Networks: Fall 2025	**Homework 1**
Rohit Lamba and Yian Yin	**Due 11:59pm, Sunday, September 7, 2025**

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope **be sure to assign your answers to the correct question**.

To be eligible for full credit, your homework must come in by 11:59pm Sunday (Sept 7th). We will also accept late homeworks after 11:59pm Sunday until 11:59pm Monday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 11:59pm Monday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and approved by the TA in charge of this homework prior to the due date of the homework. (These include illness, family emergencies, and travel associated with university activities.)

The TA in charge of this homework is Farnoosh Hashemi, sh2574@cornell.edu

Reading: The questions below are primarily based on the material in Chapters 2 and 3.

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(1) [4 points] In class, we discussed the ARPANET as an example of how the Internet can be represented as a graph (network). In this representation, nodes correspond to computing hosts, and edges (links) connect two nodes if a direct communication link exists between them. In this problem, we will investigate whether it is possible to build a mini-Internet that satisfies specific structural properties.

(a) Is it possible to construct a mini-Internet with exactly four hosts (nodes) and seven links (edges) connecting them? If your answer is "yes", provide an example of such a network, by listing the set of nodes and set of edges. If your answer is "no", explain why it is not possible.

(b) Is it possible to build a mini-Internet of four hosts such that (i) One host is connected to exactly two links (edges), and (ii) each of the other three hosts is connected to exactly one link? If your answer is "yes", provide an example of such a network, by listing the set of nodes and set of edges. If your answer is "no", explain why it is not possible.

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(2) [8 points]

A group of economists are studying the interactions between 10 banks. They drew the graph in Figure 1 to describe these interactions with nodes representing the banks and edges between two nodes representing the idea that these two banks interact with each other. So for example, banks A and D interact, but banks A and B do not interact. These "interactions" are meant to describe economic transactions between the banks.

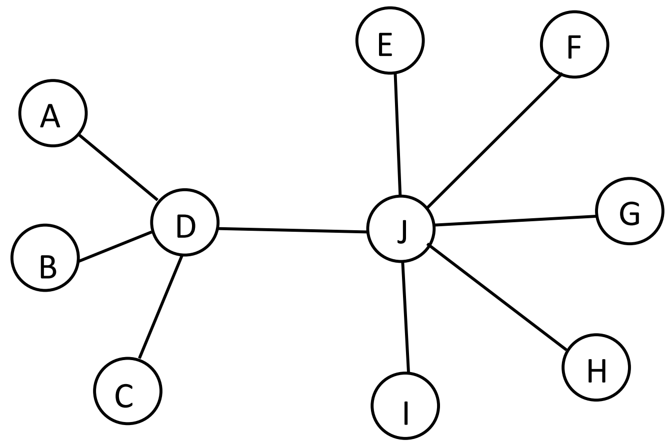

Figure 1: Qusetion 2

These economists are interested in measuring how important individual banks are purely as a function of their location in this network. One way they are considering is to count the degree of each bank. The *degree* of a node in an undirected graph such as Figure 1 is the number of edges attached to the node.

(a) For each node in Figure 1, what is it's degree?

An alternative way to ask about importance is to ask how many connected components the graph would have after removing one of the banks.

(b) For each node in Figure 1 how many connected components would the graph have if this bank was removed from the graph?

Another way to measure the importance of a bank would be to consider how close it is to all the other banks. Recall that the length of a path in a graph is the number of edges it contains, and the distance between two nodes is the length of the shortest path between them. (For example, the distance between A and J is 2, since there are two edges on the path from A though D to J.) Using the notion of distance, we'd like to consider what it means for a node to be "centrally located" in the bank network. One option for defining this notion of centrality is that the number of hops that are necessary to get to any other bank should be as small as possible.

(c) Is there a bank (or banks) in Figure 1 with the property that the maximum distance from this bank to any other bank at most 2? If there is, name all such banks and say why they have this property. If there isn't, give a brief explanation why not.

(d) In the example given in Figure 1 there is a close connection between a node have a high degree and being centrally located (using the definition of "centrally located" given above). Is this always true? That is, are there graphs such that a node with the " highest degree" is not centrally located? Either give an example in which "highest degree" and "centrally located" are different or give an argument for whey they agree for all graphs.

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(3) [6 points] One application of networks is to encode visibility in a complex scene. This question arises in a number of areas of computing, including robotics – where we might want have a set of mobile robots exploring an environment and want to understand which ones are able to see each other – and graphics – where we might be animating a digital scene and would like to know which objects can be seen from which other objects.

With this in mind, we can define a visibility graph as follows. Suppose we have a geometric arrangement of points and filled-in polygons in the plane, as shown in the example in Figure 2. We'll call this collection of points and polygons a "scene". In the scene depicted in Figure 2 (left), we have five points labeled A, B, C, D, and E, and two rectangles. We think of the points as taking up no space and not obstructing any views; we draw them as dots just to convey where they are. The rectangles are solid, and they obstruct visibility. (Although we've used rectangles here, we could use any polygon.)

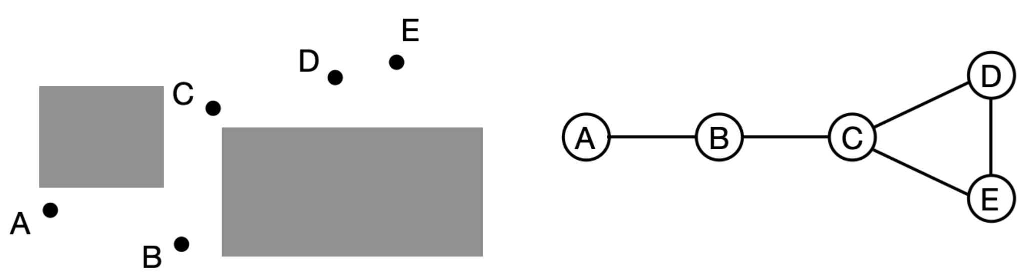

Figure 2: Question 3

We say the visibility graph for this scene has the points as its nodes, and we connect two nodes by an edge if they can see each other — that is, if their direct line of sight is not blocked by any of the polygons. (Note that visibility is mutual — in these types of scenes as defined, if I can see you, then you can see me.) In our example, A can see only B; B can see A and C; and C, D, and E can mutually see each other. (This also illustrates how the points in a scene don't obstruct visibility: C and E can see each other even though D is on the line of sight between them.) Therefore the visibility graph is the graph shown in Figure 2 (left). (It's important to remember that the way we draw the graph, as in Figure 2 (right), is designed only to record which node is connected to which; the physical layout of the nodes in the drawing is not intended to be related to their physical layout in the scene.)

When we construct this graph, it can provide us with information about visibility in a much more compact format than we get from the full scene. (Of course, the graph also fails to convey other information about position.)

(a) Construct the visibility graph for the scene in Figure 3. You can either list the edges or draw the network with labeled nodes.

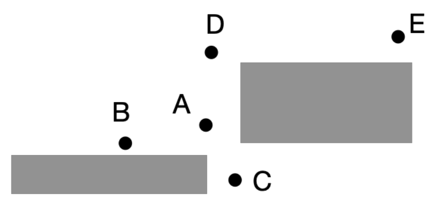

Figure 3: Question 3a

(b) An interesting question that comes up in this topic is a kind of "inverse" problem: given a graph G, can you find a scene of points and polygons for which G is the visibility graph of the scene? An example of this type of problem would be the case in which you were given the graph in Figure 2 (right) and asked whether there's a scene for which it's the visibility graph. A valid answer would be the scene in Figure 2 (left).

Consider the graph in Figure 4. Can you find a scene for which it's the visibility graph? If your answer is "yes", provide a drawing of such a scene (including a label for where each point is). If your answer is "no", explain why it is not possible.

(c) Consider the graph in Figure 5. Can you find a scene for which it's the visibility graph? If your answer is "yes", provide a drawing of such a scene (including a label for where each point is). If your answer is "no", explain why it is not possible.

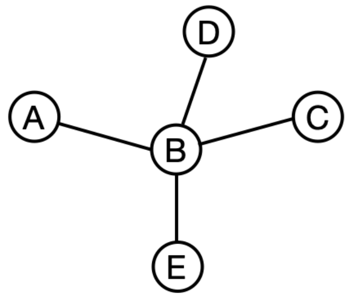

Figure 4: Question 3b

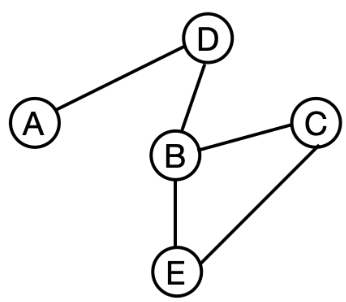

Figure 5: Question 3c

(4) [6 points]

We've looked at examples from large online social platforms in our lectures. However, due to privacy settings, these platforms might not show all friendships. Only some of the friendships are publicly visible.

(a) Suppose you are interested in the social network structure on a popular website named Y. Figure 6 shows such a friendship network based on publicly available information from Y, where solid lines represent strong ties and dashed lines represent weak ties. Identify all bridges and local bridges in the network.

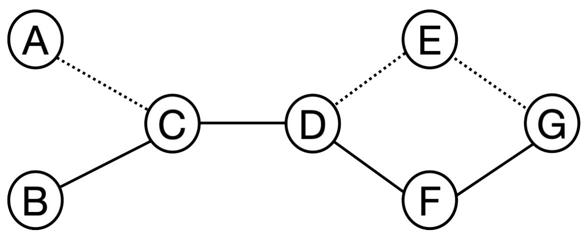

Figure 6: Question 4

(b) Use the Strong Triadic Closure Property to find the potential 'hidden' friendships that aren't shown in Figure 3 but likely exist in the real world.

(c) Add all hidden ties identified in **(b)** to the original network (treat them as weak ties). Revisit the bridges and local bridges you listed in **(a)**: are they still bridges or local bridges?

(5) [6 points] Some social psychologists are studying the friendships within a dorm at a boarding school, classifying each as a strong tie or a weak tie. The dorm has four floors and each floor contains 20 students.

Let's suppose that there is a strong tie between each pair of people who live on the same floor. Additionally, there is a strong tie between each student on the first floor and each student on the second floor, and also a strong tie between each student on the first floor and each student on the third floor. Finally, there is a weak tie between each student on the third floor and each student on the fourth floor. There are no other ties (strong or weak) aside from the ones described here.

(a) In this network, which students satisfy the Strong Triadic Closure Property, and which violate it? Provide an explanation for your answer.

The social psychologists who are analyzing this compare the results to what they found in another dorm, also with four floors and 20 students per floor. In this other dorm, it is again the case that there is a strong tie between each pair of people who live on the same floor. Additionally, there is a strong tie between each student on the first floor and each student on the second floor, and also a strong tie between each student on the third floor and each student on the fourth floor. Finally, there is a weak tie between each student on the second floor and each student on the third floor. There are no other ties (strong or weak) aside from the ones described here.

(b) In this network, which students satisfy the Strong Triadic Closure Property, and which violate it? Provide an explanation for your answer.