

# Sample-based path finding

## Lecture 3



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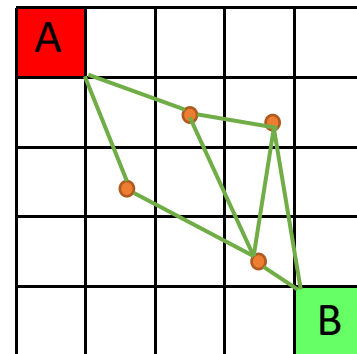
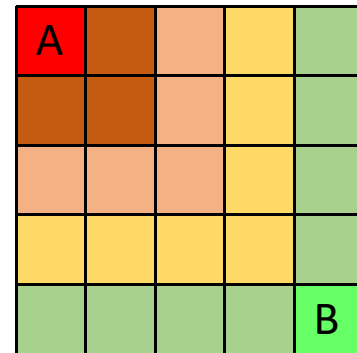




# Preliminaries

## Sampling Based-Planners

- Do not attempt to explicitly construct the C-Space and its boundaries
- Simply need to know if a single robot configuration is in collision
- Exploits simple tests for collision with full knowledge of the space
- Collision detection is a separate module- can be tailored to the application
- As collision detection improves, so do these algorithms
- Different approaches for single-query and multi-query requests





# Preliminaries

## Notion of Completeness in Planning

- Complete Planner: always answers a path planning query correctly in bounded time
- Probabilistic Complete Planner: if a solution exists, planner will eventually find it, using random sampling (e.g. Monte Carlo sampling)
- Resolution Complete Planner: same as above but based on a deterministic sampling (e.g. sampling on a fixed grid).



# Content



1. Probabilistic Road Map



2. Rapidly-exploring Random Tree



3. Optimal sampling-based path planning methods



4. Advanced path planning methods



5. Implementation



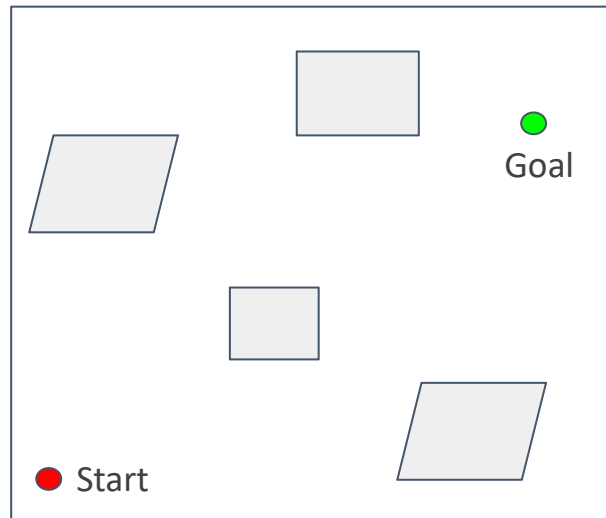
# Probabilistic Road Map



# Probabilistic Road Map

## What is PRM?

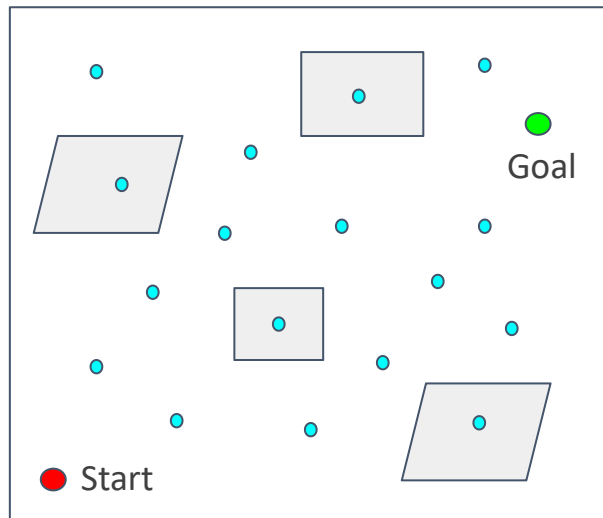
- A graph structure
- Divide planning into two phases:
  - Learning phase:
  - Query phase:
- Checking sampled configurations and connections between samples for collision can be done efficiently.
- A relatively small number of milestones and local paths are sufficient to capture the connectivity of the free space





# Probabilistic Road Map

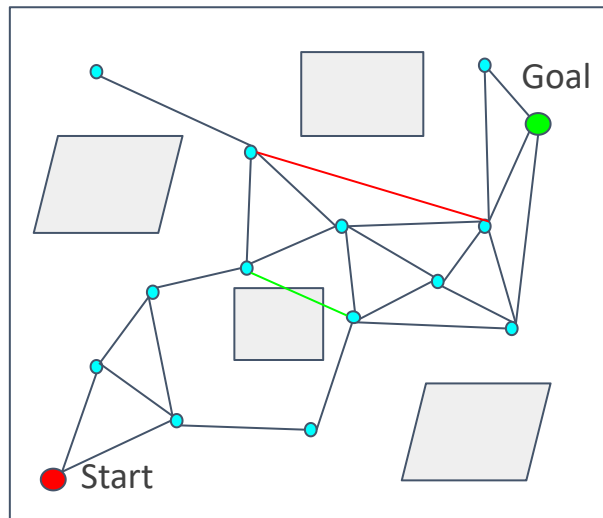
- Learning phase:
  - Sample N points in C-space
  - Delete points that are not collision-free
- Detect the c-space with random points





# Probabilistic Road Map

- Learning phase:
  - Connect to nearest points and get collision-free segments.
  - Delete segments that are not collision free.

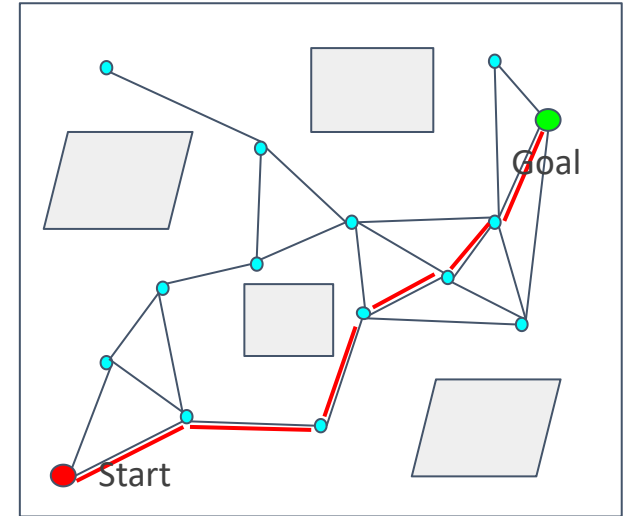






# Probabilistic Road Map

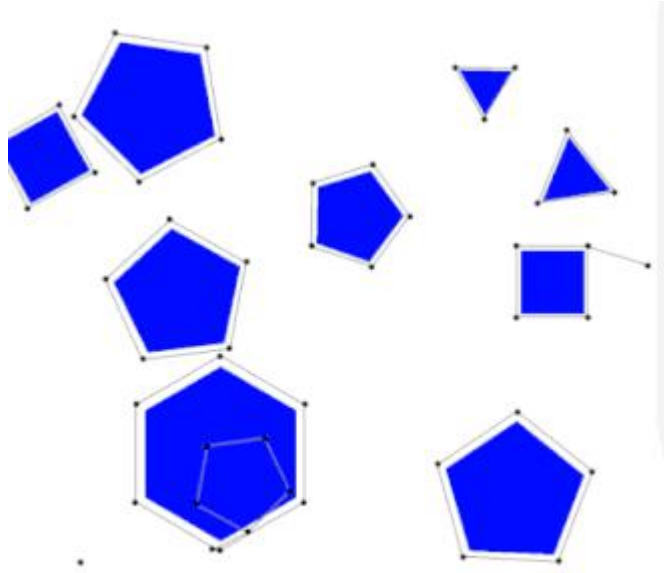
- Query phase:
  - Search on the road map to find a path from the start to the goal (using Dijkstra's algorithm or the A\* algorithm).
  - Road map is now similar with the grid map (or a simplified grid map)



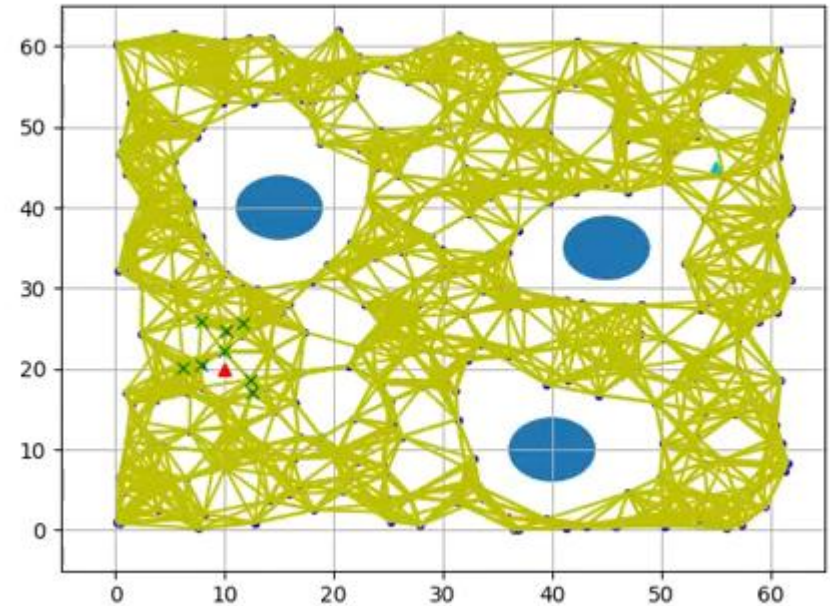


# Probabilistic Road Map

Learning phase[1]



Query phase[2]



[1] [https://en.wikipedia.org/wiki/Probabilistic\\_roadmap](https://en.wikipedia.org/wiki/Probabilistic_roadmap)

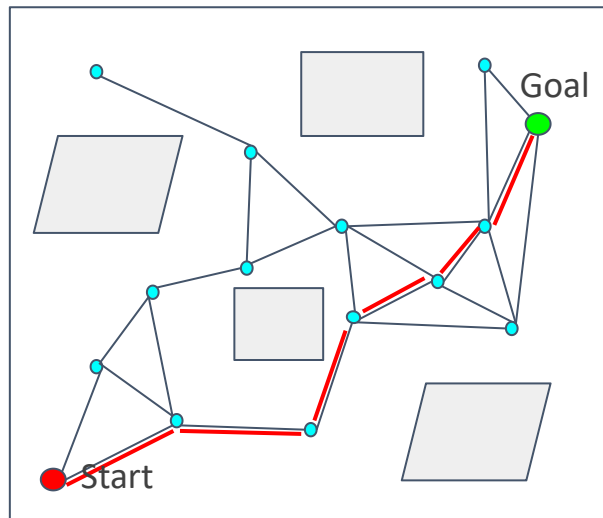
[2] [https://www.youtube.com/watch?v=8Dln3sS\\_p4Q](https://www.youtube.com/watch?v=8Dln3sS_p4Q)



# Probabilistic Road Map

## Pros and Cons

- Pros
  - Probabilistically complete
- Cons
  - Required to solve 2 point boundary value problem
  - Build graph over state space but no particular focus on generating a path
  - Not efficient

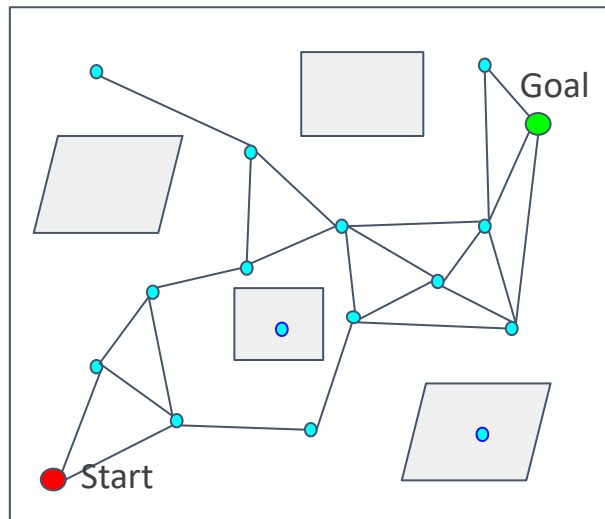




# Probabilistic Road Map

## Note: towards improving efficiency

- Lazy collision-checking
  - Collision-checking process is time-consuming, especially in complex or high-dimensional environments.
  - Sample points and generate segments without considering the collision (**Lazy**).

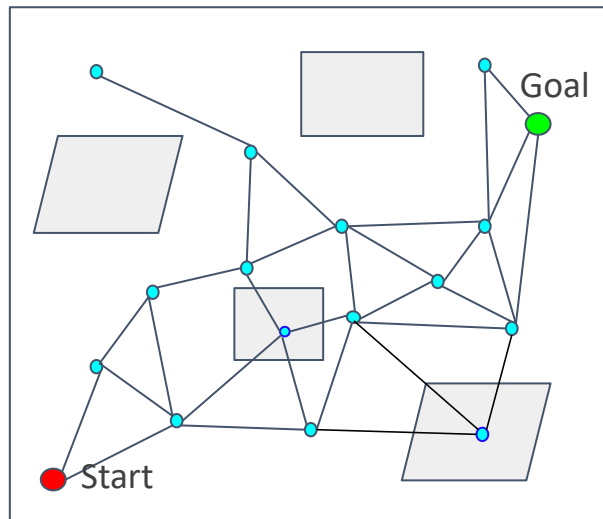




# Probabilistic Road Map

Lazy collision-checking

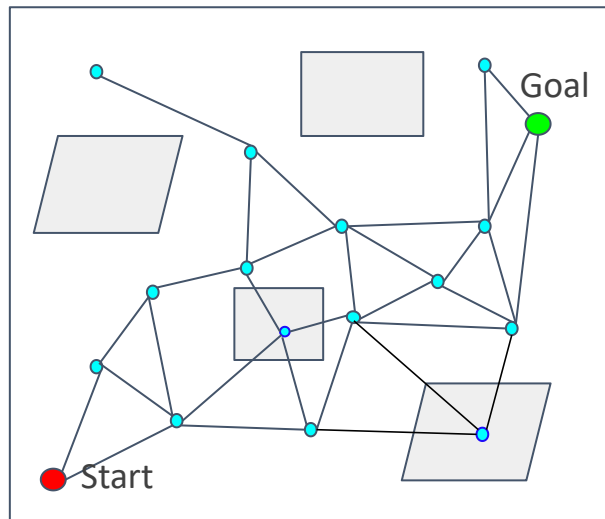
Sample points and generate segments without considering the collision (**Lazy**).





Collision-checking if necessary:

Find a path on the road map generated without collision-checking



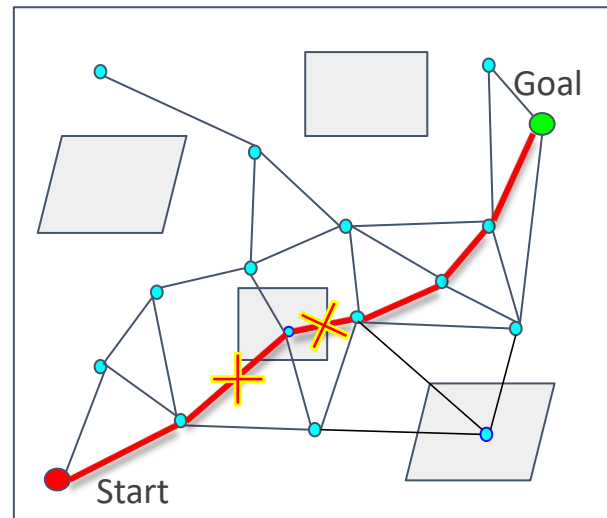


# Probabilistic Road Map

Lazy collision-checking

Collision-checking if necessary:

Delete the corresponding edges and nodes if the path is not collision free.



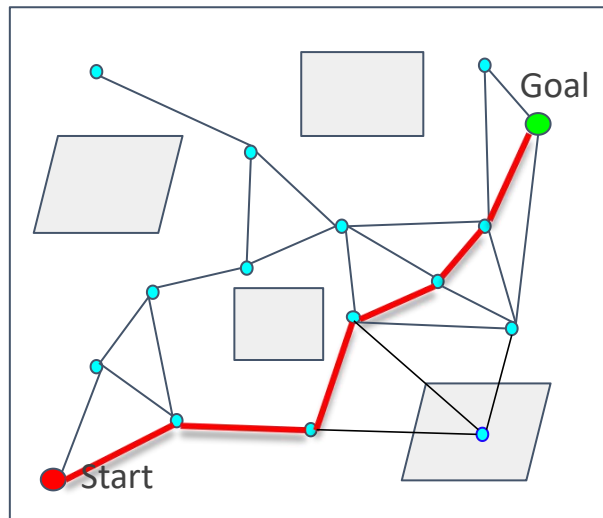


# Probabilistic Road Map

Lazy collision-checking

Collision-checking if necessary:

- Delete the corresponding edges and nodes if the path is not collision free.
- Restart path finding.







# Probabilistic Road Map

Note:

- Learning phase
- Query phase
- Lazy collision-checking

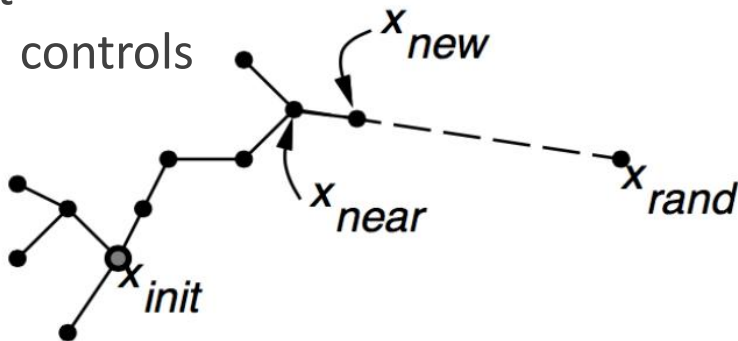


# Rapidly-exploring Random Tree



## Rapidly-exploring Random Trees

- Build up a tree through generating “next states” in the tree by executing random controls





# Rapidly-exploring Random Trees

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## Algorithm 1: RRT Algorithm

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**Input:**  $\mathcal{M}, x_{init}, x_{goal}$

**Result:** A path  $\Gamma$  from  $x_{init}$  to  $x_{goal}$

$\mathcal{T}.init();$

**for**  $i = 1$  **to**  $n$  **do**

$x_{rand} \leftarrow Sample(\mathcal{M});$

$x_{near} \leftarrow Near(x_{rand}, \mathcal{T});$

$x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize);$

$E_i \leftarrow Edge(x_{new}, x_{near});$

**if**  $CollisionFree(\mathcal{M}, E_i)$  **then**

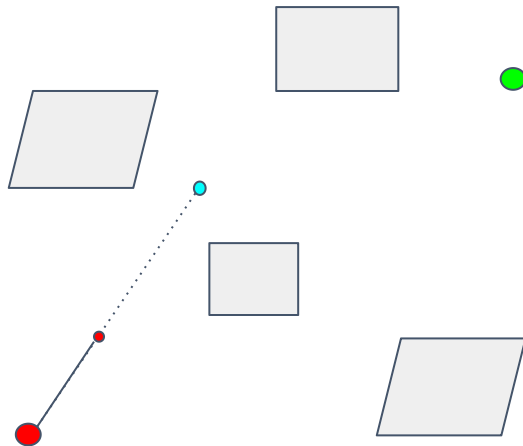
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**if**  $x_{new} = x_{goal}$  **then**

**Success** $();$

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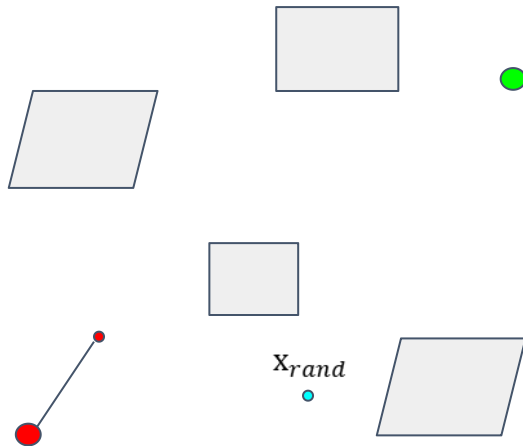
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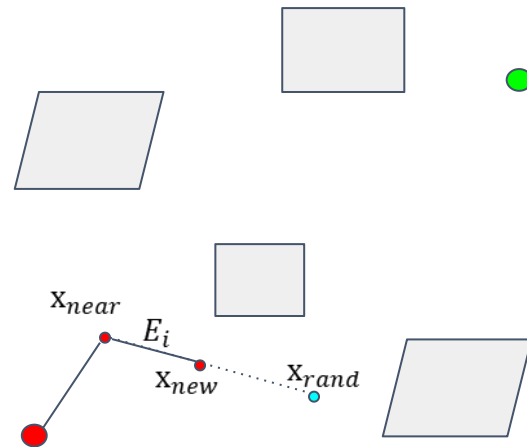
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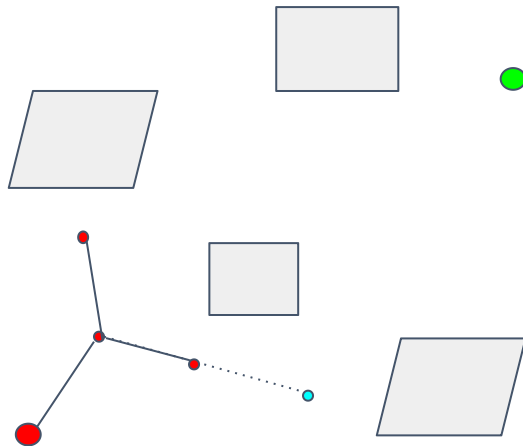
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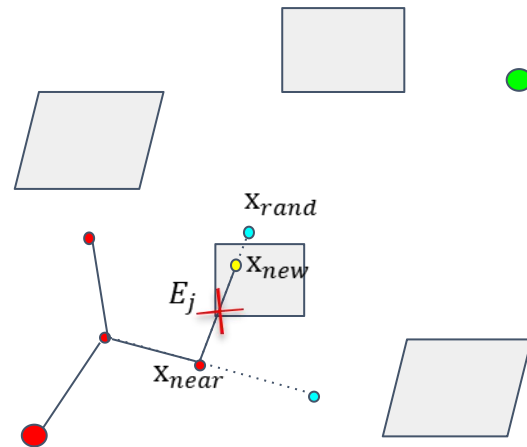
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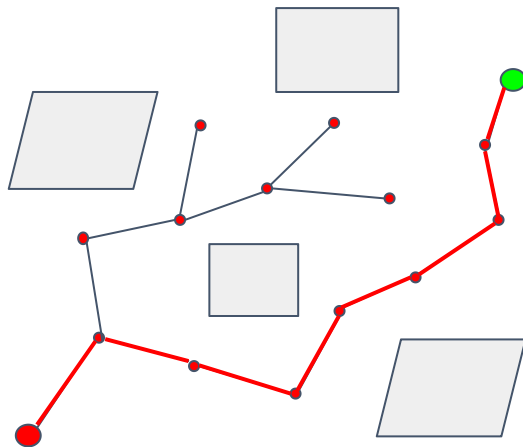
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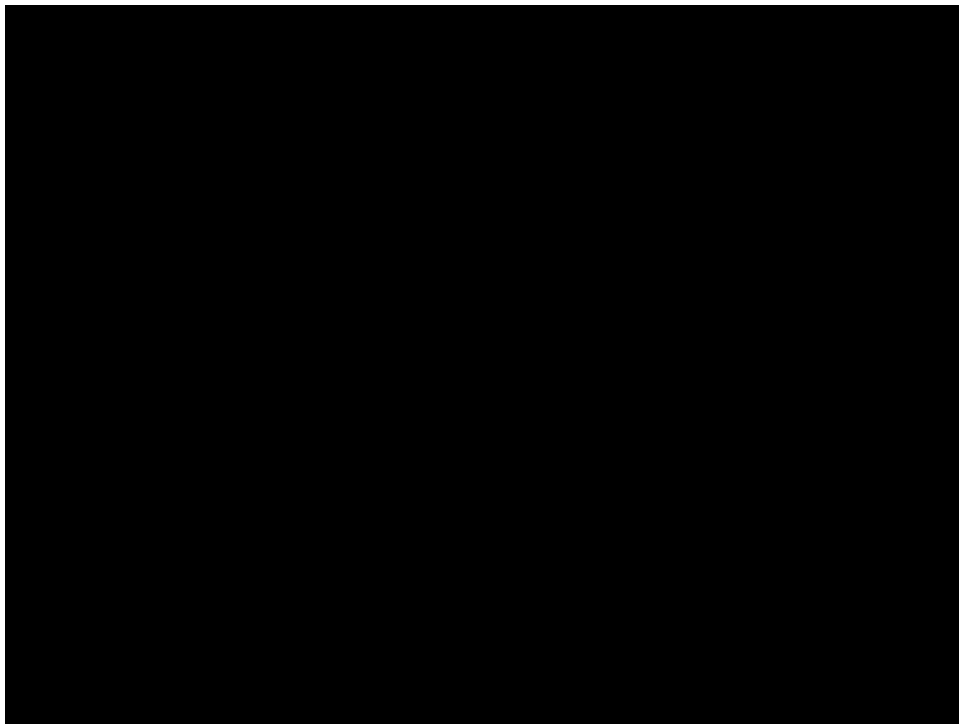
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# Rapidly-exploring Random Trees

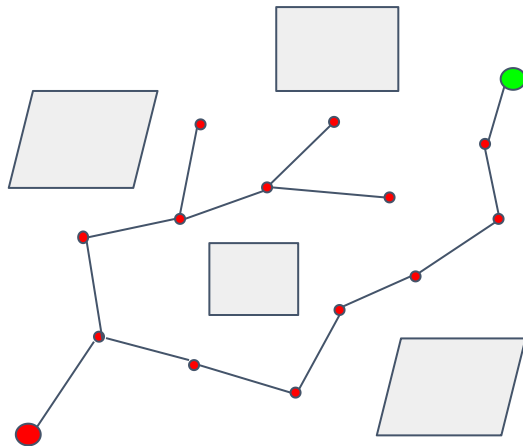
Demonstration of RRT[1]



[1] [https://www.youtube.com/watch?v=pOFtvZ\\_qVsA](https://www.youtube.com/watch?v=pOFtvZ_qVsA)



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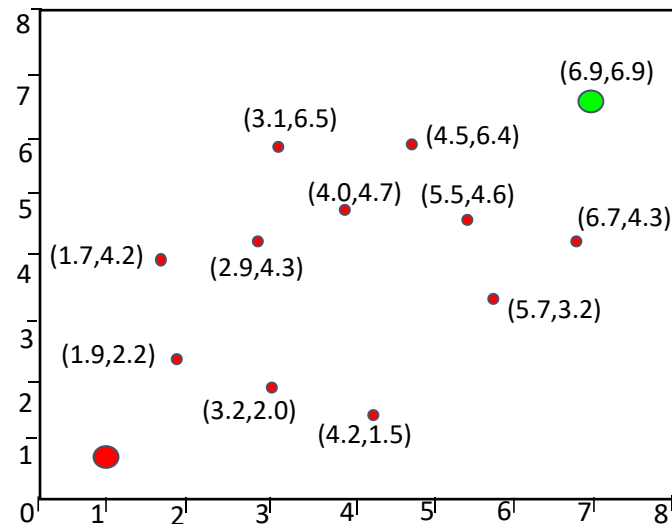
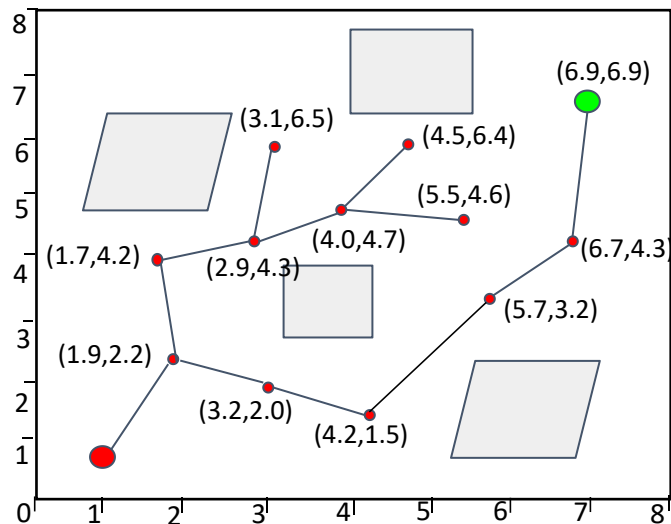




# Rapidly-exploring Random Trees

Note: towards improving efficiency

Kd-tree

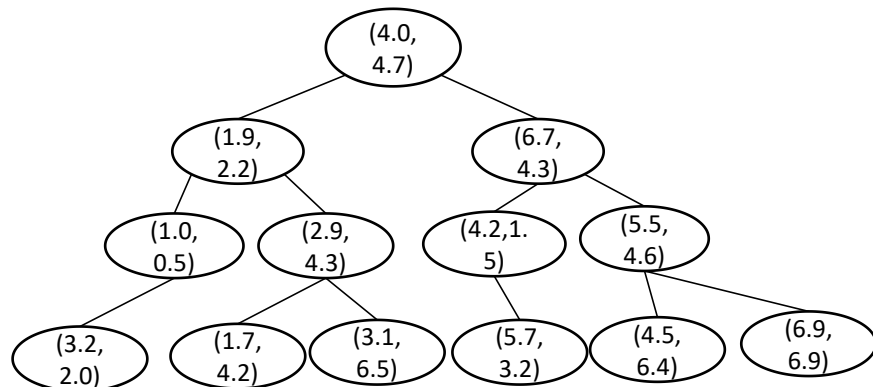
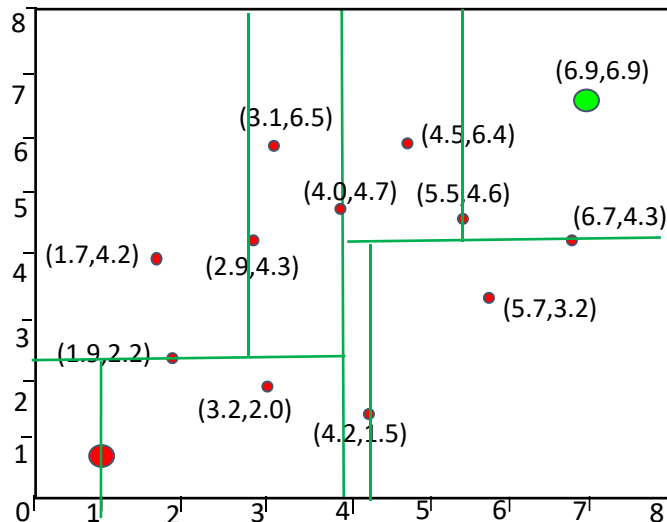




# Rapidly-exploring Random Trees

Note: towards path planning efficiency

- Kd-tree



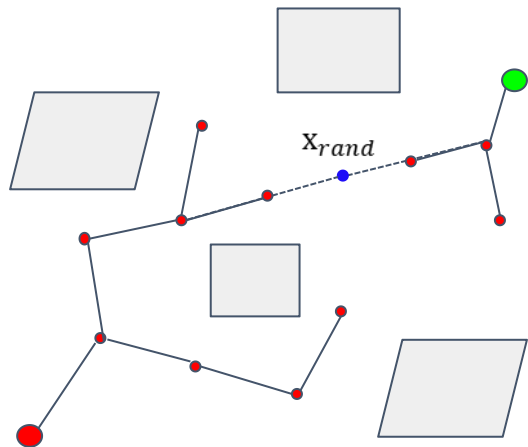
- Other variants: Spatial grid, hill climbing, etc
- 参考: <https://blog.csdn.net/junshen1314/article/details/51121582>



# Rapidly-exploring Random Trees

Note: towards improving efficiency

Bidirectional RRT / RRT Connect



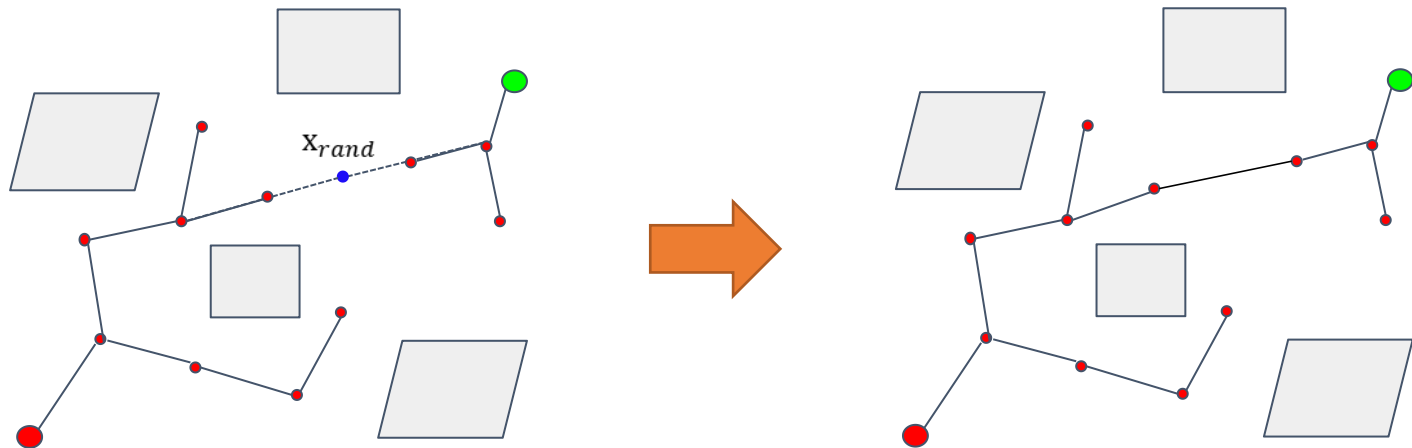
- Grow a tree from both the start point and the goal point
- Path finding when two trees are connected



# Rapidly-exploring Random Trees

Note: towards improving efficiency

Bidirectional RRT / RRT Connect

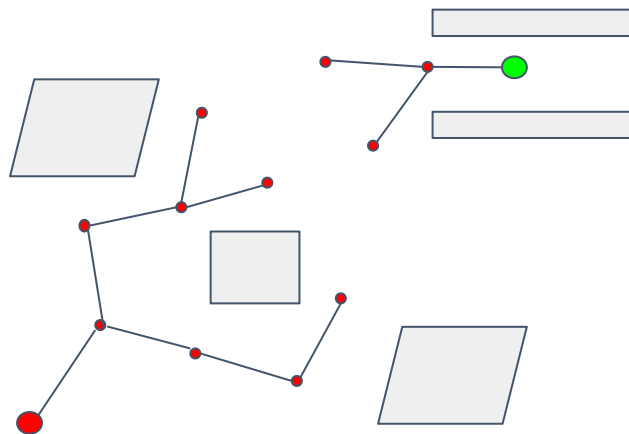
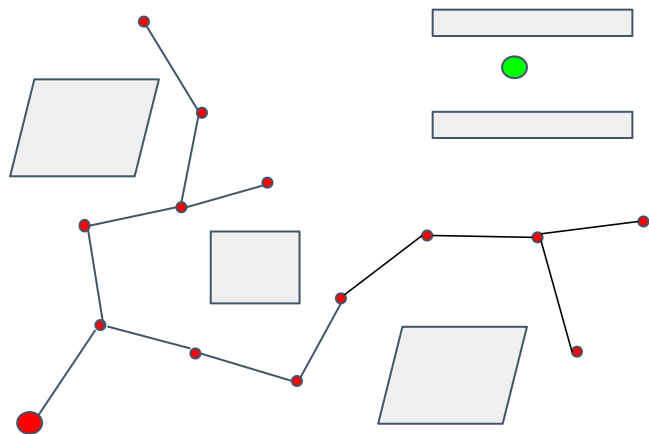




# Rapidly-exploring Random Trees

Note: towards path planning efficiency

Bidirectional RRT / RRT Connect







## Rapidly-exploring Random Trees

- Incrementally build
- Rapidly searching
- Key functions: Sampling, Nearest, Collision-checking



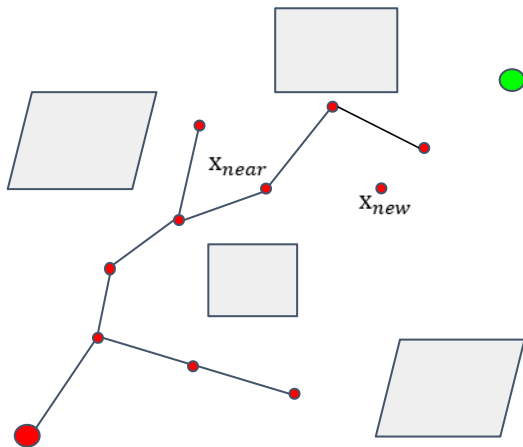


# **Optimal sampling-based path planning methods**



# Optimal sampling-based path planning methods

## Rapidly-exploring Random Tree\*



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### Algorithm 2: RRT Algorithm

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$\mathcal{T}.init()$ ;

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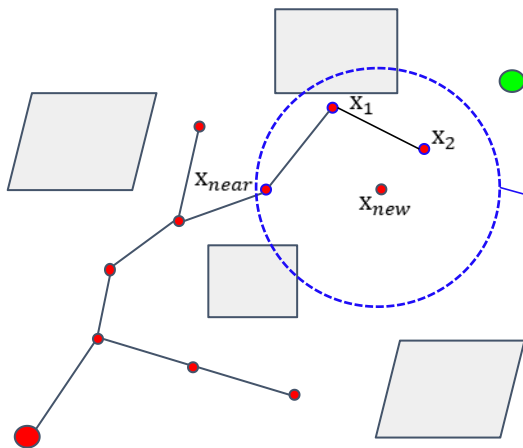
$\mathcal{T}.rewire()$ ;

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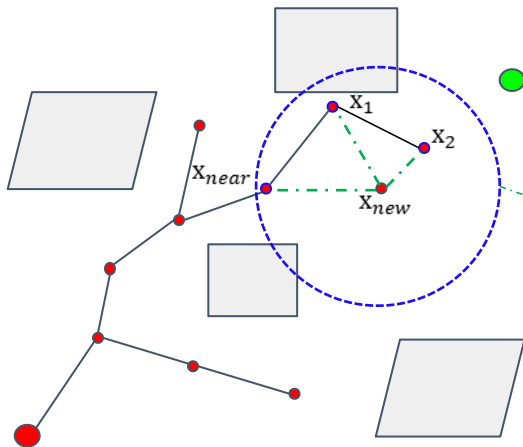
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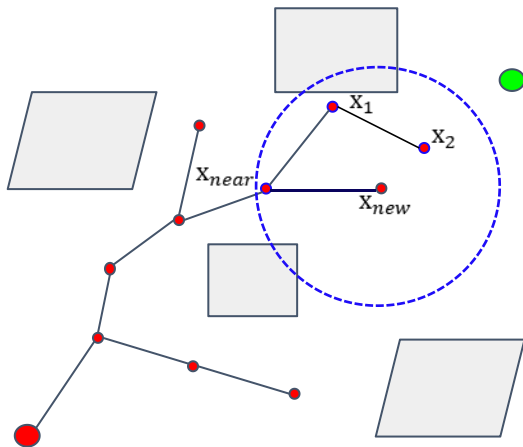
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## Rapidly-exploring Random Tree\*



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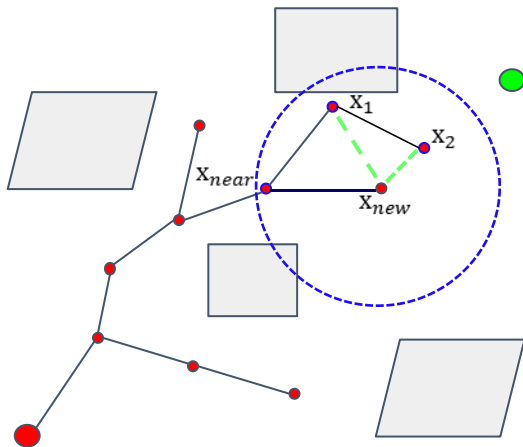
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## Rapidly-exploring Random Tree\*



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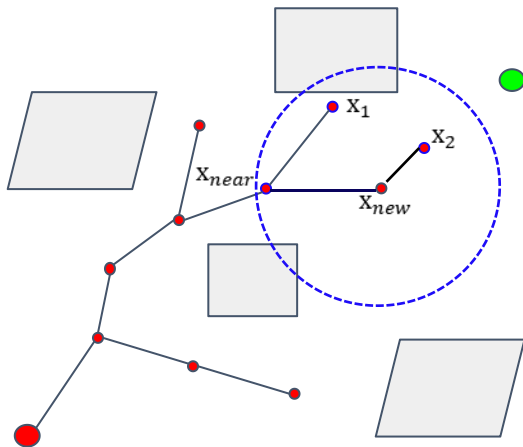
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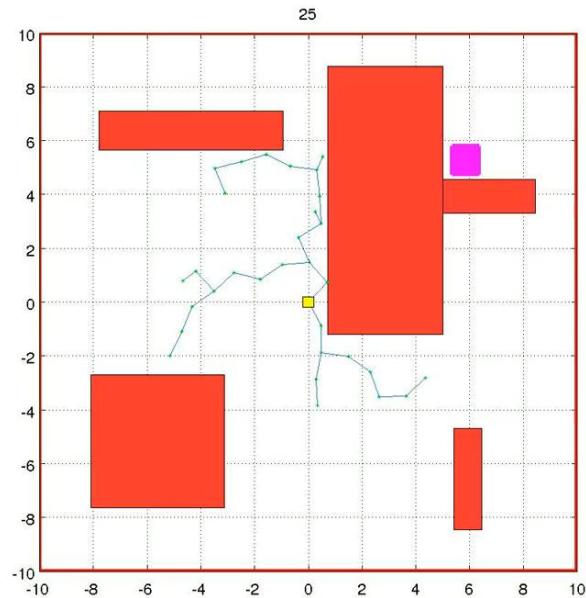
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## Rapidly-exploring Random Tree\*

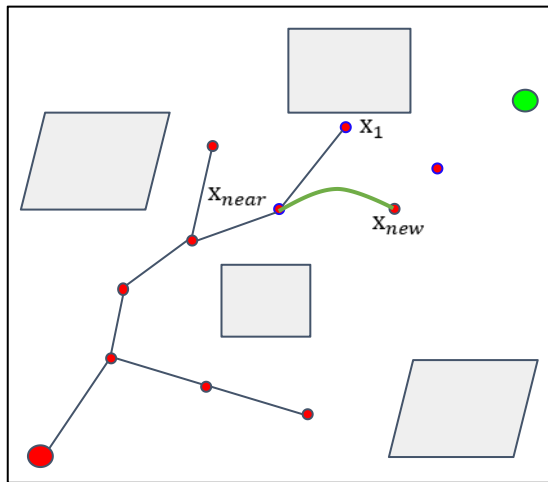
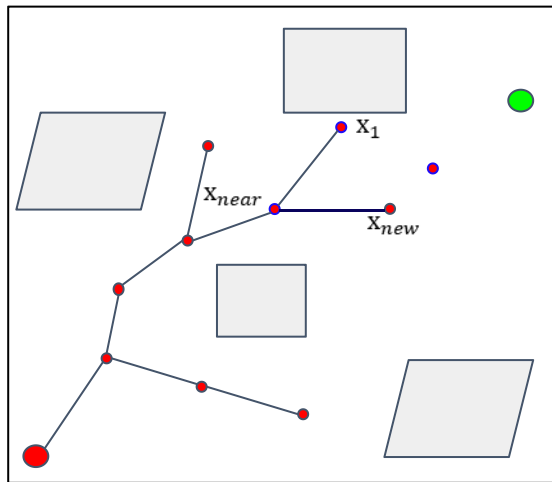


<https://www.youtube.com/watch?v=YKiQTJpPFkA>



# Optimal sampling-based path planning methods

## Kinodynamic-RRT\*

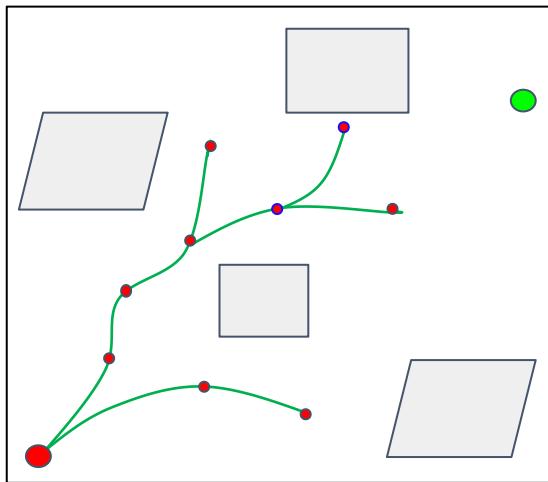
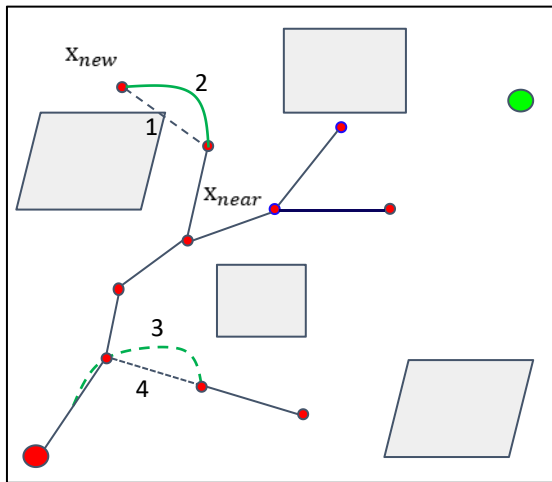


Change `Steer()` function to fit with motion or other constraints in robot navigation.



# Optimal sampling-based path planning methods

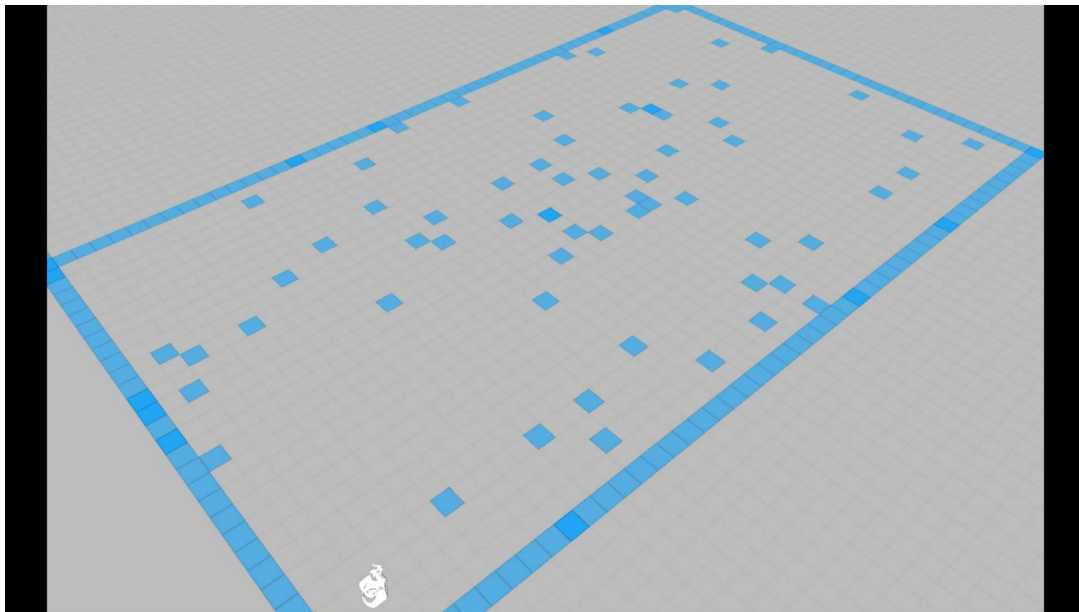
## Kinodynamic-RRT\*



Change **Steer()** function to fit with motion or other constraints in robot navigation.

# Optimal sampling-based path planning methods

## Kinodynamic-RRT\*

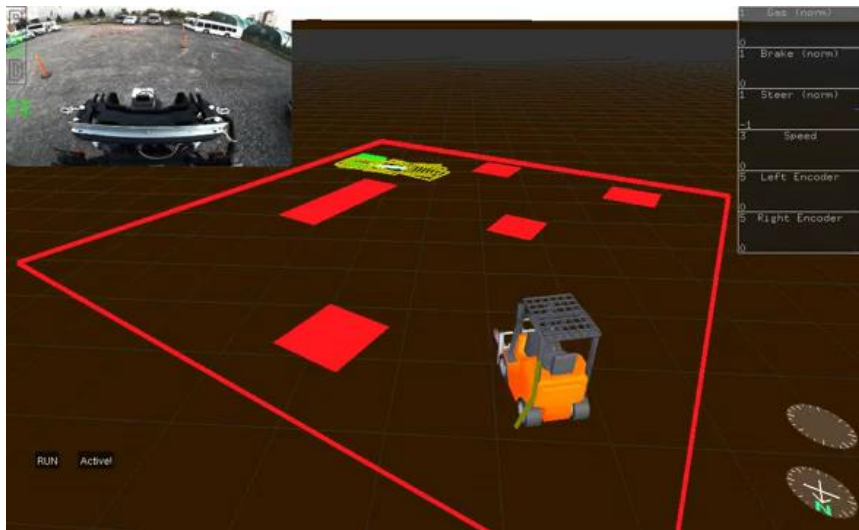


Change Steer() function to fit with motion or other constraints in robot navigation.



# Optimal sampling-based path planning methods

## Anytime-RRT\*



Keep optimizing the leaf RRT tree when the robot executes the current trajectory Anytime Fashion

[Anytime Motion Planning using the RRT\\*](#)



## Optimal sampling-based path planning methods

- Rewire function
- RRT\*
- Kino-dynamic RRT\*
- Anytime RRT\*



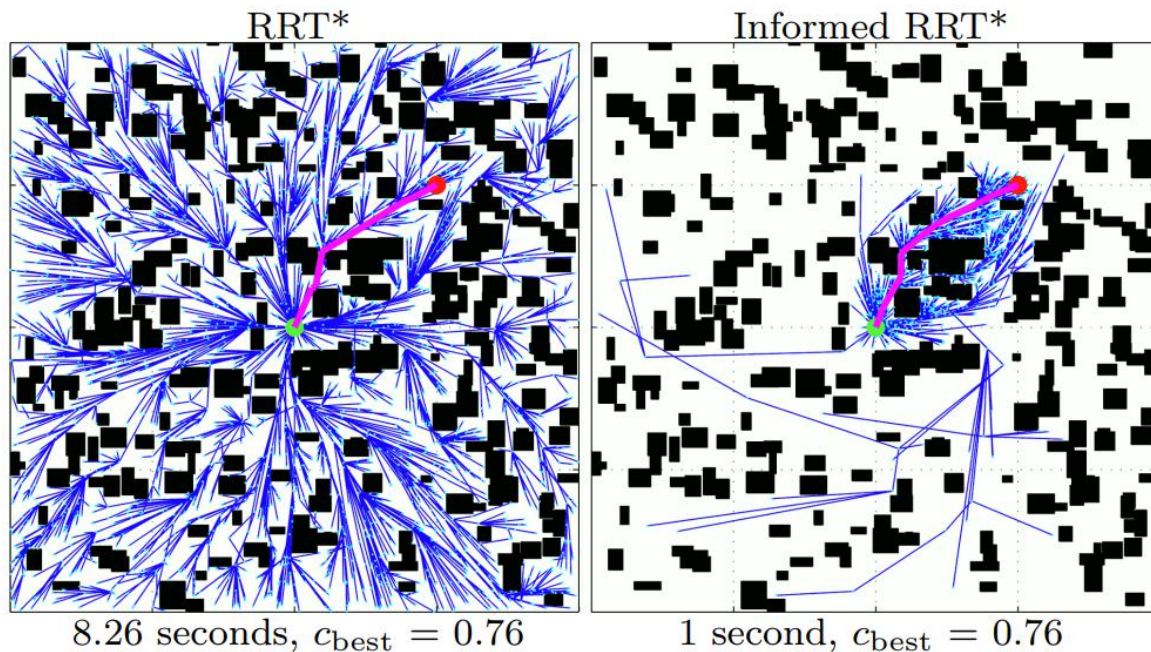


# **Advanced Sampling-based Methods**



## Advanced Sampling-based Methods

### Informed RRT\*



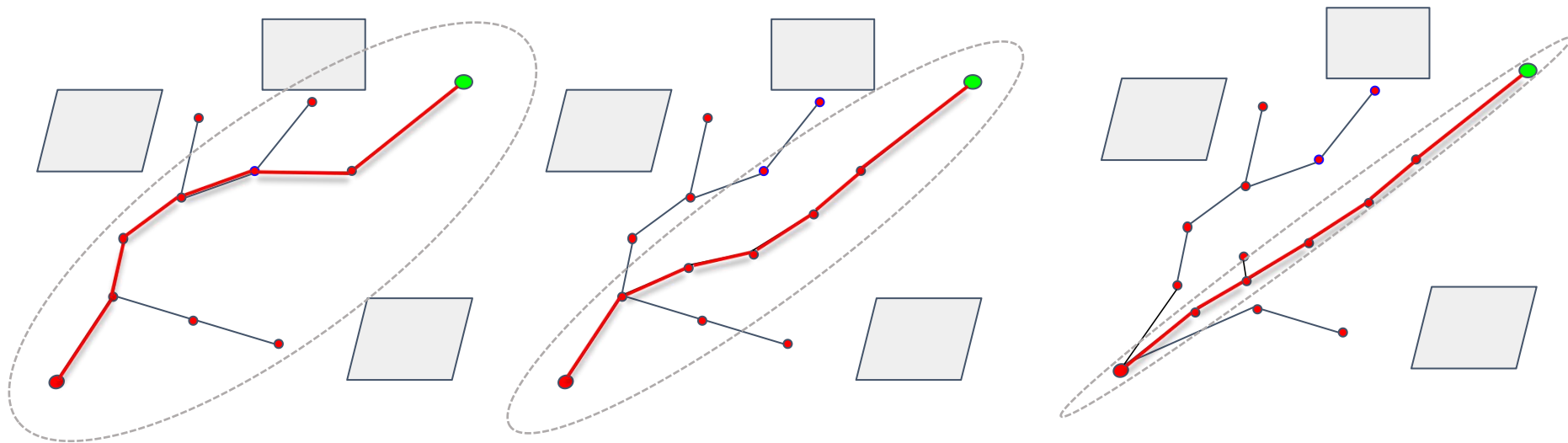
[Informed RRT\\*: Optimal sampling-based path planning focused via direct sampling of an admissible ellipsoidal heuristic](#)





# Advanced Sampling-based Methods

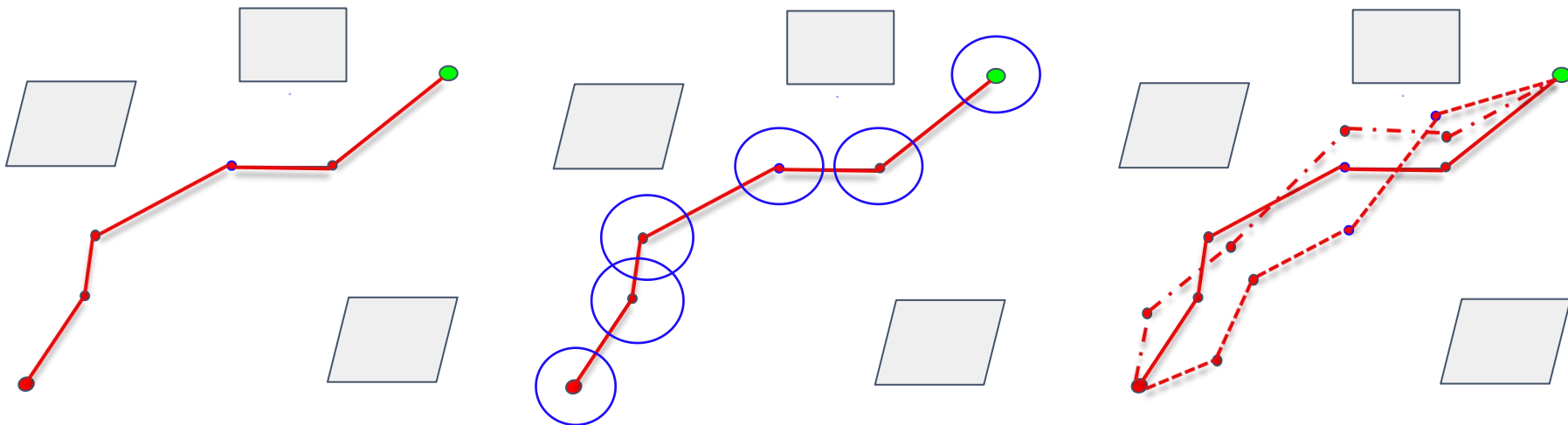
## Informed RRT\*





# Advanced Sampling-based Methods

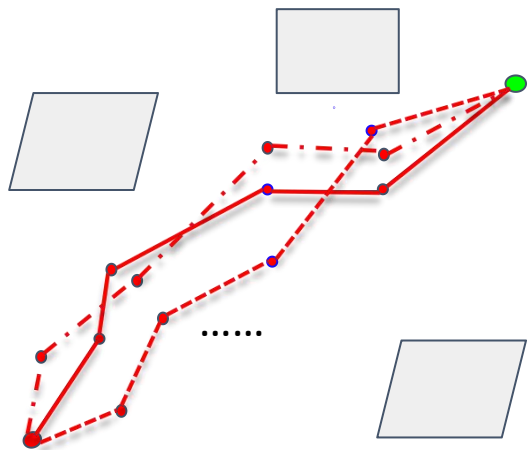
## Cross-entropy motion planning



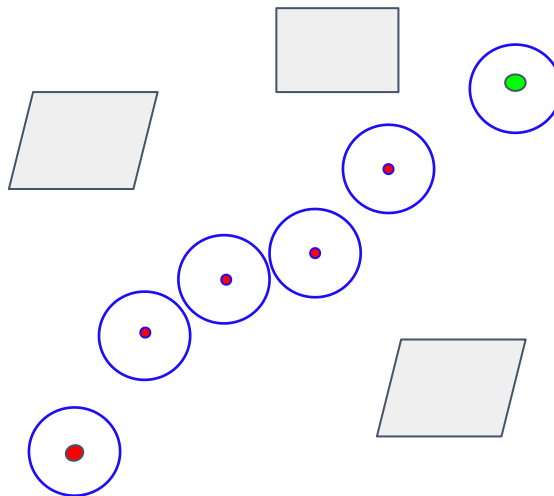


## Advanced Sampling-based Methods

### Cross-entropy motion planning



Select elite paths



Update sample region

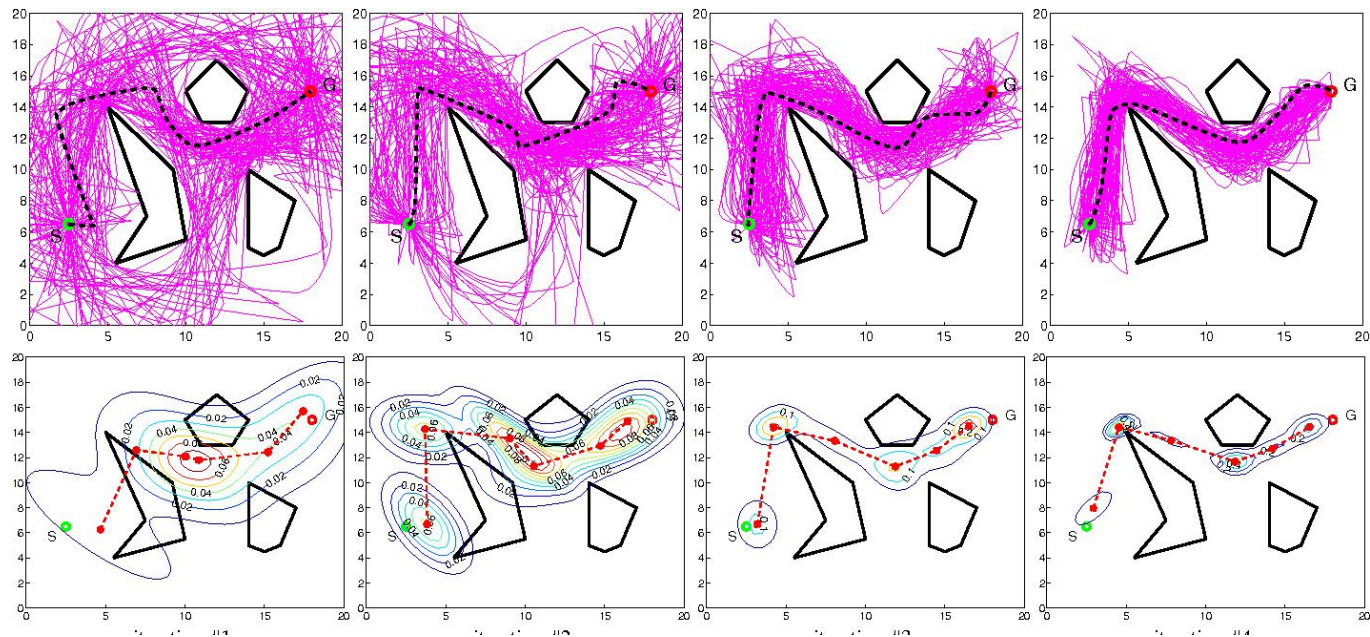


Next sample round



# Advanced Sampling-based Methods

## Cross-entropy motion planning



Link of implementation on github:



# Advanced Sampling-based Methods

## Other variants

- Lower Bound Tree RRT (LBTRRT)[a]
- Sparse Stable RRT[b]
- Transition-based RRT (T-RRT)[c]
- Vector Field RRT[d]
- Parallel RRT (pRRT)[e]
- Etc.[f]

[1] An Overview of the Class of Rapidly-Exploring Random Trees

[2] <http://msl.cs.uiuc.edu/rrt/>

[a] <https://arxiv.org/pdf/1308.0189.pdf>

[b] [http://pracsyslab.org/sst\\_software](http://pracsyslab.org/sst_software)

[c] [http://homepages.laas.fr/jcortes/Papers/jaillet\\_aaaiWS08.pdf](http://homepages.laas.fr/jcortes/Papers/jaillet_aaaiWS08.pdf)

[d] <https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6606360>

[e] [https://robotics.cs.unc.edu/publications/lchnowski2012\\_IROS.pdf](https://robotics.cs.unc.edu/publications/lchnowski2012_IROS.pdf)

[f] <https://github.com/zychaoqun>

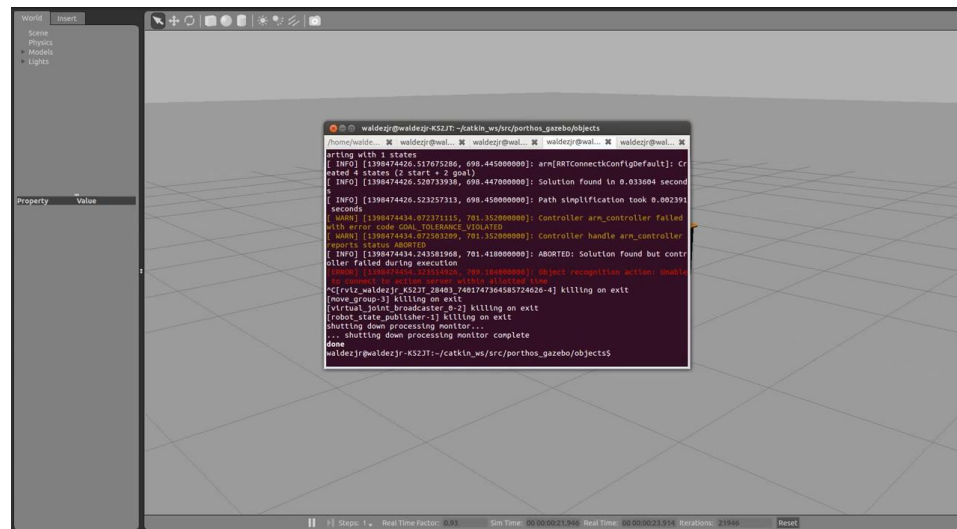


# Implementation



# Implementation

- Open Motion Planning Library [1]
- Moveit with ROS [2]
- Tutorials[3]



[1] <https://ompl.kavrakilab.org/>

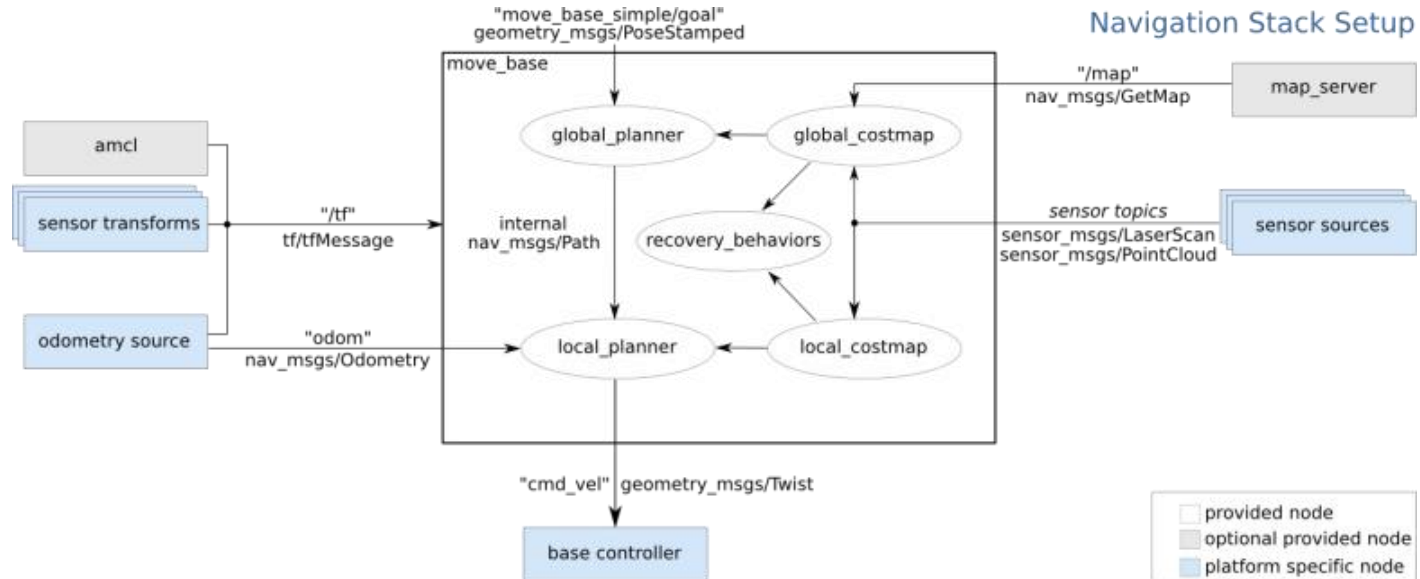
[2] <https://moveit.ros.org/>

[3] [https://industrial-training-master.readthedocs.io/en/melodic/\\_source/session4/Motion-Planning-CPP.html](https://industrial-training-master.readthedocs.io/en/melodic/_source/session4/Motion-Planning-CPP.html)



# Implementation

- Navigation stick - ROS

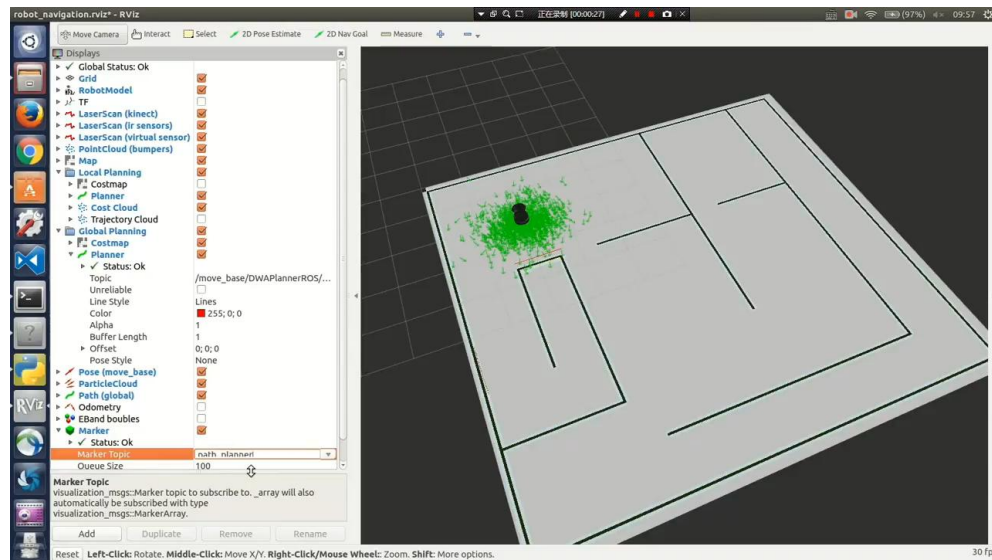






# Implementation

- Navigation stack - ROS
  - Global planner  
A\*,D\*, RRTs,etc
  - Local planner  
Dwa,eband, Teb,etc



Video demonstration of RRT implemented on ROS [1]

[1] <https://youtu.be/FsZ9b6fsQUg>



# Homework

- Implementation of RRT
  - You can either use MATLAB or C++
  - Hints: write RRT as a global planner in ROS
- Bonus: Implementation of Informed-RRT\*



在线问答

Q&A



结语

感谢各位聆听!

Thanks for Listening ●

