Pre-lab

Submitted as a separate PDF

Lab Part 1: Aligning Square Wave with Cosine Wave

This section implements a multiplier to produce a rectified signal. The sampling rate is set to 48kSample/s with a fundamental frequency of 400Hz. One of the signals generates a bipolar square wave with an amplitude of +/- 50. The second signal generates a cosine wave with the same fundamental frequency and an amplitude of 1. Because the square wave is not centered at t=0, there is a delay inserted on the square wave so that its transition coincides with the cosine wave at t=0. The required delay is a quarter of a period in samples, or:

$$Delay = \frac{N}{4} \; where \; N = \frac{48kHz}{400Hz} = 30$$

After the delay, the signals are multiplied, producing a full-wave rectified cosine that is scaled by the amplitude of the square wave. The Figures below include the block diagram from the GUI as well as the rectified output.

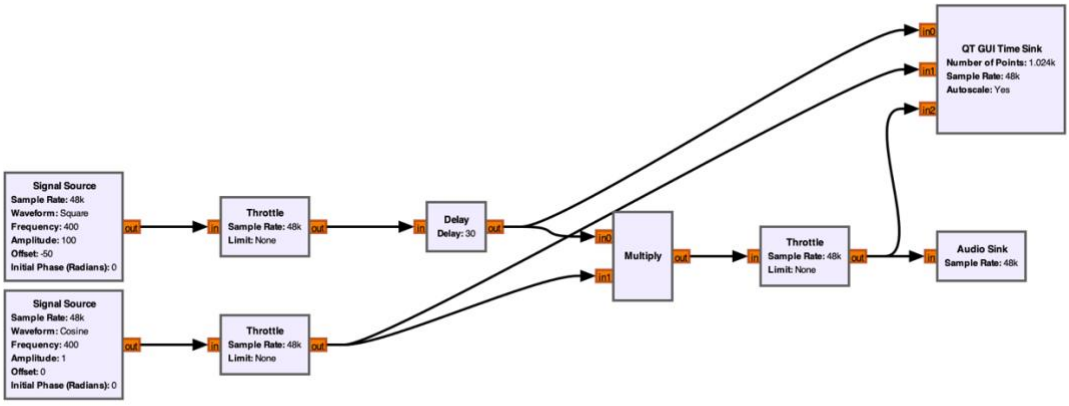

Figure 1: Block diagram of square and cosine waves being aligned

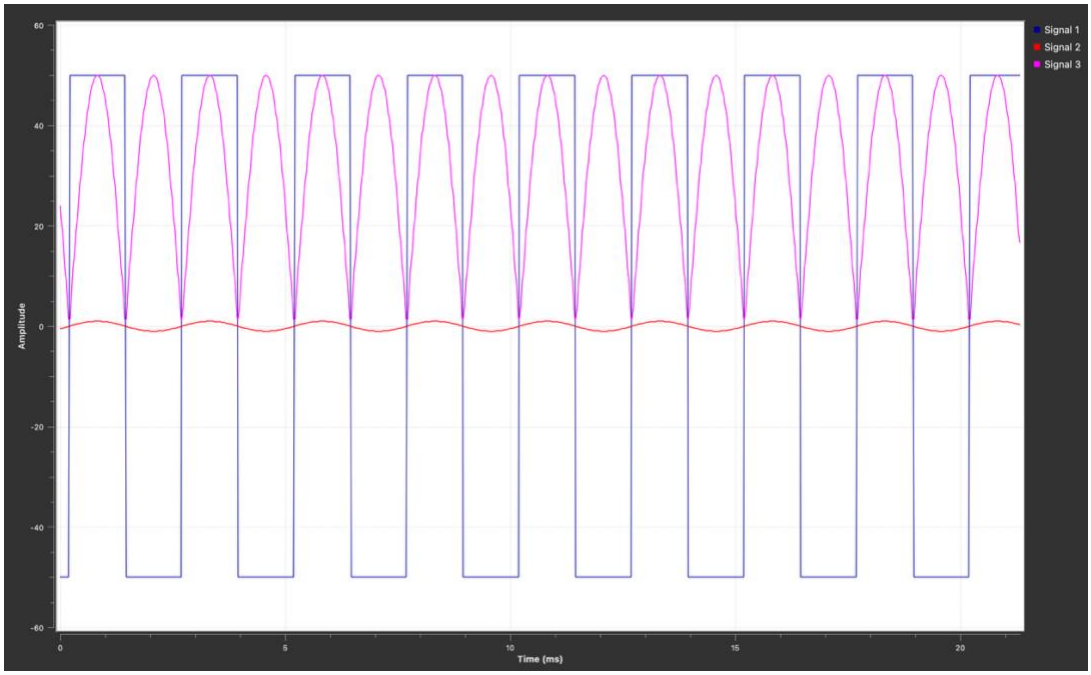

Figure 2: Output of the aligned wave

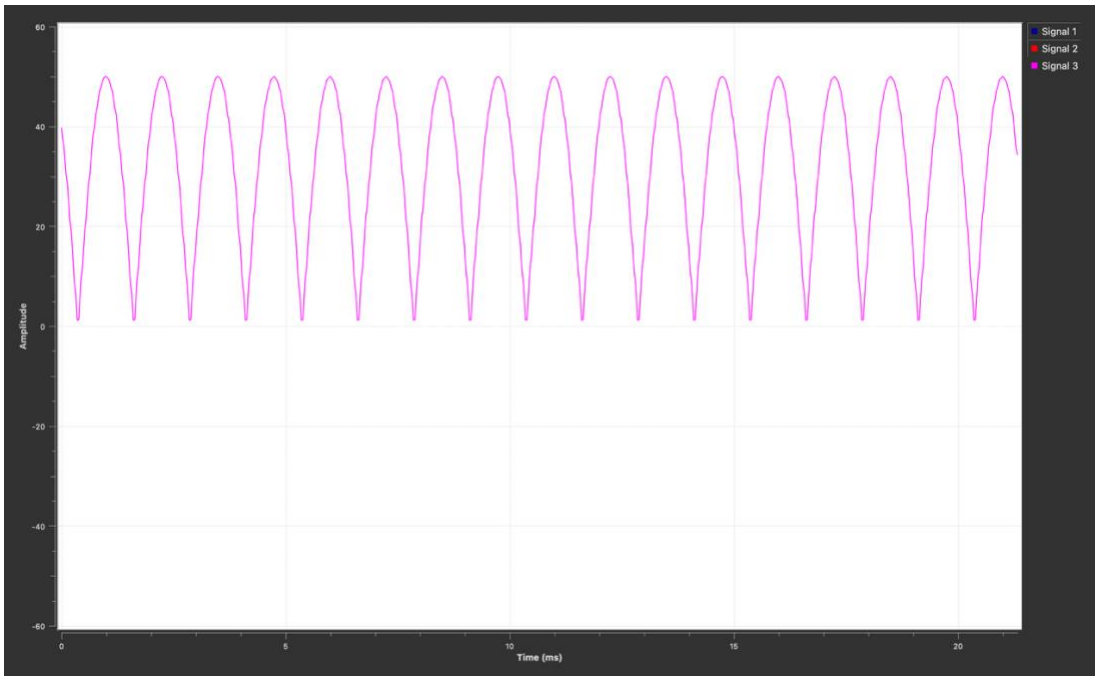

Figure 3: Rectified wave isolated for clarity

Lab Part 2: Determining Fourier Series Coefficients

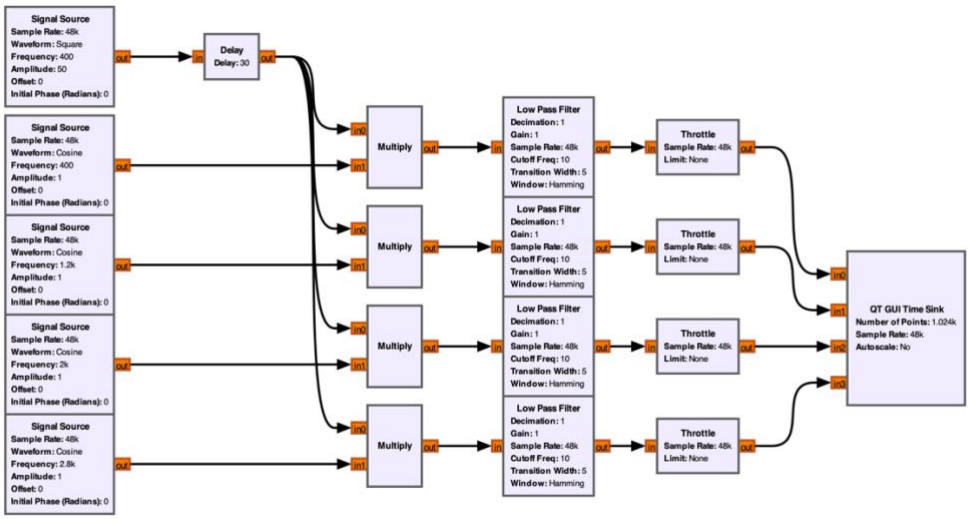

Figure 4: Block diagram for determining the coefficients of 4 signals

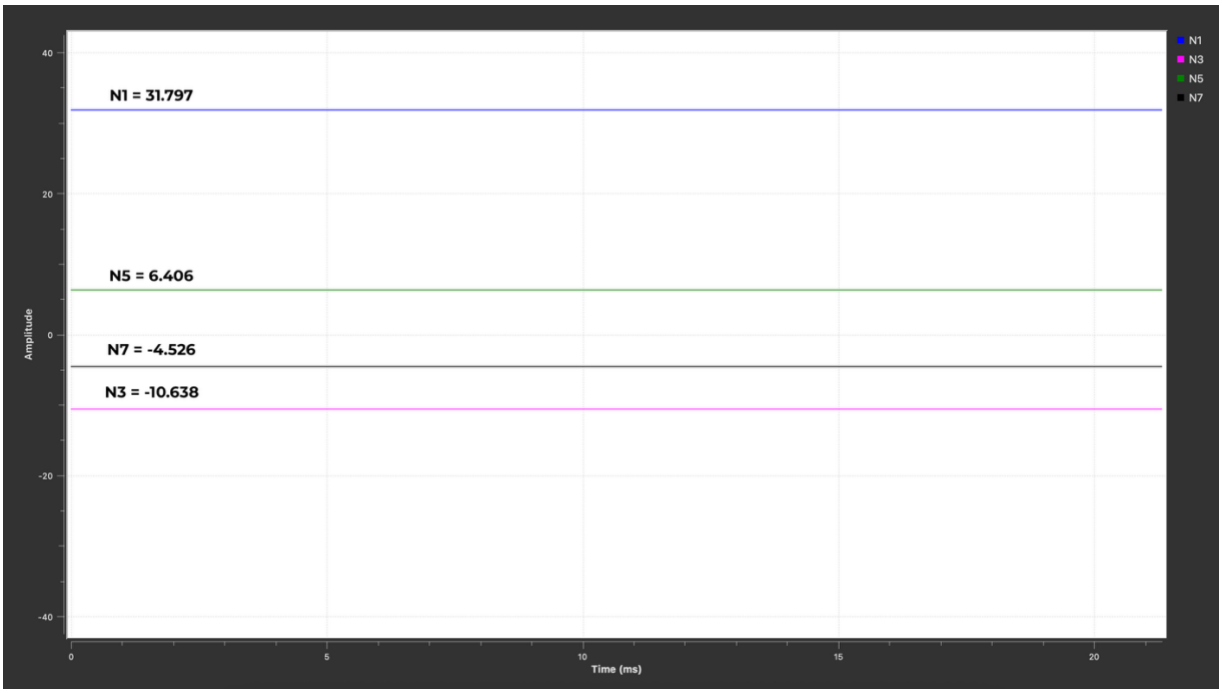

Figure 5: Output of the fourier series coefficients

The following table shows the scaling between the coefficients found in GNUradio and from the pre-lab. All the even harmonics are zero. The odd harmonics are scaled by a factor of 25 in the GNUradio, all within a 1% tolerance which we can attribute to rounding.

Harmonic (N)	Fourier Coefficients	From Pre-Lab (question 2)
1	31.79	1.27
2	0	0
3	-10.64	-0.42
4	0	0
5	6.41	0.25
6	0	0
7	-4.53	-0.18

Lab Part 3: Reconstruct Approximation of Square Wave Using Fourier Series Coefficients

The ripple count as seen in the figures below:
Note I am considering a ripple as a full oscillation

- 4 cosines (1,3,5,7) = 3 ripples per half-period
- 3 cosines (1,3,5) = 2 ripples per half-period
- 2 cosines (1,3) = 1 ripple per half-period
- 1 cosine (1) = no ripples

We see with more harmonics, the closer to the square wave shape.

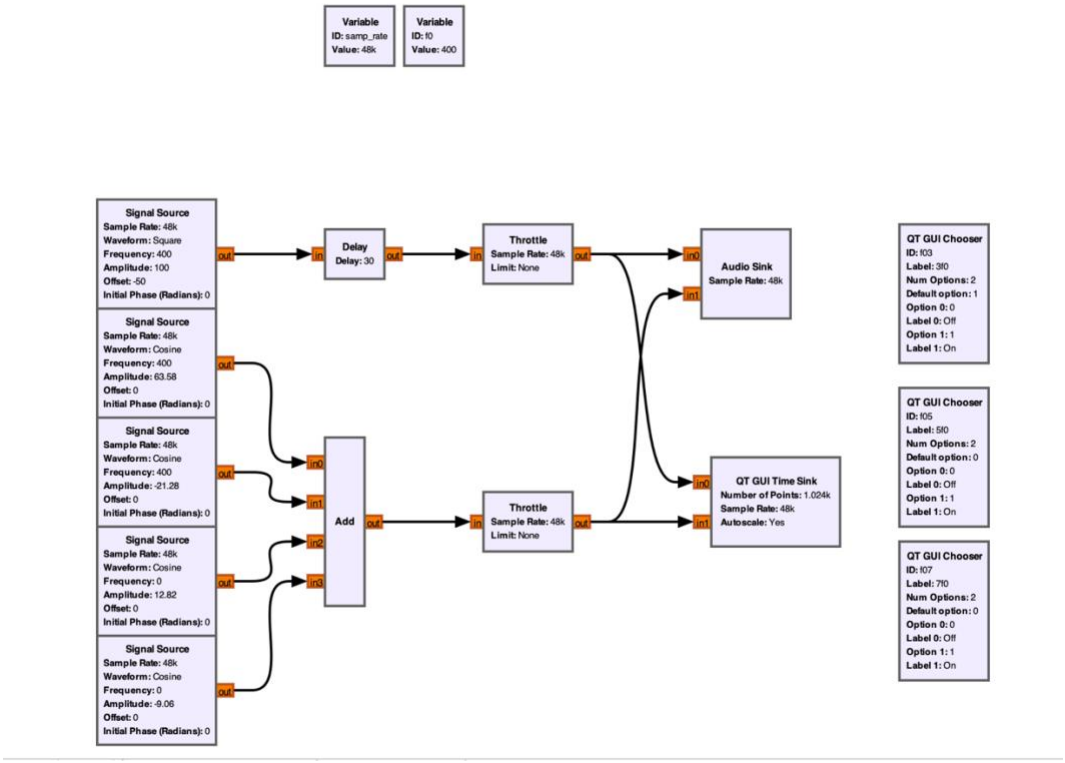

Figure 6: Block diagram of the square wave reconstruction

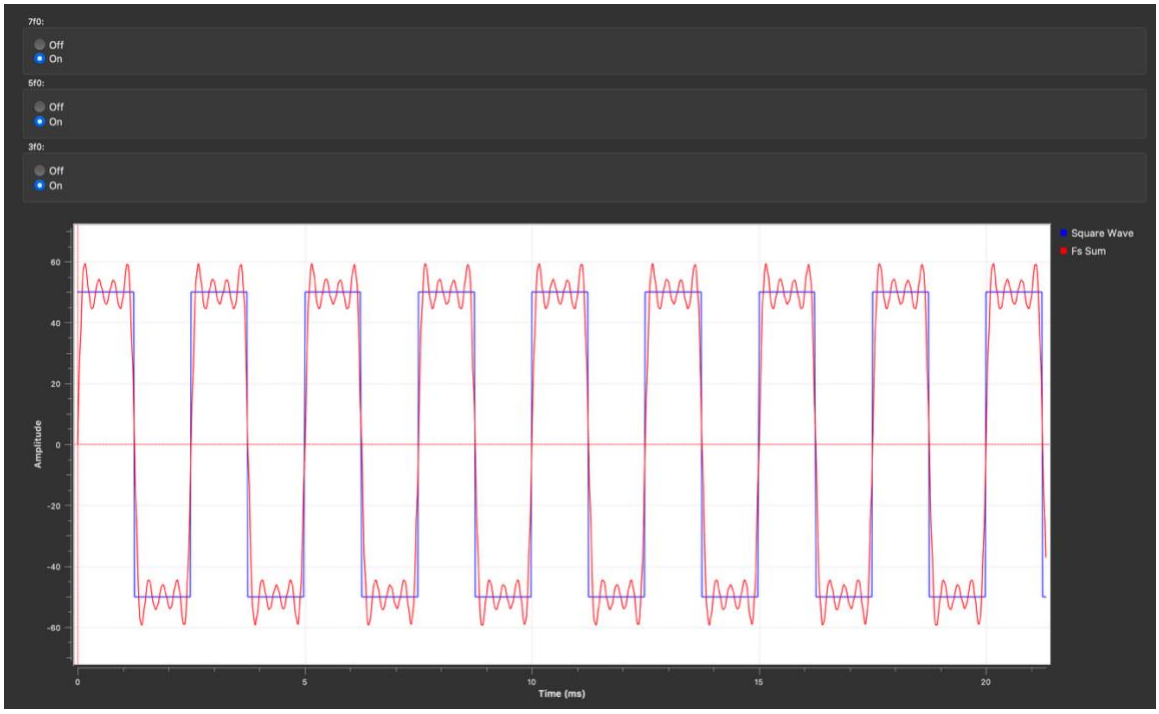

Figure 7: Three ripples with four cosine waves summed

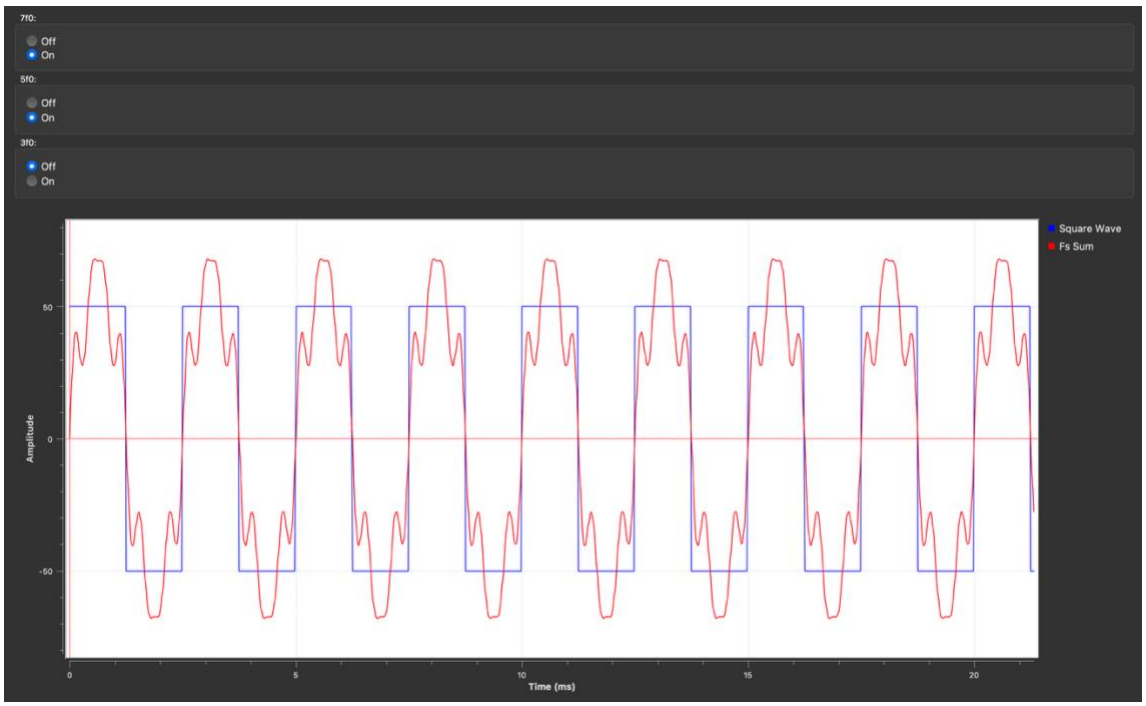

Figure 8: Two ripples with 3 cosine waves summed

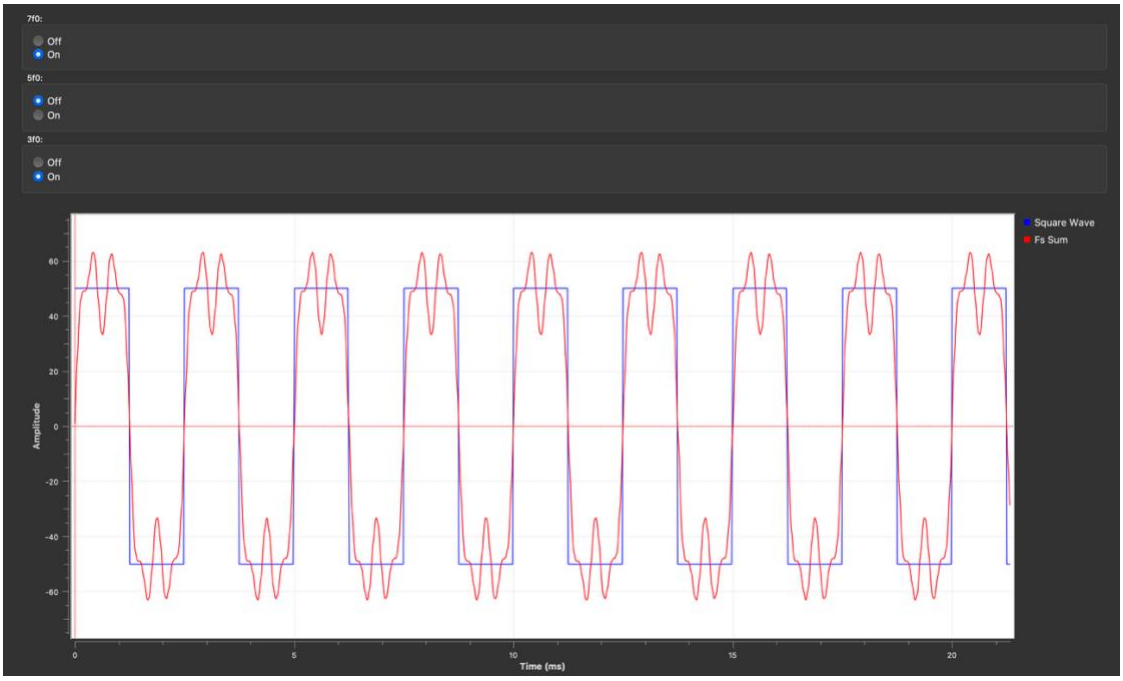

Figure 9: One ripple with two cosine waves summed

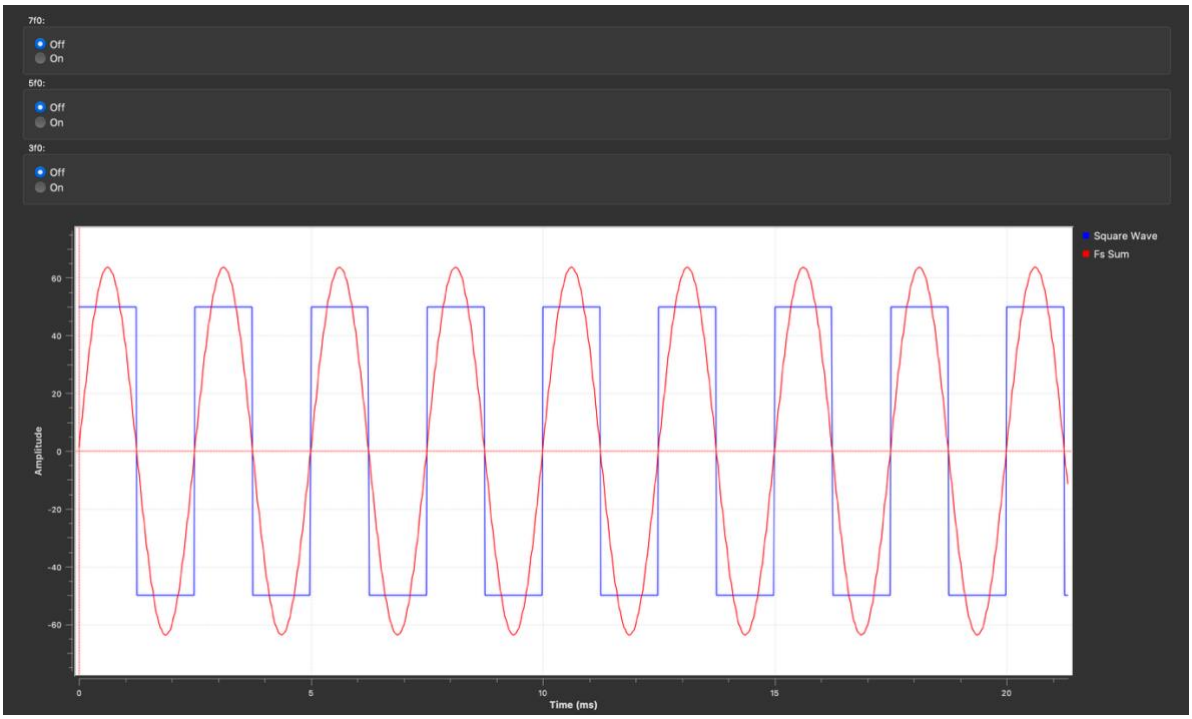

Figure 10: No ripples with a single cosine

Lab Part 4: Plotting the Error Signal

The figures below show the block diagram in GNUradio as well as the output plots. The QT GUI range acts as the gain by scaling the sum of the cosine waves. One multiply const applies a positive gain (+1) for the overlay plot. A second multiply const applies a negative gain (-1) and is added to the delayed square to give the error: $Error\ (t) = x(t) - Gain * \hat{x}(t)$. The top time sink shows that the scaled harmonic sum closely aligns with the square wave with gain ≈ 1. The bottom sink shows the error is near zero and has symmetric ringing at the edges.

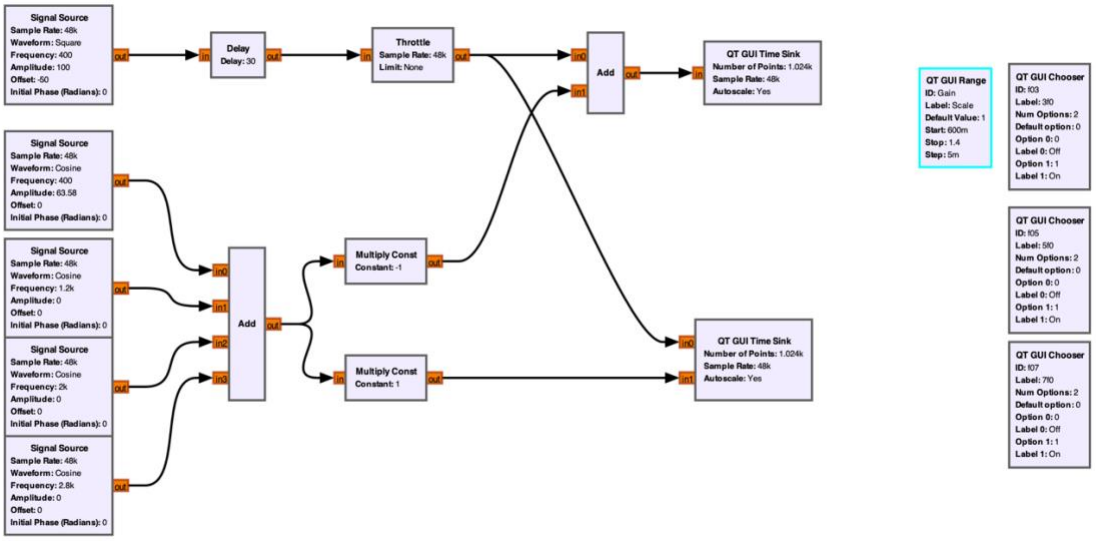

Figure 11: Block diagram for plotting the error signal

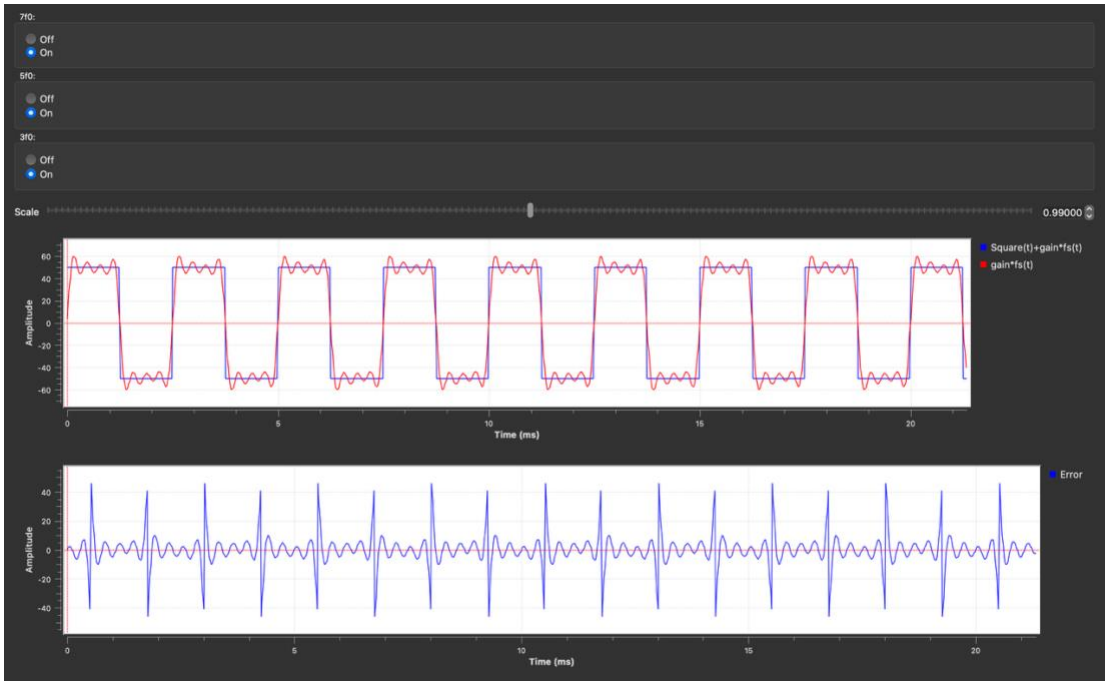

Figure 12: Output of the error signal is in the lower plot (bottom) and the overlay plot functions as a reference (top)