CS 2850

By Tony Song

August 2025

Contents

1	Mon 8/25	2
	1.1 Course info	2
	1.2 Networks	2
2	$\mathrm{Wed}\ 8/27$	3
	2.1 Graph Theory	3
	2.1.1 Degree	3
	2.1.2 Path	
3	Fri 8/29	4
	3.1 Strong and Weak Ties	4
4	$\mathrm{Wed}\ 9/3$	5
	4.1 Strong and Weak Ties	5
	4.2 Labeled Networks	
5	Fri 9/5	6

$1 \quad Mon \ 8/25$

1.1 Course info

One review class before before each prelim. One review class before final.

Suggest going to at least one TA sessions

Not punished for using AI, but def won't help learn

Exams are completely closed book

Usually, homeworks are out on Monday. Need to submit homework by **11:59 PM on Sundays**. HW is still taken 24 hours after but with 10% off. HW is rejected after 24 hours

1.2 Networks

Network:

Nodes (representing a person)

Link (representing connections between two people)

Inferences based on network info:

- 1. Who is the leader (people with the most connections)
- 2. Which one of the two parties an individual is more likely to side with

Networks can model the progression of (the spread of) a flu contagion

The shortest route is technically a straight line. GMaps gives the user a detour, however.

But the user has to travel on the network, a composition of a roads and paths.

Every intersection is a **node**. Every main road is one type of **link**. Every *other* road is another type of **link**

Systemic risk: If a node (A) is a large net seller, and (A) goes bankrupt, then every node in the network (even linked indirectly) can go bust.

Don't have to be a large node yourself. You are also systemic if you are linked with the big nodes.

If a node is dominant in connections, then they are **central**. There is some sort of **centrality** to it.

Besides finance, networks are also very relevant in politics.

Each node is a blog. Each link indicates a reference to a connected blog.

$2 \quad \text{Wed } 8/27$

2.1 Graph Theory

Graph: a way of specifying relationships among a collection of items Collection of items: nodes Relationships: edges/links

Collection of items	relationship
People	friendship
Institutions	borrow-lender
Products	proximity
Organism	predator-prey

Symmetric relationship: A is B's friend = B is A's friend

A graph that has **symmetric** relationship is an **undirected** network

Assymetric relationship: A follows $B \neq B$ follow A

A graph that has assymmetric relationship is an undirected network

2.1.1 Degree

Degree: the number of connections (edges) a node has to other nodes

In a network of n nodes and m edges, $0 \le \text{Degree} \le n-1$

Sum of degrees = 2m

Think about handshakes.

Imagine a classroom of 10 people. Each person shakes hands with exactly 3 others:

So $10 \times 3 = 30$ total degrees = $30 \div 2 = 15$ edges / handshakes

Could a graph exist where every node has degree 3, but the total number of nodes is 5? No. Total degrees = $3 \times 5 = 15$. Links = 15/2 = 7.5 edges. Have to be integer.

2.1.2 Path

Path: a sequence of nodes, where each consecutive pair of nodes is connected by an edge

You are technically allowed to go back and forth between two nodes.

Length of a path: number of steps it contains from beginning to end.

We are interested in the shortest path between two nodes

3 Fri 8/29

3.1 Strong and Weak Ties

A graph is "connected" if each pair of nodes can be connected through a path

It can still be decomposed into **connected components**.

Two nodes are connected through a path **if and only** if they are in the same connected component.

If two nodes are in the same connected component, there is always a path connecting the two

If two nodes are in different connected components, there is never a path connecting the two

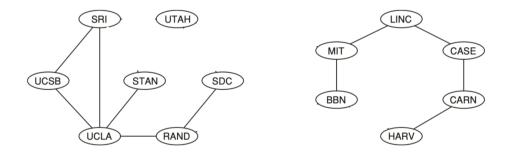


Figure 1: An isolated node is also a connected component

Bridge: An edge connecting x and y is called a bridge if deleting it would cause x and y to lie in two different components — the only path between x and y.

Local bridge: An edge connecting x and y is called a local bridge if deleting it would cause x and y to have a distance i 2 — no common friends between x and y.

Bridges are always local bridges

$4 \quad \text{Wed } 9/3$

4.1 Strong and Weak Ties

Strong Tradic closure (STC): If node a has strong ties to nodes B and C, then the B-C edge if *very* likely to form

A node **satisfies** STC property if every two of its strong tie friends are also friends (all of strong-tie friends must know each other), OR if it has less than two strong tie friends.

Otherwise it violates STC property

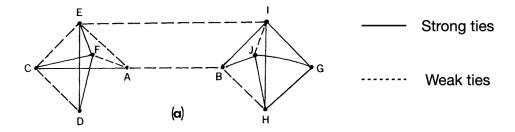


Figure 2: A and D satisfy the STC property, while J does not

Any local bridge must be a weak tie.

Weak ties are not necessarily worse than strong tie. Strong ties likely result in receiving the same information.

When trying to find a job, weak ties might would be more helpful. What is important is a diverse collection of strong and weak ties.

4.2 Labeled Networks

Does not make sense to label enemies with the same edge as friends

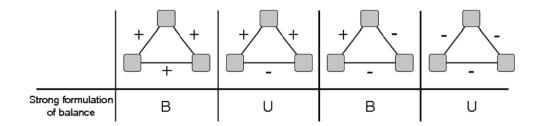


Figure 3: Second and fourth triangles are unbalanced (uncommon in society). In triangle 2, the common friend would likely have to take a side, resulting in triangle 3. In triangle 4, having common enemies can result in a mutual friend, as in triangle 3

5 Fri 9/5