1.). $w(t) = 1$ for $|t| < \pi/2$ & $w(t) = 0$ otherwise

$T_0 = 2\pi$

- $w(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jw_0 \cdot kt}$ where $w_0 = \frac{2\pi}{T_0} = 1$

- $c_k = \frac{1}{T_0} \int_{T_0} w(t) \cdot e^{-jkw_0 t} dt = \frac{1}{2\pi} \int_{2\pi}^{\pi} w(t) e^{-jkt} dt$

- Since $w(t) = 1$ only on $[-\pi/2, \pi/2]$

$$c_k = \begin{cases} \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{-jkt} dt = \frac{1}{\pi k} \cdot \sin\left(\frac{k\pi}{2}\right), & k \neq 0 \\ \frac{1}{2\pi} \int_{-\pi}^{\pi} w(t) dt = \frac{1}{2} & k = 0 \end{cases}$$

↳ Complex Coefficients are:

$c_0 = \frac{1}{2}$, $c_k = \frac{1}{\pi k} \cdot \sin\left(\frac{k\pi}{2}\right)$ $(k \neq 0)$

- Recovering Cosine Coefficients:

  - $c_n e^{jnt} + c_{-n} e^{-jnt} = 2c_n \cos(nt)$

- Given the expression for c_n:

  - $2c_n = \frac{2}{\pi n} \cdot \sin\left(\frac{n\pi}{2}\right) = \begin{cases} \frac{2}{\pi n}, & n = 1, 5, 9 \ldots \\ -\frac{2}{\pi n}, & n = 3, 7, 11 \ldots \\ 0, & \text{when } n \text{ even} \end{cases}$

$\therefore w(t) = c_0 + \sum_{n=1}^{\infty} (c_n e^{jnt} + c_{-n} e^{-jnt}) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n} (-1)^{\frac{n-1}{2}} \cos(nt)$

$= w(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \ldots\right)$

2.) • Unipolar square wave has levels 0 & A

bipolar square wave has levels −A & +A

→ in order to map together use linear scaling & shift

∴ linear transform: $y(t) = \alpha x(t) + \beta$ ←

Note:
$y(t)$ = unipolar signal
$x(t)$ = bipolar
α = scaling
β = phase

Apply bounds : for $x(t) = 0$

$y(t) = \alpha(0) + \beta = \beta = -A$

for $x(t) = A$

$y(t) = \alpha(A) + \beta = \alpha A + \beta = +A$

$\therefore \alpha A - A = A \Rightarrow \alpha A = 2A \Rightarrow \alpha = 2$

• Given the scaling factor $\alpha = 2$ the linear transform of the unipolar square wave to the bipolar square wave is:

$$y(t) = 2x(t) - A$$

3.) • Given $\omega_0(t) = \frac{4}{\pi}\left(\cos(\omega_0 t) - \frac{1}{3}\cos(3\omega_0 t) + \frac{1}{5}\cos(5\omega_0 t) - \frac{1}{7}\cos(7\omega_0 t) \pm \dots\right)$

↳ bipolar ∴ only odd harmonics

• for nth odd harmonic we have: $C_n = \frac{4}{\pi n}, \; n = 1, 3, 5, 7, \dots$

• note the scaling : $C_n = \frac{4}{\pi n} \cdot (-1)^{\frac{n-1}{2}}$

• if even : $C_n = 0$

Harmonic n	fraction	Decimal
1	$\frac{4}{\pi}$	1.27
2	0	0
3	$-\frac{4}{3\pi}$	−0.42
4	0	0
5	$\frac{4}{5\pi}$	0.25
6	0	0
7	$-\frac{4}{7\pi}$	−0.18

4.) From lecture 5:

Fourier Coefficients are defined as

$$a_K = \frac{2}{T_0}\int_{T_0} x(t)\cos(K\omega_0 t)\,dt, \quad b_K = \frac{2}{T_0}\int_{T_0} x(t)\sin(K\omega_0 t)\,dt$$

· Symmetry of the Square wave

bipolar Square wave is even function at the vertical axis

$x(-t) = x(t)$

$\cos(-t) = \cos(t)$

↳ cosine is even : ~~cos(-t) = cos(t)~~

sine is odd : $\sin(-t) = -\sin(t)$

So, for Coefficients

a_K : $x(t)\cos(K\omega_0 t)$ (even × even) = even (non-zero)

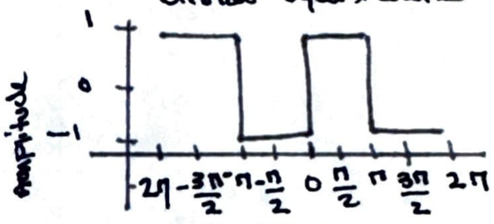
b_K : $x(t)\sin(K\omega_0 t)$ (even × odd) = odd (zero)

↳ note this is zero over a <u>symmetrical period</u>

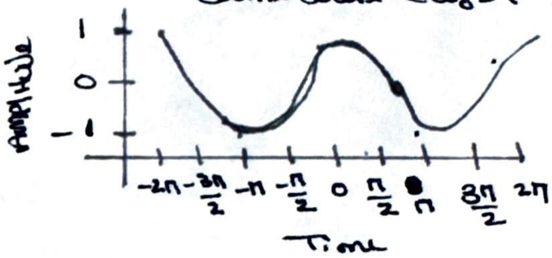
$\therefore b_K = 0$ for all K

5.)

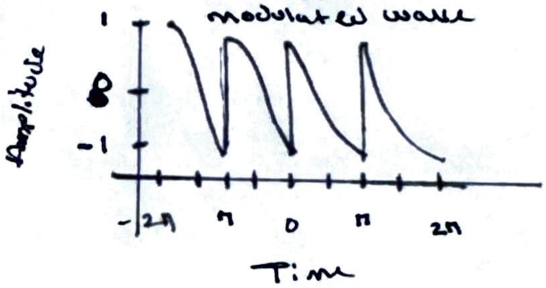
Bipolar Square wave



Cosine wave ($\omega_0 = 1$ rad/s)



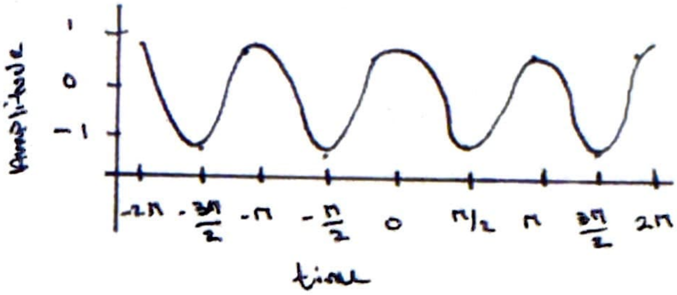
modulated wave



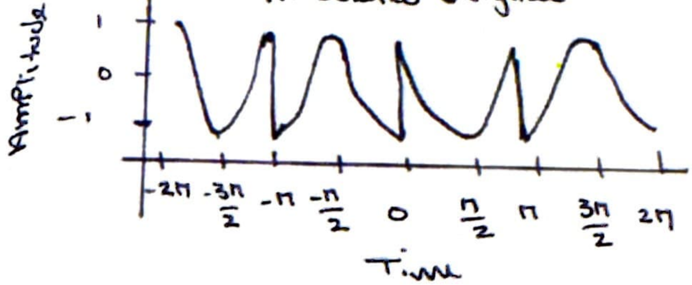
→ The Square wave is modulating the Signal

6.) • Square wave is the same

cosine wave (2^{nd} harmonic)



modulated signal



$a_2 = 0$

- because over a period the positive lobes cancel out the negative
- This is true for all even harmonics because the square wave has ½ wave symmetry: $x(t + T_0/2) = -x(t)$

∴ the fourier series of a bipolar square wave only contains <u>odd harmonics</u>

7.) Given: • $f_s = 48{,}000$ sample/sec

Signal → $f = 400Hz$

$$T = \frac{1}{f} = 2.5ms$$

• samples per period: $N = f_s \cdot T = \frac{f_s}{f}$

$$\therefore N = \frac{48K}{400} = 120$$

∴ each period of the 400Hz signal is represented by 120 samples (at 48kHz sampling rate)