Pre-Lab

Part 1:

Figures for Part 1 (P 1. 2. & 3.) were created using Miro. In figure 1, The DSB-SC correlation receiver takes the incoming RF (r(t)), multiplies it by a locally generated cosine at the carrier frequency, and the result is passed through a low-pass filters. The correlation pulls the message down to baseband while anything up near ($2f_c$) gets thrown away if the carrier-recovery loop keeps the local oscillator phase-aligned making the output essentially a scaled version of the original message. In figure 2, the standard AM (DSB-LC) envelope detector does the same job but is simplified. A diode and RC network trace the signal's envelope, which already contains the message when the carrier is big enough and there's no over-modulation. The time constant must be set so it is fast compared to the audio but slow compared to the carrier, and a DC removal block gives clean audio. In figure 3, the SSB/DSB product detector is similar the DSB-SC case but is meant for SSB or low-carrier AM. Again, a local oscillator and low-pass to baseband and mixed, which preserves both amplitude and phase provided that frequency/phase lock matter.

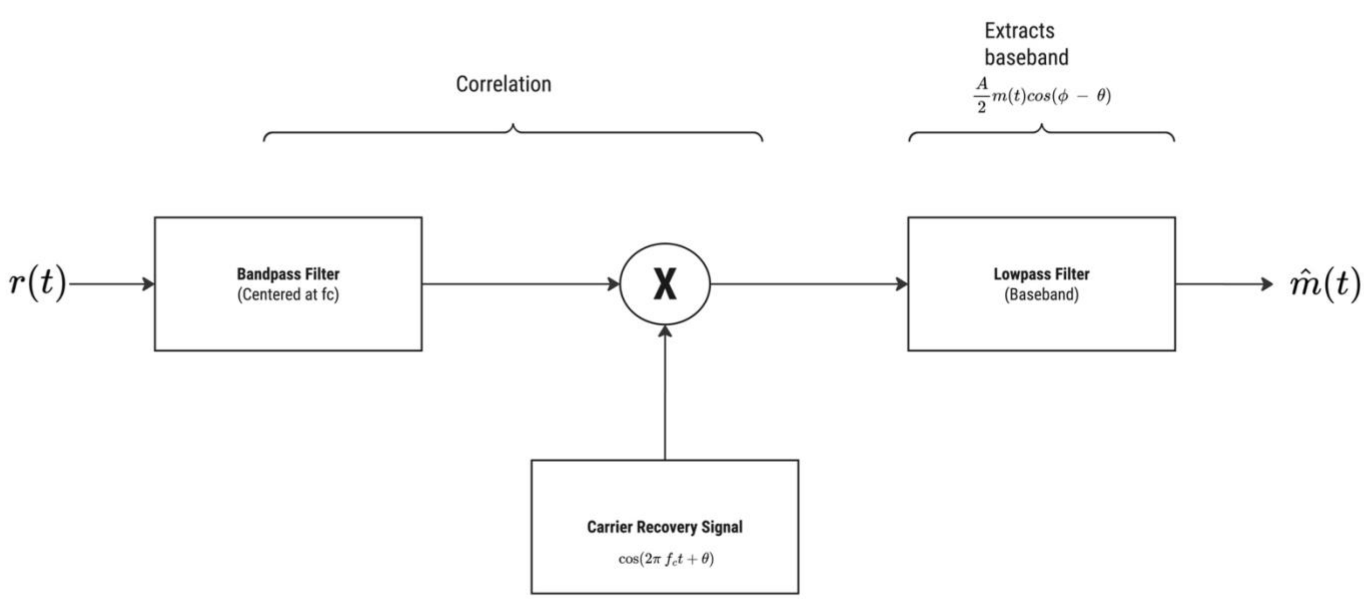


Figure 1: DSB-SC correlation receiver block diagram

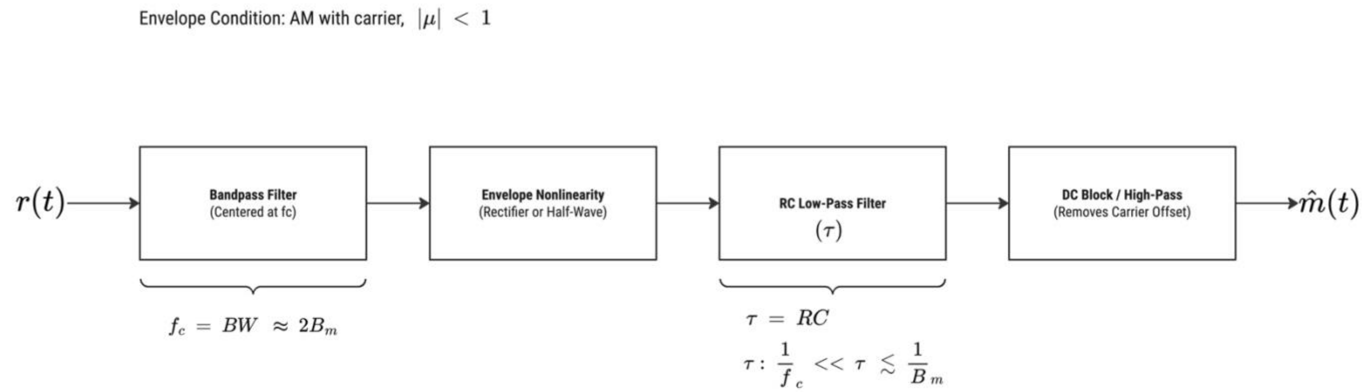


Figure 2: Envelope detector for AM block digram

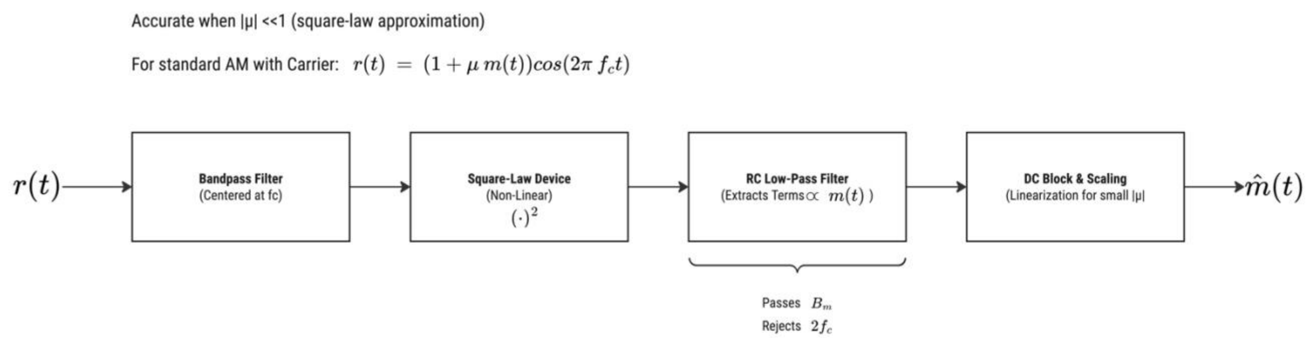


Figure 3: Squaring detector for AM block diagram

Part 2:

P 4. & 5.

4. & 5.) lets say:

- message $- m(t) = \cos(2\pi f_m t + \phi_m)$, $f_m = 1\text{KHz}$, $\phi_m = \frac{\pi}{4}$
- Carrier $- \cos(2\pi f_c t + \phi_c)$, $f_c = 10\text{KHz}$, $\phi_c = \frac{\pi}{4}$
- Lo at receiver is phase matched: $\cos(2\pi f_c t + \theta)$, $\theta = \phi_c$

- For DSB-SC Signal

$$s(t) = m(t)\cos(2\pi f_c t + \phi_c)$$
$$= \frac{1}{2}\cos(2\pi(f_c+f_m)t + (\phi_c+\phi_m)) + \frac{1}{2}\cos(2\pi(f_c-f_m)t + (\phi_c - \phi_m))$$

$$\rightarrow S(f) = \frac{1}{4}e^{j(\phi_c+\phi_m)}\delta(f-(f_c+f_m)) + \frac{1}{4}e^{-j(\phi_c+\phi_m)}\delta(f+(f_c+f_m))$$
$$+ \frac{1}{4}e^{j(\phi_c-\phi_m)}\delta(f-(f_c-f_m)) + \frac{1}{4}e^{-j(\phi_c-\phi_m)}\delta(f+(f_c-f_m))$$

- given that $\phi_c = \phi_m = \frac{\pi}{4}$: $\phi_c + \phi_m = \frac{\pi}{2} \Rightarrow \cos = 0, \sin = 1$
$\phi_c - \phi_m = 0 \Rightarrow \sin = 0, \cos = 1$

$\therefore$ at $\pm(f_c+f_m) = \pm 11\text{KHz}$
$Re\{s\} = \frac{1}{4}\cdot 0 = 0$, $Im\{s\} = \pm\frac{1}{4}$ (at $\pm f$)
- at $\pm(f_c - f_m) = \pm 9\text{KHz}$
$Re\{s\} = \frac{1}{4}$, $Im\{s\} = 0$

- PSD of DSB-SC signal

) $A_c = \frac{1}{2} \rightarrow A_c^2/4 = \frac{1}{16}$

$- S(f) = \frac{1}{16}[\delta(f - 11\text{KHz}) + \delta(f + 11\text{KHz}) + \delta(f - 9\text{KHz}) + \delta(f + 9\text{KHz})]$

- mixer output with phase matched LO

$$y(t) = s(t)\cos(2\pi f_c t + \theta)$$
$$= \frac{1}{2}\cos(2\pi f_m t + \phi_m) + \frac{1}{4}\cos(2\pi(2f_c + f_m)t + (2\phi_c + \phi_m))$$
$$\pm \frac{1}{4}\cos(2\pi(2f_c - f_m)t + (2\phi_c - \phi_m))$$

$$\hookrightarrow Y(f) = \frac{1}{4}e^{j\phi_m}\delta(f - f_m) + \frac{1}{4}e^{-j\phi_m}\delta(f + f_m)$$
$$+ \frac{1}{8}e^{j(2\phi_c + \phi_m)}\delta(f - (2f_c + f_m)) + \frac{1}{8}e^{-j(2\phi_c + \phi_m)}\delta(f + (2f_c + f_m))$$
$$+ \frac{1}{8}e^{j(2\phi_c - \phi_m)}\delta(f - (2f_c + f_m)) + \frac{1}{8}e^{-j(2\phi_c - \phi_m)}\delta(f + (2f_c - f_m))$$

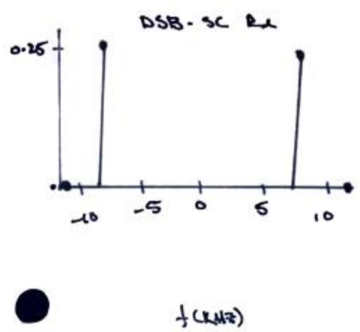
$$\hookrightarrow \phi_m = \frac{\pi}{4},\ 2\phi_c + \phi_m = \frac{3\pi}{4},\ 2\phi_c - \phi_m = \frac{\pi}{4}$$

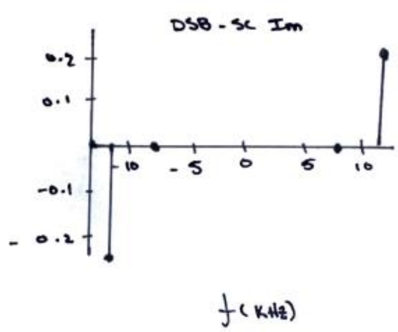
- Baseband $\pm f_m$
  $$Re\{Y\} = \frac{1}{4}\cos(\pi/4) = \frac{\sqrt{2}}{8}$$
  $$Im\{Y\} = \pm\frac{1}{4}\sin(\pi/4) = \pm\frac{\sqrt{2}}{8}$$
- $\pm(2f_c + f_m) = \pm 21\ \text{kHz},\ \phi = 3\pi/4$
  $$Re\{Y\} = \frac{1}{8}\cos(\frac{3\pi}{4}) = -\sqrt{2}/16$$
  $$Im\{Y\} = \pm\frac{1}{8}\sin(\frac{3\pi}{4}) = \pm\frac{\sqrt{2}}{16}$$
- $\pm(2f_c - f_m) = \pm 9\ \text{kHz},\ \phi = \pi/4$
  $$Re\{Y\} = \frac{1}{8}\cos(\pi/4) = \frac{\sqrt{2}}{16}$$
  $$Im\{Y\} = \pm\frac{1}{8}\sin(\pi/4) = \pm\sqrt{2}/16$$

- PSD at mixer

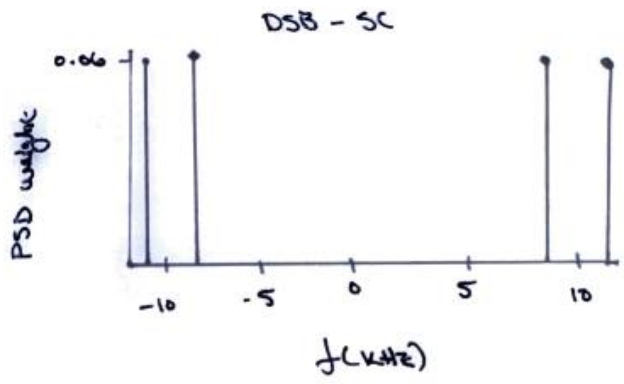
Baseband $c = \frac{1}{2}$, HF $c = \frac{1}{4}$

$$S_y(f) = \frac{1}{16}[\delta(f - f_m) + \delta(f + f_m)] + \frac{1}{64}\sum_{\sigma = \pm 1}[\delta(f - (2f_c + \sigma f_m)) + \delta(2f_c + \sigma f_m)]$$





2.6



P 6. & 7.

6. & 7.) $f_m = 1\text{kHz}$, $f_c = 10\text{kHz}$, $\theta = \phi_c$ (phase matched LO)

LO $= \cos(2\pi f_c t + \theta)$

$\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$ (at each sideband)

↳ the product of s(t) & LO gives:

$$\frac{A_k}{2}\left[\cos(2\pi(f_k - f_c)t + (\varphi_k - \theta)) + \cos(2\pi(f_k + f_c)t + (\varphi_k + \theta))\right]$$

↳ $A_k = \frac{1}{2}$, $\varphi_k = \phi_c \pm \phi_m$, $f_k = f_c \pm f_m$

- ∴ · Low freq. terms: $\pm f_m$ with phase $(\phi_c \pm \phi_m) - \theta$ & coefficient $\frac{1}{4}$
  · High freq. terms: $2f_c \pm f_m$ with phase $(\phi_c \pm \phi_m) + \theta$ & $\frac{1}{4}$

↳ sum low freq ⇒ $y_{lf}(t) = \frac{1}{2}\underbrace{\cos(\phi_c - \theta)}_{\text{phase error just scales base band}}\cos(2\pi f_m t + \phi_m)$

- with phase matched $\theta = \phi_c$

$$y(t) = \frac{1}{2}\cos(2\pi f_m t + \phi_m) + \frac{1}{4}\cos(2\pi(2f_c + f_m)t + (2\phi_c + \phi_m)) + \frac{1}{4}\cos(2\pi(2f_c - f_m)t + 2(\phi_c - \phi_m))$$

↳ $\phi_m = \phi_c = \frac{\pi}{4}$

∴ · Baseband is $\frac{1}{2}\cos(2\pi f_m t + \frac{\pi}{4})$
· HF at $2f_c + f_m = 21\text{kHz}$ with phase $2\phi_c + \phi_m = \frac{3\pi}{4}$
· HF at $2f_c - f_m = 19\text{kHz}$ with phase $2\phi_c - \phi_m = \frac{\pi}{4}$

• FT of mixer output

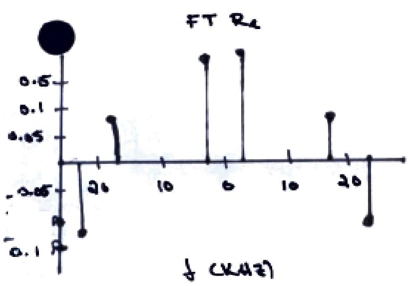
$$\mathcal{F}\{y(t)\} = \frac{A}{2}e^{j\varphi}\delta(f - f_0) + \frac{A}{2}e^{-j\varphi}\delta(f + f_0)$$

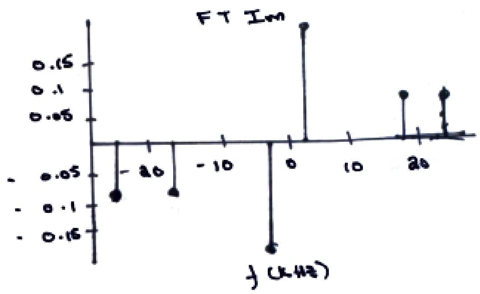
∴ at $\pm f_m$: $A = \frac{1}{2}$, $\phi_m = \frac{\pi}{4}$
· $\frac{A}{2} = \frac{1}{4}$
· $\text{Re}\{Y\} = \frac{1}{4}\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{8}$
· $\text{Im}\{Y\} = \frac{1}{4}\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{8}$

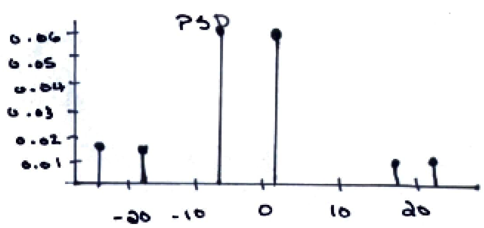
- at $\pm(2f_c+f_m) = \pm 21\text{kHz}$: $A = \frac{1}{4}$, $\phi = \frac{3\pi}{4}$
  - $\frac{A}{2} = \frac{1}{8}$
  - $Re = \frac{1}{8}\cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{16}$
  - $Im = +\frac{\sqrt{2}}{16}$ at 21kHz , $-\frac{\sqrt{2}}{16}$ at -21kHz
- at $\pm(2f_c-f_m) = \pm 19\text{kHz}$: $A = \frac{1}{4}$, $\phi = \frac{\pi}{4}$
  - $\frac{A}{2} = \frac{1}{8}$
  - $Re = \frac{\sqrt{2}}{16}$
  - $Im = \frac{\sqrt{2}}{16}$ at 19kHz, $-\frac{\sqrt{2}}{16}$ at -19kHz

- PSD at mixer output
  - At $\pm f_m$: $A = \frac{1}{2} \Rightarrow \frac{A^2}{4} = \frac{1}{16}$
    $\pm(2f_c+f_m)$: $A = \frac{1}{4} \Rightarrow \frac{1}{64}$

$\hookrightarrow S_y(f) = \frac{1}{16}[\delta(f \pm f_m)] + \frac{1}{64}[\delta(f \pm (2f_c+f_m)) + \delta(f \pm (2f_c-f_m))]$







P 8. 9. & 10.

8. 9. & 10.

$f_m = 1\,\text{kHz}$, $f_c = 10\,\text{kHz}$, $\phi_m = \phi_c = \frac{\pi}{4}$, LO is $90°$ off or $\theta = \phi_c + \frac{\pi}{2}$

· from DSB - SC signal

$m(t) = \cos(2\pi f_m t + \phi_m)$

$s(t) = m(t) \cdot \cos(2\pi f_c t + \phi_c)$

- mix with LO

use cosa cosb again

↳ $f_1 = f_c + f_m$. $\varphi_1 = \phi_c + \phi_m$, $f_2 = f_c - f_m$, $\varphi_2 = \phi_c - \phi_m$

↳ $y(t) = s(t)\cos(2\pi f_c t + \theta)$

$= \frac{1}{4}\left[\cos(2\pi(f_1 - f_c)t + (\varphi_1 - \theta)) + \cos(2\pi(f_1 + f_c)t + (\varphi_1 + \theta))\right]$

$+ \frac{1}{4}\left[\cos(2\pi(f_2 - f_c)t + (\varphi_2 - \theta)) + \cos(2\pi(f_2 + f_c) + (\varphi_2 + \theta))\right]$

$y(t) = \underbrace{\frac{1}{2}\cos(\phi_c - \theta)\cos(2\pi f_m t + \phi_m)}_{\text{Baseband}} + \frac{1}{4}\cos(2\pi(2f_c + f_m)t + (\phi_c + \phi_m + \theta))$

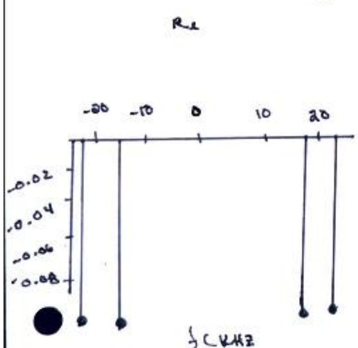
$+ \frac{1}{4}\cos(2\pi(2f_c - f_m)t + (\phi_c - \phi_m + \theta))$

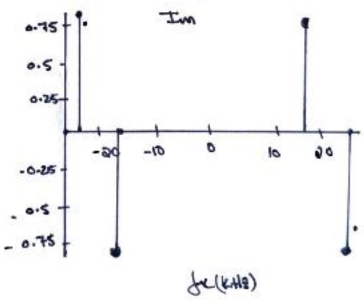
Term vanishes since LO is in quadrature &:

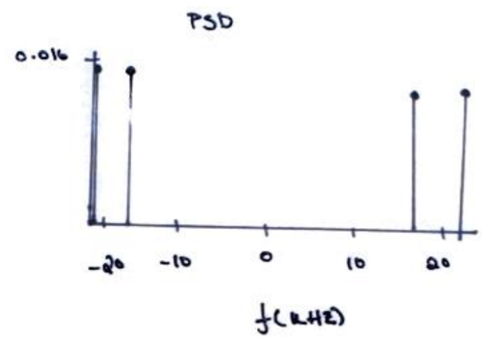
$\phi_c - \theta = \frac{\pi}{2} \Rightarrow \cos(\phi_c - \theta) = 0$

$\therefore$ ϕ at $(2f_c + f_m)$: $\phi_c + \phi_m + \theta = \frac{\pi}{4} + \frac{\pi}{4} + \frac{3\pi}{4} = \frac{5\pi}{4}$

ϕ at $(2f_c - f_m)$: $\phi_c - \phi_m + \theta = \frac{3\pi}{4}$







10.) in phase LO puts power at $\pm f_m$ at the mixer output so an LPF cleanly recovers the signal m(t). with a 90° LO error, the baseband lines vanish, only leaving power near $2f_c \pm f_m$. An LPF around baseband essentially outputs zero, so coherent recovery fails unless also using an orthogonal branch.

Lab Part 1: Correlation Receiver

First, we have the simple block diagram and frequency plots to check that the receiver is working as expected (Problem 3). Figure 5 shows the frequency-domain results show a DSB-SC receiver. In the sink after the mixer (top plot), we see two features - a baseband cluster centered at 0 kHz extending to about ±2 kHz (giving the 4 kHz message bandwidth), and lobes near ±20 kHz. After the low-pass filter (bottom plot), the ($2f_c$) images are rejected and only the baseband remains, producing the flat-topped spectrum out to ±2 kHz. So, the mixer generates baseband plus a high-frequency image, then the filter removes the image and passes the message.

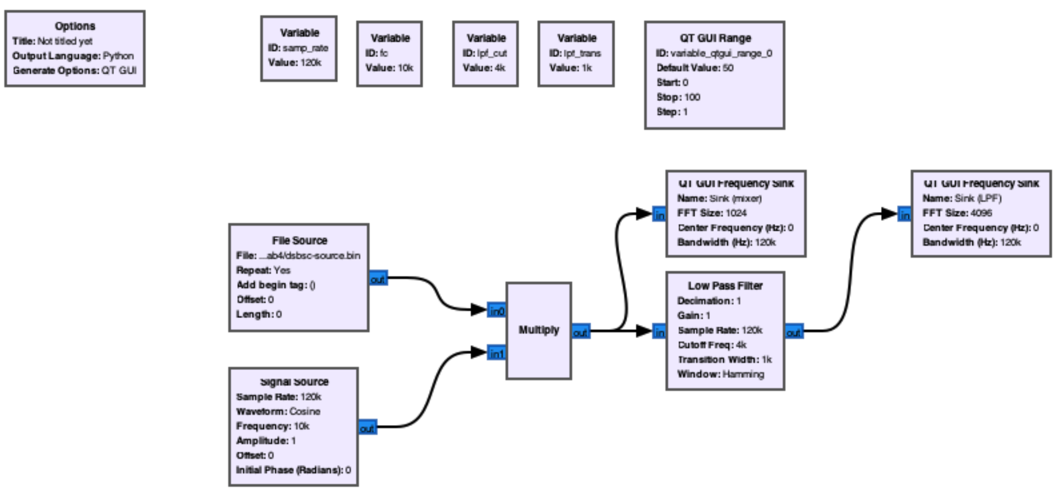


Figure 4: Block diagram for correlation receiver (note the blocks were changed to float)

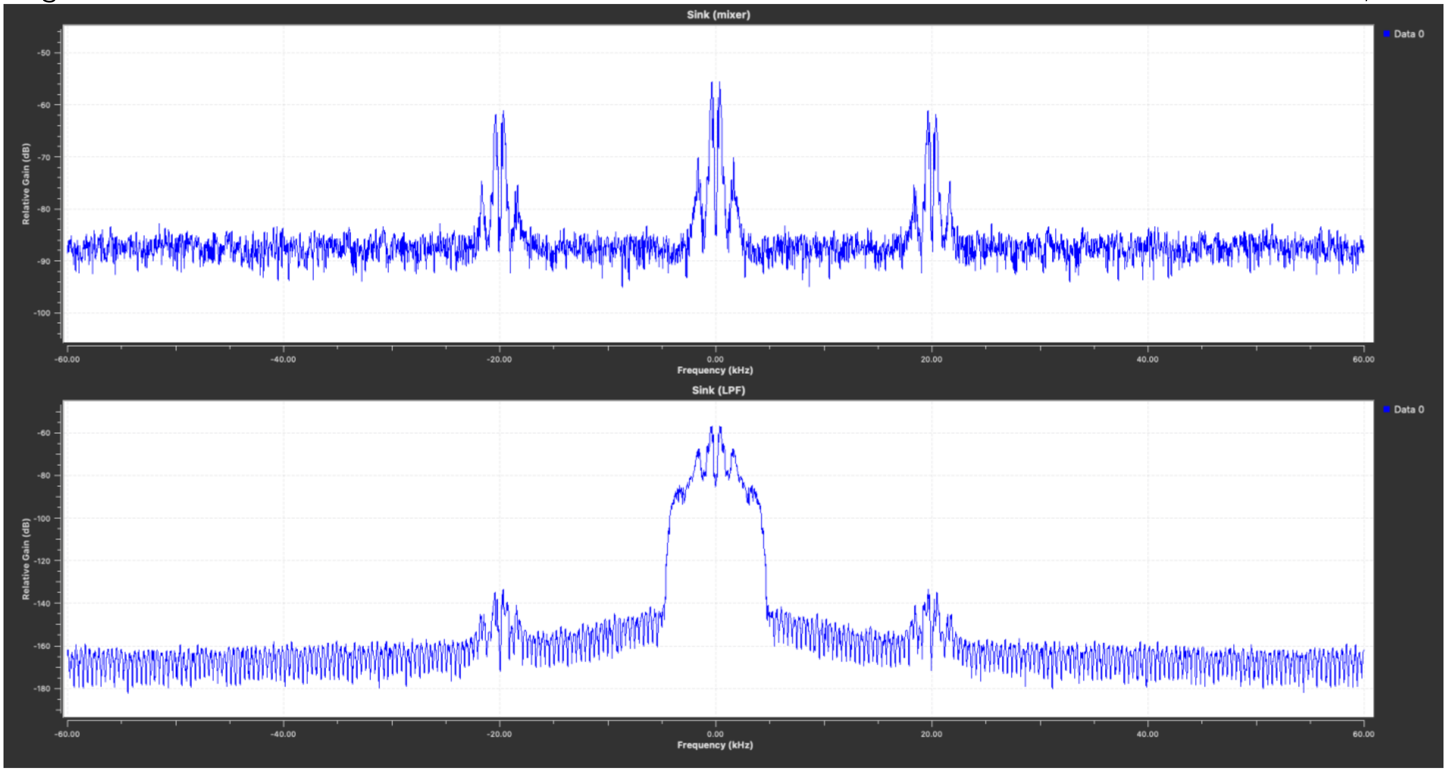


Figure 5: Spectral output at the mixer (top) and filter (bottom)

The block diagram in figure 6 reflects problems 4 and 5. In terms of defining the interpolation and decimation values for the rational resampler:

$$120{,}000 * \frac{interpolation}{decimation} = 48{,}000 \rightarrow \frac{48}{200} = \frac{2}{5}$$

The audio output can be heard here (wav file also submitted in case you do not want to use the link).

For problem 5, we add a delay block. Delaying the file source signal introduces a phase shift between the received carrier and the signal source (LO)

$$\Delta\phi = 2\pi f_c \frac{D}{samp\ rate} = 2\,\pi * 10k * \frac{D}{120k} = 2\pi * \frac{D}{12}$$

So, the correlator output scales as $cos(\Delta\phi)$ and the baseband amplitude changes according to $cos(2\pi D\ /\ 12)$

Therefore as: $D = 0 \rightarrow \Delta\phi = 0°$, $D = 1 \rightarrow \Delta\phi = 30°$, $D = 2 \rightarrow \Delta\phi = 60°$, $D = 3 \rightarrow \Delta\phi = 90°$, $D = 6 \rightarrow \Delta\phi = 180°$

The audio gets quieter at D1 and D2 until it effectively collapses at D3 with the 90° phase shift.

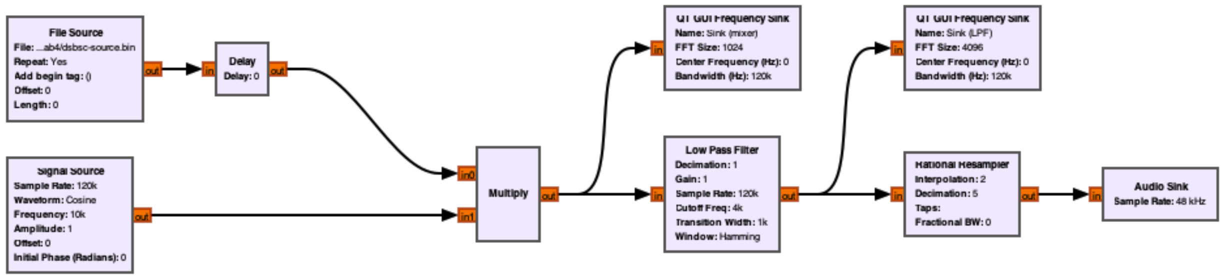


Figure 6: Block diagram of the overall system

Figure 7 shows the frequency sink outputs when the delay is 3. The image shows a fixed phase error between the received carrier and the LO:

$$\Delta\phi = \frac{\pi}{2}(90^{\circ})$$

The baseband term collapses at 90˚. Compared to when delay = 0, the delay = 3 plot shows the LPF output collapse toward the noise floor. Note that the mixer still maintains the images near the ±20 kHz range since they don't depend on correlation gain.

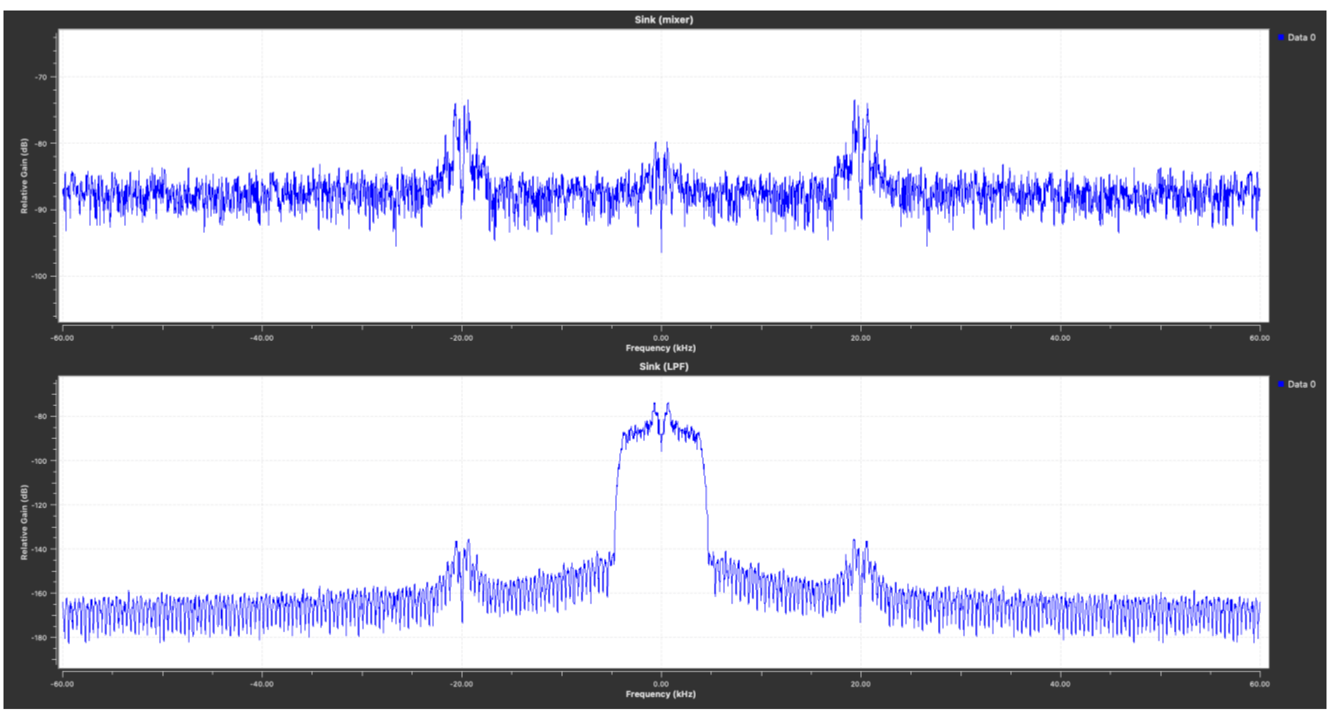


Figure 7: Collapse when delay is set to 3

Lab Part 2: Envelope Receiver

In the envelope-detector receiver, changing the delay doesn't change what you hear because the detector is noncoherent. It doesn't use a local oscillator, so it is not sensitive to carrier phase like correlation receiver. The Abs and LPF chain effectively outputs the envelope $|1 + \mu m(t)|$ (where the AM signal is $r(t) = (1 + \mu m(t) cos(2pif_c t))$ which is independent of the cosine's phase. Introducing a delay of samples at (f_s=120) kS/s only time shifts the entire waveform by delay / f_s (for 0–6 samples → 0–50 µs), which is negligible compared with the audio periods (1 kHz → 1 ms) and does not alter the envelope shape. That's why the spectrums look the same and the audio quality is unchanged when the delay is varied. This contrasts with the coherent DSB-SC correlator, where delay creates a phase error and the recovered audio scales (which lead to collapse at 90˚ phase shift). For P6 – switching the file source to the DBS-SC source affects the audio because the DSB-SC file doesn't have a carrier, so there isn't an envelope to follow. Passing the DSB-SC through a rectifier creates new components, and the filter passes through a distorted mix instead of the original signal. Figure 10 shows the output of the DSB-SC signal – the bottom plot (filter) shows spikes that are much lower than in the AM case (figure 9)). The audio is also included [here](#).

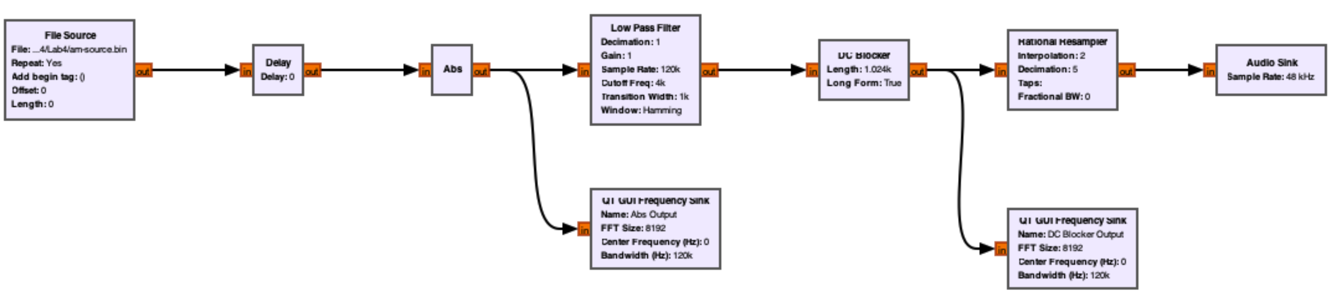


Figure 8: Block diagram of envelope receiver

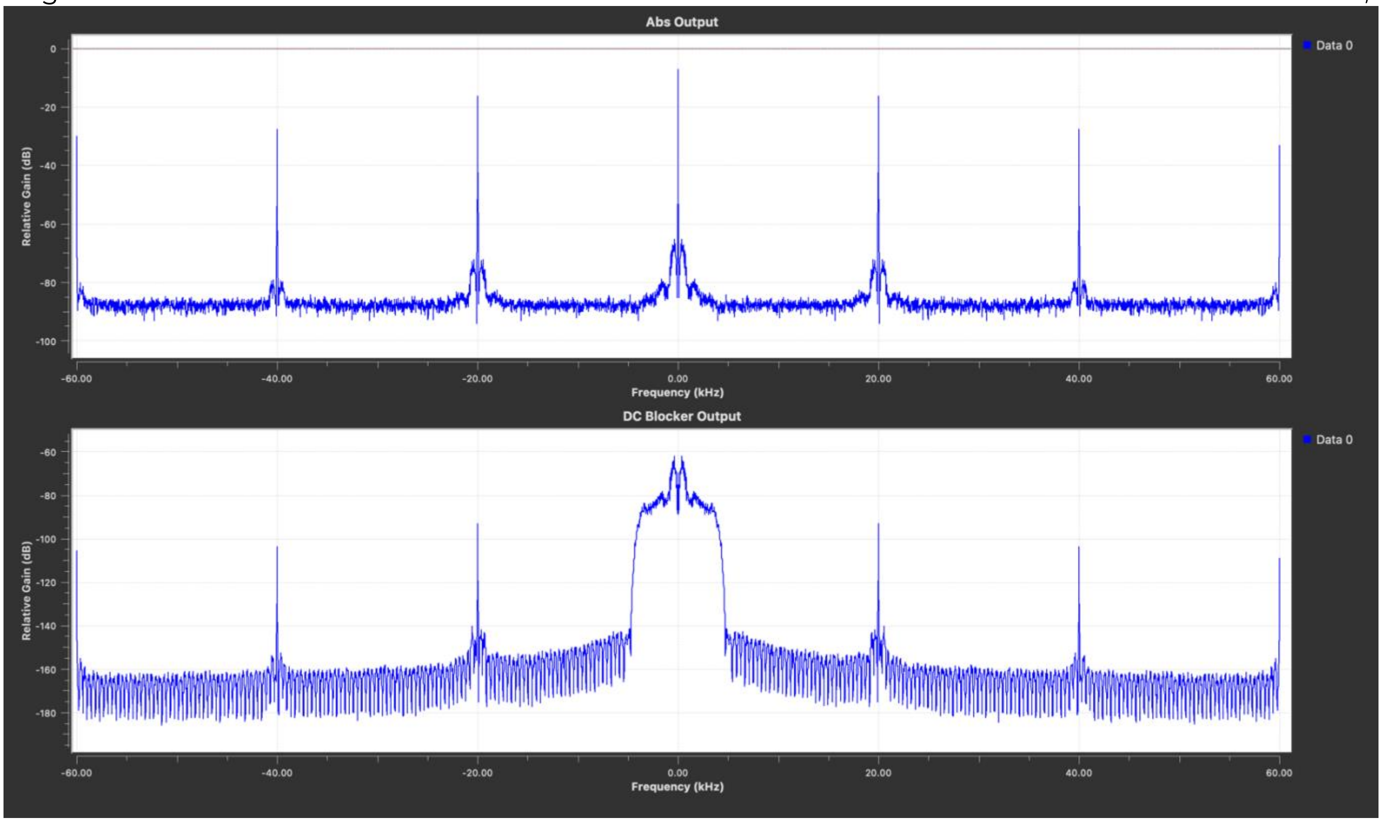


Figure 9: Envelope receiver spectra at Abs and DC blocker

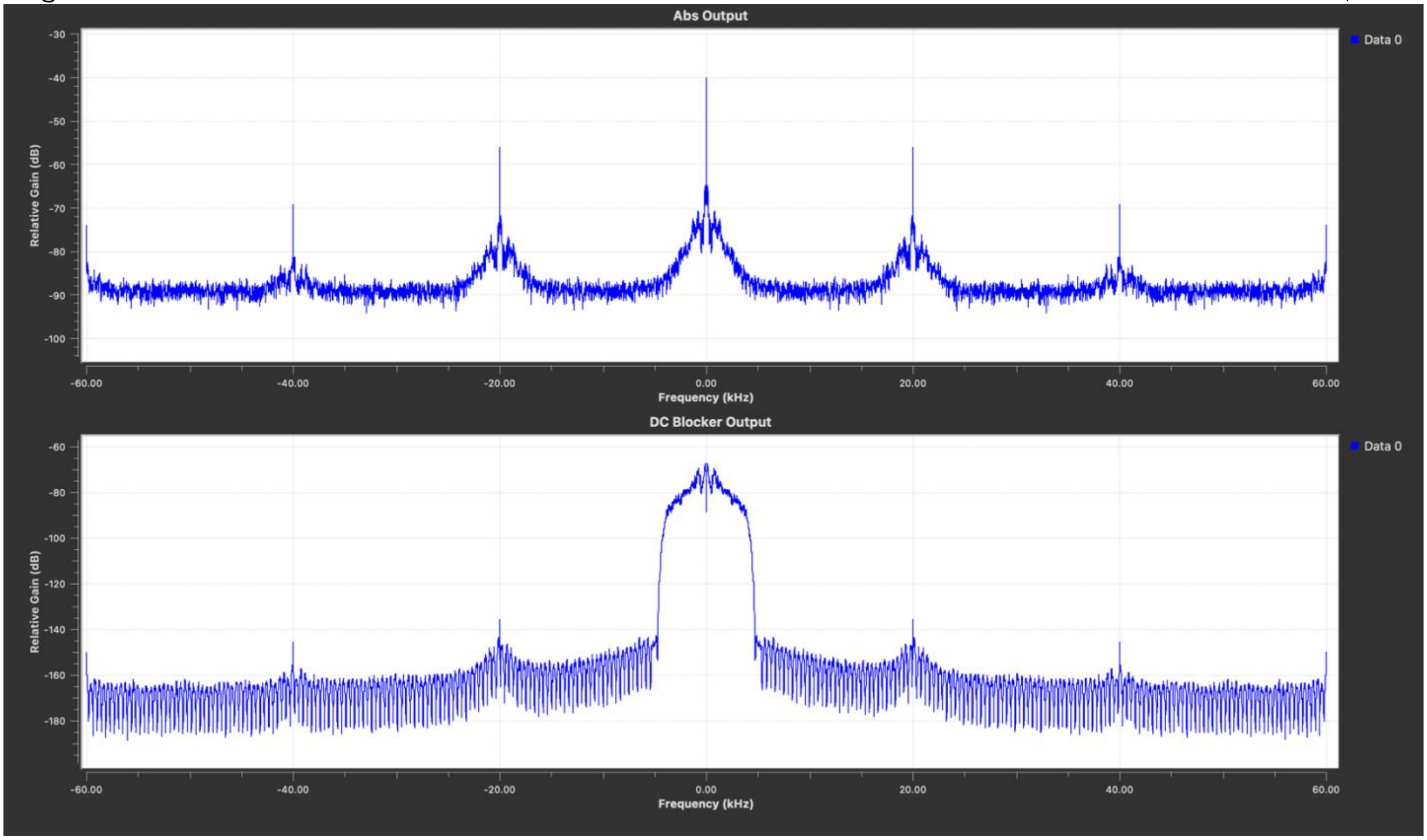


Figure 10: Spectra of DSB-SC

Lab Part 3: Squaring Receiver

After implementing the square-law AM receiver, the time-domain trace at the LPF/sqrt output showed a nearly flat line ≈1.5 with only a small ripple. Squaring an AM signal produces a large DC term and a smaller baseband term. The LPF removes the ($2f_C$) component, leaving the DC pedestal plus the message. Using the time plot to check the DC offset, I inserted an Add Const = −1.5 to re-center the waveform around 0. After this DC removal, the audio became clear. Spectrally, the frequency sink before DC removal showed a dominant spike at 0 Hz; after subtraction the spike vanished and only the 0–4 kHz baseband remained. Varying the delay for the noncoherent detector had no audible effect.

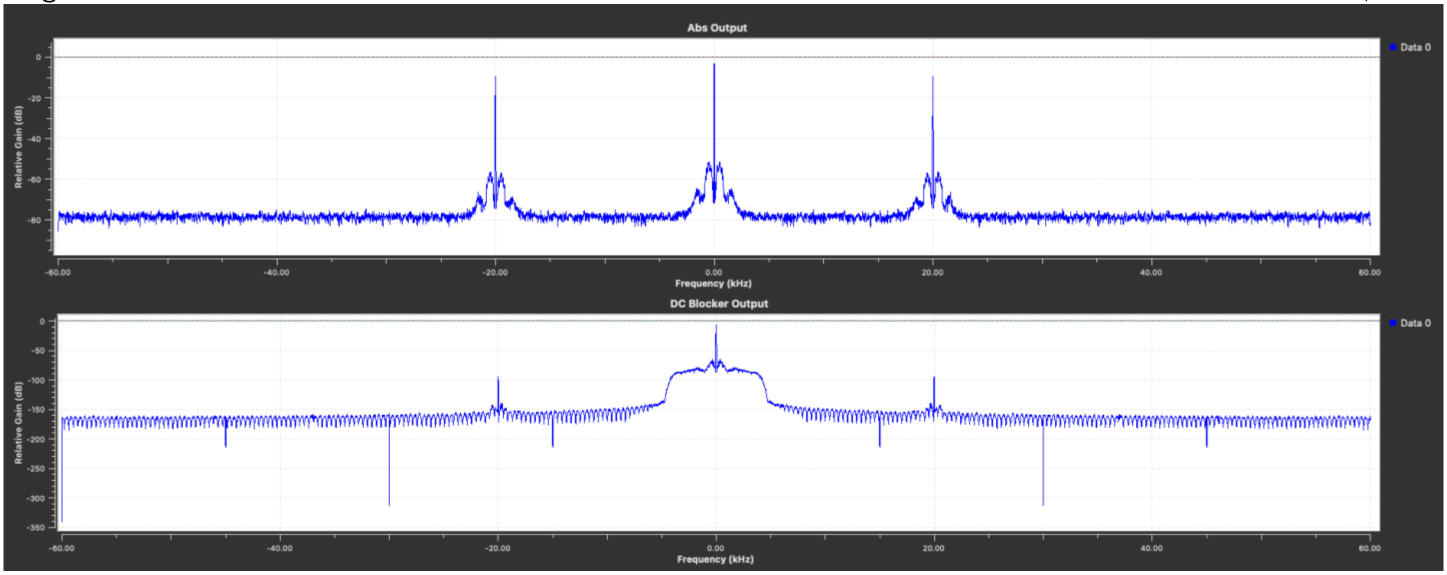


Figure 11: Spectral output before DC offset

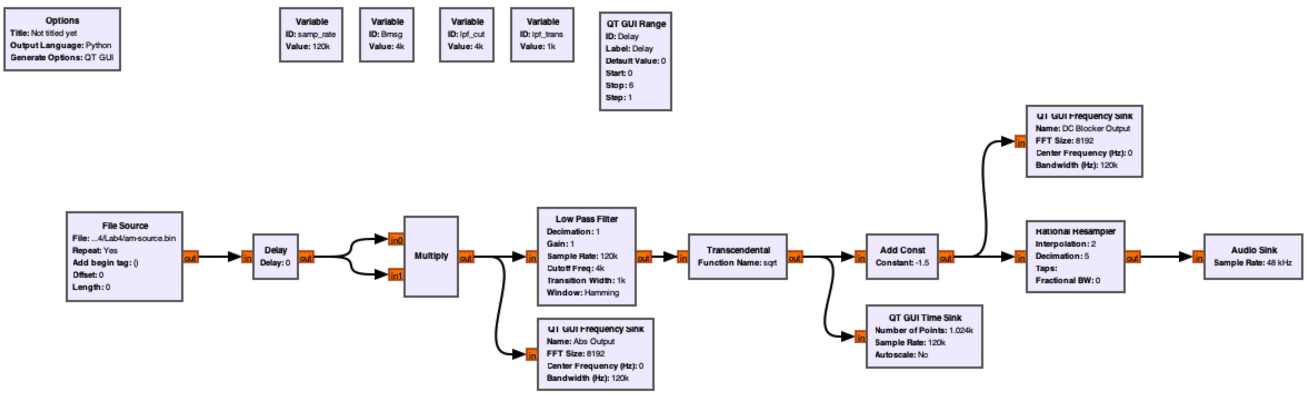


Figure 12: Block diagram for squaring receiver

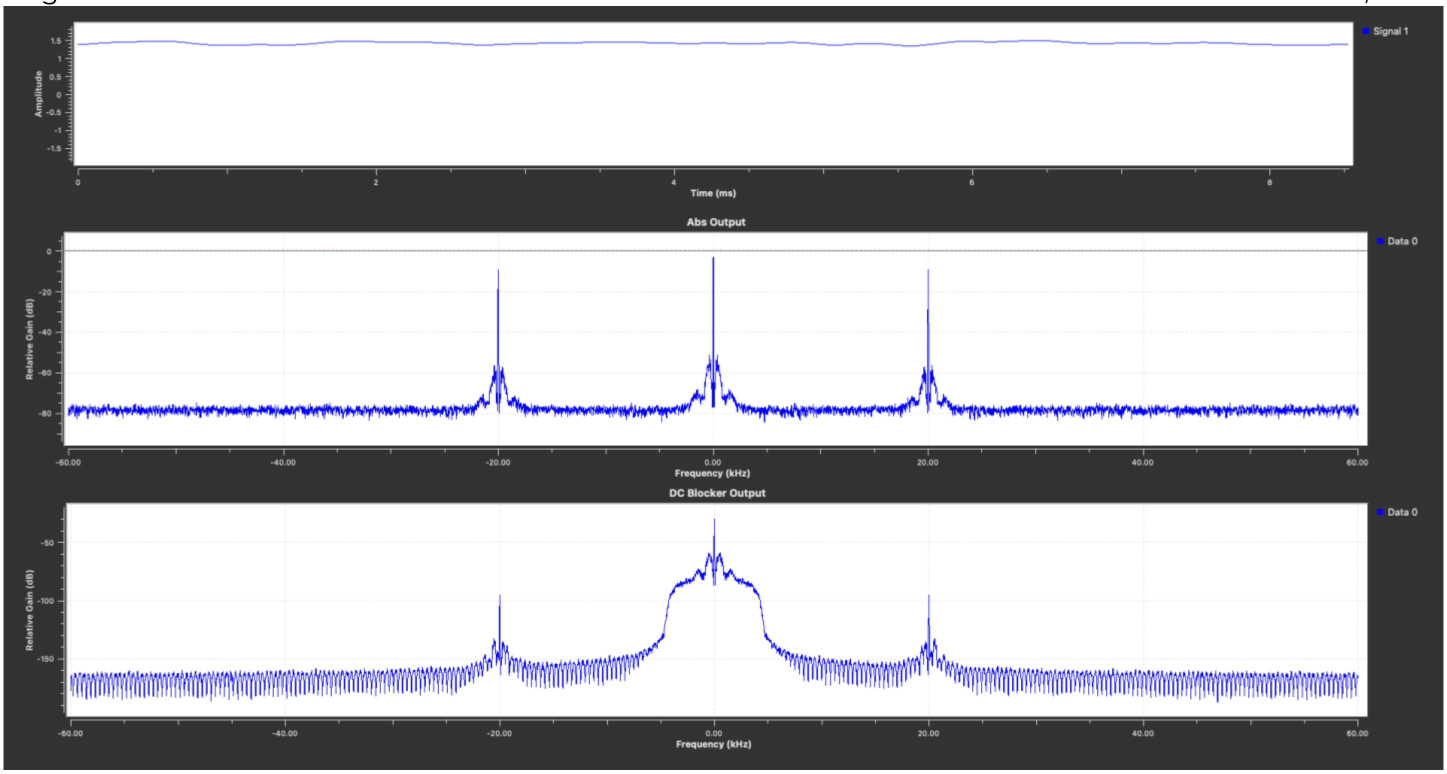


Figure 13: Spectral output after removing the DC offset

Lab Part 4: Quality Comparison

With Delay = 0 on all three, the DSB-SC gave the cleanest, most natural audio once the LPF was set to ~4 kHz. The output level scaled with mixer gain but had minimal distortion. For AM with carrier, the envelope detector produced strong, intelligible audio without extra processing - its noise floor is a bit higher (rectified) and it sounds a little harsher during fades, but it didn't require DC offset. The squaring detector also recovered intelligible audio, but only after DC removal. Once centered, it sounded comparable in loudness. In short - the DSB-SC had the best fidelity, the envelope receiver was simple with good quality, and the squaring receiver worked but needs DC correction and can be a touch more distorted**.**