Homework 1

修改了 alpha 的值，增加了 J 的表达式，J 的表达式由 Mathematica 求得。

Homework 1.

$$J=\int_0^T \frac{1}{T} j(t)^2\,dt$$

$$\dot{s}=f_s(s,u)=(v,a,j)^T$$

$$H(s,u,\lambda)=\frac{1}{T}j^2+\lambda^T\cdot f_s(s,u)=\frac{1}{T}j^2+\lambda_1\cdot v+\lambda_2\cdot a+\lambda_3\cdot j$$

$$\dot{\lambda}(t)=-\nabla_s H(s^*(t),u^*(t),\lambda(t))=-\begin{bmatrix}0\\ \lambda_1\\ \lambda_2\end{bmatrix}=\begin{bmatrix}0 & -\lambda_1 & -\lambda_2\end{bmatrix}^T$$

Velocity and acceleration are free,

=> boundary condition.

$$\lambda(T)=-\nabla h(s^*(T)),\quad h=0$$

$$\Rightarrow \lambda_2(T)=0,\quad \lambda_3(T)=0.$$

combine with. $\lambda(t)=\frac{1}{T}\begin{bmatrix}-2\alpha\\ 2\alpha+2\beta\\ -\alpha t^2-2\beta t-2\gamma\end{bmatrix}$

$$\Rightarrow \beta=\alpha T,\quad \gamma=\frac{1}{2}\alpha T^2$$

$$\Rightarrow \lambda(t)=\frac{1}{T}\begin{bmatrix}-2\alpha\\ 2\alpha(t+T)\\ \alpha(-t^2+2tT-T^2)\end{bmatrix}$$

the optimal control input:

$$u^*(t)=\underset{u(t)}{\arg\min}\, H(s^*(t),u(t),\lambda(t))$$

$$\frac{\partial H}{\partial j}=\frac{2}{T}j-\frac{1}{T}\alpha(t^2-2tT+T^2)\overset{!}{=}0$$

$$u^*=j^*=\frac{\alpha}{2}(t^2-2tT+T^2)$$

the initial state: $s=[p_0, v_0, a_0]^T$

$$\Rightarrow a=\int u^*(t)\,dt+a_0=\frac{1}{6}\alpha t^3-\frac{1}{2}\alpha Tt^2+\frac{1}{2}\alpha T^2t+a_0$$

$$v=\int a\,dt+v_0=\frac{1}{24}\alpha t^4-\frac{1}{6}\alpha Tt^3+\frac{1}{4}\alpha T^2t^2+a_0t+v_0$$

$$p=\int v\,dt+p_0=\frac{1}{120}\alpha t^5-\frac{1}{24}\alpha Tt^4+\frac{1}{12}\alpha T^2t^3+\frac{1}{2}a_0t^2+v_0t+p_0$$

So the optimal state is.

$$s^*(t)=\begin{bmatrix}\frac{1}{120}\alpha t^5-\frac{1}{24}\alpha Tt^4+\frac{1}{12}\alpha T^2t^3+\frac{1}{2}a_0t^2+v_0t+p_0\\ \frac{1}{24}\alpha t^4-\frac{1}{6}\alpha Tt^3+\frac{1}{4}\alpha T^2t^2+a_0t+v_0\\ \frac{1}{6}\alpha t^3-\frac{1}{2}\alpha Tt^2+\frac{1}{2}\alpha T^2t+a_0\end{bmatrix}$$

Because of the final state:

$$p(T)=p_T$$

$$p_T=\frac{1}{120}\alpha T^5-\frac{1}{24}\alpha T^5+\frac{1}{12}\alpha T^5+\frac{1}{2}a_0T^2+v_0T+p_0$$

$$=\frac{1}{20}\alpha T^5+\frac{1}{2}a_0T^2+v_0T+p_0$$

$$\alpha=\frac{20}{T^5}\left[p_T-\frac{1}{2}a_0T^2-v_0T-p_0\right]$$

$$\alpha=\frac{20}{T^2}\left[p_T-\frac{1}{2}a_0T^2-v_0T-p_0\right]$$

$$J=5(2p_0-2p_t+T(a_0T+2v_0))^2$$

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In[5]:= j = (alpha / 2) * ((t - T) ^ 2)
        alpha = (20 / T ^ 2) (pt - p0 - (1 / 2) * a0 * (T ^ 2) - v0 * T)
        J = (1 / T) * Integrate[j ^ 2, {t, 0, T}]
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Out[5]= $\frac{1}{2}\,\text{alpha}\,(t-T)^2$

Out[6]= $\frac{20\left(-p0+pt-\frac{a0\,T^2}{2}-T\,v0\right)}{T^2}$

Out[7]= $5\,(2\,p0-2\,pt+T\,(a0\,T+2\,v0))^2$

Homework 2

根据作业描述中的表达式，假设最终速度等于 0，使用 Mathematica 求得 J 对于 T 的导数，再使用伴随矩阵计算此四次方程的根，也就是 T*。

problem2.nb

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In[1]:= dPx = -Vx0 * T + dX;
        dPy = -Vy0 * T + dY;
        dPz = -Vz0 * T + dZ;
        dVx = -Vx0;
        dVy = -Vy0;
        dVz = -Vz0;
        alpha1 = (-12 / (T ^ 3)) * dPx + (6 / (T ^ 2)) * dVx;
        alpha2 = (-12 / (T ^ 3)) * dPy + (6 / (T ^ 2)) * dVy;
        alpha3 = (-12 / (T ^ 3)) * dPz + (6 / (T ^ 2)) * dVz;
        beta1 = (6 / (T ^ 2)) * dPx + (-2 / T) * dVx;
        beta2 = (6 / (T ^ 2)) * dPy + (-2 / T) * dVy;
        beta3 = (6 / (T ^ 2)) * dPz + (-2 / T) * dVz;
        J = T + ((1 / 3) * (alpha1 ^ 2) * (T ^ 3) + alpha1 * beta1 * (T ^ 2) + ((beta1 ^ 2) * T)) +
            ((1 / 3) * (alpha2 ^ 2) * (T ^ 3) + alpha2 * beta2 * (T ^ 2) + ((beta2 ^ 2) * T)) +
            ((1 / 3) * (alpha3 ^ 2) * (T ^ 3) + alpha3 * beta3 * (T ^ 2) + ((beta3 ^ 2) * T));

In[14]:= dJ = Simplify[D[J, T]]
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Out[14]= $\frac{-36\,dX^2-36\,dY^2-36\,dZ^2+T^4+24\,dX\,T\,Vx0-4\,T^2\,Vx0^2+24\,dY\,T\,Vy0-4\,T^2\,Vy0^2+24\,dZ\,T\,Vz0-4\,T^2\,Vz0^2}{T^4}$

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In[15]:= Collect[dJ, T]
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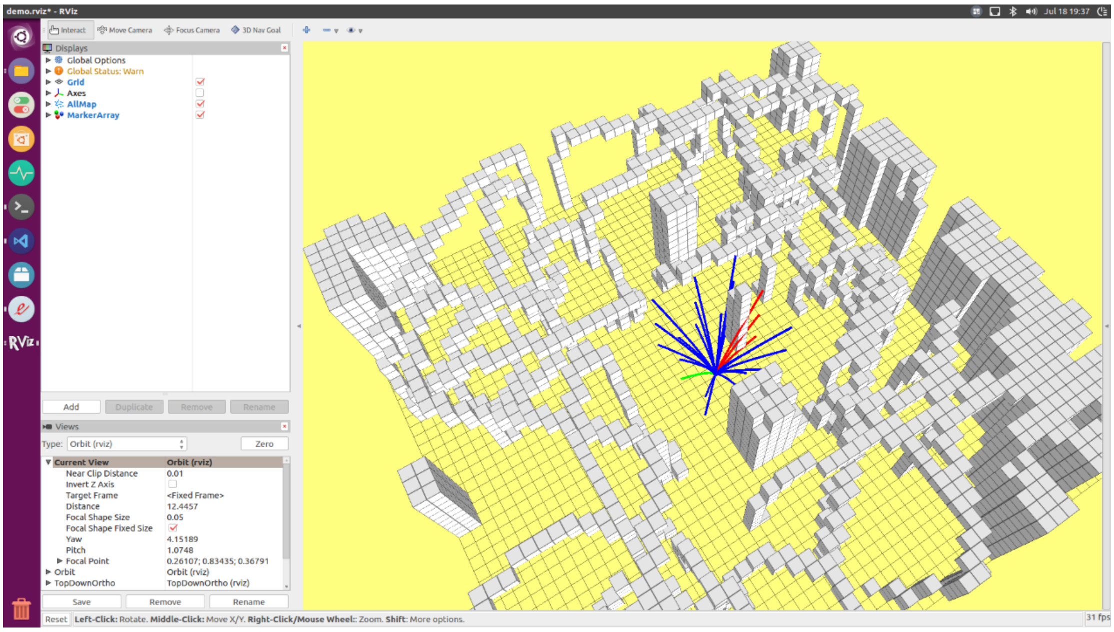
Out[15]= $1+\frac{-36\,dX^2-36\,dY^2-36\,dZ^2}{T^4}+\frac{24\,dX\,Vx0+24\,dY\,Vy0+24\,dZ\,Vz0}{T^3}+\frac{-4\,Vx0^2-4\,Vy0^2-4\,Vz0^2}{T^2}$



demo.rviz* - RViz
Interact
Move Camera
Focus Camera
3D Nav Goal
Displays
Global Options
Global Status: Warn
Grid
Axes
AllMap
MarkerArray
Add
Duplicate
Remove
Rename
Views
Type: Orbit (rviz)
Zero
Current View
Orbit (rviz)
Near Clip Distance
0.01
Invert Z Axis
Target Frame
<Fixed Frame>
Distance
12.4457
Focal Shape Size
0.05
Focal Shape Fixed Size
Yaw
4.15189
Pitch
1.0748
Focal Point
0.26107; 0.83435; 0.36791
Orbit
Orbit (rviz)
TopDownOrtho
TopDownOrtho (rviz)
Save
Remove
Rename
Reset
Left-Click: Rotate. Middle-Click: Move X/Y. Right-Click/Mouse Wheel:: Zoom. Shift: More options.
31 fps