## Pre-lab

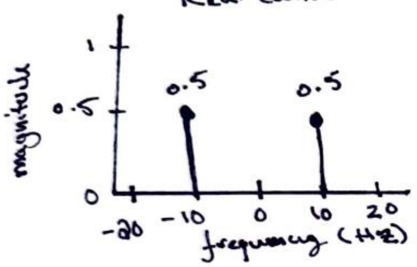
1.) · Fourier Transform : $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} \, dt$

· for Carrier Cosine : $x(t) = \cos(2\pi f_c t)$

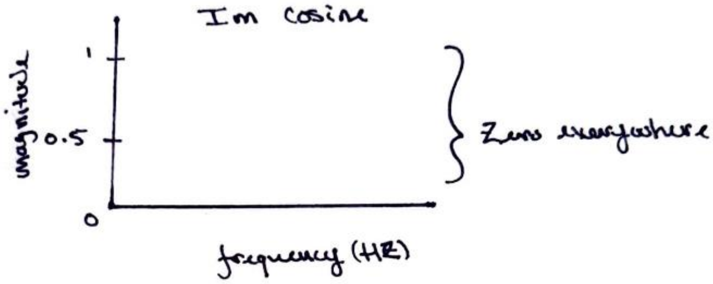
$$\mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)$$

· Re : Impulse at $\pm f_c$ w $\frac{1}{2}$ magnitude
· Im : Identically Zero

Real cosine




Im cosine



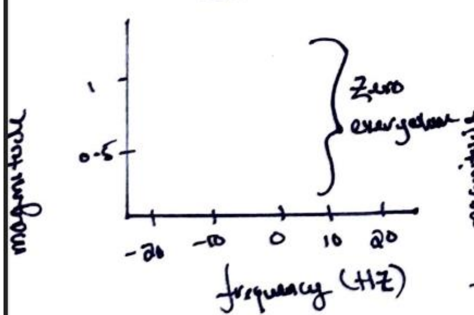

· for Carrier Sine $x(t) = \sin(2\pi f_c t)$

$$\mathcal{F}\{\sin(2\pi f_c t)\} = \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$$

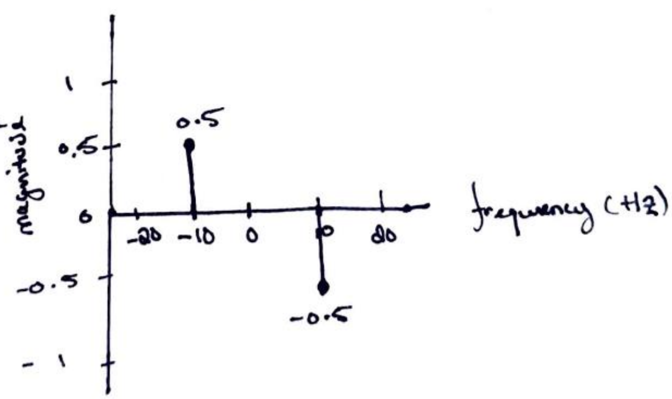
$$= -\frac{j}{2}\delta(f-f_c) + \frac{j}{2}\delta(f+f_c)$$

· Re : Identically Zero
· Im : Impulses at $f = \pm f_c$ with $-\frac{1}{2}$ and at $f = -f_c$

Real ~~cosine~~ sine




Im sine



2.) a.) • $s(t) = m(t)\cos(2\pi f_c t) = \cos(2\pi f_i t) \cdot \cos(2\pi f_c t)$

$\hookrightarrow X(f) = \int x(t)\, e^{-j2\pi f t}\, dt$

a.) $\mathcal{F}\{\cos(2\pi f_i t)\} = \frac{1}{2}[\delta(f-f_i) + \delta(f+f_i)]$

$\mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$

$S(f) = M(f) * C(f) = \frac{1}{4}[\delta(f-(f_c+f_i)) + \delta(f-(f_c-f_i)) + \delta(f-(-f_c+f_i)) + \delta(f-(-f_c-f_i))]$

$\rightarrow S(f) = \frac{1}{4}[\delta(f-(f_c+f_i)) + \delta(f-(f_c-f_i)) + \delta(f+(f_c-f_i)) + \delta(f+(f_c+f_i))]$

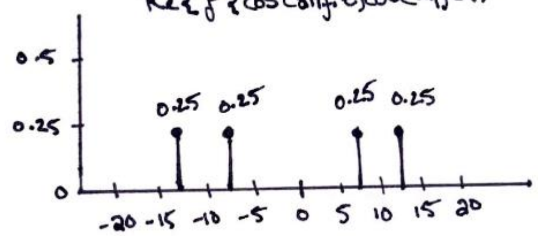
• $\mathrm{Im}\{S(f)\} = 0$

$\mathrm{Re} = \pm(f_c \pm f_i)$, $A = 1/4$

$\rightarrow \cos(2\pi f_i t)\cos(2\pi f_c t) = \frac{1}{2}\cos(2\pi(f_c+f_i)t) + \frac{1}{2}\cos(2\pi(f_c-f_i)t)$

each cosine contributes magnitude $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ at $\pm(f_c \pm f_i)$

$\mathrm{Re}\{\mathcal{F}\{\cos(2\pi f_i t)\cos(2\pi f_c t)$




b.) $s(t) = \cos(2\pi f_i t)\cos(2\pi f_c t)$

$\hookrightarrow \cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$\rightarrow s(t) = \frac{1}{2}\cos(2\pi(f_c+f_i)t) + \frac{1}{2}\cos(2\pi(f_c-f_i)t)$

$\mathcal{F}\{\cos(2\pi f_0 t)\} = \frac{1}{2}[\delta(f-f_0) + \delta(f+f_0)]$

$\rightarrow S(f) = \frac{1}{4}(\delta(f-(f_c+f_i)) + \delta(f+(f_c+f_i)) + \delta(f-(f_c-f_i)) + \delta(f+(f_c-f_i)))$

Re : 4 impulses above 1/4 mag.

Im: Zero (Spectrum even & real)

c.) $m(t) \rightarrow s(t) = m(t)\cos(2\pi f_c t) \Rightarrow S(f) = \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$

if $\mathrm{Re}\{M(f)\}$ is positive symmetric triangle centered at $f=0$ with $B \ll f_c$ then $\mathrm{Re}\{S(f)\}$ is two identical triangles with one centered at $+f_c$ & one at $-f_c$. Each triangle has same width $B$ as $M(f)$ & half of the peak height.

$\mathrm{Re}\{S(f)\} = \frac{1}{2}T(f-f_c) + \frac{1}{2}T(f+f_c)$

3.) a.) Given the modulated signal s(t)'s Fourier Transform from part 2.) there are two groups of frequency components

- one at $f_c + f_i$ (upper side band)
- one at $f_c - f_i$ (Lower side band)

↳ Together they form two sidebands that mirror around the carrier frequency
& Since both sidebands are transmitted it is called a double sideband

b.) In standard AM there is a spectral line at $f = \pm f_c$ representing the unmodulated carrier

↳ in the case of: ($s(t) = m(t)\cos(2\pi f_c t)$) there isnt a standalone carrier term at $f = \pm f_c$
The carrier is only present through the sidebands, not as a separate transmitted tone

4.) DSB-SC:

$S(f) = \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c)$

a.) $m(t) = \cos(2\pi f_i t)$

$M(f) = \frac{1}{2}[\delta(f-f_i) + \delta(f+f_i)]$

$\therefore$ $S(f)$ has impulses at $f = \pm(f_c \pm f_i)$

• $S_2(f) = S(f) * \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)] = \frac{1}{2}S(f-f_c) + \frac{1}{2}S(f+f_c)$

Shift impulses:

• from $S(f-f_c)$: $f = \{2f_c \pm f_i, \pm f_i\}$

• from $S(f+f_c)$: $f = \{\pm f_i, -2f_c \pm f_i\}$

At $f = \pm f_i$:

impulses at $f = \pm f_i$ (with magnitude 1/4)

$f = \pm(2f_c \pm f_i)$ (with magnitude 1/8)

$\therefore$ after mixing the baseband term is at $\pm f_i$ & images around $\pm 2f_c$

b.) Bandlimited $M(f)$

• $S(f) = \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c)$

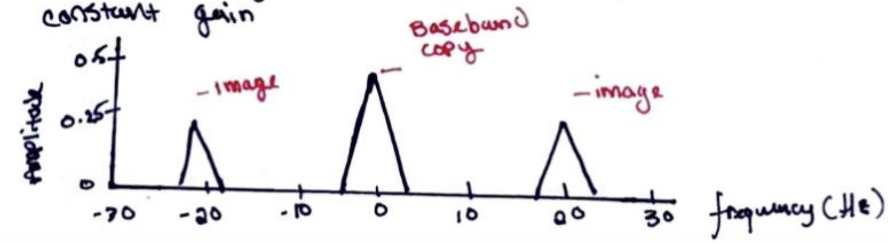
↳ mix again by $\cos(2\pi f_c t)$

$S_2(f) = \frac{1}{2} S(f-f_c) + \frac{1}{2} S(f+f_c)$

$= \frac{1}{2}[\frac{1}{2} M(f-2f_c) + \frac{1}{2} M(f)] + \frac{1}{2}[\frac{1}{2} M(f) + \frac{1}{2} M(f+2f_c)]$

$\therefore$ we should have a copy of the baseband spectrum $M(f)$ centered at 0 & scaled by 1/2 with two images centered at $\pm 2f_c$ scaled by 1/4 & since $B \ll f_c$ the lobes wont overlap

↳ so lowpass filtering $S_2(f)$ keeps $\frac{1}{2}M(f)$ & rejects the $\pm 2f_c$ images, recovering the message up to a constant gain



## Lab Part 1: First simple mixer (analog modulator)

Figure 1 shows the block diagram of the system for part 1. From the top plot in Figure 2 – we can see that the carrier oscillations (red 'DSB-SC') are being shaped by the message (blue 'message') which is we expect from a double-sideband suppressed carrier (in time). This verifies that the multiply is working correctly as a mixer. In the frequency domain we can see two spikes at +/- 500 Hz from the message spectrum and two pairs of spikes at +/- 10 kHz (+/- 500 Hz) – representing the upper and lower sidebands of the carrier. It is also worth noting that the impulse spikes are not directly at +/- 10KHz, which is expected from the suppressed carrier.

The throttle blocks are not necessary if there is an audio sink, as it functionally controls the rate of the flowgraph. The audio sink is tied to the systems sound card, which accepts data at a specific rate (44.1 kHz). So, when the audio sink is present, it acts as a rate governor that limits the flowgraph to run at 44.1 kHz.

Figure 3 shows the system with a noise input. In the time domain plot (top) the blue line (message) shows the message which raw Gaussian noise at baseband. And the red line (carrier) shows the noise multiplied by the cosine carrier - it is riding on the high-frequency oscillations of the carrier. In the frequency domain (bottom), the blue line gives a somewhat flat band centered at 0 Hz. Gaussian noise gives a broad, roughly uniform power across frequencies. The red line shows the two shifted copies of that noise band, centered around +/- $f_c$.

For DSB-SC : $s(t) = m(t)\cos(2\pi f_C t) \Rightarrow S(f) = \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c)$ there is no M(f) term at f=0. The transmitted signal does not contain the message at baseband unless m(t) has DC. You can only see the upper and lower sidebands centered around +/- $f_c$. Mixing again by $\cos(2\pi f_C t)$ and a low-pass would recover $\frac{1}{2}M(f)$.

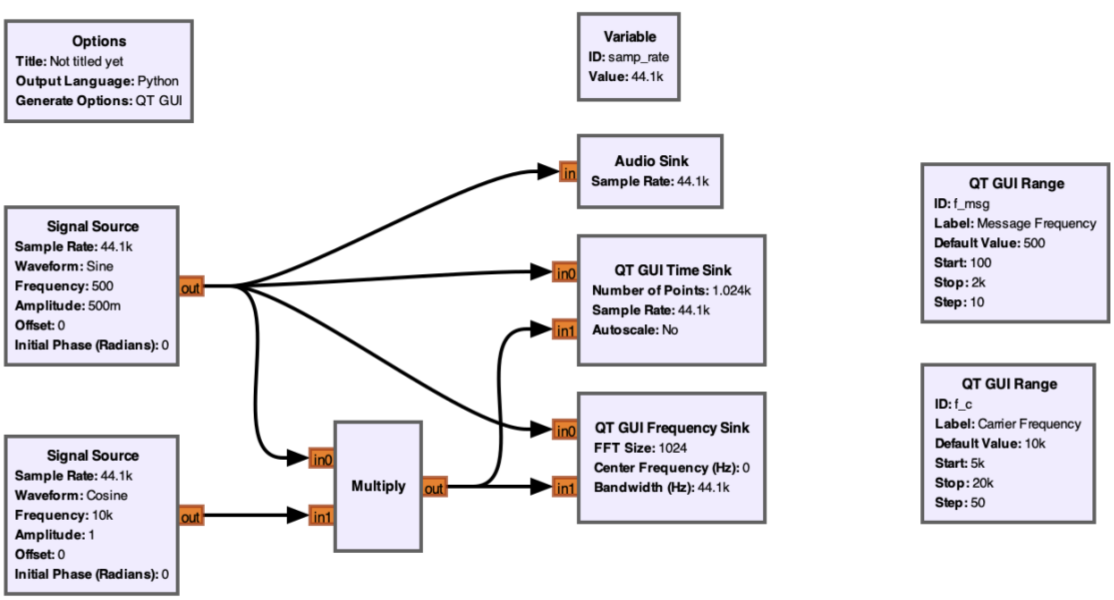



*Figure 1: Block diagram of the simple mixer*

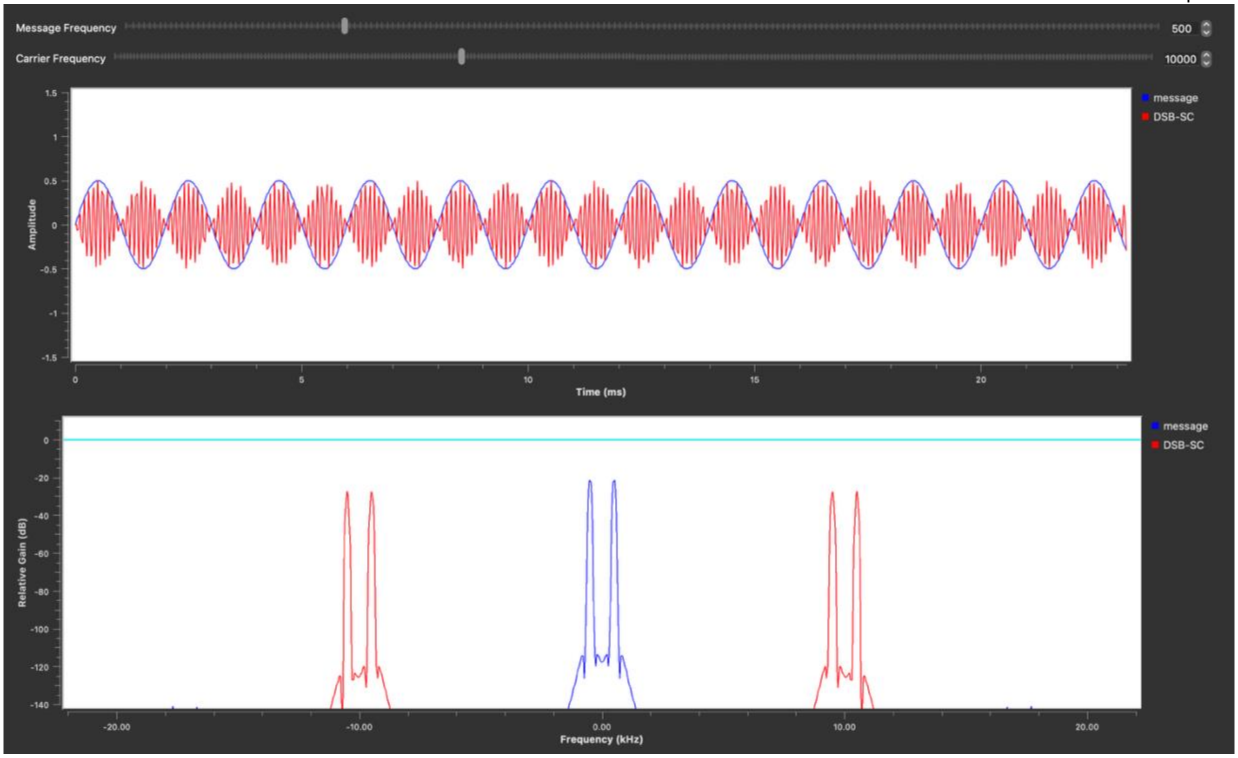



*Figure 2: Time and frequency output from the simple mixer*

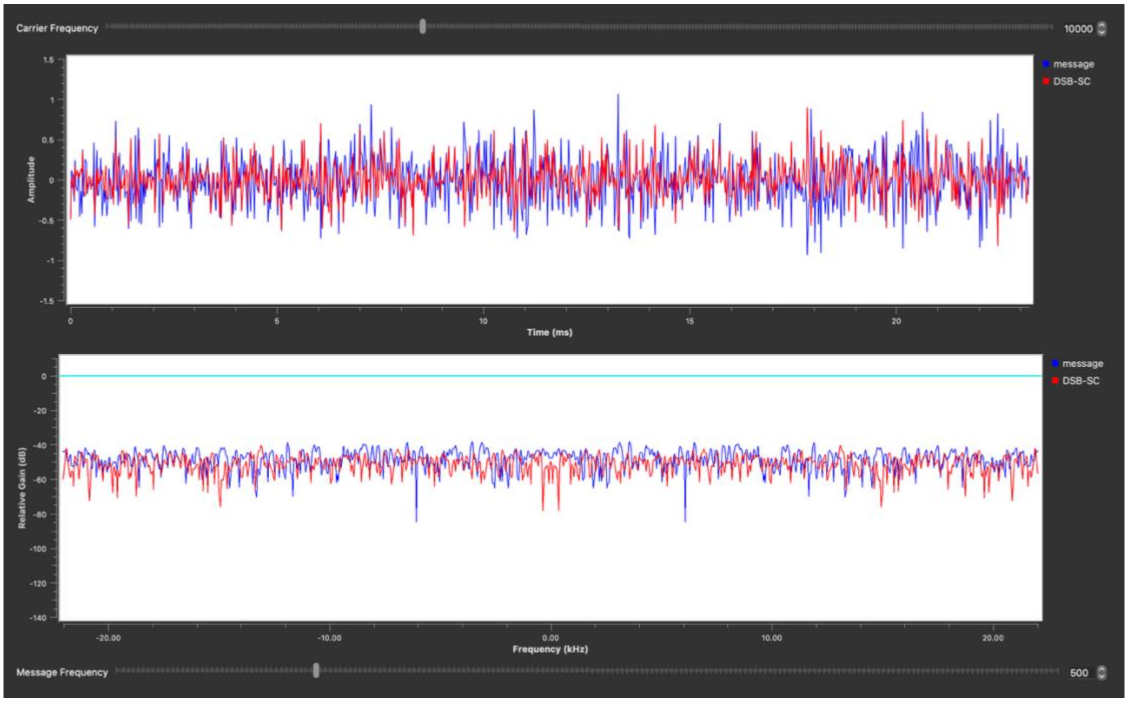



*Figure 3: Time and frequency output using noise input*

## Lab Part 2: Mixer with non-sinusoidal signals

In Figure 4 we have the square wave. When the message is on the square wave, the time-domain modulated signal shows the carrier being turned on and off with the square envelope. In the frequency domain, the square wave contributes odd harmonics only, so the DSB-SC spectrum contains sidebands at Fc +/- kFm for odd k. Because the square wave is zero-mean, no carrier line appears at Fc, so the carrier remains suppressed. In Figure 5 we have the triangle wave. With the triangle wave message, the carrier envelope ramps along linearly. The spectrum only contains odd harmonics with an amplitude that decays at $1/k^2$. This results in the higher harmonics being much weaker. The signal is a double-sideband suppressed carrier since the triangle is zero-mean and no line at $f_c$ is present.

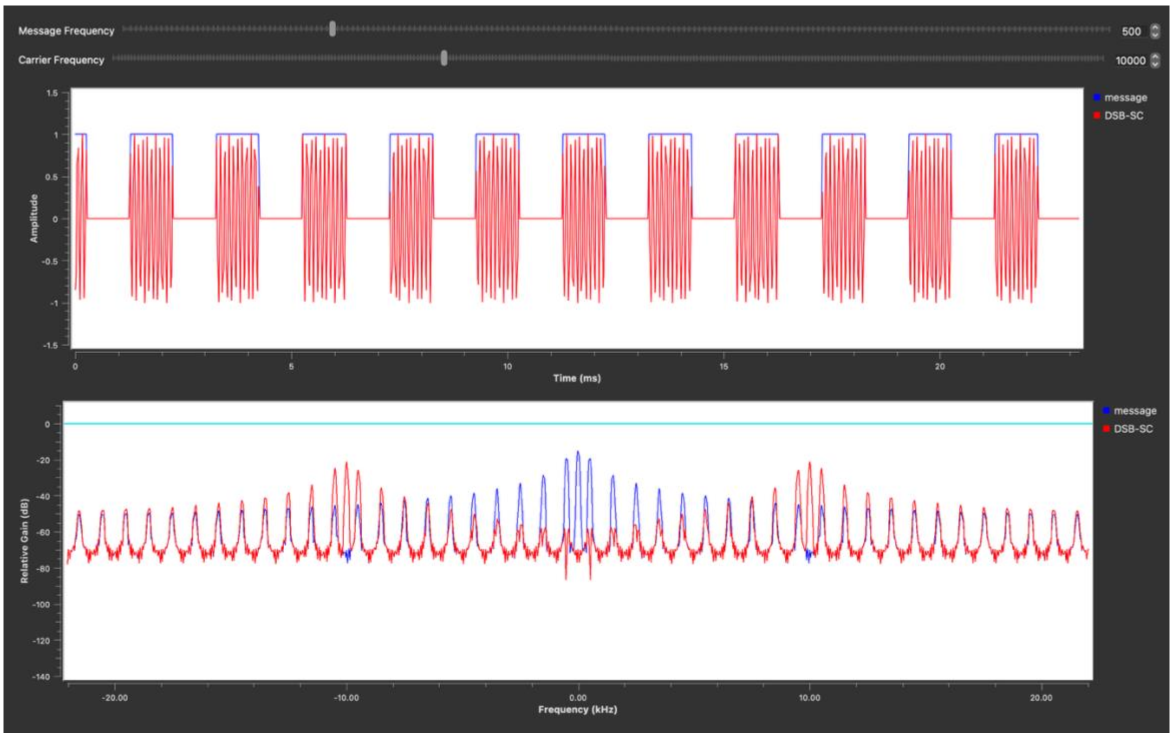



*Figure 4: Output from square wave message*

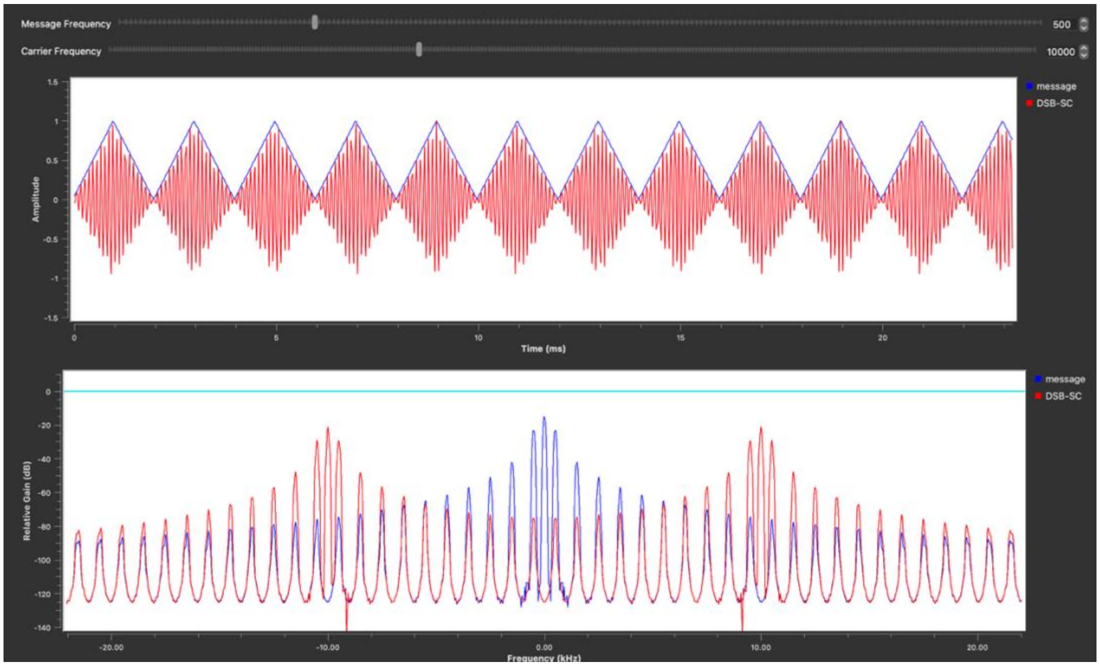



*Figure 5: Output from triangle wave message*

## Lab Part 3: Mixing a DSB-SC signal with a carrier

Here we added a second cosine source at the same carrier frequency and multiplied it with the DSB-SC output. The baseband message reappeared clearly with small high-frequency components from the images. The second mixer successfully recovers the message at baseband. However, low-pass filtering would isolate the baseband and remove the images at +/- 20 kHz.

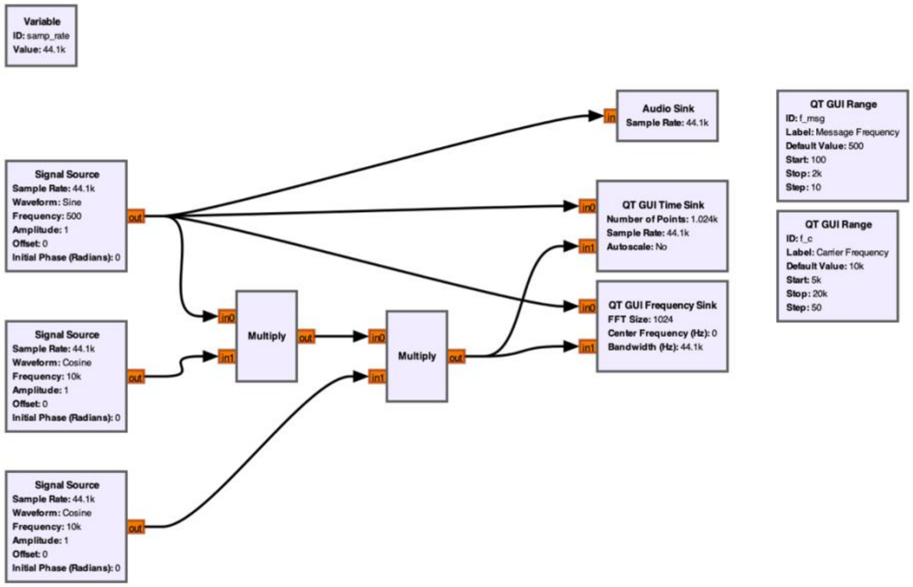



*Figure 6: Block Diagram for DSB-SC signal with carrier*

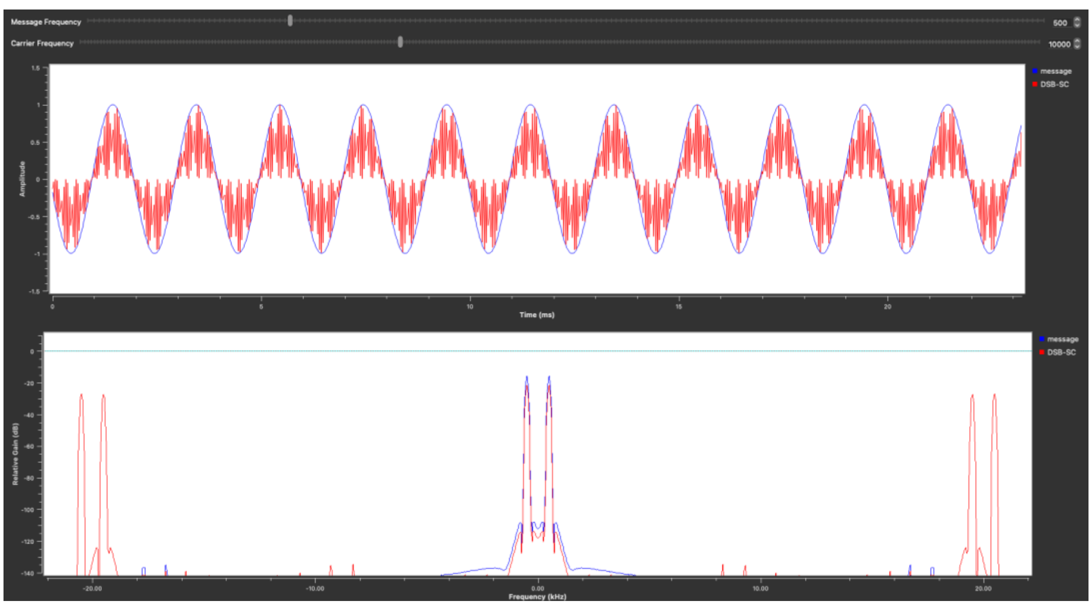



*Figure 7: Time and frequency output from signal*

## Lab Part 4: Recovering a message signal from a DSB-SC signal

Here we placed a low pass filter after the second mixer to isolate the baseband copy of the message. The LPF uses a gain of 2 to compensate for the scaling factor. The cutoff of the LPF is 2.5 kHz with a transition width of 500 Hz. In Figure 9 we can see that in the time domain, the filter output closely matches the original message. There is a small delay from the FIR filter. In the frequency domain, the LPF output only shows the baseband components while the higher frequency images are being suppressed. The low pass filter successfully recovers the original message from the DSB-SC signal.

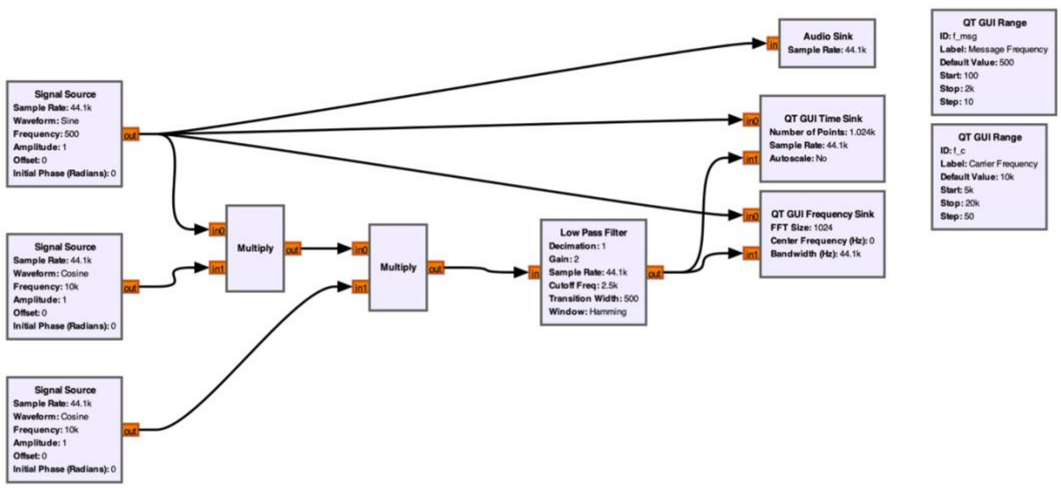



*Figure 8: Block diagram for the message recovery signa*

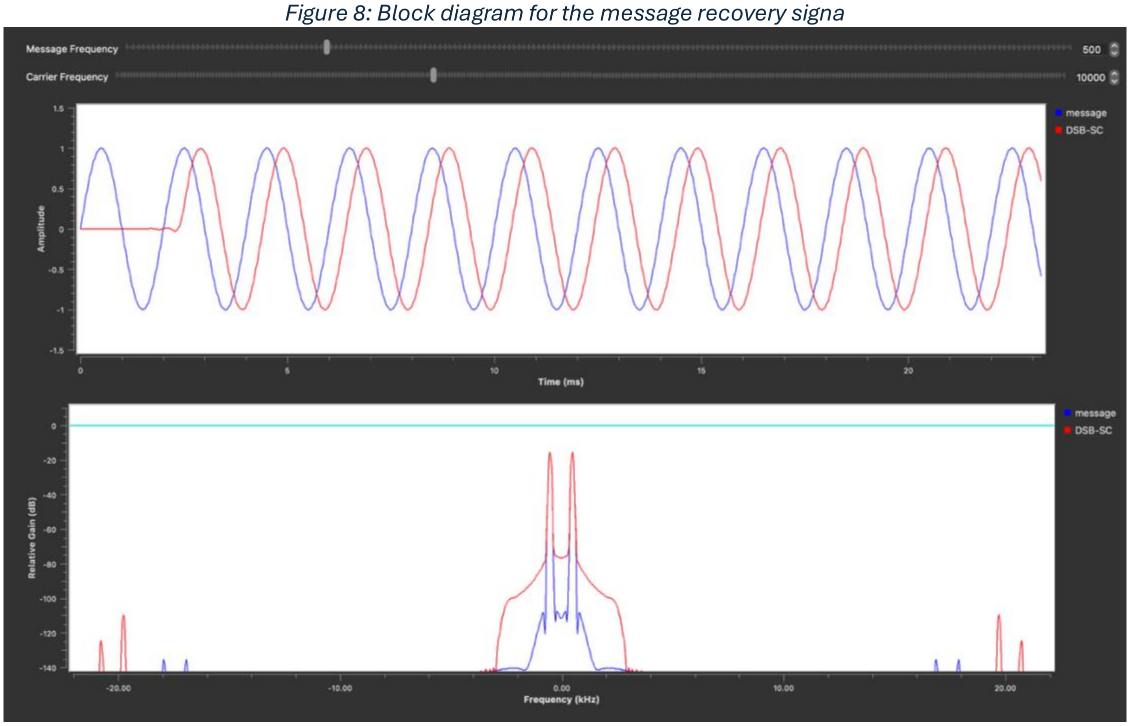



*Figure 9: Output plot from the signals*