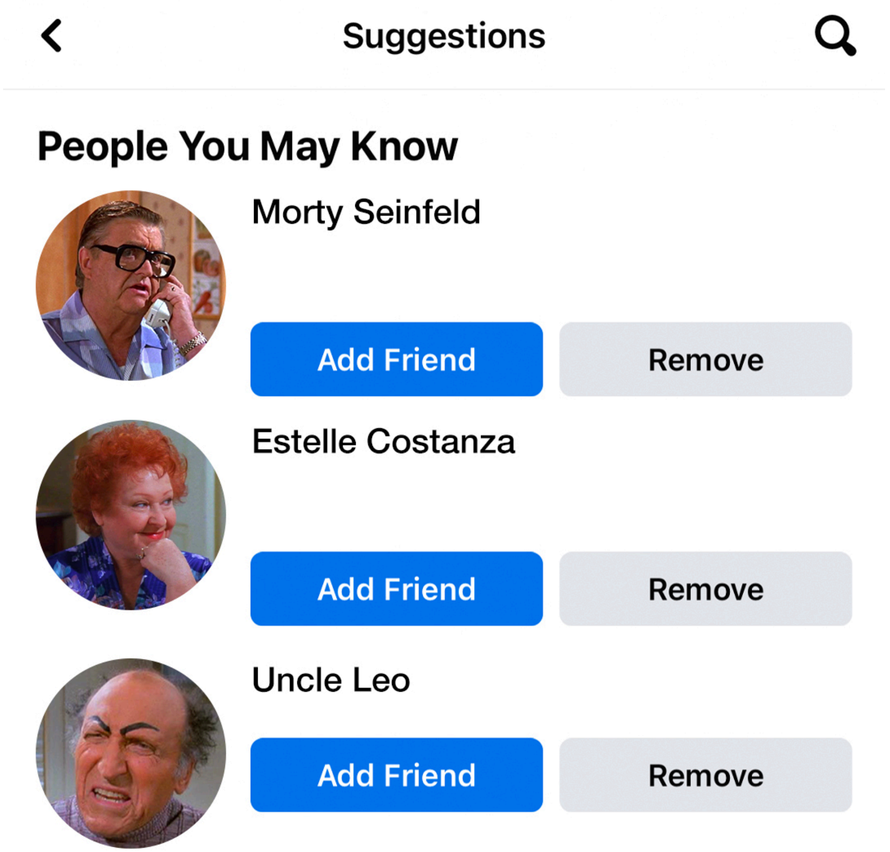

People you may know (PYMK)

You may want to connect with someone if you…

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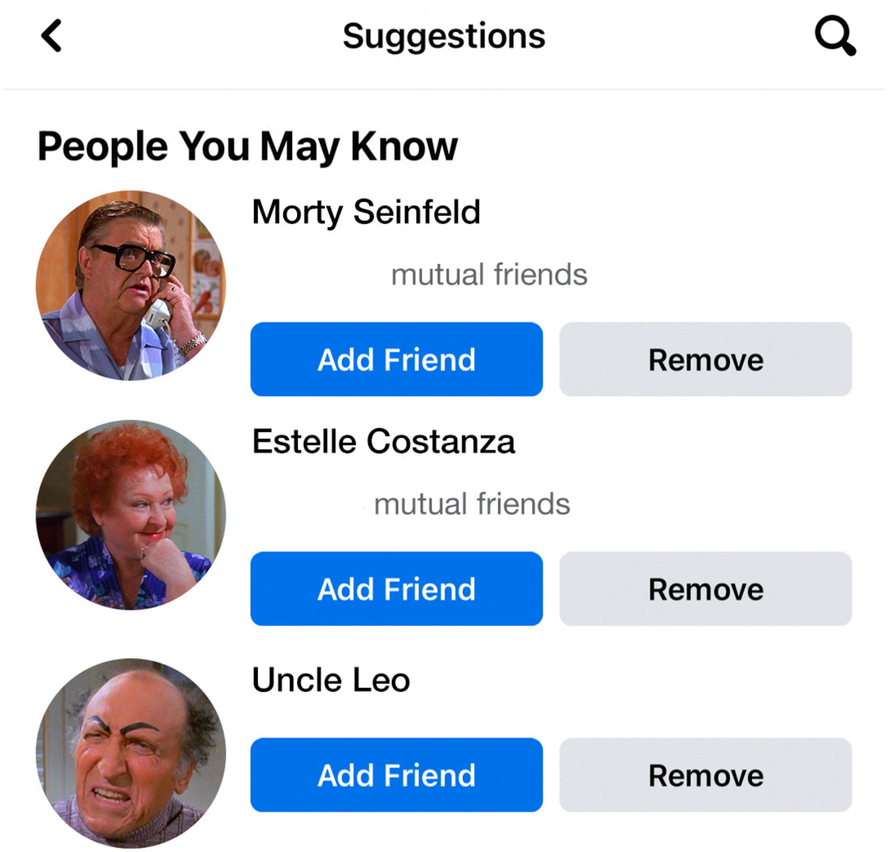


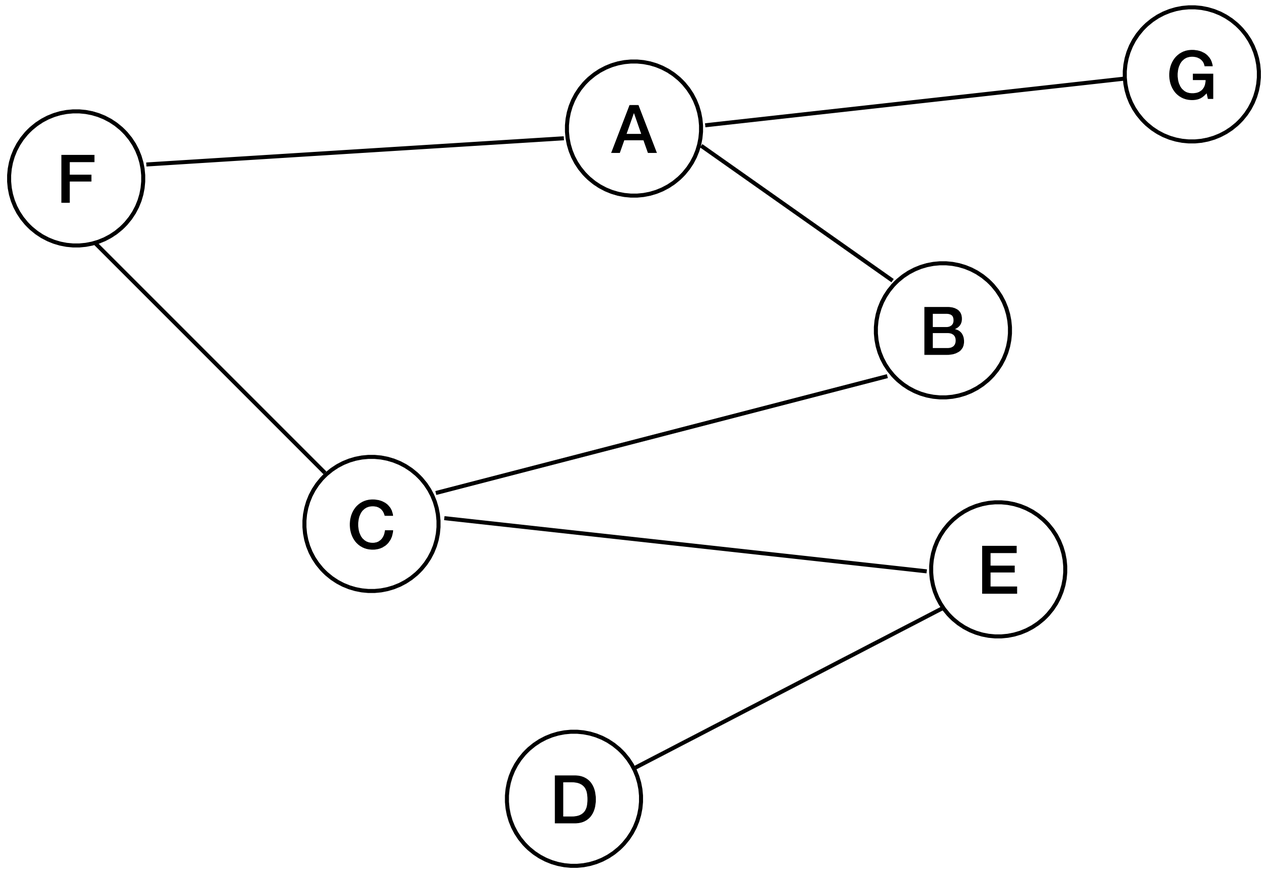
People you may know (PYMK)

You may want to connect with someone if you…

share some mutual friends

Triadic closure

Can you recommend someone that C may want to connect with?



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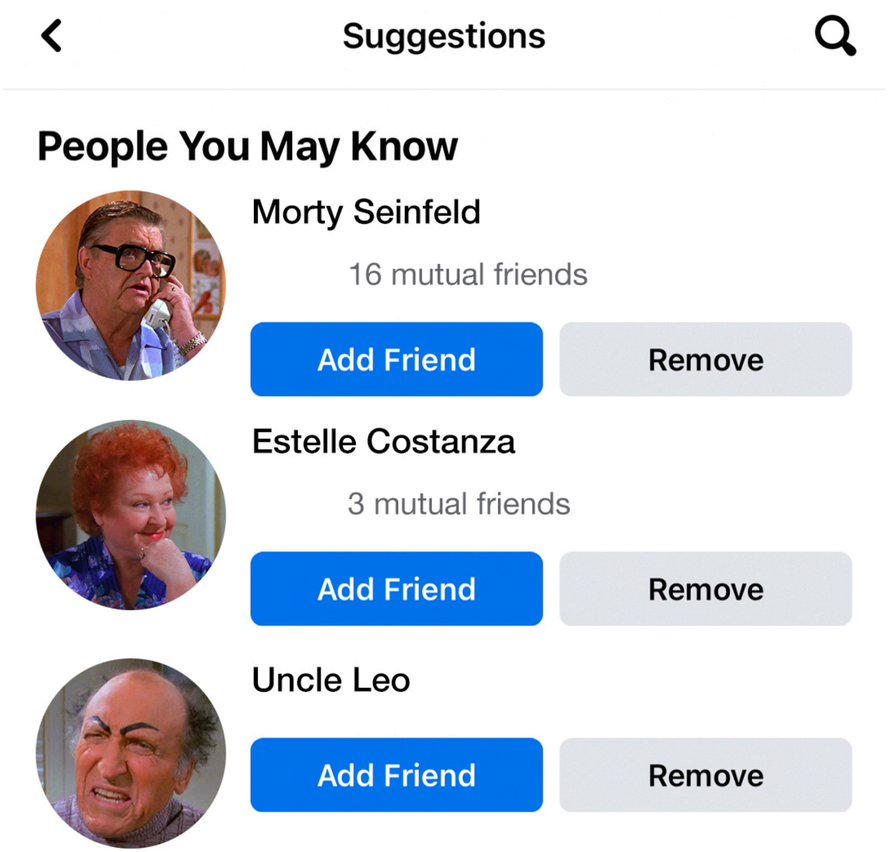

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share ***more*** mutual friends

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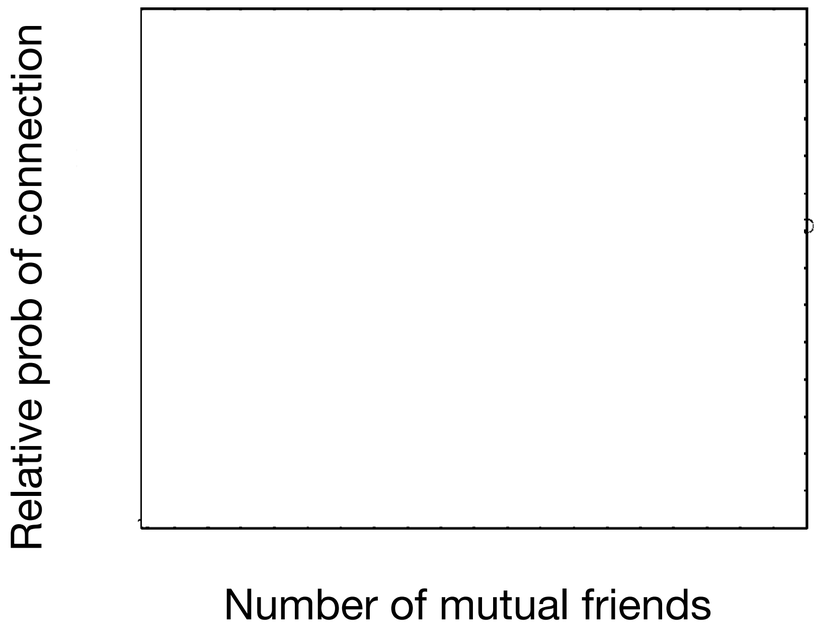
What's the likelihood for you to connect with…

someone with 0 mutual friends

someone with 5 mutual friends

someone with 10 mutual friends



Evolution of a scientific collaboration network (Newman 2001, Heiberger & Wieczorek 2016)

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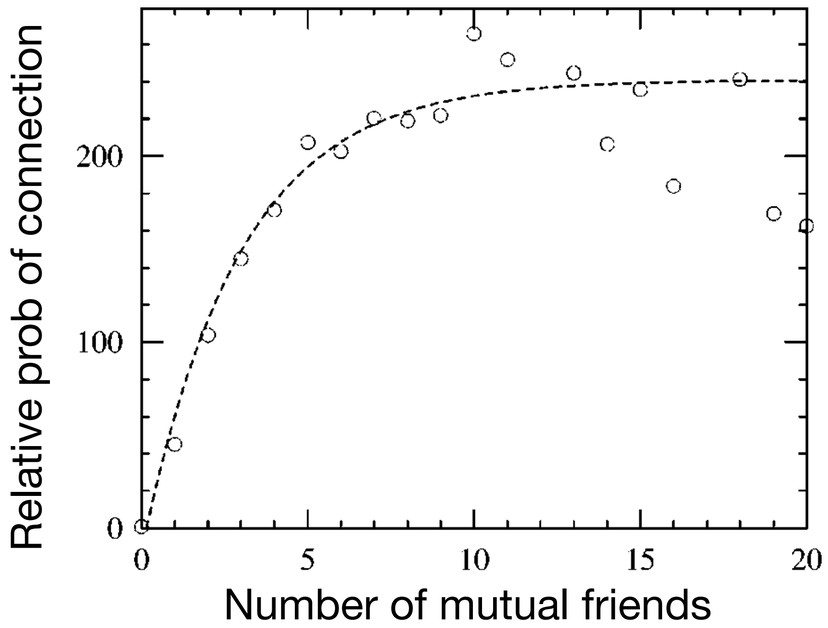

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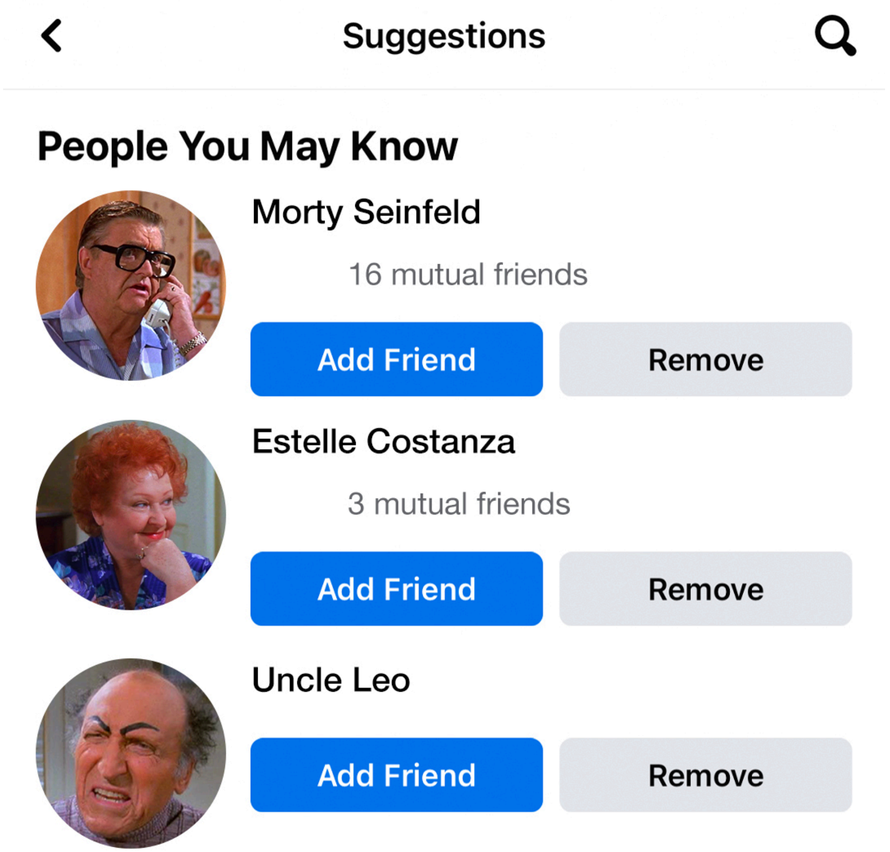

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Strong and weak ties

Close Friend / Acquaintance

Distant Cousin / Immediate Family

Long-term Neighbor / New Resident

University Roommate / Alumni Mixer Contact

Guest Lecturer / High School Mentor

Strong and weak ties

Strong ties

Weak ties

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Triadic closure and the forbidden triangle

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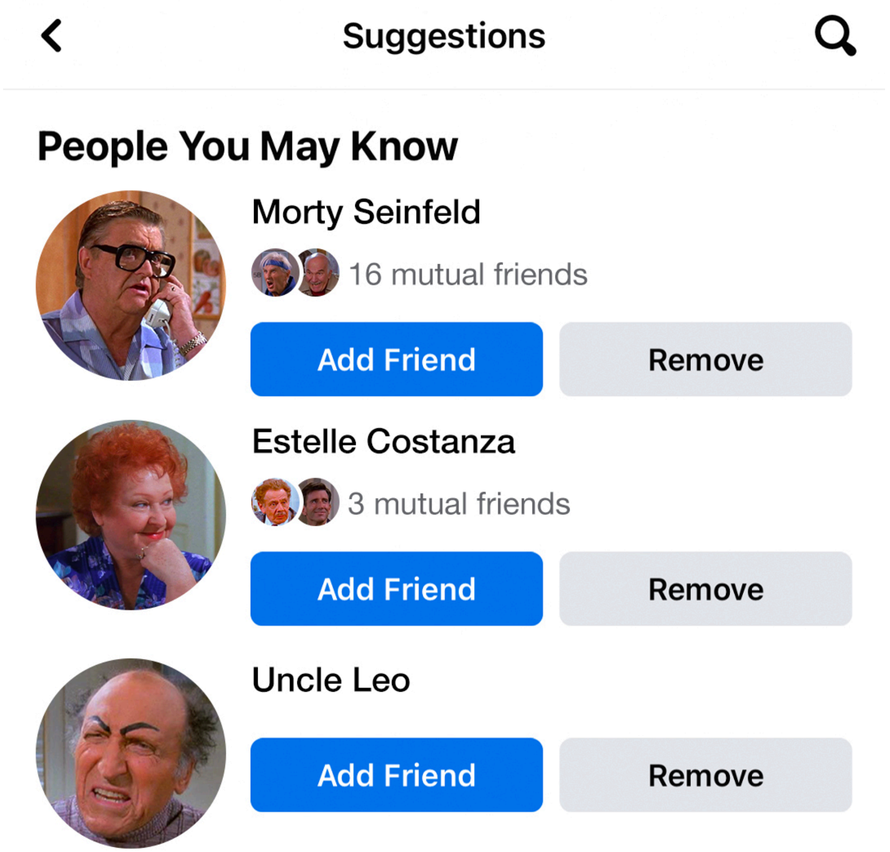

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You may want to connect with someone if you…

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share mutual ***strong-tie*** friends

Triadic closure and the forbidden triangle

Strong Triadic closure

If a node A has *strong ties* to nodes B and C, then the B-C edge is *very* likely to form.

Triadic closure and the forbidden triangle

Strong Triadic closure (STC)
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We say a given node

satisfies STC property, if every two of its strong tie friends are also friends

(or if it has less than two strong tie friends)

violates STC property, if two of its strong tie friends are not friends

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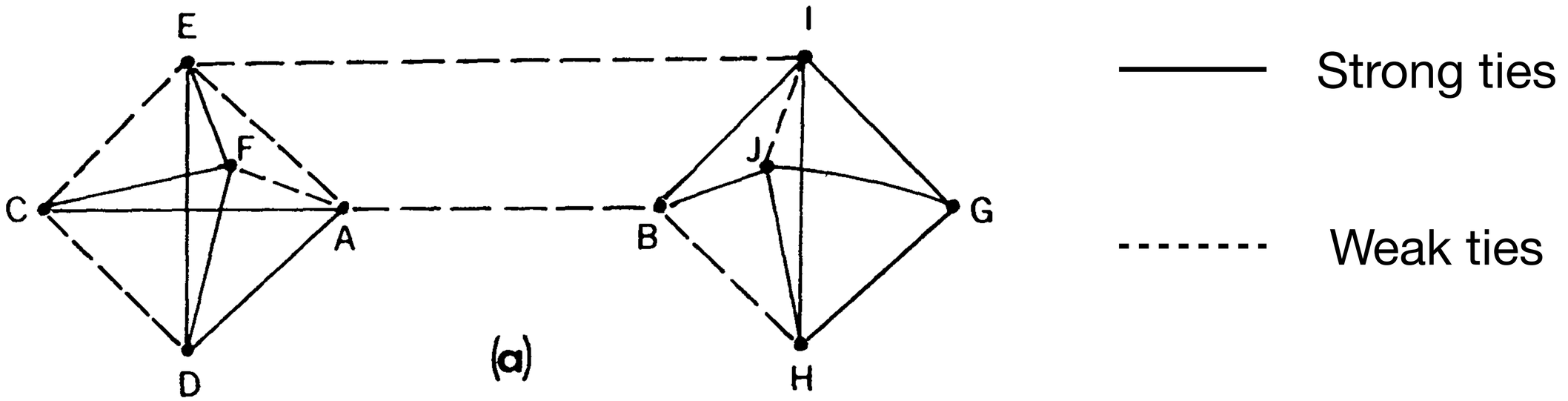
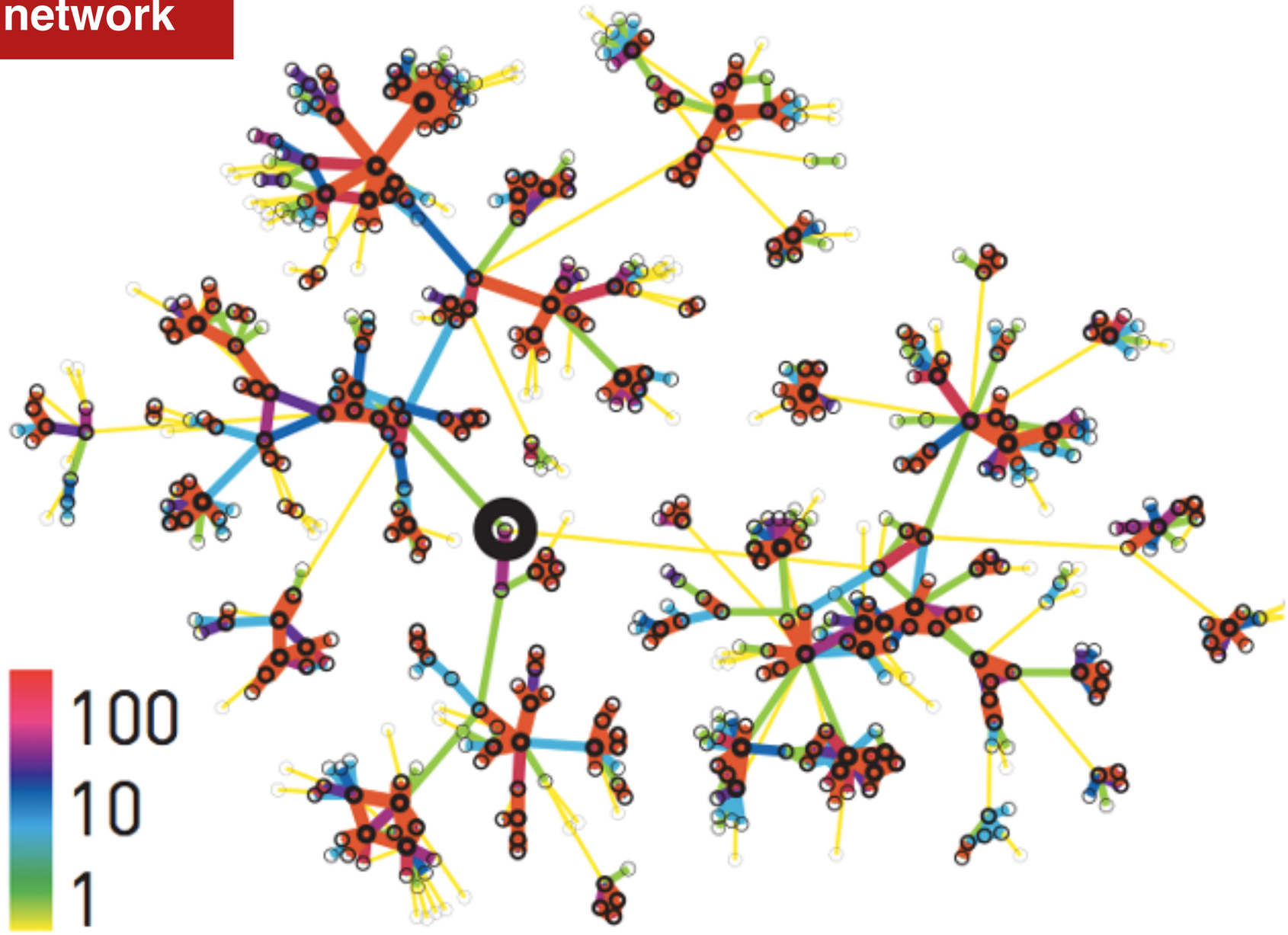


Weak ties in the network



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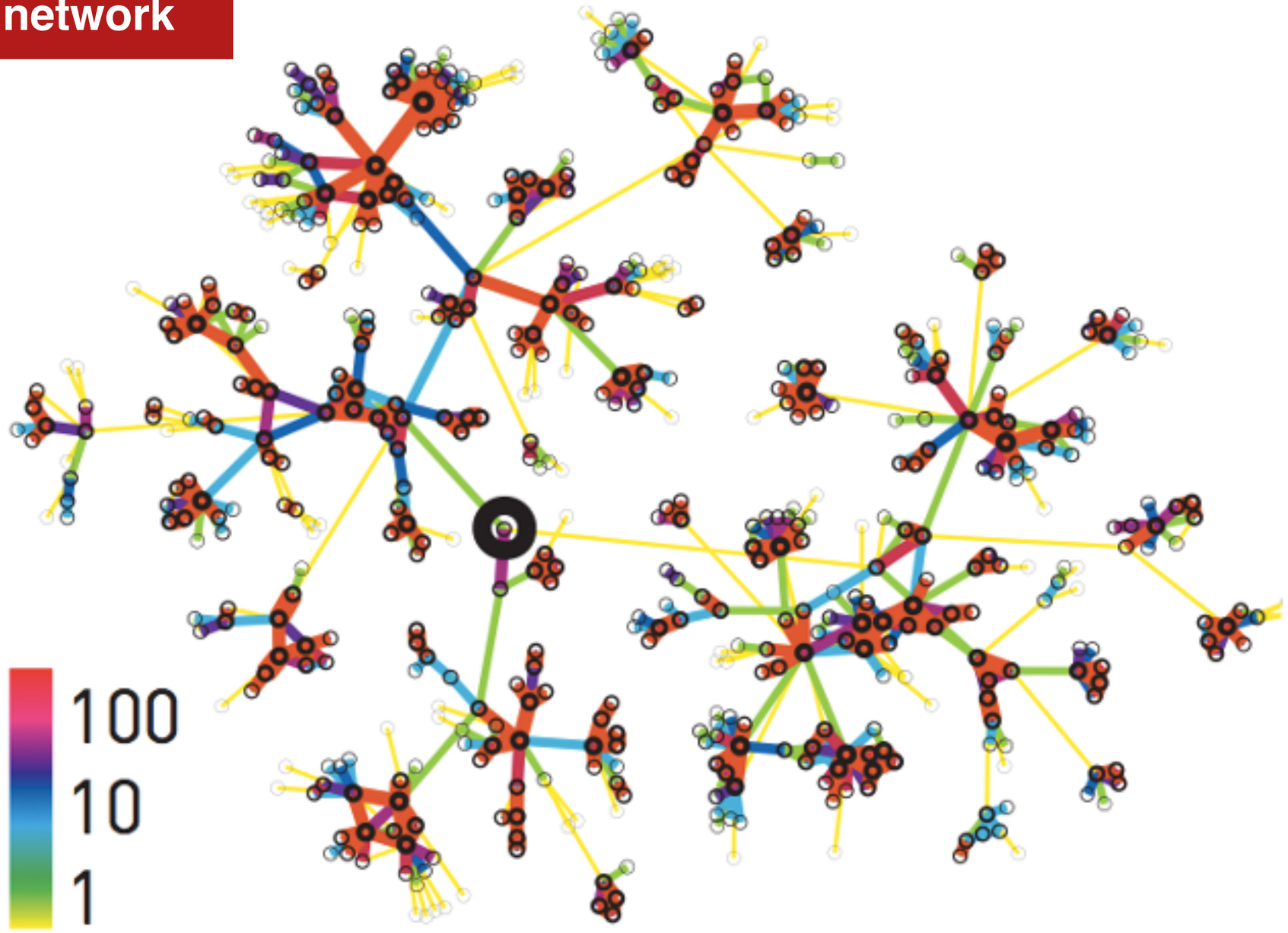

“Bridging” small communities within the network

A few definitions

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By definition, a bridge is always a local bridge

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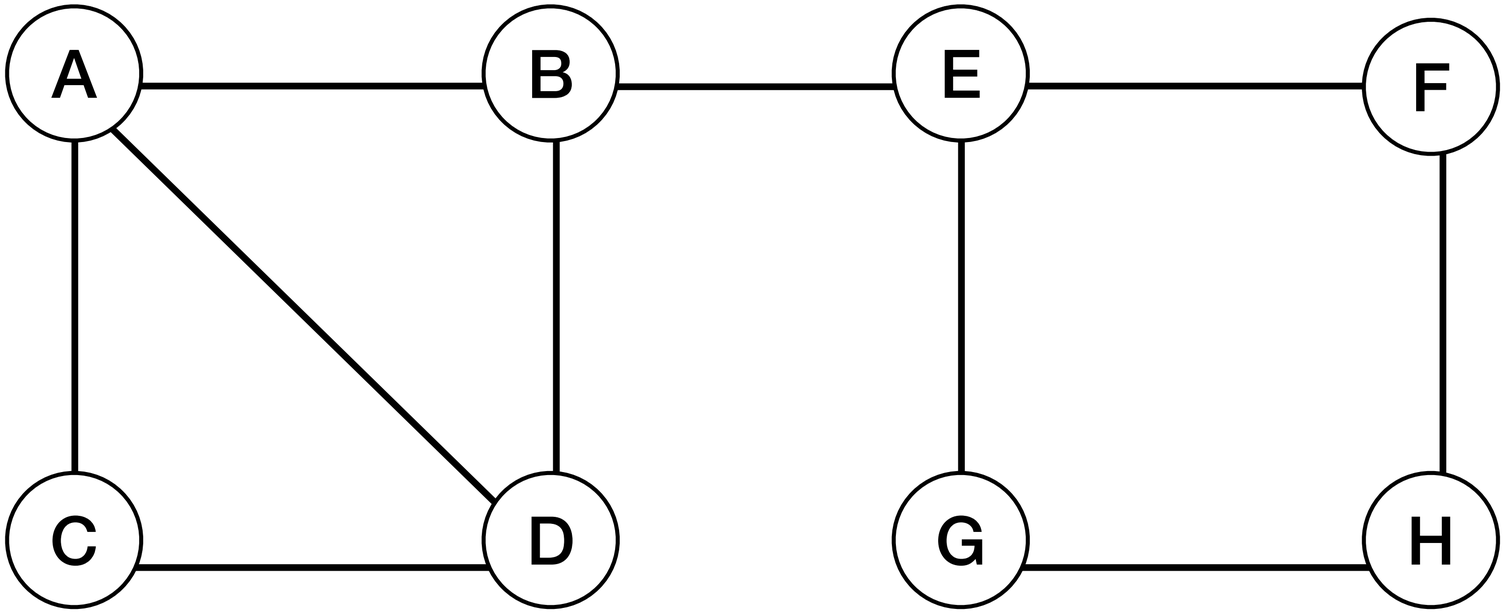

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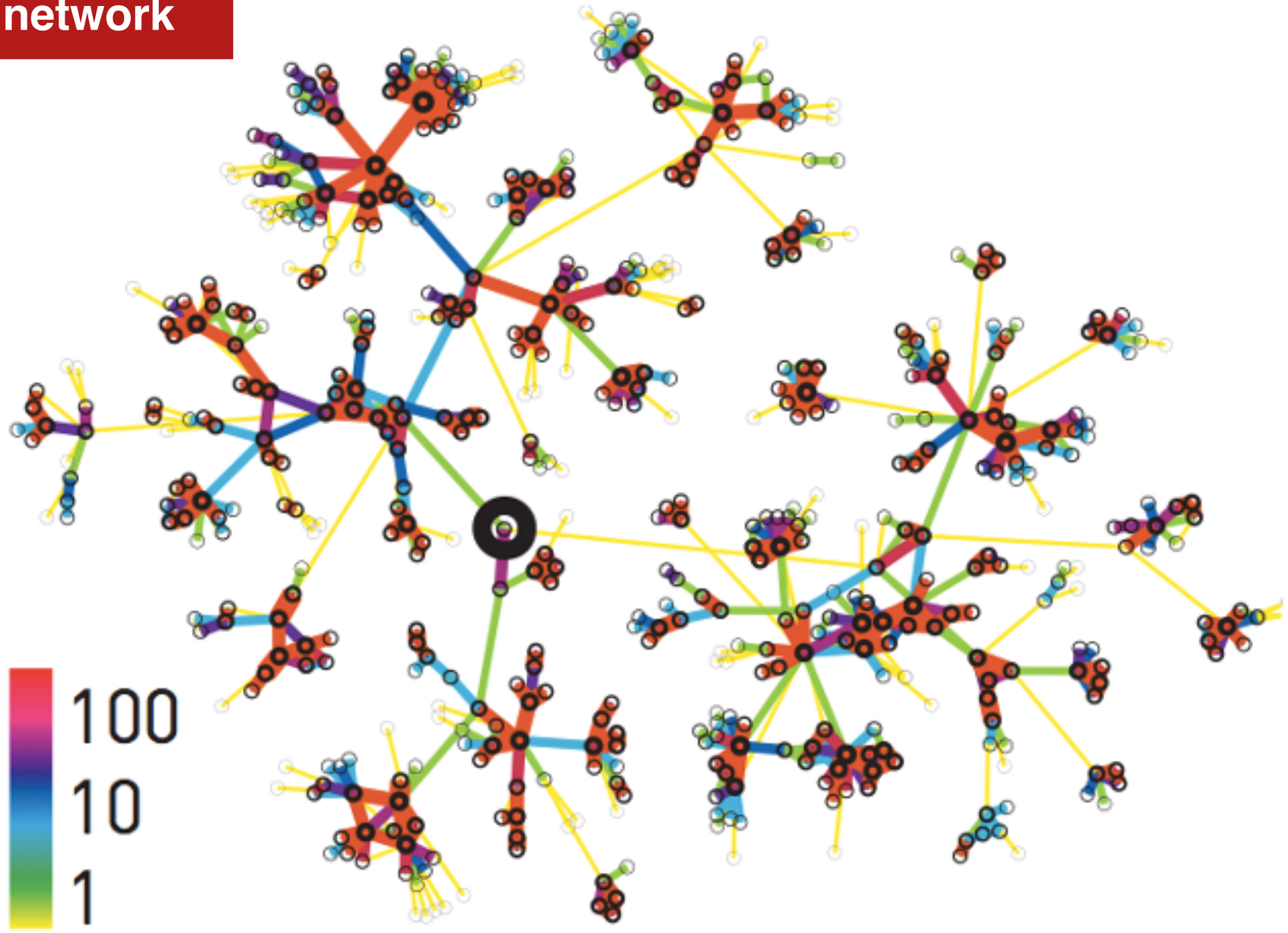

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Weak ties and local bridges

If a node A in a network

(1) satisfies the Strong Triadic Closure property; and

(2) is involved in at least two strong ties;

Then **any local bridge it is involved in must be a weak tie**.

In other words, **any strong tie it is involved in cannot be a local bridge**

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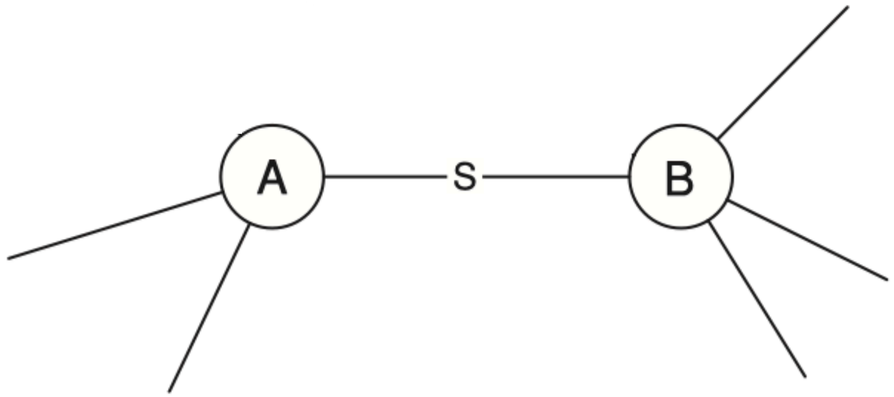

Consider a strong tie AB

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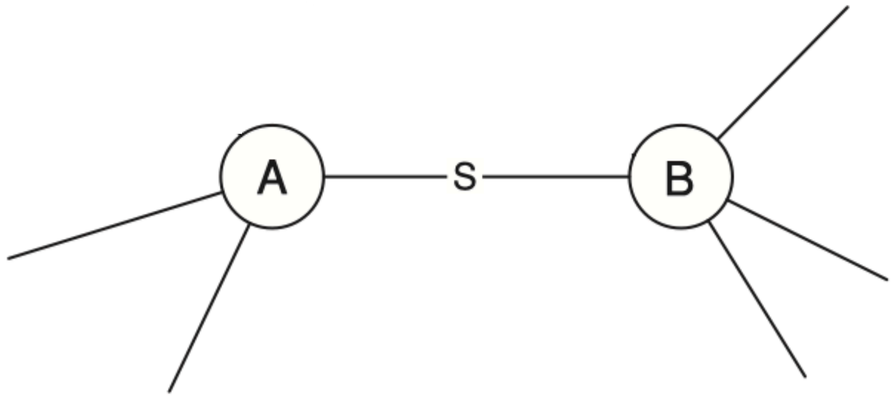

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From (2): there's another strong tie — AC

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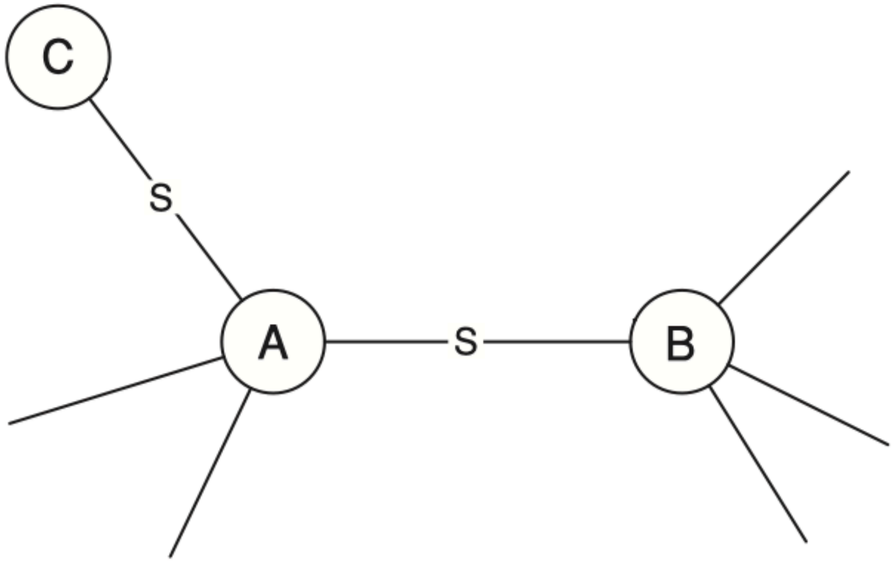

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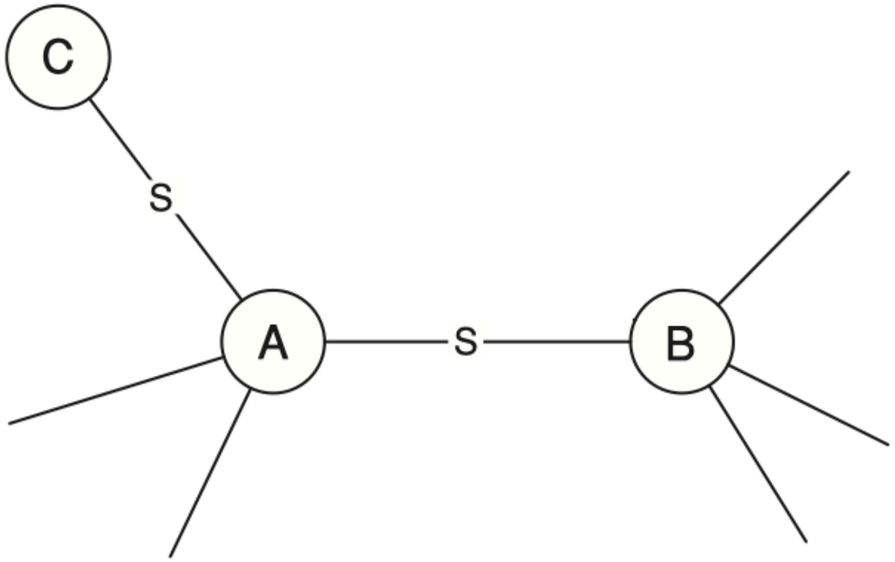

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From (1): there's a link between B and C

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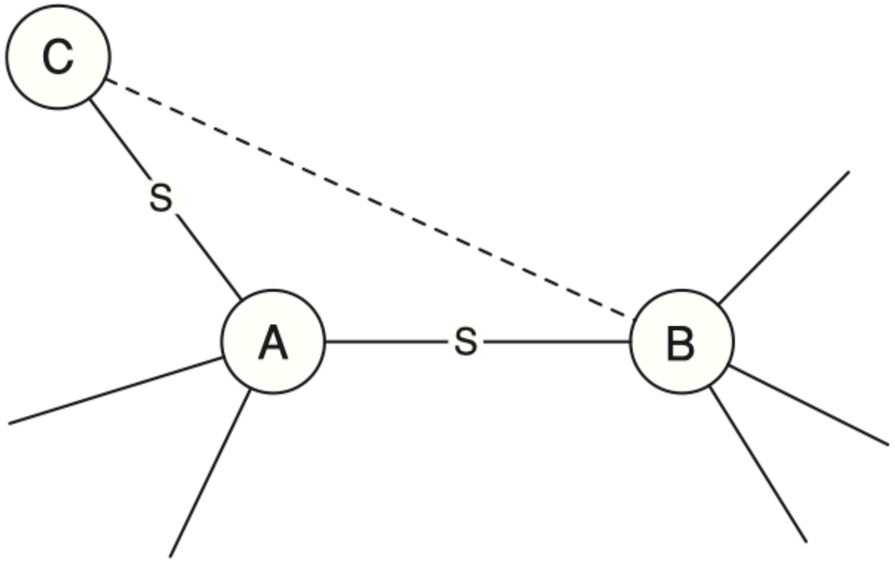

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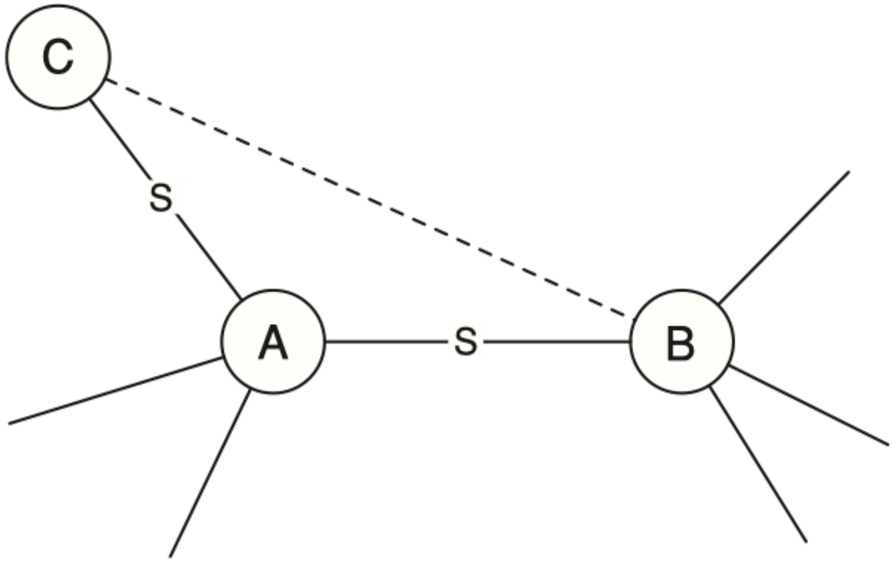


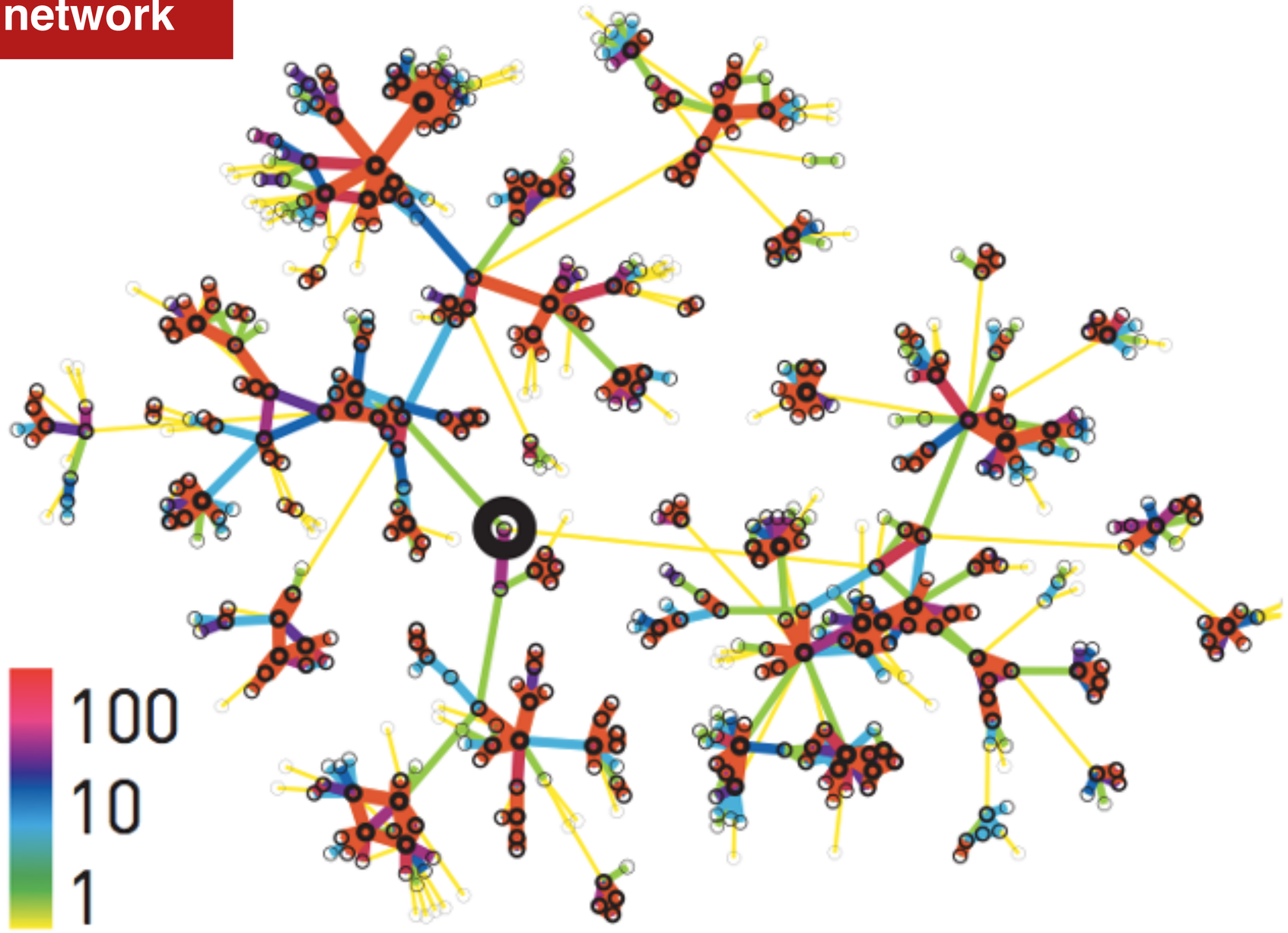
Consider a strong tie A and B

From (2): there's another strong tie — AC

From (1): there's a link between B and C

A and B has a common friend C: hence AB is NOT a local bridge

Weak ties in the network



Passing new information and knowledge