

Pre-Lab

All diagrams were created using Miro

1.) For $r(t) = m(t)\cos(2\pi f_c t + \theta)$

The I arm (Cos Lo):

$$\begin{aligned} LPF\{r(t)\cos(2\pi f_c t)\} &= LPF\{m(t)\cos(2\pi f_c t + \theta)\cos(2\pi f_c t)\} \\ &= LPF\left\{\frac{m(t)}{2}[\cos\theta + \cos(4\pi f_c t + \theta)]\right\} \\ &= \frac{m(t)}{2}\cos\theta \\ \mathbf{m}_I(t) &= \mathbf{m}(t)\cos\theta \end{aligned}$$

Q arm (sin Lo):

$$\begin{aligned} LPF\{r(t)\sin(2\pi f_c t)\} &= LPF\{m(t)\cos(2\pi f_c t + \theta)\sin(2\pi f_c t)\} \\ &= LPF\left\{\frac{m(t)}{2}[\sin(4\pi f_c t + \theta) - \sin\theta]\right\} \\ &= -\frac{m(t)}{2}\sin\theta \\ \mathbf{m}_Q(t) &= -m(t)\sin\theta \end{aligned}$$

2.)

$$m_I(t) = m(t)\cos\theta \quad \& \quad m_Q(t) = -m(t)\sin\theta$$

If we want $am_I(t) + bm_Q(t) = m(t)$

Then if we use $a = \cos\theta, b = -\sin\theta$

We get $\cos\theta m_I - \sin\theta m_Q = m(t)(\cos^2\theta + \sin^2\theta) = m(t)$

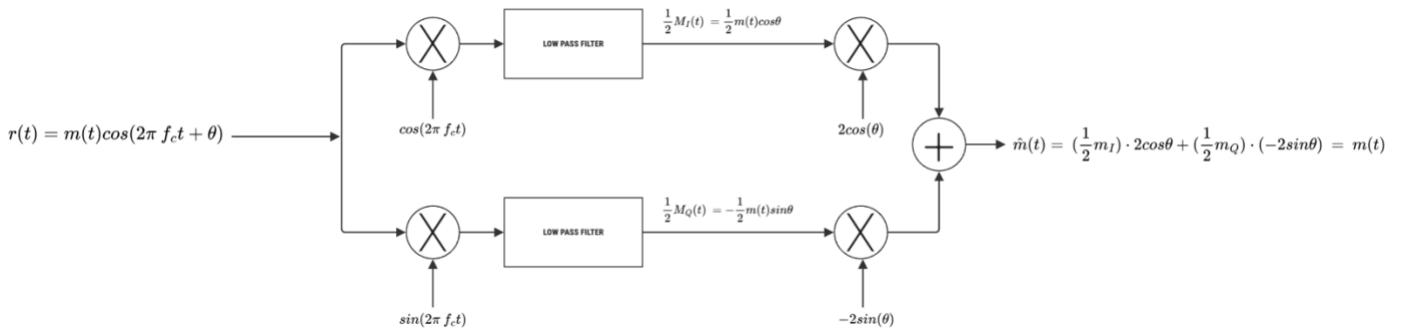


Figure 1: Quadrature coherent modulator (real LO)

$$3.) r(t) = [A + m(t)]\cos(2\pi f_c t + \theta)$$

After the mixing and LPF: $I(t) = \frac{1}{2}[A + m(t)]\cos\theta, Q(t) = -\frac{1}{2}[A + m(t)]\sin\theta$

Square and add: $I^2(t) + Q^2(t) = \frac{1}{4}[A + m(t)]^2(\cos^2\theta + \sin^2\theta) = \frac{1}{4}[A + m(t)]^2$

Square root (envelope): $E(t) = \sqrt{I^2 + Q^2} = \frac{1}{2}|A + m(t)| = \frac{1}{2}[A + m(t)]$

Remove DC: $y(t) = E(t) - \frac{A}{2} = \frac{1}{2}m(t) = km(t), \text{ where } k = \frac{1}{2}$

$$m(t) = 2\sqrt{I^2 + Q^2}$$

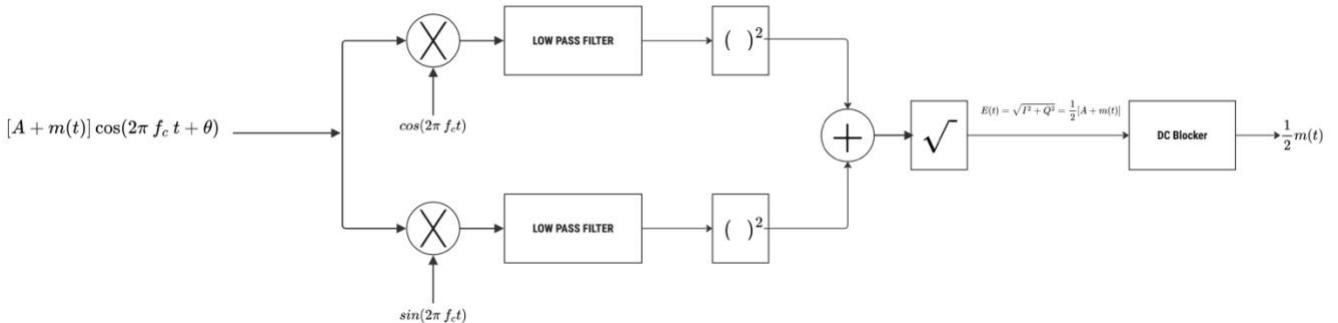


Figure 2: I/Q envelope detector for AM (real LO)

4.) Given $r(t) = m(t)\cos(2\pi f_c t + \theta)$

$$\begin{aligned} r(t)e^{j2\pi f_c t} &= m(t) \frac{e^{j2\pi f_c t + \theta} + e^{-j2\pi f_c t}}{2} e^{j2\pi f_c t} \\ &= \frac{m(t)}{2} (e^{j4\pi f_c t} + e^{-j\theta}) \end{aligned}$$

After the LPF, the complex baseband output is as follows:

$$\begin{aligned} m_c(t) &= \frac{1}{2} m(t) e^{-j\theta} \\ m_{REAL}(t) &= \frac{1}{2} m(t) \cos \theta, m_{IMAG} = \frac{1}{2} m(t) \sin \theta \end{aligned}$$

5.)

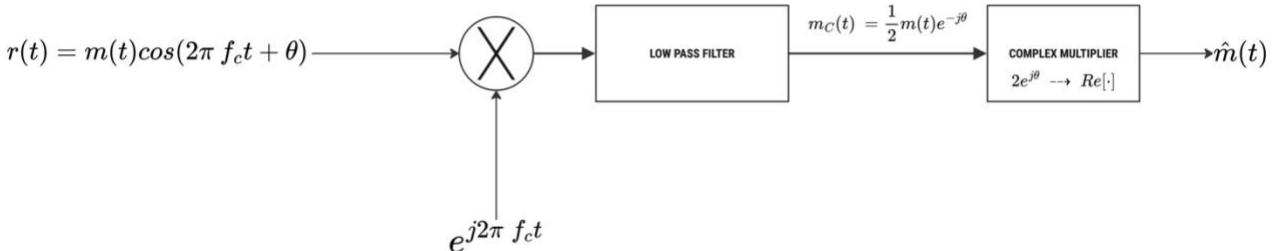


Figure 3: Coherent demodulator in complex baseband

6.)

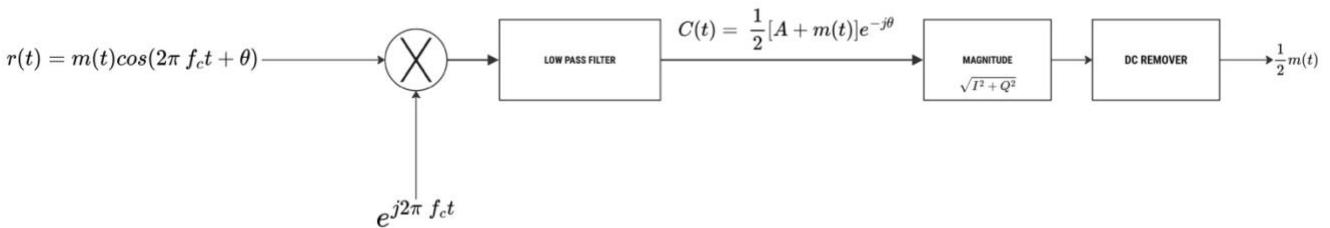


Figure 4: Complex signal envelope detector

Lab

In this lab I built an envelope receiver for a broadcast AM signal in the 915 MHz ISM band using an RTL-SDR with the osmosdr Source. The source produced I/Q samples at 264.6 kS/s while a tunable LO (\approx 916.99–917.01 MHz) mixed the RF to complex baseband with an unknown phase. To make the detector phase independent, I took the magnitude of the complex baseband (equivalent to $\sqrt{I^2 + Q^2}$) via complex to mag, then used a low pass filter with adjustable cutoff (\approx 1–50 kHz) and gain to pass the audio band. A DC Blocker removed the carrier/DC residue, and a rational resampler (decimating to 44.1 kS/s) fed the audio sink for listening. The PSD of the “Abs Output” showed a carrier at baseband with symmetric sidebands whose width tracked the audio bandwidth, and the time-domain plot displayed a clean audio waveform after filtering and gain adjustment. Overall, the quadrature-based envelope detection worked as expected without needing carrier phase recovery, and the adjustable LPF/gain allowed for balancing fidelity and noise while successfully demodulating the AM transmission.

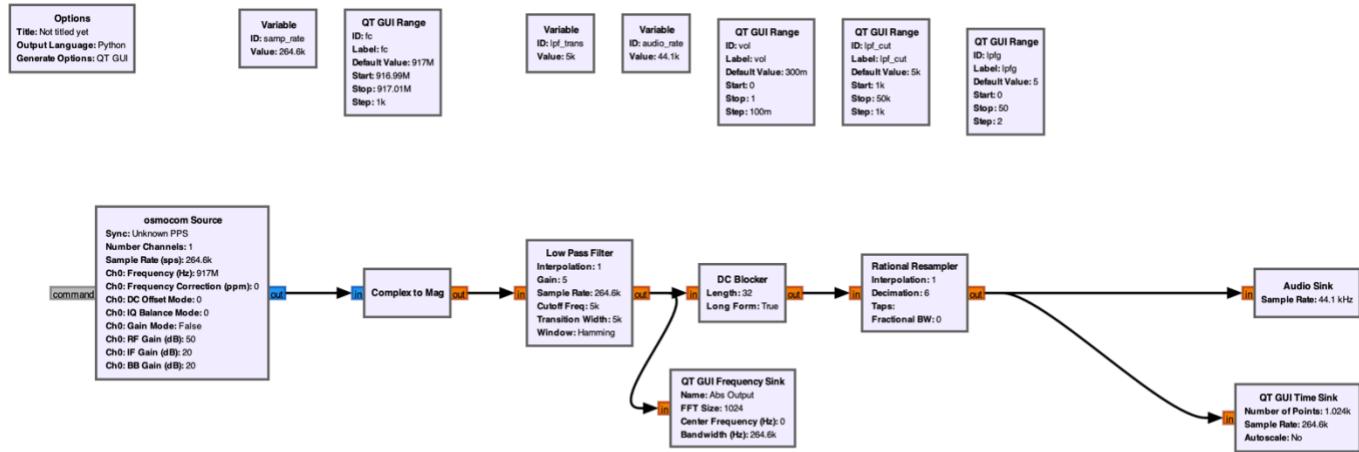


Figure 5: Block diagram of the envelope receiver

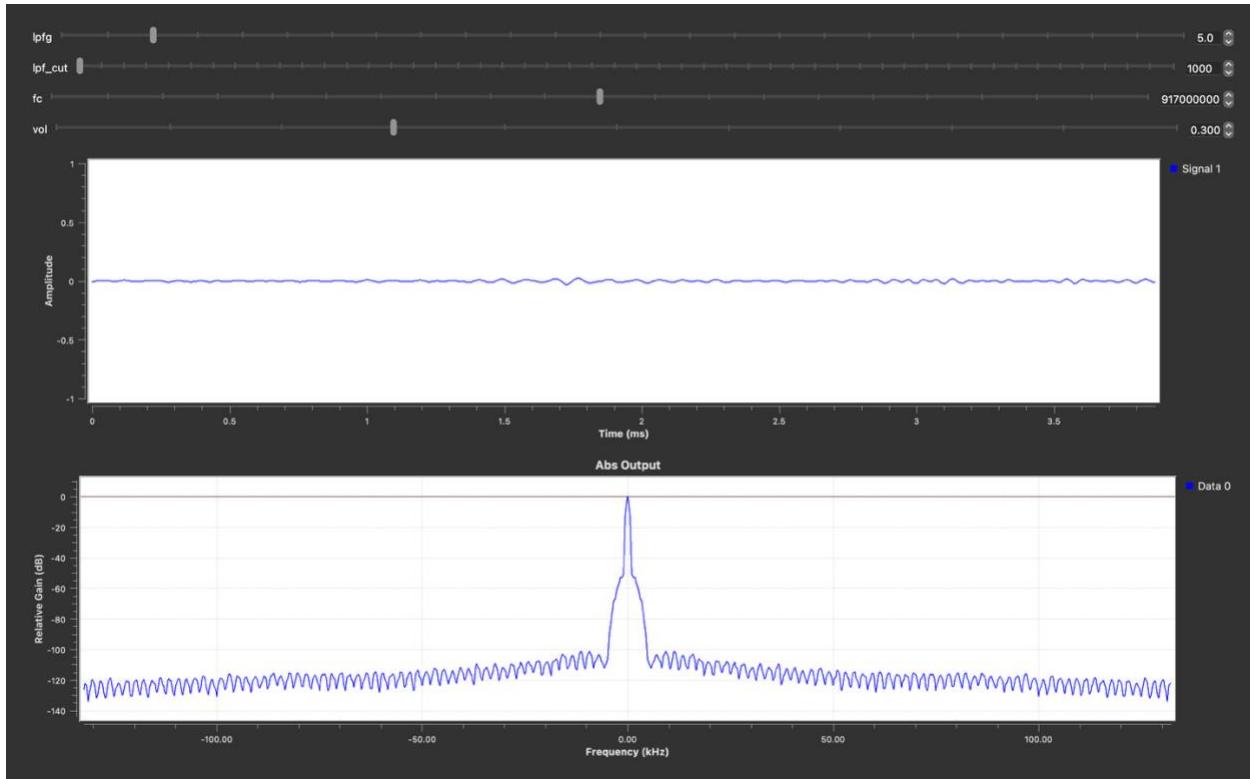
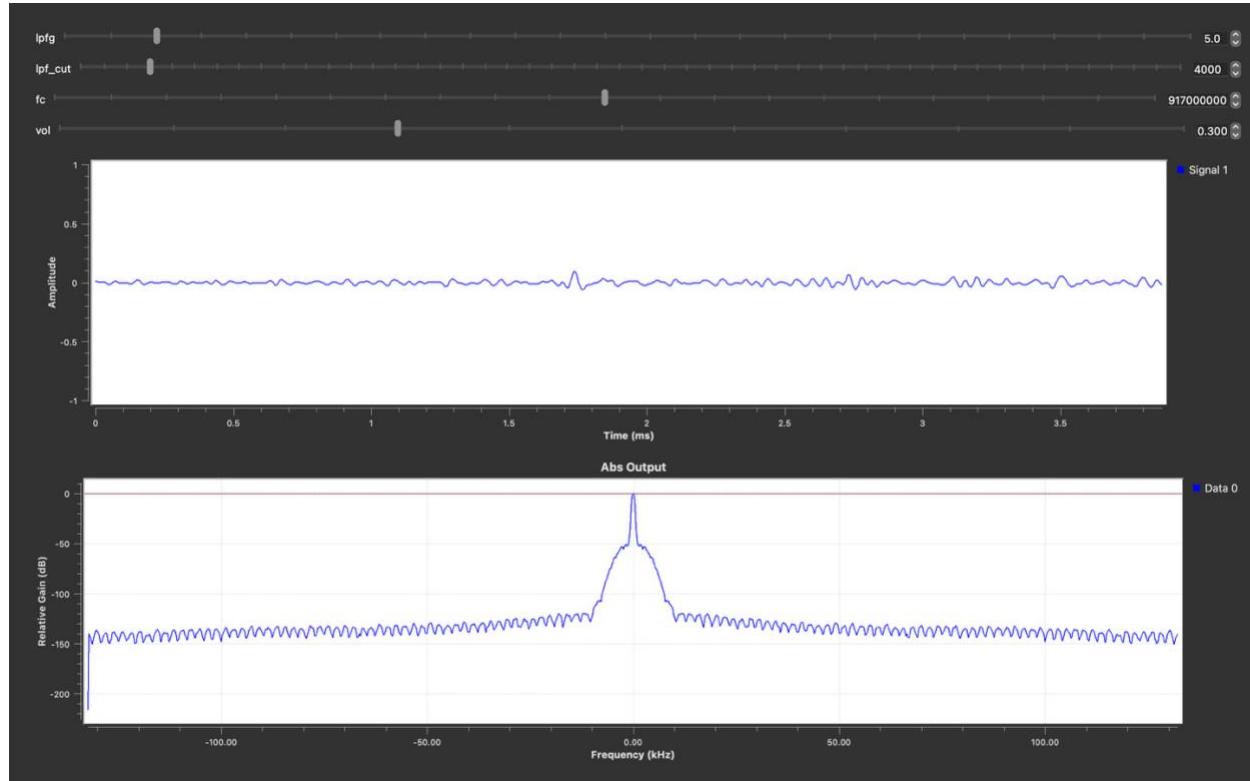


Figure 6: Envelope detector with tunable LPF — 4 kHz vs 10 kHz cutoff.